Dilaton: Saving Conformal Symmetry

Alexander Monin

Ecole Polytechnique Fédérale de Lausanne

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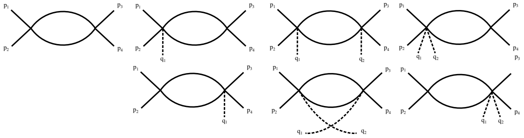
$$\int d^4x\sqrt{-\hat{g}}(\partial\hat{\Phi})^2 = \int d^4x\sqrt{g}\left\{(\partial\Phi)^2 + \Phi^2\left[(\nabla\tau)^2 + \nabla^2\tau\right]\right\}, \quad e^{-\tau}\partial^2 e^{\tau} = (\nabla\tau)^2 + \nabla^2\tau \tag{1}$$

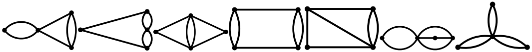
$$\delta\left[\frac{a}{d-4}\int d^n x\sqrt{-g}\,E_4\right] = \frac{a}{d-4}\int d^n x\sqrt{-g}\,\left(E_4 + (d-4)\sigma E_4\right), \quad E_4 = R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{2}$$

$$e^{i\Gamma[J_\Phi, J_X, \mathbf{g}_{\mu\nu}]} = \int(\mathcal{D}\delta\Phi)\,(\mathcal{D}\delta X)e^{iS[\Phi+\delta\Phi, X+\delta X, g_{\mu\nu}]+i\int\sqrt{-g}J_\Phi\delta\Phi+i\int\sqrt{-g}J_X\delta X}$$



(a) (b) (c) (d)

$$\Gamma[\Phi, X, g_{\mu\nu}] = W[J_\Phi, J_X, g_{\mu\nu}] - \int d^n x\sqrt{-g}\Phi J_\Phi - \int d^n x\sqrt{-g}X J_X, \quad X'(x') = \Omega^\Delta X(x), \quad \hat{\Phi}'(x') = \hat{\Phi}(x),$$



$$\hat{g}_{\lambda\sigma}(x)\frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} = \eta_{\mu\nu}e^{2\tau(x)}\Omega^2(x)\; X'(x') = \Omega^\Delta X(x) \tag{3}$$

$$\delta\left[\frac{a}{d-4}\int d^n x\sqrt{-g}\,E_4\right] = \frac{a}{d-4}\int d^n x\sqrt{-g}\,\left(E_4 + (d-4)\sigma E_4\right)\mathcal{L}_{SB} = \frac{e^{-2\tau}}{2}\left[(\partial\hat{\Phi})^2 + (\partial\tau)^2 - m^2\hat{\Phi}^2e^{-2\tau}\right]$$

$$, \tag{4}$$

$$T^\mu_\mu = (4-d)\frac{\lambda_0\phi_0^4}{4!} = -\beta(\lambda)\frac{[\phi^4]}{4!}, \quad \langle T_{\mu\nu}T_{\lambda\sigma}\rangle = \frac{\#}{2-d}\frac{1}{p^2}\left[\frac{d-1}{2}\left(P_{\mu\lambda}P_{\mu\lambda} + P_{\mu\lambda}P_{\mu\lambda}\right) - P_{\mu\lambda}P_{\mu\lambda}\right] \tag{5}$$

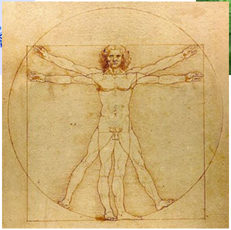
Motivation

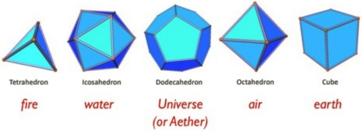
- Purely academic
- Anomalies and certain properties of QFT
- Dilaton as a spectator (background) field has already been used
- Naturalness?

