A proposal of a New Charged Lepton Flavor Violation Experiment: $\mu^- e^- \rightarrow e^- e^-$ in muonic atom

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+ arXiv: 1609??????
In Standard Model (SM)

Charged Lepton Flavor Violation (cLFV) via neutrino oscillation

But ... \[ \text{BR}(\mu \to e\gamma) \sim \left( \frac{\delta m^2}{m_W^2} \right)^2 < 10^{-54} \]

Forever invisible

Discovery of the cLFV signal

One of the evidence for beyond the SM
Introduction

Comparing LFV signals and their rates

Discrimination of new physics models

Prove for structure of new physics

Desire for many detectable cLFV processes

Supersymmetric model

Extra dimension model


Introduction

New idea for cLFV search

\[ \mu^- e^- \rightarrow e^- e^- \text{ in muonic atom} \]

What is target?

Flavor violation between \( \mu \) and \( e \)

What is advantage?

- Sensitive to both photonic dipole cLFV operator and 4-Fermi contact cLFV operator
- Clean signal [back-to-back dielectron]
Basic Idea (PRL)

$$\mu^- e^- \rightarrow e^- e^-$$
A diagram of a muonic atom labeled with electron 1S orbit and muon 1S orbit. The diagram includes a nucleus, a muon, and an electron.
\( \mu e \rightarrow e e \)

\[ \mu^- e^- \rightarrow e^- e^- \text{ in muonic atom} \]

**LFV vertex**

**Interaction rate**

\[
\Gamma(\mu^- e^- \rightarrow e^- e^- ; Z) = 2\sigma v_{\text{rel}} |\psi_{1S}^{(e)}(0; Z - 1)|^2
\]
\[ \mu e \rightarrow e e \]

\[ \mu^- e^- \rightarrow e^- e^- \text{ in muonic atom} \]

**Interaction rate**

\[ \Gamma(\mu^- e^- \rightarrow e^- e^- ; Z) = 2\sigma_{\text{inel}} |\psi_{1S}^{(e)}(0; Z - 1)|^2 \]

Overlap of wave function of \( \mu \) and \( e \)
\[ \mu e \rightarrow ee \]

Approximation

Muon localization at nucleus position

\[ \therefore m_e \ll m_\mu \]

Overlap = electron wave function at nucleus
\[ \psi_{1S}^{(e)}(r; Z) = \frac{(Z \alpha m_e)^{3/2}}{\sqrt{\pi}} \exp(-Z \alpha m_e r) \]

\[ r : \text{radial coordinate (distance from nucleus)} \]
\[ Z : \text{atomic number of nucleus in muonic atom} \]
\[ \mu^+ e^- \rightarrow e^- e^- \] in muonic atom

**LFV vertex**

**Interaction rate**

\[ \Gamma(\mu^- e^- \rightarrow e^- e^- ; Z) = 2 \sigma v_{rel} |\psi_{1S}^{(e)}(0; Z - 1)|^2 \]

**Cross section for elemental interaction**
Effective Lagrangian \[ \mathcal{L}_{\mu e \to e e} = -\frac{4G_F}{\sqrt{2}} \left[ m_{\mu} A_R \bar{\mu} R \sigma^{\mu \nu} e_L F_{\mu \nu} + m_{\mu} A_L \bar{\mu} L \sigma^{\mu \nu} e_R F_{\mu \nu} \right. \\
+ g_1 (\bar{\mu} R e_L) (\bar{e} R e_L) + g_2 (\bar{\mu} L e_R) (\bar{e} L e_R) \\
+ g_3 (\bar{\mu} R \gamma^\mu e_R) (\bar{e} R \gamma_\mu e_R) + g_4 (\bar{\mu} L \gamma^\mu e_L) (\bar{e} L \gamma_\mu e_L) \\
+ g_5 (\bar{\mu} R \gamma^\mu e_R) (\bar{e} L \gamma_\mu e_L) + g_6 (\bar{\mu} L \gamma^\mu e_L) (\bar{e} R \gamma_\mu e_R) + (\text{H.c.}) \left. \right] \]

