

Diagrammar for leptogenesis

Peter Maták

In collaboration with T. Blažek, J. Heeck, J. Heisig, and V. Zaujec



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Particle and Astroparticle Theory Seminar
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Plan for this seminar

- Introduction to unitarity constraints and holomorphic cutting rules
- Dirac leptogenesis from scatterings of massless particles
- Further (unexpected) consequences of unitarity

Part I.

CPT and unitarity constraints

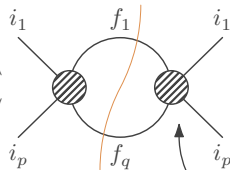
Unitarity and optical theorem

$$S^\dagger S = 1 \quad \rightarrow \quad i T^\dagger - i T = T^\dagger T \quad \text{for} \quad i T = S - 1 \quad (1)$$

Unitarity and optical theorem

$$S^\dagger S = 1 \quad \rightarrow \quad iT^\dagger - iT = T^\dagger T \quad \text{for} \quad iT = S - 1 \quad (1)$$

$$2 \operatorname{Im} T_{ii} = \sum_f |T_{fi}|^2 = - \sum_f iT_{if}^\dagger iT_{fi} = - \sum_f \quad (2)$$

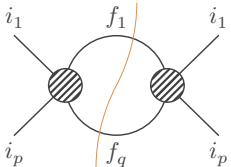


$$\int \frac{d^3 \mathbf{k}}{(2\pi)^3 2E_k} = \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta_+(k^2 - m^2) \quad \text{for each cut line}$$

$i\epsilon$ replaced by $-i\epsilon$
for each propagator

Unitarity and optical theorem

$$S^\dagger S = 1 \quad \rightarrow \quad iT^\dagger - iT = T^\dagger T \quad \text{for} \quad iT = S - 1 \quad (1)$$

$$2 \operatorname{Im} T_{ii} = \sum_f |T_{fi}|^2 = - \sum_f iT_{if}^\dagger iT_{fi} = - \sum_f \quad (2)$$


$$|T_{i \rightarrow f}|^2 = \int \prod_{\forall f} \frac{d^3 \mathbf{k}_f}{(2\pi)^3 2E_f} \sum_{\text{spin}} |T_{fi}|^2 \quad (3)$$

Imaginary kinematics in Feynman diagrams

$$\mathrm{i} T^\dagger - \mathrm{i} T = T^\dagger T \quad \rightarrow \quad \mathrm{i} T_{if}^* - \mathrm{i} T_{fi} = \sum_n T_{nf}^* T_{ni} \quad (4)$$

1. Real couplings with $T_{fi} = T_{if}$

$$2 \operatorname{Im} T_{fi} = \sum_n T_{fn}^* T_{ni} \quad (5)$$

Imaginary kinematics in Feynman diagrams

$$iT^\dagger - iT = T^\dagger T \quad \rightarrow \quad iT_{if}^* - iT_{fi} = \sum_n T_{nf}^* T_{ni} \quad (4)$$

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$$2 \operatorname{Im} T_{fi} = \sum_n T_{fn}^* T_{ni} \quad (5)$$

2. Imaginary kinematics

$$T_{fi} = \sum_{\text{diagrams}} C_{fi} K_{fi} \quad \leftarrow \quad C_{fi} = C_{if}^*, K_{fi} = K_{if} \quad (6)$$

$$2 \operatorname{Im} K_{fi} = \sum_n K_{fn}^* K_{ni} \quad (7)$$

CP asymmetric reactions

$$CPT \text{ symmetry implies } |T_{\bar{f}i}|^2 = |T_{if}|^2 \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 \quad (8)$$

$$\left. \begin{aligned} T_{fi} &= C_{fi}^{\text{tree}} K_{fi}^{\text{tree}} + C_{fi}^{\text{loop}} K_{fi}^{\text{loop}} \\ T_{if} &= C_{if}^{\text{tree}} K_{if}^{\text{tree}} + C_{if}^{\text{loop}} K_{if}^{\text{loop}} \end{aligned} \right\} \quad \Delta |T_{fi}|^2 = -4 \operatorname{Im} \left[C_{fi}^{\text{tree}} C_{fi}^{\text{loop}*} \right] \operatorname{Im} \left[K_{fi}^{\text{tree}} K_{fi}^{\text{loop}*} \right]$$

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CP asymmetric reaction – the ingredients

- loop kinematics with nonzero imaginary part
- model with irreducible complex phases

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$$S^\dagger S = S S^\dagger \quad \rightarrow \quad 1 + i T - i T^\dagger + T T^\dagger = 1 + i T - i T^\dagger + T^\dagger T \quad (9)$$

CP asymmetric reactions

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$$S^\dagger S = S S^\dagger \quad \rightarrow \quad \cancel{1 + iT - iT^\dagger} + T T^\dagger = \cancel{1 + iT - iT^\dagger} + T^\dagger T \quad (9)$$

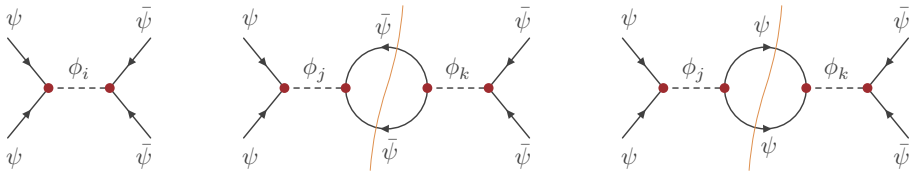
$$\sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \rightarrow \quad \sum_f \Delta |T_{fi}|^2 = 0 \quad (10)$$

[Dolgov '79; Kolb, Wolfram '80; see also Hook '11; Baldes *et al.* '14]

A toy model for **non-obtaining** the asymmetry

$$L_{\text{int}} = - \sum_i \lambda_i \phi_i \bar{\psi} \psi^c + \text{H.c.} \quad (11)$$

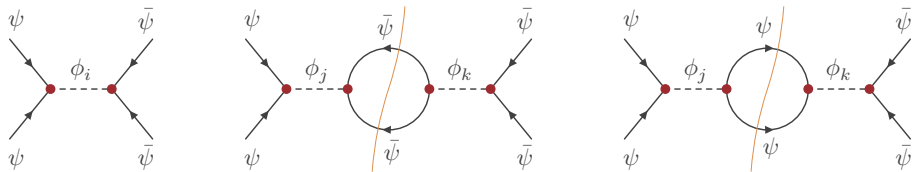
- irreducible complex phases in couplings
- imaginary kinematics in $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$ amplitude at $\mathcal{O}(\lambda^4)$ order



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- irreducible complex phases in couplings
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For $(p_1 + p_2)^2 < M_i^2$ the scalars ϕ_i cannot be produced and

$$\Delta |T_{\psi\psi \rightarrow \bar{\psi}\bar{\psi}}|^2 + \Delta |T_{\psi\psi \rightarrow \psi\psi}|^2 = 0 \quad \rightarrow \quad \Delta |T_{\psi\psi \rightarrow \bar{\psi}\bar{\psi}}|^2 = 0. \quad (12)$$

CPT and unitarity constraints

$$\text{CPT symmetry implies } |T_{\bar{f}i}|^2 = |T_{if}|^2 \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 \quad (8)$$

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***CP* asymmetric reaction – the ingredients**

- loop kinematics with nonzero imaginary part
- model with irreducible complex phases
- at least two nonvanishing asymmetries with the same initial state

CPT and unitarity constraints

$$S^\dagger S = 1 \quad \rightarrow \quad T = T^\dagger + i T^\dagger T$$

$$\Delta |T_{fi}|^2 = \left| T_{if}^* + i \sum_n T_{fn}^\dagger T_{ni} \right|^2 - |T_{if}|^2 = -2 \operatorname{Im} \left[\sum_n T_{if} T_{fn}^\dagger T_{ni} \right] + \left| \sum_n T_{fn}^\dagger T_{ni} \right|^2 \quad (13)$$

[Kolb, Wolfram '80]

CPT and unitarity constraints

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[Kolb, Wolfram '80]

No further on-shell cuts implies $T_{if}^* = T_{fi} \quad \rightarrow \quad \Delta |T_{fi}|^2 = -2 \operatorname{Im} \left[\sum_n T_{if} T_{fn} T_{ni} \right] \quad (14)$

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[Kolb, Wolfram '80]

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$$\Delta |T_{fi}|^2 = \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_n i T_{if} i T_{fn} i T_{ni} \quad (15)$$

[Covi, Roulet, Vissani '98]

CPT and unitarity constraints



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On the CP asymmetries in Majorana neutrino decays

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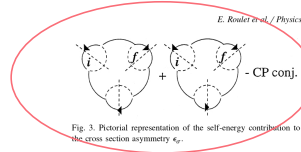


Fig. 3. Pictorial representation of the self-energy contribution to the cross section asymmetry ϵ_γ .

