Diagrammar for leptogenesis

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In collaboration with T. Blažek, J. Heeck, J. Heisig, and V. Zaujec

COMENIUS UNIVERSITY BRATISLAVA

Particle and Astroparticle Theory Seminar

27 May 2024, Max-Planck-Institut für Kernphysik

Plan for this seminar

- Introduction to unitarity constraints and holomorphic cutting rules
- Dirac leptogenesis from scatterings of massless particles
- Further (unexpected) consequences of unitarity

Part I.

CPT and unitarity constraints

Unitarity and optical theorem

$$S^\dagger S = 1 \quad \rightarrow \quad \mathrm{i}\,T^\dagger - \mathrm{i}\,T = T^\dagger T \quad \text{for} \quad \mathrm{i}\,T = S - 1 \tag{1}$$

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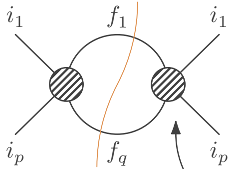
$$2\,\mathrm{Im}\;T_{ii} = \sum_f \left|T_{fi}\right|^2 = -\sum_f \mathrm{i}\,T^\dagger_{if}\mathrm{i}\,T_{fi} = -\sum_f \tag{2}$$



$$\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2E_k} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} 2\pi\delta_+(k^2 - m^2) \text{ for each cut line}$$

iϵ replaced by $-$iϵ for each propagator

Unitarity and optical theorem

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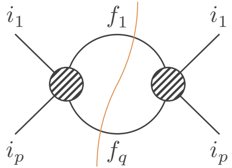

$$\left| T_{i\to f} \right|^2 = \int \prod_{\forall f} \frac{\mathrm{d}^3 \boldsymbol{k}_f}{(2\pi)^3 2E_f} \sum_{\text{spin}} \left| T_{fi} \right|^2 \tag{3}$$

Imaginary kinematics in Feynman diagrams

$$\mathrm{i}\,T^\dagger - \mathrm{i}\,T = T^\dagger T \quad \to \quad \mathrm{i}\,T^*_{if} - \mathrm{i}\,T_{fi} = \sum_n T^*_{nf}\,T_{ni} \tag{4}$$

1. Real couplings with $T_{fi} = T_{if}$

$$2\,\mathrm{Im}\;T_{fi} = \sum_n T^*_{fn}\,T_{ni} \tag{5}$$

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$$2\,\mathrm{Im}\,T_{fi} = \sum_n T^*_{fn}\,T_{ni} \tag{5}$$

2. Imaginary kinematics

$$T_{fi} = \sum_{\text{diagrams}} C_{fi} K_{fi} \quad \leftarrow \quad C_{fi} = C^*_{if},\, K_{fi} = K_{if} \tag{6}$$

$$2\,\mathrm{Im}\,K_{fi} = \sum_n K^*_{fn} K_{ni} \tag{7}$$

[Cutkosky '60; Veltman '63]

CP asymmetric reactions

$$CPT \text{ symmetry implies } |T_{\overline{fi}}|^2 = |T_{if}|^2 \quad \rightarrow \quad \Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 \tag{8}$$

$$\left.\begin{aligned} T_{fi} &= C_{fi}^{\text{tree}} K_{fi}^{\text{tree}} + C_{fi}^{\text{loop}} K_{fi}^{\text{loop}} \\ T_{if} &= C_{if}^{\text{tree}} K_{if}^{\text{tree}} + C_{if}^{\text{loop}} K_{if}^{\text{loop}} \end{aligned}\right\} \quad \Delta|T_{fi}|^2 = -4\,\text{Im}\left[C_{fi}^{\text{tree}} C_{fi}^{\text{loop}*}\right] \text{Im}\left[K_{fi}^{\text{tree}} K_{fi}^{\text{loop}*}\right]$$

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CP asymmetric reaction – the ingredients

- loop kinematics with nonzero imaginary part
- model with irreducible complex phases

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$$S^\dagger S = S S^\dagger \quad \rightarrow \quad 1 + \mathrm{i}\,T - \mathrm{i}\,T^\dagger + T T^\dagger = 1 + \mathrm{i}\,T - \mathrm{i}\,T^\dagger + T^\dagger T \tag{9}$$