\text{cLFV effective coupling constant}

\begin{align*}
A_R & \quad A_L & \quad g_1 & \quad g_2 & \quad g_3 & \quad g_4 & \quad g_5 & \quad g_6
\end{align*}

\text{Sensitive to the structure of new physics}
Effective Lagrangian

\begin{equation}
\mathcal{L}_{\mu e \rightarrow e e} = -\frac{4G_F}{\sqrt{2}} \left[ m_\mu A_R \bar{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu} 
+ g_1 (\bar{\mu_R} e_L)(\bar{e_R} e_L) + g_2 (\bar{\mu_L} e_R)(\bar{e_L} e_R) 
+ g_3 (\bar{\mu_R} \gamma^\mu e_R)(\bar{e_R} \gamma_\mu e_R) + g_4 (\bar{\mu_L} \gamma^\mu e_L)(\bar{e_L} \gamma_\mu e_L) 
+ g_5 (\bar{\mu_R} \gamma^\mu e_R)(\bar{e_L} \gamma_\mu e_L) + g_6 (\bar{\mu_L} \gamma^\mu e_L)(\bar{e_R} \gamma_\mu e_R) + \text{(H.c.)} \right]
\end{equation}

4-Fermi interaction type
Branching ratio

\[ \text{Br}(\mu^- e^- \rightarrow e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \rightarrow e^- e^-) \]

\[ = 24\pi (Z - 1)^3 \alpha^3 \left( \frac{m_e}{m_\mu} \right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G \]

\[ = (3.31 \times 10^{-12})(Z - 1)^3 (\frac{\tilde{\tau}_\mu}{\tau_\mu}) G \]

Lifetime of free muon \((2.197 \times 10^{-6} \text{ s})\)

Lifetime of bound muon

\[
\begin{cases}
2.19 \times 10^6 \text{ s} & \text{for } ^1\text{H} \\
(7 - 8) \times 10^{-8} \text{ s} & \text{for } ^{238}\text{U}
\end{cases}
\]
\[ \mu e \rightarrow e e \]

4-Fermi interaction dominant case

Branching ratio

\[
\text{Br}(\mu^- e^- \rightarrow e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \rightarrow e^- e^-) \\
= 24\pi(Z - 1)^3 \alpha^3 \left( \frac{m_e}{m_\mu} \right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G \\
= (3.31 \times 10^{-12})(Z - 1)^3 (\tilde{\tau}_\mu/\tau_\mu) G 
\]

\[
G \equiv G_{12} + 16G_{34} + 4G_{56} + 8G'_{14} + 8G'_{23} - 8G'_{56} 
\]

\[
G_{ij} \equiv |g_i|^2 + |g_j|^2 \\
G'_{ij} \equiv \text{Re} \left( g_i^* g_j \right) 
\]
**μ e → e e**

4-Fermi interaction dominant case

Branching ratio

\[
\text{Br}(\mu^+ \rightarrow e^+ e^+ e^-) = \frac{1}{8}(G_{12} + 16G_{34} + 8G_{56})
\]

Comparison two BRs → probe for CP violating

\[
G \equiv G_{12} + 16G_{34} + 4G_{56} + 8G'_{14} - 8G'_{23} - 8G'_{56}
\]

\[
G_{ij} \equiv |g_i|^2 + |g_j|^2
\]

\[
G'_{ij} \equiv \text{Re}(g_i^* g_j)
\]
Branching ratio

\[
Br(\mu^- e^- \rightarrow e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \rightarrow e^- e^-) \\
= 24\pi (Z - 1)^3 \alpha^3 \left( \frac{m_e}{m_\mu} \right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G \\
= (3.31 \times 10^{-12})(Z - 1)^3 \left( \frac{\tilde{\tau}_\mu}{\tau_\mu} \right) G
\]

Enhancement factor from overlap of wave functions

- Positive charge attracts muon and electron toward the nucleus position.