Therefore, one can pictorially represent the self-energy contributions to $\sigma(\ell^+ H^+ \rightarrow \ell^+ H)$ as in Fig. 2, where the cut blobs are the initial (i) and final (f) states, while the remaining blob stands for the one-loop self-energy. This last is actually the sum of two contributions, one with a lepton and one with an antilepton.

Now, in the computation of ϵ_γ , only the absorptive part of the loop will contribute, and the Cutkoski

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Turning now to the vertex contributions, to see the cancellations we need to add the three contributions shown in Fig. 4 (including the tree level u -channel interfering with the one-loop self-energy diagram). In terms of cut diagrams, this can be expressed as in Fig. 5, where CP-conjugate stands for the same four diagrams with all the arrows reversed. Hence, the CP-conjugate contribution will exactly cancel the four diagrams, since changing the directions of the arrows just exchanges among themselves the first and fourth diagrams, as well as the second and third ones. We then see explicitly that the absorptive part of the self-energies are also playing here a crucial role, enforcing the cancellation of the CP violation produced by the vertex diagrams.

The only remaining diagrams to be considered are the interference of the one-loop vertex diagrams with the tree level u -channel. Pictorially, they are represented in Fig. 6, and they again cancel since the two

$$\Delta |T_{fi}|^2 = \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_n i T_{if} i T_{fn} i T_{ni} \quad (15)$$

[Covi, Roulet, Vissani '98]

Holomorphic cutting rules

$$S^\dagger S = 1 \quad \rightarrow \quad i T^\dagger = i T - i T i T^\dagger \quad (16)$$

$$|T_{fi}|^2 = -i T_{if}^\dagger i T_{fi} = -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_{n,k} i T_{in} i T_{nk} i T_{kf} i T_{fi} + \dots \quad (17)$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

Holomorphic cutting rules

$$S^\dagger S = 1 \quad \rightarrow \quad iT^\dagger = iT - iTiT^\dagger \quad (16)$$

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[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

$$\begin{aligned} \Delta|T_{fi}|^2 &= |T_{fi}|^2 - |T_{if}|^2 = \sum_n \left(iT_{in}iT_{nf}iT_{fi} - iT_{if}iT_{fn}iT_{ni} \right) \\ &\quad - \sum_{n,k} \left(iT_{in}iT_{nk}iT_{kf}iT_{fi} - iT_{if}iT_{fk}iT_{kn}iT_{ni} \right) \\ &\quad + \dots \end{aligned} \quad (18)$$

[Blažek, Maták '21]

Holomorphic cutting rules

$$S^\dagger S = 1 \quad \rightarrow \quad iT^\dagger = iT - iTiT^\dagger \quad (16)$$

$$|T_{fi}|^2 = -iT_{if}^\dagger iT_{fi} = -iT_{if} iT_{fi} + \sum_n iT_{in} iT_{nf} iT_{fi} - \sum_{n,k} iT_{in} iT_{nk} iT_{kf} iT_{fi} + \dots \quad (17)$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

$$\begin{aligned} \Delta|T_{fi}|^2 &= |T_{fi}|^2 - |T_{if}|^2 = \sum_n \left(iT_{in} iT_{nf} iT_{fi} - iT_{if} iT_{fn} iT_{ni} \right) \\ &\quad - \sum_{n,k} \left(iT_{in} iT_{nk} iT_{kf} iT_{fi} - iT_{if} iT_{fk} iT_{kn} iT_{ni} \right) \\ &\quad + \dots \quad \leftarrow \quad \sum_f \Delta|T_{fi}|^2 = 0 \end{aligned} \quad (18)$$

[Blažek, Maták '21]

Holomorphic cutting rules

$$\Delta|T_{fi}|^2 = \sum_n \left(i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) - \dots \quad (19)$$

$$\Delta|T_{\psi\psi \rightarrow \bar{\psi}\bar{\psi}}|^2 \ni \quad (20)$$

The diagrams illustrate the holomorphic cutting rules for the process $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$. The first row shows two terms separated by a minus sign. The first term has vertices ϕ_i, ϕ_j, ϕ_k with a $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$ vertex at ϕ_i . The second term has vertices ϕ_k, ϕ_j, ϕ_i with a $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$ vertex at ϕ_k . The second row shows two terms separated by a minus sign, preceded by a plus sign. The first term has vertices ϕ_i, ϕ_j, ϕ_k with a $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$ vertex at ϕ_i . The second term has vertices ϕ_k, ϕ_j, ϕ_i with a $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$ vertex at ϕ_k . Orange vertical lines indicate cuts in the diagrams.

Holomorphic cutting rules

$$\Delta|T_{fi}|^2 = \sum_n \left(i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) - \dots \quad (19)$$

$$\Delta|T_{\psi\psi \rightarrow \bar{\psi}\bar{\psi}}|^2 \ni \quad (20)$$

The diagrams illustrate the holomorphic cutting rules for the process $\psi\psi \rightarrow \bar{\psi}\bar{\psi}$. They show two rows of terms, each representing a different cut configuration in the complex plane. The top row consists of two terms separated by a minus sign. The first term shows a sequence of three vertices ϕ_i, ϕ_j, ϕ_k with external legs $\psi, \psi, \bar{\psi}, \bar{\psi}$. Red diagonal lines indicate cuts, and orange vertical lines represent the cuts in the complex plane. The second term is similar but with different cut placements. The bottom row also consists of two terms separated by a minus sign, starting with a plus sign. These diagrams show different cut configurations for the same process.

Holomorphic cutting rules

$$\Delta|T_{fi}|^2 = \sum_n \left(i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) - \dots \quad (19)$$

$$\Delta|T_{\psi\psi \rightarrow \bar{\psi}\bar{\psi}}|^2 \ni \quad (20)$$

The diagrams show the following cut configurations:

- Top-left (+):** Cuts on the top-left and bottom-right external lines.
- Top-right (-):** Cuts on the top-right and bottom-left external lines.
- Bottom-left (+):** Cuts on the top-left and bottom-left external lines.
- Bottom-right (-):** Cuts on the top-right and bottom-right external lines.

The sum of these four terms is equal to zero.

CP asymmetry in the Boltzmann equation

Change in # of particles $\leftrightarrow$ average # of their interactions

$$\dot{n}_{f_1} + 3Hn_{f_1} = \sum_{\text{all reactions}} \gamma_{fi} - \gamma_{if} \quad \leftarrow \quad \gamma_{fi} = \int \prod_{\forall i} [d\mathbf{p}_i] f_i(\mathbf{p}_i) \int \prod_{\forall f} [d\mathbf{p}_f] \frac{1}{V_4} |T_{fi}|^2 \quad (21)$$

$$[d\mathbf{p}] = \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p}$$

$$|T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\mathbf{p}_f - \mathbf{p}_i) |M_{fi}|^2$$

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Boltzmann equation for the asymmetry in f_1 particle density

$$\left. \begin{aligned} \Delta n_{f_1} &= n_{f_1} - n_{\bar{f}_1} \\ \Delta \gamma_{fi} &= \gamma_{fi} - \gamma_{i\bar{f}} \end{aligned} \right\} \quad \Delta \dot{n}_{f_1} + 3H \Delta n_{f_1} = \sum_{\text{all reactions}} \Delta \gamma_{fi} - \Delta \gamma_{if} \quad (22)$$

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Kinetic equilibrium as a simplifying assumption

$$f_i(\mathbf{p}_i) \propto \exp\left\{-\frac{E_i}{T}\right\} \quad \gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \gamma_{fi}^{\text{eq}} \quad (23)$$

CP asymmetry in the Boltzmann equation

Change in # of particles $\leftrightarrow$ average # of their interactions

$$\dot{n}_{f_1} + 3Hn_{f_1} = \sum_{\text{all reactions}} \gamma_{fi} - \gamma_{if} \quad \leftarrow \quad \gamma_{fi} = \int \prod_{\forall i} [d\mathbf{p}_i] f_i(\mathbf{p}_i) \int \prod_{\forall f} [d\mathbf{p}_f] \frac{1}{V_4} |T_{fi}|^2 \quad (21)$$

$$[d\mathbf{p}] = \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \quad |T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\mathbf{p}_f - \mathbf{p}_i) |M_{fi}|^2$$

Detailed balance condition

$$\prod_{\forall i} f_i^{\text{eq}}(\mathbf{p}_i) = \prod_{\forall f} f_f^{\text{eq}}(\mathbf{p}_f) \quad \Delta\gamma_{fi}^{\text{eq}} = -\Delta\gamma_{if}^{\text{eq}} \quad (24)$$

CP asymmetry in the Boltzmann equation

$$\Delta\gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \Delta\gamma_{fi}^{\text{eq}} + \left(\frac{\Delta n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} + \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{\Delta n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} + \cdots \right) \times \gamma_{fi}^{\text{eq}} \quad (25)$$