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$$S^\dagger S = S S^\dagger \quad \to \quad \cancel{1 + \mathrm{i}T - \mathrm{i}T^\dagger} + TT^\dagger = \cancel{1 + \mathrm{i}T - \mathrm{i}T^\dagger} + T^\dagger T \tag{9}$$

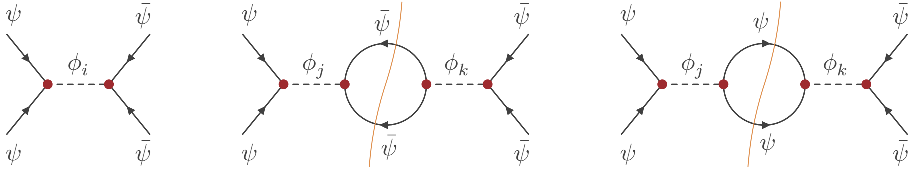
$$\sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \to \quad \sum_f \Delta|T_{fi}|^2 = 0 \tag{10}$$

[Dolgov '79; Kolb, Wolfram '80; see also Hook '11; Baldes *et al.* '14]

A toy model for **non-obtaining** the asymmetry

$$L_{\text{int}} = -\sum_i \lambda_i \phi_i \bar{\psi}\psi^c + \text{H.c.} \tag{11}$$

- irreducible complex phases in couplings
- imaginary kinematics in $\psi\psi \to \bar{\psi}\bar{\psi}$ amplitude at $\mathcal{O}(\lambda^4)$ order



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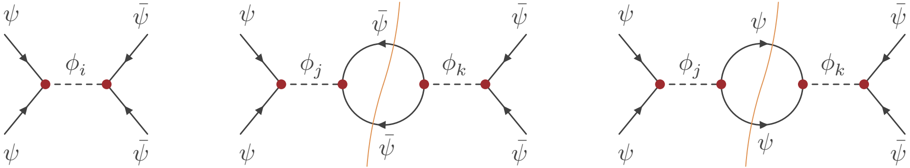

For $(p_1 + p_2)^2 < M_i^2$ the scalars ϕ_i cannot be produced and

$$\Delta |T_{\psi\psi\to\bar{\psi}\bar{\psi}}|^2 + \Delta |T_{\psi\psi\to\psi\psi}|^2 = 0 \quad \to \quad \Delta |T_{\psi\psi\to\bar{\psi}\bar{\psi}}|^2 = 0. \tag{12}$$

CPT and unitarity constraints

$$CPT \text{ symmetry implies } |T_{\overline{fi}}|^2 = |T_{if}|^2 \quad \rightarrow \quad \Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 \tag{8}$$

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- loop kinematics with nonzero imaginary part
- model with irreducible complex phases
- at least two nonvanishing asymmetries with the same initial state

CPT and unitarity constraints

$$S^\dagger S = 1 \quad \rightarrow \quad T = T^\dagger + \mathrm{i}\, T^\dagger\, T$$

$$\Delta \left| T_{fi} \right|^2 = \left| T^*_{if} + \mathrm{i} \sum_n T^\dagger_{fn}\, T_{ni} \right|^2 - \left| T_{if} \right|^2 = -2\, \mathrm{Im} \left[\sum_n T_{if}\, T^\dagger_{fn}\, T_{ni} \right] + \left| \sum_n T^\dagger_{fn}\, T_{ni} \right|^2 \qquad (13)$$

[Kolb, Wolfram ’80]

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[Kolb, Wolfram '80]

No further on-shell cuts implies $T_{if}^* = T_{fi} \quad \rightarrow \quad \Delta \left| T_{fi} \right|^2 = -2\, \mathrm{Im} \left[\sum_n T_{if} T_{fn} T_{ni} \right] \quad (14)$

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No further on-shell cuts implies $T^*_{if} = T_{fi}$ $\quad \rightarrow \quad$ $\Delta \left| T_{fi} \right|^2 = -2\, \mathrm{Im} \left[\sum_n T_{if}\, T_{fn}\, T_{ni} \right]$ (14)

$$\Delta \left| T_{fi} \right|^2 = \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \sum_n \mathrm{i}\, T_{if} \mathrm{i}\, T_{fn} \mathrm{i}\, T_{ni} \quad (15)$$