Notable advantage for heavy nuclei
Effective Lagrangian

\[
\mathcal{L}_{\mu^- e^- \rightarrow e^- e^-} = \frac{-4G_F}{\sqrt{2}} \left[ m_\mu A_R \overline{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \overline{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} 
+ g_1 (\overline{\mu}_R e_L) (\overline{e}_R e_L) + g_2 (\overline{\mu}_L e_R) (\overline{e}_L e_R) \right]
\]

Photonic interaction type
$\mu e \rightarrow e e$

Branching ratio

$$\text{Br}(\mu^− e^− \rightarrow e^− e^−)$$

$$= 1536\pi^2 (Z - 1)^3 \alpha^4 (|A_R|^2 + |A_L|^2) \frac{m_e}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$

$$= 2.08 \times 10^{-9} (Z - 1)^3 (|A_R|^2 + |A_L|^2) \left(\frac{\tilde{\tau}_\mu}{\tau_\mu}\right)$$

Photon propagator in non-relativistic limit

$$\frac{1}{q^2} \simeq \frac{1}{m_\mu m_e}$$

Enhancement factor compared with 4-Fermi case

$$\frac{m_\mu^2}{m_e^2}$$
$\mu e \rightarrow e e$

Case: same order cLFV coupling

$$A_{L(R)} \equiv g_i \left( i = 1, 2, \ldots 6 \right)$$

Ratio between photonic and 4-Fermi cross section

$$\sigma v_{\text{photonic}} \sim \alpha \frac{m_\mu^2}{m_e^2} \times \sigma v_{4\text{Fermi}} \sim 10^3 \times \sigma v_{4\text{Fermi}}$$

One of the distinct features for the process
Discovery reach with the naïve estimate
Discovery reach

How to get upper limit for $\text{BR}(\mu^- e^- \rightarrow e^- e^-)$

- Calculate ratio of the BR to other limited cLFV BR

**4-Fermi interaction dominant case**

$$\frac{\text{Br}(\mu^- e^- \rightarrow e^- e^-)}{\text{Br}(\mu^+ \rightarrow e^+ e^+ e^-)}$$

**Photonic interaction dominant case**

$$\frac{\text{Br}(\mu^- e^- \rightarrow e^- e^-)}{\text{Br}(\mu^+ \rightarrow e^+ e^+ e^-)} \quad \text{and} \quad \frac{\text{Br}(\mu^- e^- \rightarrow e^- e^-)}{\text{Br}(\mu^+ \rightarrow e^+ \gamma)}$$

These ratios are independent on cLFV effective coupling
Discovery reach

- $\text{Br (}\mu \rightarrow 3e, \text{ Photonic}) < 1.0 \times 10^{-12}$
- $\text{Br (}\mu^+ \rightarrow e^+\gamma) < 1.2 \times 10^{-11}$
- $\text{Br (}\mu \rightarrow 3e, 4\text{Fermi}) < 1.0 \times 10^{-12}$
- $\text{Br (}\mu^+ \rightarrow e^+\gamma) < 1.7 \times 10^{-13}$

Branching Ratio to $\mu^e \rightarrow e^-e^-$

<table>
<thead>
<tr>
<th>Al</th>
<th>Ti</th>
<th>Au</th>
<th>Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
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<tr>
<td>90</td>
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</table>
Discovery reach

Current experimental bound in 4-Fermi interaction dominant case
Discovery reach

Current experimental bound in photonic interaction dominant case
### Discovery reach

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Searching for</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEG</td>
<td>$\mu \to e\gamma$</td>
<td>$10^{7.5} \mu/s$</td>
</tr>
<tr>
<td>MUSIC</td>
<td>$\mu \to 3e$</td>
<td>$10^8 \mu/s$</td>
</tr>
<tr>
<td>COMET</td>
<td>$\mu^-N \to e^-N$</td>
<td>$10^{11} \mu/s$</td>
</tr>
<tr>
<td>Mu2E (E973)</td>
<td>$\mu^-N \to e^-N$</td>
<td>$10^{11} \mu/s$</td>
</tr>
<tr>
<td>PRISM</td>
<td>$\mu^-N \to e^-N$</td>
<td>$10^{12} \mu/s$</td>
</tr>
</tbody>
</table>