Plan

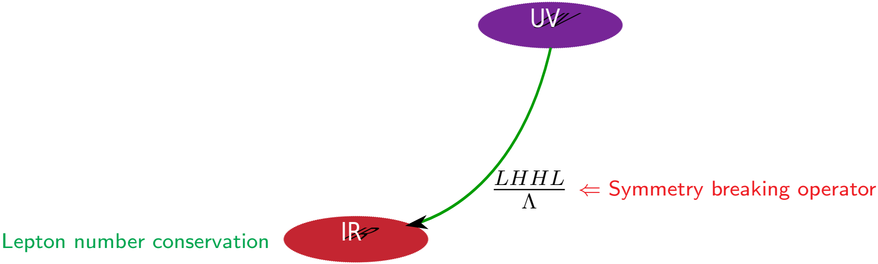
- Symmetries and conservation laws
- Spontaneous symmetry breaking and coset construction
- Anomalies
- Conformal symmetry at the quantum level?
- Toy model up to 3 loops
- General argument
- Conclusions

Symmetries are ubiquitous





Are symmetries fundamental?



"Purification" by the RG flow

Within the Wilsonian approach global symmetries can be understood as accidents

Symmetry $\Leftrightarrow$ conservation law

Space-time translations $\Rightarrow \partial_\mu T_{\mu\nu} = 0$
$U(1)$-phase rotation $\Rightarrow \partial_\mu j_\mu = 0$

Symmetries provide building blocks
and selection rules

$$A_\mu j^\mu,\ \bar{L} H e_R,\ V(H^\dagger H),\ \text{etc.}$$

Conformal symmetry

Group of transformations leaving Minkowski metric invariant up to an overall factor

$$g'_{\mu\nu}(x) = \Omega(x)\eta_{\mu\nu}$$

$$x'^{\mu} = x^{\mu} + \xi^{\mu} \Rightarrow \partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{d}\eta_{\mu\nu}\partial\xi$$

Solution

$$\xi^\mu = \underbrace{a^\mu + \omega^{\mu\nu}x_\nu}_{\text{Poincaré: } P_\mu, M_{\mu\nu}} - \overbrace{\alpha x^\mu}^{\text{dilatation: } D} + \underbrace{2(bx)x^\mu - b^\mu x^2}_{\text{special conformal: } K_\mu} .$$

$$j^\mu = \xi^\nu T^\mu_\nu, \quad \partial_\mu j^\mu_C = 0 \Rightarrow T^\mu_\mu = 0$$

CFTs

Commutation relations for $SO(2,d)$

$$[J_{\mu\nu}, J_{\lambda\sigma}] = i\left(J_{\mu\sigma}\eta_{\nu\lambda} + J_{\nu\lambda}\eta_{\mu\sigma} - J_{\nu\sigma}\eta_{\mu\lambda} - J_{\mu\lambda}\eta_{\nu\sigma}\right)$$

Conformal group puts severe restrictions on correlators

Poincaré + dilatations fix two point functions

$$\langle\phi_\Delta(x)\phi_\Delta(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

Special conformal transformations

$$|x'_\mu - y'_\mu| = \frac{|x_\mu - y_\mu|}{\sqrt{(1-2bx+b^2x^2)(1-2by+b^2y^2)}}$$

allow to fix 3-point functions.

Not about that!

Symmetry breaking

Massive states exist!

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Phenomenological viability implies symmetry breaking

Spontaneous symmetry breaking implies existence of Goldstone modes

Promoting all dimensionful couplings to a dynamical field X realizes the scale symmetry nonlinearly

$$m^2\Phi^2 \to \lambda_{\Phi X}\Phi^2 X^2$$

It is enough to introduce only one additional degree of freedom – the dilaton – to realize nonlinearly the conformal symmetry

Nonlinear realization: coset construction

Group H can be promoted to G by adding Goldstone modes π^A
for a given symmetry breaking pattern $G \to H$