CP asymmetry in the Boltzmann equation

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Source terms

- through $\Delta\gamma_{fi}^{\text{eq}}$ depend on $\Delta|T_{fi}|^2$
- symmetric part of particle densities

CP asymmetry in the Boltzmann equation

$$\Delta\gamma_{fi} = \frac{n_{i_1}^{\text{eq}}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}^{\text{eq}}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}^{\text{eq}}}{n_{i_p}^{\text{eq}}} \times \Delta\gamma_{fi}^{\text{eq}} + \left(\frac{\Delta n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}^{\text{eq}}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}^{\text{eq}}}{n_{i_p}^{\text{eq}}} + \frac{n_{i_1}^{\text{eq}}}{n_{i_1}^{\text{eq}}} \frac{\Delta n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}^{\text{eq}}}{n_{i_p}^{\text{eq}}} + \cdots \right) \times \gamma_{fi}^{\text{eq}} \quad (25)$$

Source terms

- through $\Delta\gamma_{fi}^{\text{eq}}$ depend on $\Delta|T_{fi}|^2$
- symmetric part of particle densities

Wash-out terms

- depend on density asymmetries
- symmetric part of reaction rates
- typically lead to asymmetry suppression

Which reactions?

$$\sum_{\text{all reactions}} \Delta\gamma_{fi} = \sum_i \sum_{f \ni f_1} \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \Delta\gamma_{fi}^{\text{eq}} + \text{wash-out} \quad (26)$$

- In the f_1 asymmetry source-term, f_1 must be present in the final state of the contributing reactions.
- Out-of-equilibrium particles must appear in the initial state.

[See also Racker '19]

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[See also Racker '19]

$$\sum_{\text{all reactions}} \Delta\gamma_{if} = \sum_i \sum_{f \ni f_1} \frac{n_{f_1}}{n_{f_1}^{\text{eq}}} \frac{n_{f_2}}{n_{f_2}^{\text{eq}}} \cdots \frac{n_{f_q}}{n_{f_q}^{\text{eq}}} \times \Delta\gamma_{if}^{\text{eq}} + \text{wash-out} \quad (27)$$

$$\left(\Delta\dot{n}_{f_1} + 3H\Delta n_{f_1} \right)_{\text{source}} = \sum_i \sum_{f \ni f_1} \left(\frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} - 1 \right) \times \Delta\gamma_{fi}^{\text{eq}} \quad (28)$$

Summary I.

- Feynman integrals may get imaginary kinematics from on-shell intermediate states.
- Irreducible complex phases may lead to CP -violating processes.
- Unitarity and CPT symmetry imply

$$\begin{aligned}\Delta|T_{fi}|^2 &= \sum_n \left(i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) \\ &\quad - \sum_{n,k} \left(i T_{in} i T_{nk} i T_{kf} i T_{fi} - i T_{if} i T_{fk} i T_{kn} i T_{ni} \right) \\ &\quad + \dots \quad \leftarrow \quad \sum_f \Delta|T_{fi}|^2 = 0\end{aligned}$$

Part II.

Dirac leptogenesis from scatterings

Leptogenesis with Dirac neutrinos

- Originally introduced in Phys. Rev. Lett. **84**, 4039. [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles.
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons.

$$Y_B = \frac{28}{79} Y_{B-L_{\text{SM}}} = \frac{28}{79} \Delta_{\nu_R} \quad (29)$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

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$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \quad (30)$$

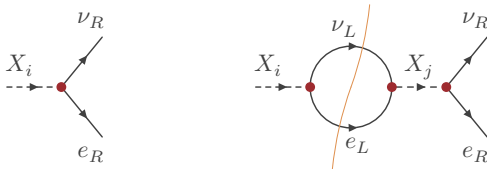
[Heeck, Heisig, Thapa '23a]

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[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

$$\Delta |T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta |T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (31)$$

Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 = \text{Diagram 1} - \text{Diagram 2} \quad (32)$$

Diagram 1: Two vertices, \$X_i\$ and \$X_j\$. \$X_i\$ emits a right-handed neutrino (\$\nu_R\$) and a right-handed electron (\$e_R\$). \$X_j\$ emits a left-handed neutrino (\$\nu_L\$) and a left-handed electron (\$e_L\$).

Diagram 2: Two vertices, \$X_i\$ and \$X_j\$. \$X_i\$ emits a left-handed neutrino (\$\nu_L\$) and a left-handed electron (\$e_L\$). \$X_j\$ emits a right-handed neutrino (\$\nu_R\$) and a right-handed electron (\$e_R\$).

$$\Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = \text{Diagram 3} - \text{Diagram 4} \quad (33)$$

Diagram 3: Two vertices, \$X_i\$ and \$X_j\$. \$X_i\$ emits a left-handed neutrino (\$\nu_L\$) and a left-handed electron (\$e_L\$). \$X_j\$ emits a right-handed neutrino (\$\nu_R\$) and a right-handed electron (\$e_R\$).

Diagram 4: Two vertices, \$X_i\$ and \$X_j\$. \$X_i\$ emits a right-handed neutrino (\$\nu_R\$) and a right-handed electron (\$e_R\$). \$X_j\$ emits a left-handed neutrino (\$\nu_L\$) and a left-handed electron (\$e_L\$).

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (31)$$

Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 = \text{Diagram 1} - \text{Diagram 2} \quad (32)$$

Diagram 1: Two loops. The first loop has external lines X_i and e_R , and internal lines ν_R and e_R . The second loop has external lines X_j and e_L , and internal lines ν_L and e_L . Diagram 2: Identical to Diagram 1 but with red diagonal lines crossing out the internal lines of both loops.

$$\Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = \text{Diagram 3} - \text{Diagram 4} \quad (33)$$

Diagram 3: Two loops. The first loop has external lines X_i and e_L , and internal lines ν_L and e_L . The second loop has external lines X_j and e_R , and internal lines ν_R and e_R . Diagram 4: Identical to Diagram 3 but with red diagonal lines crossing out the internal lines of both loops.

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (31)$$

Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 = \text{Diagram 1} - \text{Diagram 2} \quad (32)$$

Diagram 1: A loop diagram with two vertices. The left vertex has an incoming dashed line labeled X_i and an outgoing solid line labeled ν_R . The right vertex has an incoming solid line labeled ν_L and an outgoing dashed line labeled X_i . The loop is formed by a solid line labeled e_R and a dashed line labeled X_j . The diagram is crossed out with red lines.

Diagram 2: A loop diagram with two vertices. The left vertex has an incoming dashed line labeled X_i and an outgoing solid line labeled ν_L . The right vertex has an incoming solid line labeled ν_R and an outgoing dashed line labeled X_i . The loop is formed by a solid line labeled e_L and a dashed line labeled X_j . The diagram is crossed out with red lines.

$$\Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = \text{Diagram 3} - \text{Diagram 4} \quad (33)$$

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Diagram 4: A loop diagram with two vertices. The left vertex has an incoming dashed line labeled X_i and an outgoing solid line labeled ν_R . The right vertex has an incoming solid line labeled ν_L and an outgoing dashed line labeled X_i . The loop is formed by a solid line labeled e_R and a dashed line labeled X_j . The diagram is crossed out with red lines.

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (31)$$

Leptogenesis with Dirac neutrinos

$$\Delta|T_{\nu_R e_R \rightarrow X_i}|^2 = \text{Diagram 1} - \text{Diagram 2} \quad (34)$$

$$\Delta|T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = \text{Diagram 3} - \text{Diagram 4} \quad (35)$$

$$\Delta|T_{\nu_R e_R \rightarrow X_i}|^2 + \Delta|T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = 0 \quad (36)$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \quad (37)$$

[Heeck, Heisig, Thapa '23b]

Leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \quad (37)$$

[Heeck, Heisig, Thapa '23b]

$$\cancel{\Delta |T_{\nu_R e_R \rightarrow X_i}|^2} + \Delta |T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = 0$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \quad (38)$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \quad (38)$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

- Baryon and lepton number conservation

$$\Delta_{d_R} = -\Delta_Q - \Delta_{u_R}, \quad \Delta_{\nu_R} = -\Delta_L - \Delta_{e_R}. \quad (39)$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \quad (38)$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

- Baryon and lepton number conservation

$$\Delta_{d_R} = -\Delta_Q - \Delta_{u_R}, \quad \Delta_{\nu_R} = -\Delta_L - \Delta_{e_R}. \quad (39)$$

- Further global symmetries

$$\Delta_{\nu_R} = \Delta_{d_R}, \quad \Delta_L = \Delta_Q, \quad \Delta_{e_R} = \Delta_{u_R}. \quad (40)$$

Leptogenesis without heavy particles?