**For run-time 1 year $\sim 3 \times 10^7$s**

$10^{18} - 10^{19}$ muon at COMET experiment
Discovery reach

\[\mu^+ \rightarrow e^+\gamma\] can be a first signal of LFV!?
Discovery reach

<table>
<thead>
<tr>
<th>Project</th>
<th>Intensity Reach</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMET / PRISM</td>
<td>$10^{18} - 10^{19} \mu$/year</td>
</tr>
<tr>
<td>$\nu$ factories</td>
<td>$10^{21} \mu$/year</td>
</tr>
</tbody>
</table>

With these number of muons the process will be seen !!
Precise Estimate (PRD +) 1

Total Rate
Precise Estimate

- High Z nucleus is preferable
- All leptons are under strong Coulomb potential
- Distorted wave function
- Out-going electron is very energetic $\gg m_e$
- & High Z means high velocity for bound leptons
- Relativistic treatment
- Nuclei is not a point charge
- Numerical solution for Wave fns, integration for amplitude

Solve Dirac Eq., integrate Wave fns, numerically

For trial, uniform charge density is assumed (not so important)

\[ V(r) = -\frac{Z\alpha}{R}\left(\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2}\right) \quad \text{for} \quad r < R \]

\[ = -\frac{Z\alpha}{r} \quad \text{for} \quad r > R \quad \text{with} \quad R = 1.2A^{1/3}\text{fm} \]
Calculating method

Decay rate $\Gamma$

\[
\Gamma = 2\pi \sum_f \sum_{\bar{i}} \delta(E_f - E_i) \left| \langle \psi^{s_1}_e(p_1)\psi^{s_2}_e(p_2) | H | \psi^{s\mu}_\mu(1s)\psi^{s\epsilon}_e(1s) \rangle \right|^2
\]

use partial wave expansion to express the distortion

\[
\psi^s_e(p) = \sum_{\kappa,\mu,m} 4\pi i^{l_\kappa}(l_\kappa, m, 1/2, s | j_\kappa, \mu) Y_{l_\kappa,m}(\hat{p}) e^{-i\delta_\kappa} \psi^\kappa,\mu_p
\]

get radial functions by solving Dirac eq. numerically

\[
\begin{align*}
\frac{d g_\kappa(r)}{dr} + \frac{1 + \kappa}{r} g_\kappa(r) - (E + m + e\phi(r)) f_\kappa(r) &= 0 \\
\frac{d f_\kappa(r)}{dr} + \frac{1 - \kappa}{r} f_\kappa(r) + (E - m + e\phi(r)) g_\kappa(r) &= 0
\end{align*}
\]

$\psi(r) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ if_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$

$\phi$ : nuclear Coulomb potential
Effective Lagrangian  

\[ \mathcal{L}_{\mu \bar{e} \rightarrow e e} = -\frac{4G_F}{\sqrt{2}} \left[ m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} ight. 
\]

\[ + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R) (\bar{e}_L e_R) 
\]

\[ + g_3 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_R \gamma_\mu e_R) + g_4 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_L \gamma_\mu e_L) 
\]

\[ + g_5 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_L \gamma_\mu e_L) + g_6 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_R \gamma_\mu e_R) + \text{(H.c.)} \right] 

4-Fermi interaction type
Reaction rate for contact interactions

\[ \Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi_{p_1}^e \psi_{p_2}^e | \mathcal{L}_I | \psi_B^\mu \psi_B^e \rangle \right|^2 \]

\[ = \frac{G_F^2}{\pi^3} \int dE_{p_1} |\mathbf{p}_1||\mathbf{p}_2| \sum_{J,\kappa_1,\kappa_2} (2J + 1)(2j_{\kappa_1} + 1)(2j_{\kappa_2} + 1) \]

\[ \times \left| \sum_{i=1}^6 g_i W_i (J, \kappa_1, \kappa_2, E_{p_1}) \right|^2 \]