Action of the group G is realized non-linearly on the coset $G/H \ni \Omega$

$$\Omega(\pi) = e^{i\pi^A T_A}$$

$$g: \quad \Omega(\pi') = g\Omega(\pi)h^{-1}(g,\pi)$$

The form

$$\Omega^{-1}\partial_\mu\Omega = \nabla_\mu\pi^A \underbrace{T_A}_{\text{broken}} + \omega^a_\mu \underbrace{\bar{t}_a}_{\text{unbroken}}$$

transforms not covariantly

$$\Omega^{-1}d\Omega \to \Omega^{-1}d\Omega + h^1 dh^{-1}$$

but not covariant derivatives

$$\nabla_\mu\pi^A T_A \to h\left(\nabla_\mu\pi^A T_A\right)h^{-1}$$

Lagrangian is an arbitrary H-invariant function of $\nabla\pi$!

Space-time symmetries

In this case

$$\Omega = \underbrace{e^{ix^\mu \bar{P}_\mu}}_{\text{unbroken}} e^{i\pi^A T_A}$$

The form

$$\Omega^{-1}\partial_\mu \Omega = e_\mu^\nu \Big[\bar{P}_\nu + \nabla_\mu \pi^A \underbrace{T_A}_{\text{broken}} + \omega_\mu^a \underbrace{\bar{t}_a}_{\text{unbroken}} \Big]$$

again provides building blocks: vierbein e_μ^ν, covariant derivative $\nabla\pi$

It is rather usual that some derivatives can be put consistently to zero

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Certain modes are dependent: Inverse Higgs mechanism

$$\pi^A T_A \langle O \rangle = 0$$

Example: Point particle in $d = 1 + 1$

Symmetry breaking pattern: boost and space translation are broken

$$O(1,1) \times \mathbb{R}^{1,1} \to \mathbb{R} \quad \Rightarrow \quad \Omega = e^{i\bar{P}_0 t} e^{iP_1 \pi} e^{iK\eta}$$

Transformation properties under the boost

$$g = e^{iKa}: \quad g\Omega = \underbrace{e^{iKa}\, e^{i\bar{P}_0 t + iP_1 \pi(t)} e^{-iKa}}_{\substack{t' = t\cosh a + \pi(t)\sinh a \\ \pi'(t') = t\sinh a + \pi(t)\cosh a}} \; \underbrace{e^{i(\eta + a)K}}_{\eta'(t') = \eta(t) + a}$$

$$\Omega^{-1} d\Omega = i\bar{P}_0(\cosh\eta - \dot{\pi}\sinh\eta)dt + iP_1 \underbrace{(-\sinh\eta + \dot{\pi}\cosh\eta)\, dt}_{\text{can be set to zero}} + i\dot{\eta}\, K dt$$

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$$\Omega^{-1} d\Omega = i\bar{P}_0 \underbrace{(\cosh\eta - \dot{\pi}\sinh\eta)}_{i\sqrt{1-\dot{\pi}^2}dt} dt + iP_1 \underbrace{(-\sinh\eta + \dot{\pi}\cosh\eta)\, dt}_{\tanh\eta = \dot{\pi}} + i \underbrace{\dot{\eta}}_{\sim\ddot{\pi}} K dt$$

Lagrangian for a relativistic point-like particle!

Nonlinear realization: conformal symmetry

For each spontaneously broken symmetry there is a dynamical field

Dilaton from the coset construction

An element of the coset is parametrized in a standard way

$$\Omega = e^{i\bar{P}_\mu x^\mu} e^{iK_\mu c^\mu(x)} e^{iD\tau(x)}$$

The fundamental form

$$\Omega^{-1}d\Omega = idx^\mu \left[P_\mu \underbrace{e^{-\tau}}_{\text{vierbein}} + D \underbrace{(\partial_\mu \tau - 2c_\mu)}_{\nabla_\mu \tau} + K_\nu \underbrace{(\partial_\mu c^\nu + 2c_\mu c^\nu - \delta^\nu_\mu c^2)}_{\nabla_\mu c^\nu} \ldots \right]$$