[Covi, Roulet, Vissani '98]

CPT and unitarity constraints

ELSEVIER

2 April 1998

PHYSICS LETTERS B

Physics Letters B 424 (1998) 101–105

On the CP asymmetries in Majorana neutrino decays

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Received 12 January 1998
Editor: R. Gatto

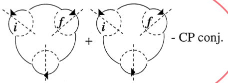

Fig. 3. Pictorial representation of the self-energy contribution to the cross section asymmetry ϵ_σ.

Therefore, one can pictorially represent the self-energy contributions to $\sigma(\ell^c H^* \to \ell H)$ as in Fig. 2, where the cut blobs are the initial (i) and final (f) states, while the remaining blob stands for the one-loop self-energy. This last is actually the sum of two contributions, one with a lepton and one with an antilepton.

Now, in the computation of ϵ_σ, only the absorptive part of the loop will contribute, and the Cutkoski

Turning now to the vertex contributions, to see the cancellations we need to add the three contributions shown in Fig. 4 (including the tree level u-channel interfering with the one-loop self-energy diagram). In terms of cut diagrams, this can be expressed as in Fig. 5, where CP-conjugate stands for the same four diagrams with all the arrows reversed. Hence, the CP-conjugate contribution will exactly cancel the four diagrams, since changing the directions of the arrows just exchanges among themselves the first and fourth diagrams, as well as the second and third ones. We then see explicitly that the absorptive part of the self-energies are also playing here a crucial role, enforcing the cancellation of the CP violation produced by the vertex diagrams.

The only remaining diagrams to be considered are the interference of the one-loop vertex diagrams with the tree level u-channel. Pictorially, they are represented in Fig. 6, and they again cancel since the two

$$\Delta |T_{fi}|^2 = \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \sum_n \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni} \qquad (15)$$

[Covi, Roulet, Vissani ’98]

Holomorphic cutting rules

$$S^\dagger S = 1 \quad \to \quad \mathrm{i}\,T^\dagger = \mathrm{i}\,T - \mathrm{i}\,T \mathrm{i}\,T^\dagger \tag{16}$$

$$|\,T_{fi}|^2 = -\mathrm{i}\,T^\dagger_{if}\mathrm{i}\,T_{fi} = -\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} + \ldots \tag{17}$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

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$$|T_{fi}|^2 = -\mathrm{i}\,T^\dagger_{if}\mathrm{i}\,T_{fi} = -\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} + \ldots \tag{17}$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

$$\begin{aligned}\Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 = &\sum_n \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\right) \\ &- \sum_{n,k}\left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni}\right) \\ &+ \ldots\end{aligned} \tag{18}$$

[Blažek, Maták '21]

Holomorphic cutting rules

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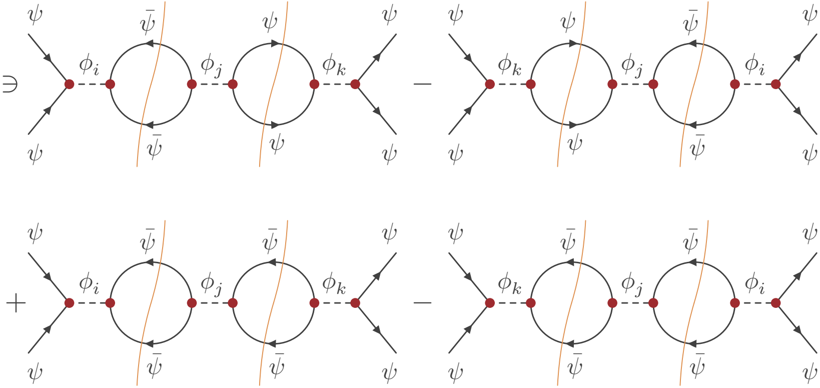
[Blažek, Maták '21]