(Amplitude)\(^2 \) up to J

\( \kappa \): (total angular + orbital) momentum for scattered electron

- cLFV interactions Physics !!
- Overlap of wave functions
High angular momentum is very important

| nuclei | $|\kappa| \leq 1$ | $|\kappa| \leq 5$ | $|\kappa| \leq 10$ | $|\kappa| \leq 20$ |
|--------|-----------------|-----------------|-----------------|-----------------|
| $^{40}$Ca | 0.141           | 0.847           | 1.11            | 1.15            |
| $^{120}$Sn | 0.731           | 2.17            | 2.21            | 2.21            |
| $^{208}$Pb | 2.89            | 6.94            | 6.96            | 6.96            |

$\Gamma_0$: previous calculation
Upper limits of BR (contact process)

\[ BR(\mu^+ \rightarrow e^+e^-e^+) < 1.0 \times 10^{-12} \]

(SINDRUM, 1988)

\[ BR(\mu^-e^- \rightarrow e^-e^-) < B_{\text{max}} \]

\[ (g_1(\bar{e}_L\mu_R)(\bar{e}_L\epsilon_R)) \]

this work (1s)

atomic #, \( Z \)

needed # of muonic atoms (\( Z = 82 \))

\[ 2.1 \times 10^{18} \]

\[ 3.0 \times 10^{17} \]

Koike et al. (1s)
Dirac Eq.: Final State

- Previous calculation (1)
  Final electrons are described by plane wave

Though Out-going electrons are highly relativistic and its wave functions are distorted by nuclear Coulomb potential, especially for high Z

- Improvement for scattered electrons
  Coulomb scattering wave function of Dirac Eq. with finite size nucleus
Precise Estimate

Dirac Eq. : Final State

\[ \psi_{\kappa}(\vec{r}) = \left( g_{\kappa}(r) \chi_{\kappa}(\hat{r}) \right) + \left( i f_{\kappa}(r) \chi_{-\kappa}(\hat{r}) \right) \]

solid line : large component, \( g \)
dotted line : small component, \( f \)
(black line : w.f. plane wave)

Wave fun. near the center position becomes larger, which leads enhancement of the overlap and hence reaction rate

Muon is located at \( r \ll O(10) \) fm

- Improvement for scattered electrons
  - Plane wave
  - Coulomb scattering wave function of Dirac Eq. with finite size nucleus
Precise Estimate

Dirac Eq. : Bound electron

- Previous calculation (2)
  Non-relativistic electron wave function for bound state

The bound electron is a relativistic Dirac particle

Electron orbit radius $\gg$ Nucleus size

- Improvement for bound electron

NR wave function $\rightarrow$ Wave function of Dirac particle in point Coulomb potential
Dirac Eq. : Bound electron

\[ \psi_\kappa(\vec{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}(\hat{r}) \end{pmatrix} \]

- solid line : large component, \( g \)
- dotted line : small component, \( f \)
- (black line : \( g \): Non-Rel , \( f = 0 \))

Muon is located at \( r < \text{O}(10) \text{ fm} \)

- More attracted, leading enhancement of the overlap

- Improvement for bound electron
  - NR wave function
  - Wave function of Dirac particle in point Coulomb potential
Precise Estimate

Dirac Eq. : Bound muon

- Previous Calculation (3)
  Muon is localized at nucleus position

- Muon is not at center but spread around nucleus

- Improvement for bound muon
  Localized muon wave function
  Wave function of Dirac particle in Coulomb potential by finite size nucleus
Density near the center position becomes smaller, leading to decline of the overlap and hence the reaction rate.