Covariant derivative $\nabla_\mu \tau$ can be consistently put to zero $2c_\mu = \partial_\mu \tau$

$$\Uparrow$$

$$\exists\ \tau = 2c_\mu x^\mu, c_\mu = \text{const}: \quad [\tau(x)D + c^\mu(x)K_\mu] \langle O \rangle = 0$$

Example

Free massive scalar field in $d = 4$:

$$\mathcal{L} = \frac{1}{2}(\partial\hat{\Phi})^2 - \frac{1}{2}m^2\hat{\Phi}^2$$

Vierbein: $e^{-\tau}$ and $2c_\mu c^\nu - \delta^\nu_\mu c^2 \to (\partial\tau)^2$

As a result the Lagrangian for the non-linear realization has the form

$$\mathcal{L}_{SB} = \frac{e^{-2\tau}}{2}\left[(\partial\hat{\Phi})^2 + (\partial\tau)^2 - m^2\hat{\Phi}^2 e^{-2\tau}\right]$$

$$X = ve^{-\tau},\ \hat{\Phi} = \Phi e^{\tau}$$

$$\mathcal{L}_{SB} = \frac{1}{2}(\partial\Phi)^2 - \frac{\Phi}{X}\partial_\mu\Phi\partial_\mu X + \frac{1}{2}(\partial\Phi)^2\left(1 + \frac{\Phi^2}{X^2}\right) - \frac{m^2}{2v^2}X^2\Phi^2$$

One degree of freedom is enough!

Alternative way to introduce the dilaton

A theory with the action $S[\hat{\Phi}]$ is coupled to a background metric $\hat{g}_{\mu\nu}$

$$\Downarrow$$

$$S[\hat{\Phi}, \hat{g}_{\mu\nu}]$$

Dilaton appears as a conformal mode of the metric

$$\hat{g}_{\mu\nu} = e^{2\tau} g_{\mu\nu}, \;\; \hat{\Phi} = e^{-\Delta\tau}\Phi$$

$$S_{SB}[\Phi, \tau] = S[\hat{\Phi}, \hat{g}_{\mu\nu}], \;\; g_{\mu\nu} = \eta_{\mu\nu}$$

Obviously the Lagrangian for Φ and τ is invariant under

$$\Phi \to \Phi e^{-\Delta\sigma}, \;\; g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}, \;\; \tau \to \tau - \sigma$$

$$\Updownarrow$$

$$\hat{\Phi} \to \hat{\Phi}, \;\; \hat{g}_{\mu\nu} \to \hat{g}_{\mu\nu}$$

Example

Free massive scalar field in $d = 4$

$$\mathcal{L} = \frac{1}{2}(\partial\hat{\Phi})^2 - \frac{1}{2}m^2\hat{\Phi}^2$$

$$\Downarrow$$

$$S[\hat{\Phi}, \hat{g}_{\mu\nu}] = \frac{1}{2}\int d^4x\sqrt{-\hat{g}}\left[(\partial\hat{\Phi})^2 - m^2\hat{\Phi}^2 + \frac{v^2}{6}\hat{R}\right]$$

Introducing $X = ve^{\tau}$

$$\mathcal{L}_{SB} = \frac{1}{2}(\partial\Phi)^2 - \frac{\Phi}{X}\partial_\mu\Phi\partial_\mu X + \frac{1}{2}(\partial\Phi)^2\left(1 + \frac{\Phi^2}{X^2}\right) - \frac{m^2}{2v^2}X^2\Phi^2$$

The metric for the σ-model is flat $\Rightarrow$ conformal invariance is manifest

From classical to quantum

Anomalies

Sometimes regularization breaks a symmetry $\partial_\mu j^\mu \neq 0$

Examples

- Chiral anomaly
  - Pauli-Villars breaks $e^{i\alpha\gamma_5}$ explicitly
  - Dim-reg breaks chiral symmetry due to specific definition of γ_5
- Trace anomaly. Regularizations introduce scale explicitly
  - Dim-reg: couplings are dimensionful in $d \neq 4$