Holomorphic cutting rules

$$\Delta |T_{fi}|^2 = \sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni} \right) - \dots \tag{19}$$

$$\Delta |T_{\psi\psi\to\bar\psi\bar\psi}|^2 \ni \tag{20}$$



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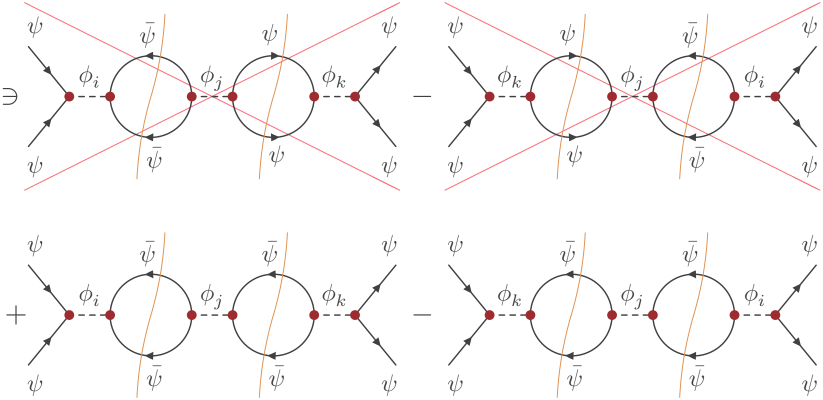

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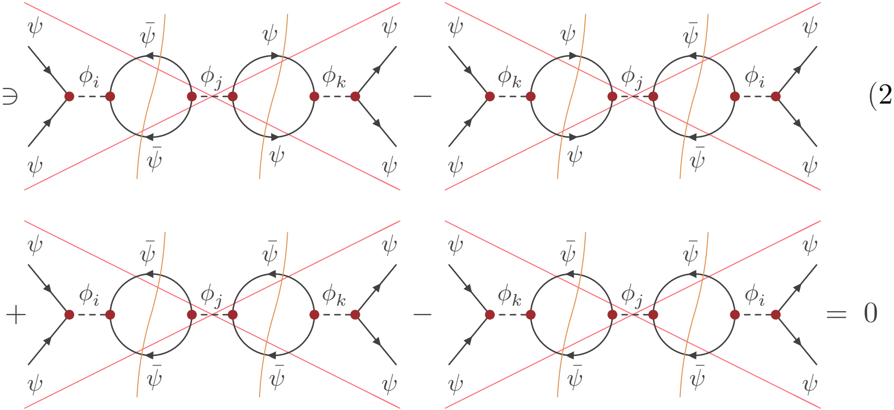

CP asymmetry in the Boltzmann equation

Change in # of particles $\leftrightarrow$ average # of their interactions

$$\dot{n}_{f_1} + 3Hn_{f_1} = \sum_{\text{all reactions}} \gamma_{fi} - \gamma_{if} \quad \leftarrow \quad \gamma_{fi} = \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i(\boldsymbol{p}_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} |T_{fi}|^2 \tag{21}$$

$$[\mathrm{d}\boldsymbol{p}] = \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3 2E_{\boldsymbol{p}}} \qquad |T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\boldsymbol{p}_f - \boldsymbol{p}_i) |M_{fi}|^2$$

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Boltzmann equation for the asymmetry in f_1 particle density

$$\left.\begin{aligned} \Delta n_{f_1} &= n_{f_1} - n_{\bar{f}_1} \\ \Delta \gamma_{fi} &= \gamma_{fi} - \gamma_{\overline{if}} \end{aligned}\right\} \quad \Delta \dot{n}_{f_1} + 3H\Delta n_{f_1} = \sum_{\text{all reactions}} \Delta\gamma_{fi} - \Delta\gamma_{if} \qquad (22)$$

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Kinetic equilibrium as a simplifying assumption

$$f_i(\boldsymbol{p}_i) \propto \exp\left\{-\frac{E_i}{T}\right\} \qquad\qquad \gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \gamma_{fi}^{\text{eq}} \tag{23}$$

CP asymmetry in the Boltzmann equation

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$$[\mathrm{d}\boldsymbol{p}] = \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3 2E_{\boldsymbol{p}}} \qquad |T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\boldsymbol{p}_f - \boldsymbol{p}_i) |M_{fi}|^2$$