**Improvement for bound muon**

Localized muon wave function → Wave function of Dirac particle in Coulomb potential by finite size nucleus.
Overlap of wave functions

\[ \rho_{tr}(r) = g^{-1}_{p_1}(r)g^{-1}_\mu(r)g^{-1}_{p_2}(r)g^{-1}_e(r) \]

\[ A = 208 \text{ if } Z = 82 \]
Not only 1S but also 2S, ..., contribute

<table>
<thead>
<tr>
<th>nuclei</th>
<th>c [fm]</th>
<th>z [fm]</th>
<th>$\Gamma/\Gamma_0$ (only 1S)</th>
<th>$1S + 2S + \cdots$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>3.51(7)</td>
<td>0.563</td>
<td>1.15</td>
<td>1.35</td>
</tr>
<tr>
<td>$^{120}\text{Sn}$</td>
<td>5.315(25)</td>
<td>0.576(11)</td>
<td>2.21</td>
<td>2.67</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>6.624(35)</td>
<td>0.549(8)</td>
<td>6.96</td>
<td>8.78</td>
</tr>
</tbody>
</table>

Rate: much larger
Upper limits of BR (contact process)

\[ BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12} \]

(SINDRUM, 1988)

\[ BR(\mu^-e^- \to e^-e^-) < B_{max} \]

\[ (g_1(\bar{e}_L\mu_R)(\bar{e}_L\bar{e}_R)) \]

Needed # of muonic atoms \((Z = 82)\)

\[ 2.1 \times 10^{18} \]  

\[ 3.0 \times 10^{17} \]
Effective Lagrangian

\[ \mathcal{L}_{\mu e \rightarrow e e} = -\frac{4G_F}{\sqrt{2}} \left[ m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\mu_L e_R) (e_L e_R) \right] \]

Photonic interaction type
Upper limits of BR (photonic process)

\[ BR(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13} \]

(MEG, 2013)

\[ BR(\mu^-e^- \rightarrow e^-e^-) < B_{\text{max}} \]

\[ (g_L \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}) \]

Koike et al. (1s)

needed # of muonic atoms \( (Z = 82) \)

\[ 1.8 \times 10^{18} \rightarrow 6.6 \times 10^{18} \]
Suppression factor for photonic interaction
Phase shift effect of distortion

(makes a momentum of $e^-$ larger effectively)

**photonic process**

- No effect on photon by coulomb force
- No distortion for photon propergater
- $\mu^-$ - $e^-$
- $e^-$ - $e^-$

Totally (combined with the effect to enhance the value near the origin),

suppressed...
Phase shift effect of distortion

(makes a momentum of $e^-$ larger effectively)

**contact process**

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

- no momentum mismatches

**photonic process**

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !! suppressed...
Precise Estimate (PRD +) 1

Interaction type dependence
How to discriminate?
Discriminating method 1

\[ \frac{\Gamma(Z)}{(Z - 1)^3 \Gamma(Z = 2)} \]

\[ Z \text{ dependence of } \Gamma \text{ except } (Z - 1)^3 \]

- The \( Z \) dependences are different between interactions.
- Compared to \((Z - 1)^3\), that of contact process is larger, while that of photonic process is smaller.
Discriminating method 2

~ energy and angular distributions ~

\[ E_1 : \text{energy of an emitted electron} \]
\[ \theta : \text{angle between two emitted electrons} \]

\[ \frac{1}{\Gamma} \frac{d^2 \Gamma}{dE_1 d\cos \theta} \text{ [MeV}^{-1}] \]

Contact process

Photonic process

Preliminary
Summary
Summary

- New LFV process: $\mu^- e^- \rightarrow e^- e^-$ in muonic atom

- Clean signal (back to back electron with $E_e \approx m_\mu/2$)

- Interaction rate: $\Gamma(\mu^- e^- \rightarrow e^- e^-; Z) \sim (Z - 1)^3$

- Advantage: Large nucleus

- Discrimination among interactions: possible

  Making use of $Z$ dependence, energy spectrum, position dependence
Summary

Detectable in on-going or future experiments

Discussion with COMET people

Unfortunately not a discovery mode ...., though
Discussion (photonic dipole Interaction)
Numerical result

- Large enhancement of the reaction rate by the improvements
- Especially the improvements of electron wave functions
- Example: upper bound for pb nucleus $\sim 3.5 \times 10^{-18}$
The reason of enhancement