Trace anomaly

Classical improved energy momentum tensor for $\lambda\phi^4$ is traceless

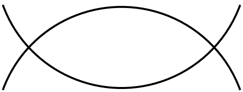
$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L} - \xi(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2)\phi^2$$

At the quantum level dimensionally regularized trace is

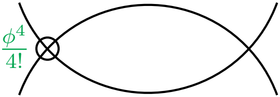
$$T^\mu_\mu = (4-d)\frac{\lambda_0\phi_0^4}{4!}$$

After renormalization

Coupling



Operator



$$T^\mu_\mu = \frac{(4-d)}{4!}\underbrace{\left(\lambda + \frac{\beta(\lambda)}{4-d}\right)}_{\lambda_0}\underbrace{\left(1 - \frac{2\beta(\lambda)}{(4-d)\lambda}\right)[\phi^4]}_{\phi_0^4} = -\beta(\lambda)\frac{[\phi^4]}{4!}$$

Conformal symmetry at the quantum level

- Regularization is a source of an explicit symmetry breaking
- Introduce regularization consistent with the symmetry

Non-linear realization gives a hint
how to introduce appropriate regularization:
All the scales are dilaton related!

Example, dim-reg ϕ^4 theory

$$\underbrace{(\partial\phi)^2 + v^2 e^{-2\tau}(\partial\tau)^2 - \lambda\phi^4}_{\text{Classical } d=4} \qquad \underbrace{(\partial\phi)^2 + v^2 e^{-2\tau}(\partial\tau)^2 - \lambda v^{\#} e^{(4-d)\tau}\phi^4}_{\text{Dim-reg } d\neq 4}$$

Another source of anomaly

Regularization is not the end of the story
Counter terms might break the symmetry!

Example: Weyl anomaly

$d = 4$: Regularized generating functional $W[g_{\mu\nu}]$ is Weyl invariant

$$e^{iW[g_{\mu\nu}]} = \int \mathcal{D}\phi e^{\frac{i}{2}\int_d (\partial\phi)^2 + \xi\phi^2 R}, \quad \xi = \frac{1}{4}\frac{d-2}{d-1}$$

The counter term with $E_4 = R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is not

$$\delta\left[\frac{a}{d-4}\int d^n x\sqrt{-g}\,E_4\right] \overset{g_{\mu\nu}\to e^{2\sigma}g_{\mu\nu}}{=} a\int d^n x\sqrt{-g}\,\sigma E_4$$

One has to make sure that counterterms respect the symmetry!

Example: 1 loop

Dimensionally regularized $d = 4 - 2\varepsilon$ Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial X)^2 - \frac{\lambda}{4!}\phi^4 X^{2\alpha}, \quad \alpha = \frac{4-d}{d-2}$$

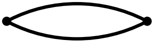
can be expanded around $X = v + \chi$

$$\phi^4 X^{2\alpha} = \phi^4 \overbrace{v^{2\alpha}}^{\mu^{2\varepsilon}} \left[1 + \underbrace{2\varepsilon} \ln\left(1 + \frac{\chi}{v}\right) + \underbrace{2\varepsilon^2} \left(\ln\left(1 + \frac{\chi}{v}\right) + \ln^2\left(1 + \frac{\chi}{v}\right)\right) + O\left(\varepsilon^3\right)\right]$$

Not important at one loop (for 2ε) and two loops (for $2\varepsilon^2$)

Leading divergence is $1/\varepsilon^{\#\text{loops}}$

Renormalization



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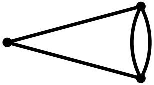
Scale/conformally invariant 1-loop counter term

$$-\frac{3}{16\pi^2}\frac{\lambda^2 v^{2\alpha}}{2\varepsilon}\frac{\phi^4}{4!} = -\frac{\lambda^2}{(4\pi)^2}\frac{3}{2\varepsilon}\frac{\phi^4}{4!}X^{2\alpha} + \text{finite}$$