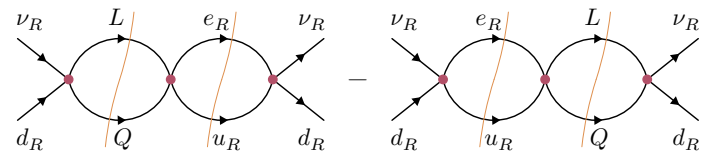
$$\begin{aligned}
 & \text{Diagram 1: } \nu_R, L, d_R, Q \text{ at a vertex} = i \sum_i \frac{F_i^\dagger G_i}{M_i^2} \\
 & \text{Diagram 2: } \nu_R, e_R, d_R, u_R \text{ at a vertex} = i \sum_i \frac{K_i^\dagger G_i}{M_i^2} \\
 & \text{Diagram 3: } e_R, L, u_R, Q \text{ at a vertex} = i \sum_i \frac{F_i^\dagger K_i}{M_i^2}
 \end{aligned}$$

$$\langle \sigma_1 v \rangle = \frac{64}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\text{Tr}(F_i^\dagger F_j) \text{Tr}(G_j^\dagger G_i)}{M_i^2 M_j^2} \equiv \frac{64}{3\pi} \frac{T^2}{T_{\text{reh}}^2} \frac{\alpha_1}{\zeta(3)^2} \quad (41)$$

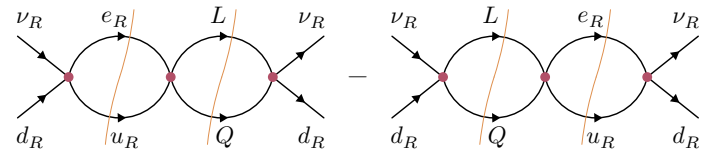
$$\langle \sigma_2 v \rangle = \frac{32}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\text{Tr}(K_i^\dagger K_j) \text{Tr}(G_j^\dagger G_i)}{M_i^2 M_j^2} \equiv \frac{32}{3\pi} \frac{T^2}{T_{\text{reh}}^2} \frac{\alpha_2}{\zeta(3)^2} \quad (42)$$

$$\langle \sigma_3 v \rangle = \frac{64}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\text{Tr}(F_i^\dagger F_j) \text{Tr}(K_j^\dagger K_i)}{M_i^2 M_j^2} \equiv \frac{64}{3\pi} \frac{T^2}{T_{\text{reh}}^2} \frac{\alpha_3}{\zeta(3)^2} \quad (43)$$

Leptogenesis without heavy particles?

$$\Delta |T_{\nu_R d_R \rightarrow L Q}|^2 =$$


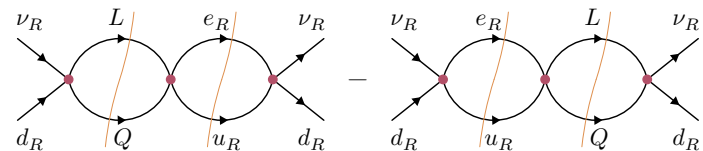
$$- \quad (44)$$

$$\Delta |T_{\nu_R d_R \rightarrow e_R u_R}|^2 =$$


$$- \quad (45)$$

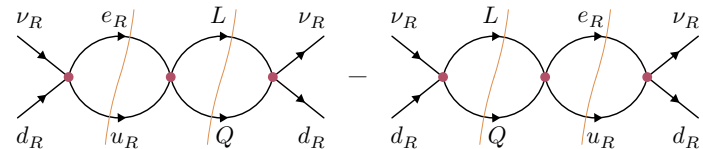
$$\Delta |T_{\nu_R d_R \rightarrow L Q}|^2 = -\Delta |T_{\nu_R d_R \rightarrow e_R u_R}|^2 \neq 0 \quad (46)$$

Leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \rightarrow L Q}|^2 =$$


The diagram consists of two loop diagrams separated by a minus sign. Each loop has two internal vertices (red dots) and two external vertices. The external lines are labeled ν_R , d_R , L , Q , e_R , and u_R . The first loop has ν_R and d_R entering from the left, and L and Q exiting to the right. The second loop has e_R and u_R entering from the left, and ν_R and d_R exiting to the right.

$$- \quad (44)$$

$$\Delta|T_{\nu_R d_R \rightarrow e_R u_R}|^2 =$$


The diagram consists of two loop diagrams separated by a minus sign. Each loop has two internal vertices (red dots) and two external vertices. The external lines are labeled ν_R , d_R , e_R , u_R , L , and Q . The first loop has ν_R and d_R entering from the left, and e_R and u_R exiting to the right. The second loop has L and Q entering from the left, and ν_R and d_R exiting to the right.

$$- \quad (45)$$

$$\Delta\langle\sigma_1 v\rangle \equiv \frac{512}{\pi^2} \frac{T^4}{T_{\text{reh}}^6} \frac{\epsilon}{\zeta(3)^2} = -\Delta\langle\sigma_2 v\rangle \quad (47)$$

Freeze-in and wash-in

$$\left(\frac{d\Delta_L}{dx}\right)_{\text{source}} = -\left(\frac{d\Delta_{e_R}}{dx}\right)_{\text{source}} \rightarrow \left(\frac{d\Delta_{\nu_R}}{dx}\right)_{\text{source}} = 0 \quad (48)$$

Freeze-in and wash-in

$$\left(\frac{d\Delta_L}{dx}\right)_{\text{source}} = -\left(\frac{d\Delta_{e_R}}{dx}\right)_{\text{source}} \rightarrow \left(\frac{d\Delta_{\nu_R}}{dx}\right)_{\text{source}} = 0 \quad (48)$$

$$\left(\frac{d\Delta_L}{dx}\right)_{\text{wash-out}} \neq -\left(\frac{d\Delta_{e_R}}{dx}\right)_{\text{wash-out}} \rightarrow \left(\frac{d\Delta_{\nu_R}}{dx}\right)_{\text{wash-in}} \neq 0 \quad (49)$$

[Domcke *et al.* '21]

Freeze-in and wash-in

$$\frac{dY_{\nu_R}}{dx} = - \frac{1}{x^4} \frac{\Gamma}{H} \Big|_{T_{\text{reh}}} \left(Y_{\nu_R} - Y_{\nu_R}^{\text{eq}} \right) \quad \leftarrow \quad \Gamma = \frac{5}{9} s Y_{\nu_R}^{\text{eq}} \left(\langle \sigma_1 v \rangle + \langle \sigma_2 v \rangle \right) \quad (50)$$

Freeze-in and wash-in

$$\frac{dY_{\nu_R}}{dx} = -\frac{1}{x^4} \frac{\Gamma}{H} \Big|_{T_{\text{reh}}} \left(Y_{\nu_R} - Y_{\nu_R}^{\text{eq}} \right) \quad \leftarrow \quad \Gamma = \frac{5}{9} s Y_{\nu_R}^{\text{eq}} \left(\langle \sigma_1 v \rangle + \langle \sigma_2 v \rangle \right) \quad (50)$$

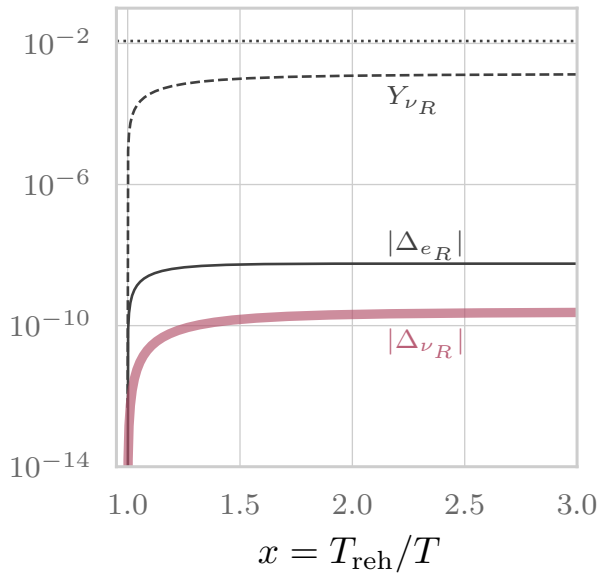
$$\begin{aligned} \frac{d\Delta_L}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{9H} \frac{ds}{dx} \Bigg\{ & \Delta \langle \sigma_1 v \rangle \left(Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) + \frac{10}{9} \langle \sigma_3 v \rangle \left(\Delta_L - 2\Delta_{e_R} \right) \\ & + \frac{8}{9} \langle \sigma_1 v \rangle \left[\Delta_L - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left(\Delta_L - \frac{3}{2} \Delta_{\nu_R} \right) \right] \Bigg\} \end{aligned} \quad (51)$$

$$\begin{aligned} \frac{d\Delta_{e_R}}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{9H} \frac{ds}{dx} \Bigg\{ & \Delta \langle \sigma_2 v \rangle \left(Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) - \frac{10}{9} \langle \sigma_3 v \rangle \left(\Delta_L - 2\Delta_{e_R} \right) \\ & + \frac{8}{9} \langle \sigma_2 v \rangle \left[2\Delta_{e_R} - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left(2\Delta_{e_R} - \frac{3}{2} \Delta_{\nu_R} \right) \right] \Bigg\} \end{aligned} \quad (52)$$