- Wave function of initial electron approaches to nucleus
  - Increase of overlap
- Wave function density of muon at nucleus becomes smaller
  - Decrease of overlap

- Wave functions of final electron also approach to nucleus
  - Increase of overlap

As a total

Enhancement
Cross section of cLFV elemental process

- cLFV effective Lagrangian

\[
\mathcal{L}_{\mu^-e^-\rightarrow e^-e^-} = -\frac{4G_F}{\sqrt{2}} \left[ m_\mu A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} 
\right. \\
\left. + g_1 (\bar{\mu}_R e_L) (\bar{e}_R e_L) + g_2 (\bar{\mu}_L e_R) (\bar{e}_L e_R) 
\right. \\
\left. + g_3 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_R \gamma^\mu e_R) + g_4 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_L \gamma^\mu e_L) 
\right. \\
\left. + g_5 (\bar{\mu}_R \gamma^\mu e_R) (\bar{e}_L \gamma^\mu e_L) + g_6 (\bar{\mu}_L \gamma^\mu e_L) (\bar{e}_R \gamma^\mu e_R) + \text{(H.c.)} \right]
\]

Dipole photonic interaction
Photonic interaction: Long-range force

Photon propagator: infrared divergent

\[ \mathcal{M}(p_1, s_1; p_2, s_2; 1S, s_\mu; n, s_e) \equiv \langle e(p_1, s_1); e(p_2, s_2) | H | \mu(1S, s_\mu); e(n, s_e) \rangle \]

\[ \propto \int d^3x d^3x' \int \frac{d^3q}{(2\pi)^3} \frac{q_v e^{-i\vec{q} \cdot (\vec{x} - \vec{x}')}}{q_0^2 - |\vec{q}|^2 + i\epsilon} \]

\[ \times \psi_{e p_1, s_1}(\vec{x}) \sigma^{\mu\nu} \psi_{e 1S, s_\mu}(\vec{x}) \psi_{e p_2, s_2}(\vec{x}') \gamma_\mu \psi_{e n, s_e}(\vec{x}') - (1 \leftrightarrow 2) \]

Scattered electrons (Coulomb distorted)  Bound leptons

Infrared divergent, especially for high Z
Backup slides
Numerical result (previous work)

- The new process and $\mu \rightarrow 3e$ are described by the same operator.

- Relation between upper bounds of the new process and $\mu \rightarrow 3e$

\[
\frac{\text{Br}(\mu^- e^- \rightarrow e^- e^-)}{\text{Br}(\mu^+ \rightarrow e^+ e^+ e^-)} \lesssim 192\pi (Z - 1)^3 \alpha^3 \left( \frac{m_e}{m_\mu} \right)^3 \frac{\tau_\mu}{\tau_\mu}.
\]

- Large enhancement by large nucleus charge.

- Example: Upper bound for Pb nucleus $\sim 5 \times 10^{-19}$
Distortion of emitted electrons

- $\kappa = -1$ partial wave

$rg_{E_{1/2}}^{\kappa=-1}(r)$

$Z = 82$

$E_{1/2} \approx 48\text{MeV}$

- 208Pb case

1. enhanced value near the origin
2. phase shift to boost momentum effectively

what the distortion makes

- distorted wave
- plane wave

overlap of w.f.
Phase shift effect of distortion

(makes a momentum of $e^-$ larger effectively)

contact process

bound $\mu^-$ emitted $e^-$

bound $e^-$ emitted $e^-$

- no momentum mismatches

photonic process

bound $\mu^-$ emitted $e^-$

bound $e^-$ emitted $e^-$

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin), enhanced !! suppressed...
Model-discriminating power

After finding CLFV transition, “which CLFV interaction exists” would be important.

Here, only 2 simple models will be considered.