Example: 2 loops

Diagrams to be taken into account

ε^{-2}



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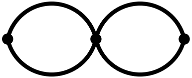
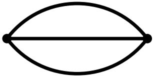


ε^{-1}



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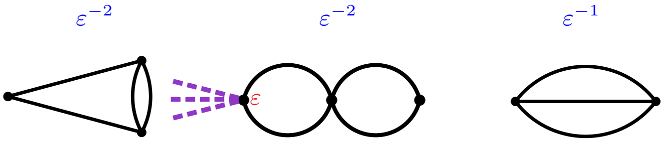

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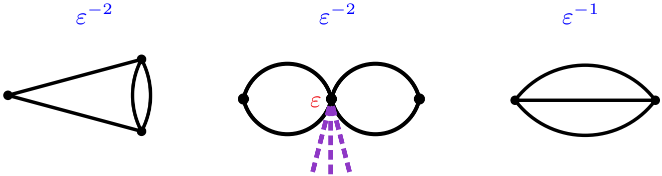

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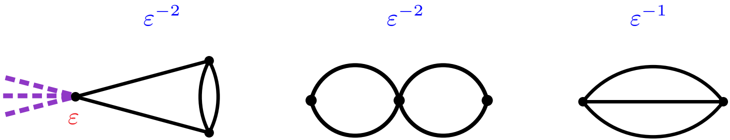

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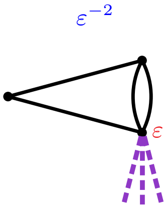

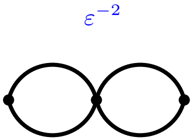

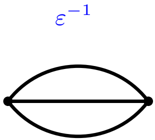

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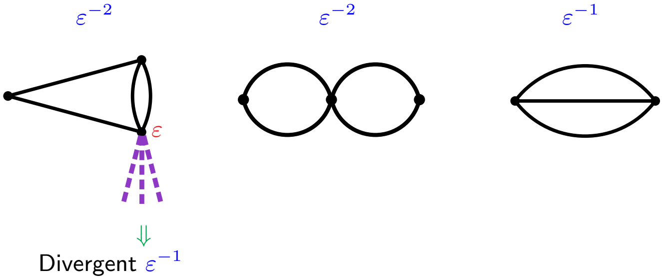

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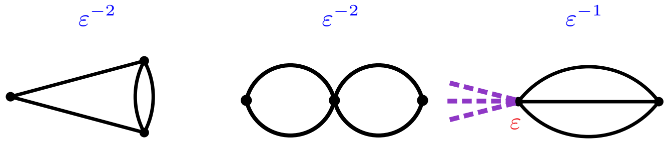


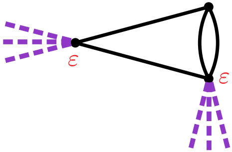
$\Downarrow$

Finite!

Example: 2 loops

Diagrams to be taken into account

ε^{-2}



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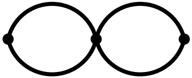

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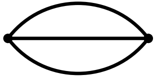

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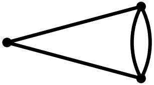
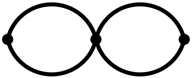
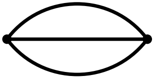


ε^{-2}



ε^{-1}



$\Downarrow$

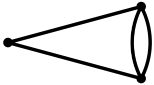
Standard renormalization of ϕ^4

$$-\frac{\lambda^2}{(4\pi)^4}\frac{1}{24\varepsilon}(\partial\phi)^2$$

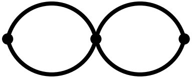
Example: 2 loops

Diagrams to be taken into account

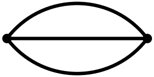
ε^{-2}



ε^{-2}



ε^{-1}



$\Downarrow$ Standard renormalization of ϕ^4

$$-\frac{\lambda^2}{(4\pi)^4}\frac{1}{24\varepsilon}(\partial\phi)^2$$

$\swarrow$ (first two diagrams)