[Blažek *et al.* '24]

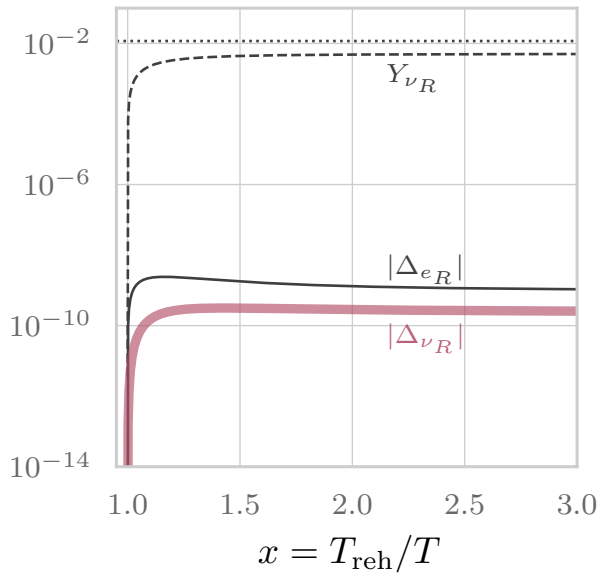
Numerical solution

$$\alpha_1 = 7.9 \times 10^{-11}, \epsilon = 2.8 \times 10^{-16}$$



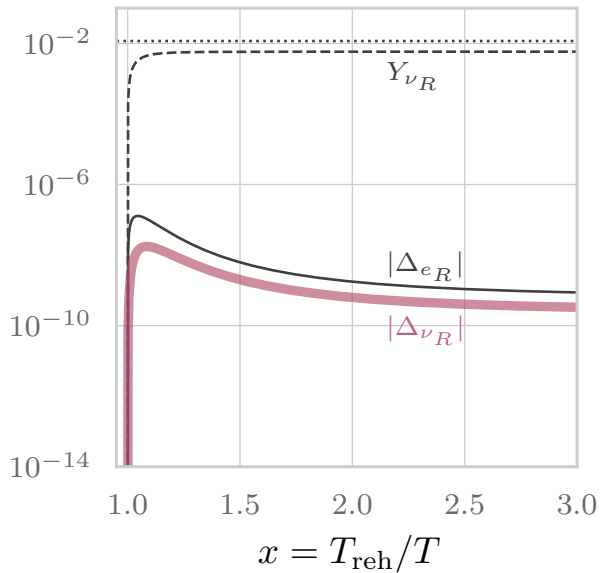
Numerical solution

$$\alpha_1 = 5.9 \times 10^{-10}, \epsilon = 3.9 \times 10^{-16}$$



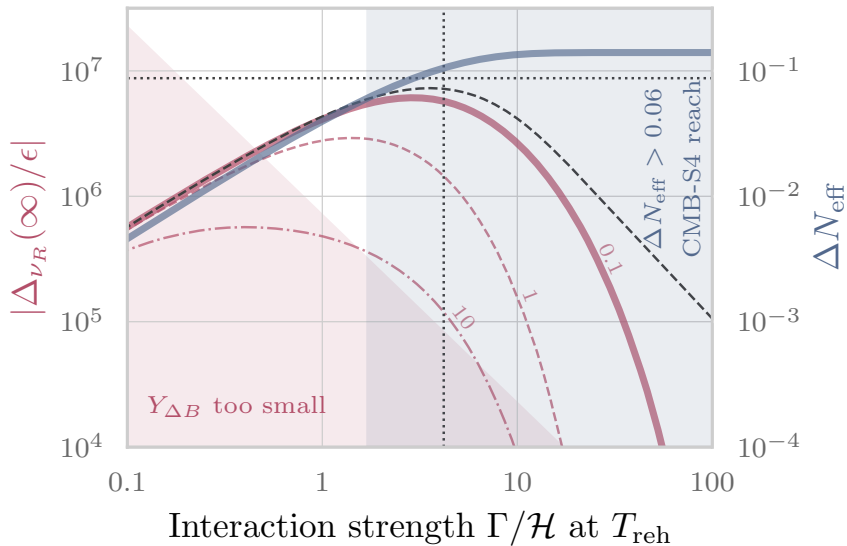
Numerical solution

$$\alpha_1 = 2.0 \times 10^{-9}, \epsilon = 6.1 \times 10^{-14}$$



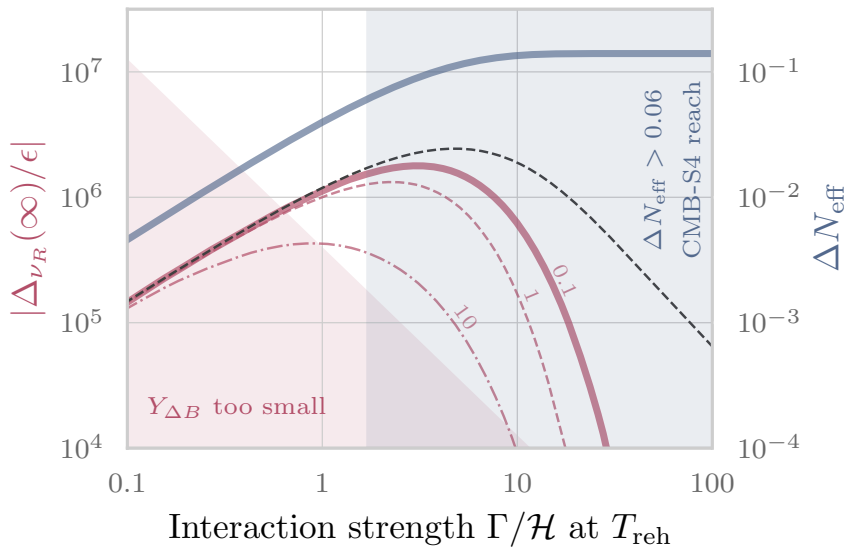
Numerical solution

$$\alpha_1 : \alpha_2 : \alpha_3 = 0.01 : 1 : q_3$$



Numerical solution

$$\alpha_1 : \alpha_2 : \alpha_3 = 1 : 2 : q_3$$



Summary II.

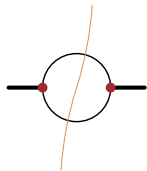
- Dirac leptogenesis through scatterings of massless particles works with sufficiently complex model.
- Right-handed neutrino source-term vanishes.
- Asymmetry is washed in from small differences in standard-model particle asymmetries.

Part III.

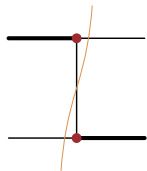
Further consequences of unitarity

Unitarity and the Boltzmann equation

$$\gamma_{fi}^{\text{eq}} = \int \prod_{\forall i} [d\mathbf{p}_i] f_i^{\text{eq}}(p_i) \int \prod_{\forall f} [d\mathbf{p}_f] \frac{1}{V_4} \left(-i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \dots \right) \quad (53)$$



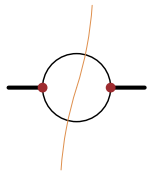
$$\int [d\mathbf{p}_1] e^{-E_{p_1}/T} \int [d\mathbf{k}_1][d\mathbf{k}_2] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$



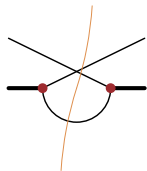
?

Unitarity and the Boltzmann equation

$$\gamma_{fi}^{\text{eq}} = \int \prod_{\forall i} [d\mathbf{p}_i] f_i^{\text{eq}}(p_i) \int \prod_{\forall f} [d\mathbf{p}_f] \frac{1}{V_4} \left(-i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \dots \right) \quad (53)$$

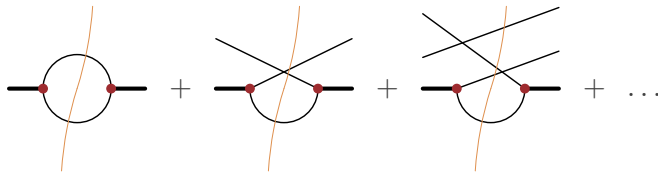


$$\int [d\mathbf{p}_1] e^{-E_{p_1}/T} \int [d\mathbf{k}_1][d\mathbf{k}_2] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$



$$\int [d\mathbf{p}_1] e^{-E_{p_1}/T} \int [d\mathbf{k}_1][d\mathbf{k}_2] e^{-E_{k_1}/T} (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$

Unitarity and the Boltzmann equation



$$\int [d\mathbf{p}_1] e^{-E_{p_1}/T} \int [d\mathbf{k}_1][d\mathbf{k}_2] \left[1 + \frac{1}{e^{E_{k_1}/T} - 1} \right] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2) \quad (54)$$

[Blažek, Maták '21b]

Vacuum diagrams and complex phases

- Used to represent the weak-basis invariants. [Botella, Nebot, Vives '06; Nilles, Ratz, Trautner, Vaudrevange '18]
- Reversing the arrows on charged-particle propagators must lead to a topologically inequivalent vacuum diagram.

$$\Delta |T_{fi}|^2 = \sum_n \left(i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) - \dots \quad (55)$$



$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.}$$

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.}$$

Vacuum diagrams and the Boltzmann equation

Diagrammatic equation (56) shows a vacuum diagram on the left, which is a triangle with vertices marked by red dots. The top vertex is connected to the bottom-left and bottom-right vertices by lines labeled L and e respectively. The bottom-left and bottom-right vertices are connected by a line labeled ν . Inside the triangle, there are two internal lines labeled Q and d , and a vertex labeled u . An arrow points to the right, where the same diagram is shown with a vertical orange line (a cut) passing through it. The cut separates the diagram into two parts: a left part with external lines $\bar{e}$ and ν , and a right part with external lines $\bar{e}$ and ν , and internal lines Q and d .