Detailed balance condition

$$\prod_{\forall i} f_i^{\mathrm{eq}}(\boldsymbol{p}_i) = \prod_{\forall f} f_f^{\mathrm{eq}}(\boldsymbol{p}_f) \qquad \Delta\gamma_{fi}^{\mathrm{eq}} = -\Delta\gamma_{if}^{\mathrm{eq}} \tag{24}$$

CP asymmetry in the Boltzmann equation

$$\Delta\gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\text{eq}}}\frac{n_{i_2}}{n_{i_2}^{\text{eq}}}\cdots\frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \Delta\gamma_{fi}^{\text{eq}} + \left(\frac{\Delta n_{i_1}}{n_{i_1}^{\text{eq}}}\frac{n_{i_2}}{n_{i_2}^{\text{eq}}}\cdots\frac{n_{i_p}}{n_{i_p}^{\text{eq}}} + \frac{n_{i_1}}{n_{i_1}^{\text{eq}}}\frac{\Delta n_{i_2}}{n_{i_2}^{\text{eq}}}\cdots\frac{n_{i_p}}{n_{i_p}^{\text{eq}}} + \ldots\right) \times \gamma_{fi}^{\text{eq}} \quad (25)$$

CP asymmetry in the Boltzmann equation

$$\Delta\gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\rm eq}}\frac{n_{i_2}}{n_{i_2}^{\rm eq}}\cdots\frac{n_{i_p}}{n_{i_p}^{\rm eq}} \times \Delta\gamma_{fi}^{\rm eq} + \left(\frac{\Delta n_{i_1}}{n_{i_1}^{\rm eq}}\frac{n_{i_2}}{n_{i_2}^{\rm eq}}\cdots\frac{n_{i_p}}{n_{i_p}^{\rm eq}} + \frac{n_{i_1}}{n_{i_1}^{\rm eq}}\frac{\Delta n_{i_2}}{n_{i_2}^{\rm eq}}\cdots\frac{n_{i_p}}{n_{i_p}^{\rm eq}} + \ldots\right) \times \gamma_{fi}^{\rm eq} \quad (25)$$

Source terms

- through $\Delta\gamma_{fi}^{\rm eq}$ depend on $\Delta|T_{fi}|^2$
- symmetric part of particle densities

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Source terms

- through $\Delta\gamma_{fi}^{\text{eq}}$ depend on $\Delta|T_{fi}|^2$
- symmetric part of particle densities

Wash-out terms

- depend on density asymmetries
- symmetric part of reaction rates
- typically lead to asymmetry suppression

Which reactions?

$$\sum_{\text{all reactions}} \Delta\gamma_{fi} = \sum_i \sum_{f \ni f_1} \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} \times \Delta\gamma_{fi}^{\text{eq}} + \text{ wash-out} \tag{26}$$

- In the f_1 asymmetry source-term, f_1 must be present in the final state of the contributing reactions.
- Out-of-equilibrium particles must appear in the initial state.

[See also Racker '19]

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$$\left(\Delta\dot{n}_{f_1} + 3H\Delta n_{f_1}\right)_{\text{source}} = \sum_i \sum_{f \ni f_1} \left(\frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} - 1 \right) \times \Delta\gamma_{fi}^{\text{eq}} \tag{28}$$

Summary I.

- Feynman integrals may get imaginary kinematics from on-shell intermediate states.
- Irreducible complex phases may lead to CP-violating processes.
- Unitarity and CPT symmetry imply

$$
\begin{aligned}
\Delta|T_{fi}|^2 = & \sum_n \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\right) \\
& - \sum_{n,k} \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni}\right) \\
& + \ldots \quad \leftarrow \quad \sum_f \Delta|T_{fi}|^2 = 0
\end{aligned}
$$

Part II.