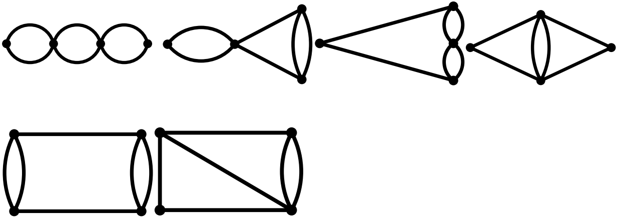
$$-\frac{\lambda^3}{(4\pi)^4}\frac{3}{4}\left[\frac{3-2\varepsilon}{\varepsilon^2}\right]\frac{\phi^4}{4!}X^{2\alpha}$$

$$\underbrace{-\frac{\lambda^3}{(4\pi)^4}\frac{3}{4}\left[\frac{3-2\varepsilon}{\varepsilon^2}\right]\frac{\phi^4}{4!}v^{2\alpha}}_{\phi^4 \text{ renormalization}}\left[1+4\varepsilon\ln\left(1+\frac{\chi}{v}\right)\right] + \text{finite}$$

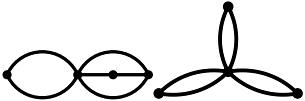
Example: 3 loops

ε^{-3} Divergent graphs

Graphs from ϕ^4



New graphs



Example: 3 loops

$$\text{Power of divergence} = \underbrace{\varepsilon^{-3}}_{\text{Divergence of a graph}} \times \varepsilon^{\#\text{ of vertices with }\chi}$$

$$\Downarrow$$

At most 2 vertices!!

New structures appear due to the dilaton in the loop!

$$\mathcal{L}_{CT} = -\frac{C_6}{6!}\frac{\phi^6}{X^2}X^{2\alpha} - \frac{C_8}{8!}\frac{\phi^8}{X^4}X^{2\alpha}$$

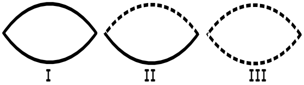
$$C_6 = \frac{\lambda^4}{(4\pi)^6}180\frac{1}{\varepsilon}, \qquad C_8 = \frac{\lambda^4}{(4\pi)^6}\frac{9625}{6}\frac{1}{\varepsilon}$$

6d example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial X)^2 - \frac{g}{4!}\underbrace{\frac{\phi^4}{X}}X^{2\alpha_6}, \qquad 2\alpha_6 = \frac{(6-d)}{(d-2)}$$

↙

Leads to ε unsuppressed vertices



Counterterms are conformally invariant!

$$\mathcal{L}_{CT} = \frac{1}{(2!)^2}\frac{C_1}{6-d}\frac{\phi^2}{X}\partial^2\frac{\phi^2}{X} + \frac{1}{(3!)^2}\frac{C_2}{6-d}\frac{\phi^3}{X^2}\partial^2\frac{\phi^3}{X^2} + \frac{1}{(4!)^2}\frac{C_3}{6-d}\frac{\phi^4}{X^3}\partial^2\frac{\phi^4}{X^3}$$

$$C_1 = \frac{g^2}{12(4\pi)^3} \qquad C_2 = \frac{g^2}{6(4\pi)^3} \qquad C_3 = \frac{g^2}{3(4\pi)^3}$$

8d example

Classically conformal invariant Lagrangian has the form

$$\mathcal{L} = \frac{1}{2}(\partial\phi) + \frac{1}{2}(\partial X)^2 - \frac{g}{3!}\frac{\phi^3}{X^{1/3}}X^{\#}$$