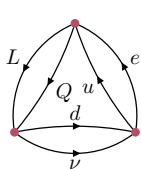
(56)

Diagrammatic equation (57) shows a vacuum diagram on the left, which is a triangle with vertices marked by red dots. The top vertex is connected to the bottom-left and bottom-right vertices by lines labeled L and e respectively. The bottom-left and bottom-right vertices are connected by a line labeled ν . Inside the triangle, there are two internal lines labeled Q and d , and a vertex labeled u . An arrow points to the right, where the same diagram is shown with a vertical orange line (a cut) passing through it. The cut separates the diagram into two parts: a left part with external lines $\bar{e}$ and ν , and a right part with external lines $\bar{e}$ and ν , and internal lines Q and d . The diagram is then expanded into a series of diagrams, each with a cut, and the series is summed with an ellipsis $+$ $\dots$.

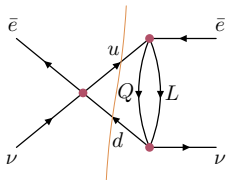
(57)

$$\frac{i}{p^2 + i\epsilon} \rightarrow 2\pi \sum_{w=1}^{\infty} (-1)^w [f^{\text{eq}}(|p^0|)]^w \delta(p^2) = -2\pi f_{\text{FD}}(|p^0|) \delta(p^2) \quad (58)$$

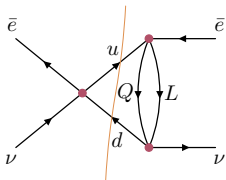
Vacuum diagrams and the Boltzmann equation



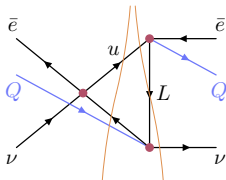
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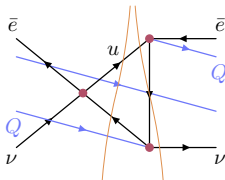
(56)



→



+



+ ...

(57)

$$\Delta\gamma_{\nu\bar{e}(Q)\rightarrow u\bar{d}(Q)}^{\text{eq}} + \Delta\gamma_{(\nu)\bar{e}Q\rightarrow(\nu)\bar{L}u}^{\text{eq}} = 0$$

(59)

The assumptions we made

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp \left\{ - \sum_p F_p a_p^\dagger a_p \right\}, Z = \prod_p Z_p \quad (60)$$

$$e^{-E_p/T} \rightarrow e^{-F_p} = \frac{f_p}{1 \pm f_p} \quad f_p = \text{Tr} [a_p^\dagger a_p \rho] \quad (61)$$

$$\rho' = S \rho S^\dagger \rightarrow (1 + iT) \rho (1 - iT + iTiT - \dots) \quad (62)$$

The collision term for the Boltzmann equation is obtained as $\text{Tr} [a_p^\dagger a_p (\rho - \rho')] / V_4$.

[McKellar, Thomson '94; Blažek, Maták '21b]

What else can be done?

- Thermal masses from anomalous thresholds. [Blažek, Maták '22]
- *CPT* and unitarity constraints generalized to thermal-corrected asymmetries. In the seesaw type-I leptogenesis at $\mathcal{O}(Y^4 Y_t^2)$ they look like

$$\Delta\gamma_{N_i Q \rightarrow lt}^{\text{eq}} + \Delta\gamma_{N_i(Q) \rightarrow lH(Q)}^{\text{eq}} + \Delta\gamma_{N_i(Q) \rightarrow \bar{l}\bar{H}(Q)}^{\text{eq}} + \Delta\gamma_{N_i Q \rightarrow \bar{l}Q Q \bar{t}}^{\text{eq}} = 0, \quad (63)$$

$$m_{H, Y_t}^2(T) \frac{\partial}{\partial m_H^2} \Big|_0 \left(\Delta\gamma_{N_i \rightarrow lH}^{\text{eq}} + \Delta\gamma_{N_i \rightarrow \bar{l}\bar{H}}^{\text{eq}} \right) = 0. \quad (64)$$

[Blažek, Maták, Zaujec '22]

- No double-counting from intermediate states. Resonant dark matter annihilation at fixed order. [Maták '24; see also Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

Summary III.

- Vacuum diagrams must not be invariant under the arrow reversal to come with a complex phase.
- Completing the Boltzmann equations by all possible unitary cuts represents thermal corrections (even when you do not notice that).

Thank you for your attention!

Backup

Description of multiparticle states

Single-particle state: $|p\rangle \leftarrow$ particle species, momentum, spin, ...

$$\rho_p = \frac{1}{Z_p} \exp \{ - F_p a_p^\dagger a_p \}, \quad Z_p = \text{Tr} \exp \{ - F_p a_p^\dagger a_p \} = \frac{\exp F_p}{\exp F_p - 1} \quad (65)$$

[Wagner '91]

Description of multiparticle states

Single-particle state: $|p\rangle \leftarrow$ particle species, momentum, spin, ...

$$\rho_p = \frac{1}{Z_p} \exp \{ - F_p a_p^\dagger a_p \}, \quad Z_p = \text{Tr} \exp \{ - F_p a_p^\dagger a_p \} = \frac{\exp F_p}{\exp F_p - 1} \quad (65)$$

[Wagner '91]

$$f_p = \text{Tr} \rho_p a_p^\dagger a_p = \frac{1}{\exp F_p - 1} \rightarrow \overset{\circ}{f}_p \stackrel{\text{def.}}{=} \exp \{ - F_p \} \quad (66)$$

Boltzmann equation from density matrix evolution

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp \left\{ - \sum_p F_p a_p^\dagger a_p \right\}, \quad Z = \prod_p Z_p \quad (67)$$

$$\rho' = S \rho S^\dagger \quad \rightarrow \quad \rho' - \rho = i T \rho - i \rho T^\dagger + T \rho T^\dagger \quad (68)$$

$$\rho' - \rho = T \rho T^\dagger - \frac{1}{2} T T^\dagger \rho - \frac{1}{2} \rho T T^\dagger + \frac{i}{2} [T + T^\dagger, \rho]$$

[McKellar, Thomson '94]

$$i T^\dagger = \frac{i}{2} (T + T^\dagger) + \frac{1}{2} T T^\dagger, \quad i T = \frac{i}{2} (T + T^\dagger) - \frac{1}{2} T T^\dagger \quad (69)$$

Boltzmann equation from density matrix evolution

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp \left\{ - \sum_p F_p a_p^\dagger a_p \right\}, \quad Z = \prod_p Z_p \quad (67)$$

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$$\rho' - \rho = T \rho T^\dagger - \frac{1}{2} T T^\dagger \rho - \frac{1}{2} \rho T T^\dagger + \frac{i}{2} [T + T^\dagger, \rho]$$

[McKellar, Thomson '94]

$$f'_p - f_p = \text{Tr} [a_p^\dagger a_p (\rho' - \rho)] \quad (70)$$

$$[a_p^\dagger a_p, \rho] = 0 \quad \rightarrow \quad f'_p - f_p = \text{Tr} [a_p^\dagger a_p (\rho i T i T^\dagger - i T \rho i T^\dagger)] \quad (71)$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \text{Tr} [a_p^\dagger a_p (\rho \mathbf{i} T \mathbf{i} T^\dagger - \mathbf{i} T \rho \mathbf{i} T^\dagger)] \quad (72)$$

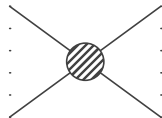
$$\begin{aligned} &= \sum_{k=1}^{\infty} (-1)^k \text{Tr} [a_p^\dagger a_p (\mathbf{i} T \rho (\mathbf{i} T)^k - \rho (\mathbf{i} T)^{k+1})] \\ &= \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) f_1^{\circ i_1} f_2^{\circ i_2} \dots (\mathbf{i} T)_{in}^k \mathbf{i} T_{ni} \end{aligned}$$

[Blažek, Maták '21b]

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) f_1^{\circ i_1} f_2^{\circ i_2} \dots (i T)_{in}^k i T_{ni} \quad (72)$$

[Blažek, Maták '21b]



$$\propto f_q^{\circ 3} \rightarrow Z_q = \sum_{i_q=0}^{\infty} f_q^{\circ i_q} \quad (73)$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) \overset{\circ}{f}_1^{i_1} \overset{\circ}{f}_2^{i_2} \dots (\overset{\circ}{i}T)_{in}^k \overset{\circ}{i}T_{ni} \quad (72)$$

[Blažek, Maták '21b]

$|q\rangle \neq |p\rangle$

1. i_q goes up by one in some $\overset{\circ}{i}T$, down in another.
2. The same in a reversed order.
3. i_q is lowered and increased in the same $\overset{\circ}{i}T$.