Dirac leptogenesis from scatterings

Leptogenesis with Dirac neutrinos

- Originally introduced in Phys. Rev. Lett. **84**, 4039. [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles.
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons.

$$Y_B = \frac{28}{79} Y_{B-L_{\mathrm{SM}}} = \frac{28}{79} \Delta_{\nu_R} \tag{29}$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

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$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \tag{30}$$

[Heeck, Heisig, Thapa '23a]

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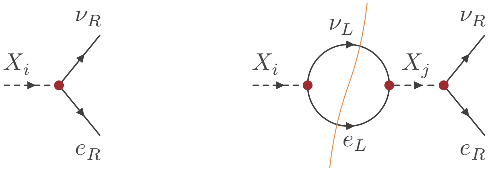

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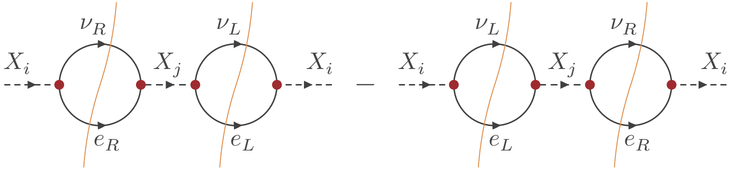
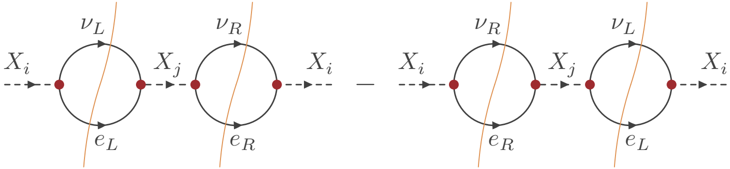
$$\Delta |T_{X_i \to \nu_R e_R}|^2 + \Delta |T_{X_i \to \nu_L e_L}|^2 = 0 \tag{31}$$

Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i\to\nu_R e_R}|^2 =$$ (32)



$$\Delta|T_{X_i\to\nu_L e_L}|^2 =$$ (33)



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Leptogenesis with Dirac neutrinos

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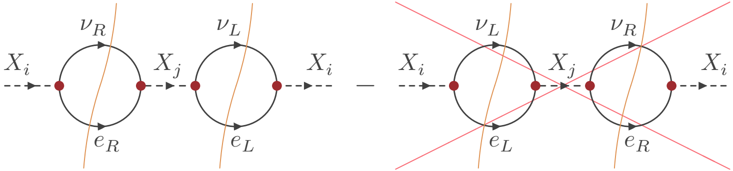

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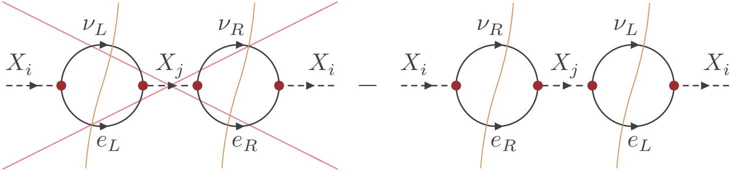

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Leptogenesis with Dirac neutrinos

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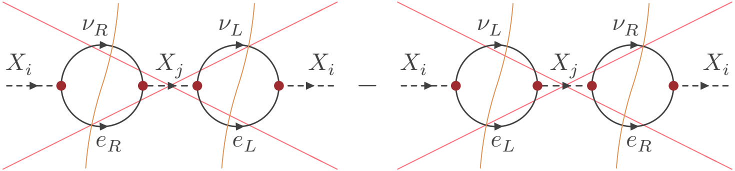

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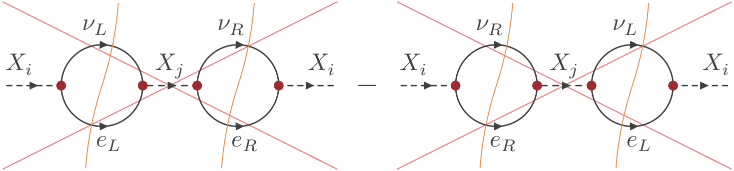