**model 1**: contact type

\[ \mathcal{L}_I = g_1 (\bar{e}_L \mu_R)(\bar{e}_L e_R) \]

**model 2**: photonic type

\[ \mathcal{L}_I = g_R \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} \]
Summary

- $\mu^- e^- \rightarrow e^- e^-$ process in a muonic atom
  - interesting candidate for CLFV search
  - Our finding
    - **Distortion** of emitted electrons
    - **Relativistic treatment** of a bound electron
      are important in calculating decay rates.
  - Distortion makes difference between 2 processes.
- contact process: decay rate **Enhanced** (7 times in $Z = 82$)
- photonic process: decay rate **suppressed** (1/4 times in $Z = 82$)

- How to identify interaction types, found by this analyses
  - atomic # dependence of the decay rate
  - energy and angular distributions of emitted electrons
Radial functions (bound $e^-$)

$g_e^{1s}(r)$

$^{208}$Pb case \hspace{1cm} Z = 81
(considering $\mu^-$ screening)

<table>
<thead>
<tr>
<th>Type</th>
<th>$B_e$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rela</td>
<td>$9.88 \times 10^{-2}$</td>
</tr>
<tr>
<td>Non-rela</td>
<td>$8.93 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Relativity enhances the value near the origin.
Upper limits of $\text{Br}(\mu^- e^- \to e^- e^-)$

- $\mu eee$ interaction
  - $\text{Br}(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$
  - $\text{Br}(\mu^- e^- \to e^- e^-) < 4.5 \times 10^{-19}$ for Pb ($Z = 82$)

- $\mu e\gamma$ interaction
  - $\text{Br}(\mu^+ \to e^+ \gamma) < 5.7 \times 10^{-13}$
  - $\text{Br}(\mu^- e^- \to e^- e^-) < 5.7 \times 10^{-19}$ for Pb ($Z = 82$)
Discriminating method 2

~ energy and angular distributions ~

Angular distributions ($\cos \theta \approx 1$)

\[
\frac{d\Gamma}{\Gamma d\cos \theta}
\]

- contact (same chirality)
- contact (opposite chirality)
- photonic

$g_5$ has larger tail than $g_1$ due to Pauli principle.
Energy and angular distribution \((g1)\)\(g_1(e_L\mu_R)(\bar{e}_L\bar{e}_R)\)

\[ Z = 82 \]

Muon is not point-like \(\rightarrow\) distribution is spread

\(E_1\) : energy of scattered electron

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dE_1} \quad [\text{MeV}^{-1}]
\]

\(E_1\) : energy of scattered electron

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta}
\]

\(\theta\) : angle between scattered electrons

\[1 \leq E_1 \leq m_\mu - B_\mu - B_e\]

- Peak at half value

- Almost back to back
Energy dependence of angular distribution ($g_1$)

$$g_1(\bar{e}_L \mu_R)(\bar{e}_L \epsilon_R)$$

$Z = 82$

$E_{p_1} = 0.57\text{MeV}$

$E_{p_1} = 23.0\text{MeV}$

$E_{p_1} = 47.7\text{MeV}$

At half value, almost back-to-back
Angular dependence of energy distribution ($g_1$)

$$g_1(\bar{e}_L\mu_R)(\bar{e}_L e_R)$$

($Z = 82$)

(Normalized by the maximum value)
Angle and energy distribution ($g_1$)

\[ g_1(\bar{e}_L \mu_R)(\bar{e}_L e_R) \]

$Z = 82$

"3D" plot
Angular dependence of energy distribution ($g_5$)

$$g_5 (\overline{e_R} \gamma_\mu \mu_R) (\overline{e_L} \gamma^\mu e_L)$$

(Normalized by the maximum value)

$Z = 82$
Angle and energy distribution ($g_5$)

\[ g_5 \left( \bar{e}_R \gamma_{\mu} \mu_R \right) \left( \bar{e}_L \gamma^\mu e_L \right) \]

"3D" plot

\[ Z = 82 \]
Angular distribution (forward)

Electron emission with same chirality to same direction is suppressed by Pauli principle

\[ Z = 82 \]

\[ g_5 \left( \bar{e}_R \gamma_\mu \mu_R \right) \left( \bar{e}_L \gamma^\mu e_L \right) \]

\[ g_1 \left( \bar{e}_L \mu_R \right) \left( \bar{e}_L e_R \right) \]