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) \overset{\circ}{f}_1^{i_1} \overset{\circ}{f}_2^{i_2} \dots (\overset{\circ}{i}T)_{in}^k \overset{\circ}{i}T_{ni} \quad (72)$$

[Blažek, Maták '21b]

$$\underline{|q\rangle \neq |p\rangle}$$

1. i_q goes up by one in some $\overset{\circ}{i}T$, down in another.

$$\left\langle \frac{1}{\sqrt{(i_q + 1)!}} a_q^{i_q+1} a_q^\dagger \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \rightarrow \sqrt{1 + i_q} \quad (74)$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) \overset{\circ}{f}_1^{i_1} \overset{\circ}{f}_2^{i_2} \dots (\overset{\circ}{i}T)_{in}^k \overset{\circ}{i}T_{ni} \quad (72)$$

[Blažek, Maták '21b]

$$\underline{|q\rangle \neq |p\rangle}$$

1. i_q goes up by one in some $\overset{\circ}{i}T$, down in another.

$$\left\langle \frac{1}{\sqrt{(i_q + 1)!}} a_q^{i_q+1} a_q^\dagger \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \rightarrow \sqrt{1 + i_q} \quad (74)$$

$$\sum_{i_q=1}^{\infty} (1 + i_q) \overset{\circ}{f}_q^{i_q} = Z_q (1 + f_q) \quad (75)$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) f_1^{\circ i_1} f_2^{\circ i_2} \dots (i T)_{in}^k i T_{ni} \quad (72)$$

[Blažek, Maták '21b]

$$\underline{|q\rangle \neq |p\rangle}$$

2. and 3.

$$\left\langle \frac{1}{\sqrt{(i_q - 1)!}} a_q^{i_q - 1} a_q \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \rightarrow \sqrt{i_q} \quad (76)$$

$$\sum_{i_q=1}^{\infty} i_q f_q^{\circ i_q} = Z_q f_q \quad (77)$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} (n_p - i_p) f_1^{\circ i_1} f_2^{\circ i_2} \dots (i T)_{in}^k i T_{ni} \quad (72)$$

[Blažek, Maták '21b]

$$\underline{|q\rangle = |p\rangle}$$

$$n_p - i_p = +1 \quad \rightarrow \quad Z_p(1 + f_p) \quad (78)$$

$$n_p - i_p = -1 \quad \rightarrow \quad -Z_p f_p \quad (79)$$

Unstable-particle intermediate states

$$|T_{lH \rightarrow \bar{l}\bar{H}}|^2 \leftarrow \left| \sum_i \left(\text{Diagram}_i \right) \right|^2 = \sum_{i,j} \left(\text{Diagram}_i \right) \left(\text{Diagram}_j \right)^* \quad (80)$$

The diagram Diagram_i represents a Feynman diagram for the process $lH \rightarrow \bar{l}\bar{H}$ via an unstable particle N_i . It consists of two vertices connected by a solid line labeled N_i . The left vertex has an incoming solid line l_α and an incoming dashed line H . The right vertex has an outgoing solid line $\bar{l}_\beta$ and an outgoing dashed line $\bar{H}$. The entire expression is squared, indicating the sum of the squared magnitudes of the amplitudes for each intermediate state N_i .

The right-hand side of the equation shows the expansion of the squared magnitude as a sum over pairs of states i and j . Each term is the product of the amplitude for state i and the complex conjugate of the amplitude for state j . The diagram for state j is identical in structure to the one for state i , but it is enclosed in large parentheses with a superscript $*$ to denote the complex conjugate.

Unstable-particle intermediate states

$$|T_{lH \rightarrow \bar{l}\bar{H}}|^2 \leftarrow \left| \sum_i \text{Diagram}_i \right|^2 = \sum_{i,j} \text{Diagram}_i \left(\text{Diagram}_j \right)^* \quad (80)$$

Terms with $i \neq j$

$$\frac{1}{s - M^2 + i\epsilon} = \mathcal{P} \frac{1}{s - M^2} + i\pi\delta(s - M^2) \quad (81)$$

$$\text{Diagram}_i = \text{Diagram}_i^{\text{principal value}} + \frac{1}{2} \text{Diagram}_i^{\text{cut}} \quad (82)$$

Unstable-particle intermediate states

Terms with $i = j$

$$\left| \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nearrow \\ H \end{array} \quad N_i \quad \begin{array}{c} \bullet \\ \nwarrow \\ \bar{l}_\beta \\ \searrow \\ \bar{H} \end{array} \right|^2 \propto \left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \left| \frac{1}{s - M^2 + iM\Gamma} \right|^2 \quad (83)$$

Unstable-particle intermediate states

Terms with $i = j$

$$\left| \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nearrow \\ H \end{array} \quad \begin{array}{c} N_i \\ \text{---} \\ \bullet \end{array} \quad \begin{array}{c} \bar{l}_\beta \\ \nwarrow \\ \bullet \\ \searrow \\ \bar{H} \end{array} \right|^2 \propto \left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \frac{\epsilon}{M\Gamma} \left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \quad (83)$$

Unstable-particle intermediate states

Terms with $i = j$

$$\left| \begin{array}{c} l_{\alpha} \\ \swarrow \\ \text{---} \bullet \text{---} N_i \text{---} \bullet \text{---} \searrow \\ \nwarrow \text{---} H \quad \text{---} \bar{H} \end{array} \right|^2 \propto \left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \frac{\pi}{M_i \Gamma} \delta(s - M^2) \quad (83)$$

$$|T_{lH \rightarrow \bar{l}\bar{H}}|^2 \approx |T_{lH \rightarrow N}|^2 \times \text{Br}(N \rightarrow \bar{l}\bar{H}) \quad (84)$$

double-counting the inverse decays

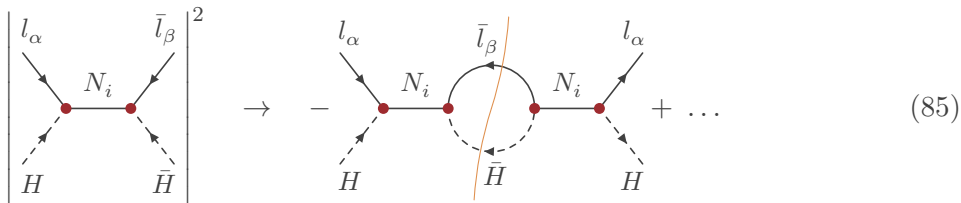
Unstable-particle intermediate states

Terms with $i = j$ from holomorphic cuts

$$\left| \begin{array}{c} l_\alpha \quad \bar{l}_\beta \\ \swarrow \quad \searrow \\ \text{---} N_i \text{---} \\ \nearrow \quad \nwarrow \\ H \quad \bar{H} \end{array} \right|^2 \rightarrow - \begin{array}{c} l_\alpha \quad \bar{l}_\beta \\ \swarrow \quad \searrow \\ \text{---} N_i \text{---} \text{---} N_i \text{---} \\ \nearrow \quad \nwarrow \quad \nearrow \quad \nwarrow \\ H \quad \bar{H} \quad H \end{array} \quad (85)$$