One of the counter terms

$$\delta_3^2 = \frac{g^3}{8-d}\frac{1}{6(4\pi)^4}\frac{1}{3!}\left\{\phi^2 X^{-2/3}\partial^2(\phi X^{-1/3}) \underbrace{-} \phi X^{-1/3}\left[\partial(\phi X^{-1/3})\right]^2\right\}$$

Only for relative minus the expression is conformally invariant

General argument

1PI effective action $\Gamma_{\text{reg}}[\Phi]$ is a Legendre transform of $W_{\text{reg}}[J_\Phi]$

$$e^{iW_{\text{reg}}[J_\Phi]} = \int_{\text{reg}} \mathcal{D}\Phi \, e^{iS[\Phi]+i\int J_\Phi \Phi}$$

If a regulator is consistent with a symmetry

$$\delta\Gamma_{\text{reg}}[\Phi] = \frac{1}{\varepsilon}\delta\Gamma_P[\Phi] + \delta\Gamma_F[\Phi] = 0$$

For internal symmetries δ does not depend on the # of dimensions

$$\delta\Gamma_P[\Phi] = \delta\Gamma_F[\Phi] = 0$$

Example: Weyl symmetry

General argument

Conformal transformations depend explicitly on d!

Regularized effective action with metric $\Gamma[\Phi, X, g_{\mu\nu}]$ is Weyl invariant:

$$\hat{\Phi}(x) = e^{-\tau(x)\Delta}\Phi(x), \qquad \hat{g}_{\mu\nu}(x) = e^{2\tau(x)} g_{\mu\nu}(x), \qquad \hat{X} = v$$

$$\Gamma[\Phi, X, g_{\mu\nu}] = \Gamma[\hat{\Phi}, v, \hat{g}_{\mu\nu}] = \frac{1}{\varepsilon}\Gamma_P[\hat{\Phi}, v, \hat{g}_{\mu\nu}] + \Gamma_F[\hat{\Phi}, v, \hat{g}_{\mu\nu}]$$

Assuming no Diff anomaly

$$\Gamma_P[\underbrace{\hat{\Phi}'(x')}_{\hat{\Phi}(x)}, v, \underbrace{\hat{g}'_{\mu\nu}(x')}_{\hat{g}_{\lambda\sigma}(x)\frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}}] = \Gamma_P[\hat{\Phi}, v, \hat{g}_{\mu\nu}]$$

Conformal symmetry

For the flat metric $\hat{g}_{\mu\nu}(x) = \eta_{\mu\nu} e^{2\tau(x)}$ conformal transformations

$$x'_\mu = \underbrace{\Omega^{-1}(x)}_{1-2(ax)+a^2x^2} \quad (x_\mu - a_\mu x^2)$$

lead to

$$\hat{g}'_{\mu\nu}(x') = \eta_{\mu\nu} e^{2\tau(x)} \Omega^2(x)$$

It can be written as transformation of Φ and X keeping $g_{\mu\nu} = \eta_{\mu\nu}$ fixed

$$e^{2\tau'(x')} \eta_{\mu\nu} \equiv \hat{g}'_{\mu\nu}(x') = \eta_{\mu\nu} e^{2\tau(x)} \Omega^2(x)$$

$$\underbrace{e^{-\Delta\tau'(x')} \Phi'(x') \equiv \overbrace{\hat{\Phi}'(x')}^{} }_{\text{Weyl}} \overbrace{= \hat{\Phi}(x)}^{\text{Diff}} \equiv \Phi(x) e^{-\Delta\tau(x)}$$

As a result we get the standard conformal transformations!

$$\Phi'(x') = \Omega^\Delta \Phi(x)$$

$$e^{2\tau'(x')} = e^{2\tau(x)} \Omega^2(x)$$

Conclusions

- Non-linearly realized conformal symmetry can be promoted to the quantum level
- Dilaton is a source for all scales in the theory
- Relevance for naturalness?
- Unitarity?

Thank you very much