Unstable-particle intermediate states

Terms with $i = j$ from holomorphic cuts

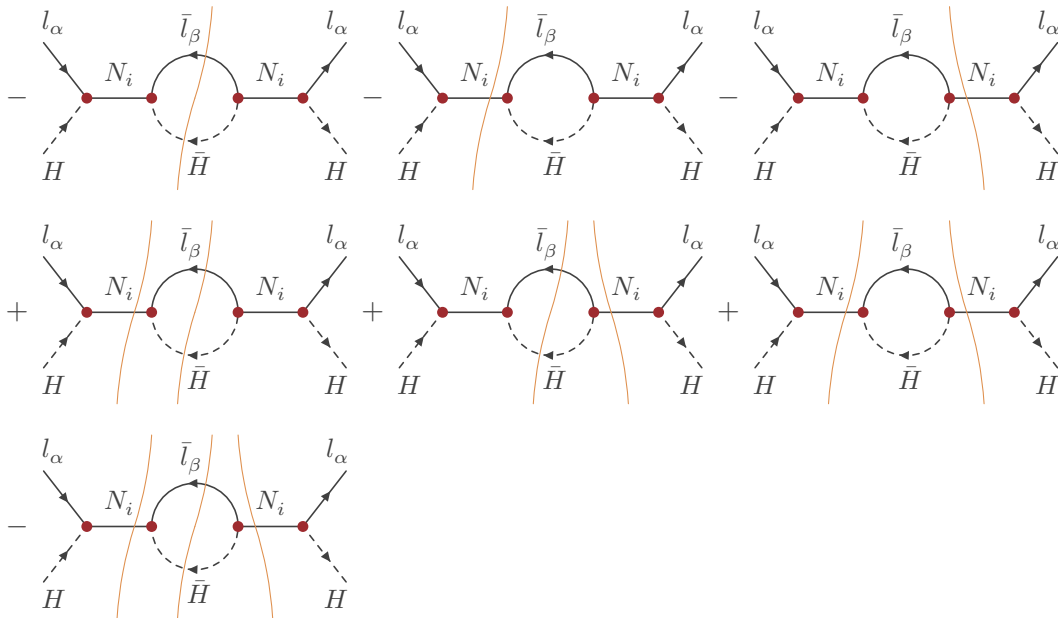


The diagram shows the expansion of the squared modulus of a transition amplitude. On the left, a vertex N_i is connected to an incoming lepton l_α and an outgoing lepton $\bar{l}_\beta$ by solid lines, and to an incoming Higgs H and an outgoing anti-Higgs $\bar{H}$ by dashed lines. This is enclosed in a vertical bar and squared. An arrow points to the right, where the expansion is shown. The first term is a similar diagram, but the internal line between the two N_i vertices is a loop. The top half of the loop is a solid line labeled $\bar{l}_\beta$ with an arrow pointing left, and the bottom half is a dashed line labeled $\bar{H}$ with an arrow pointing right. A vertical orange line is drawn through the loop. This term is preceded by a minus sign. The expansion continues with an ellipsis.

$$\left| \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nearrow \\ \bar{l}_\beta \end{array} \begin{array}{c} N_i \\ \text{---} \\ \bullet \end{array} \begin{array}{c} \swarrow \\ \bar{H} \\ \nearrow \\ H \end{array} \right|^2 \rightarrow - \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nearrow \\ \bar{l}_\beta \end{array} \begin{array}{c} N_i \\ \text{---} \\ \bullet \end{array} \begin{array}{c} \swarrow \\ \bar{l}_\beta \\ \nearrow \\ \bar{H} \end{array} \begin{array}{c} N_i \\ \text{---} \\ \bullet \end{array} \begin{array}{c} \swarrow \\ H \\ \nearrow \\ H \end{array} + \dots \quad (85)$$

$$|T_{fi}|^2 = -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_{n,k} i T_{in} i T_{nk} i T_{kf} i T_{fi} + \dots \quad (86)$$

Unstable-particle intermediate states



Unstable-particle intermediate states

$$\begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nwarrow \\ H \end{array} \begin{array}{c} \xrightarrow{N_i} \\ \bullet \end{array} \begin{array}{c} \nearrow \\ \bar{l}_\beta \\ \searrow \\ \bar{H} \end{array} = \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nwarrow \\ H \end{array} \begin{array}{c} \xrightarrow{N_i} \\ \bullet \end{array} \begin{array}{c} \nearrow \\ \bar{l}_\beta \\ \searrow \\ \bar{H} \end{array} + \frac{1}{2} \begin{array}{c} l_\alpha \\ \swarrow \\ \bullet \\ \nwarrow \\ H \end{array} \begin{array}{c} \xrightarrow{N_i} \\ \bullet \end{array} \begin{array}{c} \nearrow \\ \bar{l}_\beta \\ \searrow \\ \bar{H} \end{array} \quad (87)$$

$$-i\Sigma = -i\text{Re}\Sigma + \text{Im}\Sigma \quad \rightarrow \quad \begin{array}{c} \bar{l}_\beta \\ \curvearrowright \\ \bullet \end{array} \begin{array}{c} \curvearrowleft \\ \bar{H} \\ \bullet \end{array} = \begin{array}{c} \bar{l}_\beta \\ \curvearrowright \\ \bullet \end{array} \begin{array}{c} \curvearrowleft \\ \bar{H} \\ \bullet \end{array} + \frac{1}{2} \begin{array}{c} \bar{l}_\beta \\ \curvearrowright \\ \bullet \end{array} \begin{array}{c} \curvearrowleft \\ \bar{H} \\ \bullet \end{array} \quad (88)$$

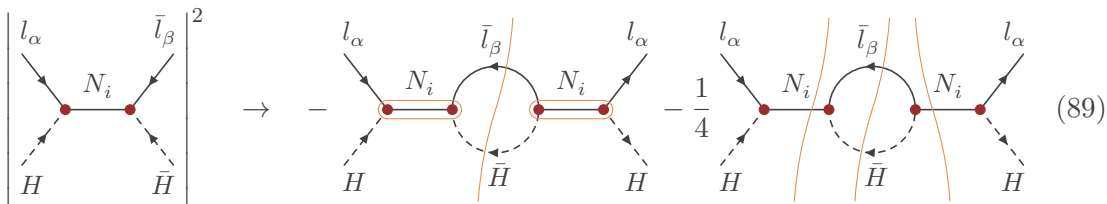
Unstable-particle intermediate states

$$\left| \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ \bar{l}_\beta \end{array} \right|^2 \rightarrow - \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ \bar{l}_\beta \end{array} \begin{array}{c} \text{---} H \text{---} \\ \text{---} \bar{H} \text{---} \end{array} \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ H \end{array} - \frac{1}{4} \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ H \end{array} \begin{array}{c} \text{---} H \text{---} \\ \text{---} \bar{H} \text{---} \end{array} \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ H \end{array} \quad (89)$$

$$\propto \text{Im} \Sigma(s) \times \frac{(s - M_i^2)^2 - \epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2} = \text{Im} \Sigma(s) \times -\frac{\partial}{\partial s} \mathcal{P} \frac{1}{s - M_i^2}$$

[Hannessdottir, Mizera '22; Maták '23]

Unstable-particle intermediate states



$$\left| \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ \bar{l}_\beta \end{array} \right|^2 \rightarrow - \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ \bar{l}_\beta \end{array} \begin{array}{c} \bar{l}_\beta \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ H \end{array} - \frac{1}{4} \begin{array}{c} l_\alpha \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ \bar{l}_\beta \end{array} \begin{array}{c} \bar{l}_\beta \\ \swarrow \\ \text{---} N_i \text{---} \\ \nwarrow \\ H \end{array} \quad (89)$$

$$\propto \text{Im} \Sigma(s) \times \frac{(s - M_i^2)^2 - \epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2} = \text{Im} \Sigma(s) \times -\frac{\partial}{\partial s} \mathcal{P} \frac{1}{s - M_i^2}$$

[Hannessdottir, Mizera '22; Maták '23]

No double-counting

$$\frac{\epsilon}{M_i \Gamma_i} \left\{ \frac{1}{(s - M_i^2)^2 + \epsilon^2} - \frac{2\epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2} \right\} \rightarrow 0 \quad (90)$$

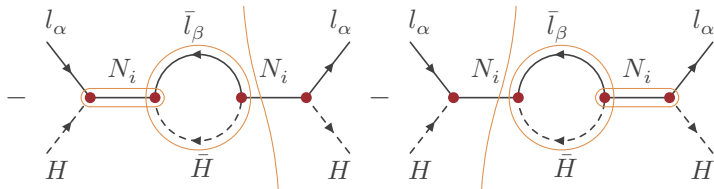
[Maták '22]

...the rest

(91)

$$\propto 2 \text{Re} \Sigma(s) \times \delta(s - M_i^2) \mathcal{P} \frac{1}{s - M_i^2} = \text{Re} \Sigma(s) \times -\frac{\partial}{\partial s} \delta(s - M_i^2) \quad (92)$$

...the rest



$$- \quad - \quad (91)$$

$$\propto 2 \operatorname{Re} \Sigma(s) \times \delta(s - M_i^2) \mathcal{P} \frac{1}{s - M_i^2} = \operatorname{Re} \Sigma(s) \times -\frac{\partial}{\partial s} \delta(s - M_i^2) \quad (92)$$

$$\rightarrow \left\{ \operatorname{Re} \Sigma(s) \frac{\partial}{\partial s} + \frac{\partial \operatorname{Re} \Sigma(s)}{\partial s} \right\} \Big|_{s=M_i^2} |T_{l_\alpha H \rightarrow N_i}|^2 \quad (93)$$

[Maták '23]