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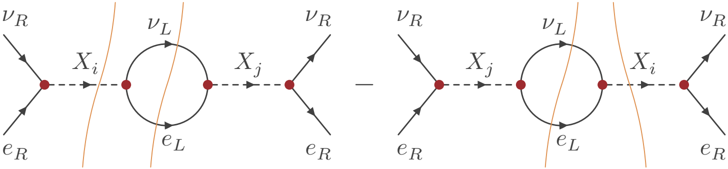
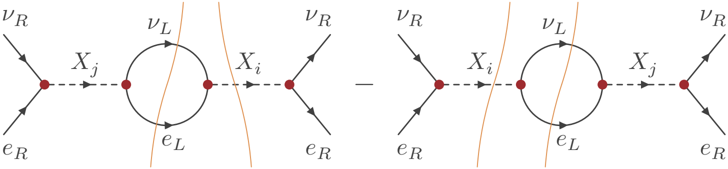
Leptogenesis with Dirac neutrinos

$$\Delta|T_{\nu_R e_R \to X_i}|^2 = \tag{34}$$



$$\Delta|T_{\nu_R e_R \to \nu_L e_L}|^2 = \tag{35}$$



$$\Delta|T_{\nu_R e_R \to X_i}|^2 + \Delta|T_{\nu_R e_R \to \nu_L e_L}|^2 = 0 \tag{36}$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2}\bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \tag{37}$$

[Heeck, Heisig, Thapa '23b]

Leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2}\bar{L}^c F_i L \bar{X}_i + \bar{e}_R^c G_i \nu_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \tag{37}$$

[Heeck, Heisig, Thapa '23b]

$$\cancel{\Delta|T_{\nu_R e_R \to X_i}|^2} + \Delta|T_{\nu_R e_R \to \nu_L e_L}|^2 = 0$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_\text{reh} \tag{38}$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \tag{38}$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

- Baryon and lepton number conservation

$$\Delta_{d_R} = -\Delta_Q - \Delta_{u_R}, \quad \Delta_{\nu_R} = -\Delta_L - \Delta_{e_R}. \tag{39}$$

Leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}_R^c G_i \nu_R \bar{X}_i + \bar{u}_R^c K_i e_R \bar{X}_i + \text{H.c.} \quad \leftarrow \quad X_i \text{ masses above } T_{\text{reh}} \tag{38}$$

[Heeck, Heisig, Thapa '23a; Blažek *et al.* '24]

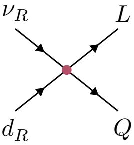
- Baryon and lepton number conservation

$$\Delta_{d_R} = -\Delta_Q - \Delta_{u_R}, \quad \Delta_{\nu_R} = -\Delta_L - \Delta_{e_R}. \tag{39}$$

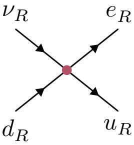
- Further global symmetries

$$\Delta_{\nu_R} = \Delta_{d_R}, \quad \Delta_L = \Delta_Q, \quad \Delta_{e_R} = \Delta_{u_R}. \tag{40}$$

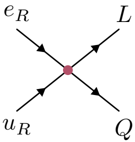
Leptogenesis without heavy particles?



$$= \mathrm{i} \sum_i \frac{F_i^\dagger G_i}{M_i^2}$$



$$= \mathrm{i} \sum_i \frac{K_i^\dagger G_i}{M_i^2}$$



$$= \mathrm{i} \sum_i \frac{F_i^\dagger K_i}{M_i^2}$$

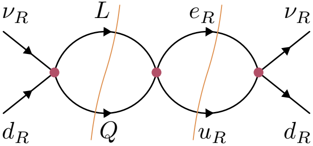
$$\langle \sigma_1 v \rangle = \frac{64}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\mathrm{Tr}\left(F_i^\dagger F_j\right) \mathrm{Tr}\left(G_j^\dagger G_i\right)}{M_i^2 M_j^2} \equiv \frac{64}{3\pi} \frac{T^2}{T_{\mathrm{reh}}^2} \frac{\alpha_1}{\zeta(3)^2} \quad (41)$$

$$\langle \sigma_2 v \rangle = \frac{32}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\mathrm{Tr}\left(K_i^\dagger K_j\right) \mathrm{Tr}\left(G_j^\dagger G_i\right)}{M_i^2 M_j^2} \equiv \frac{32}{3\pi} \frac{T^2}{T_{\mathrm{reh}}^2} \frac{\alpha_2}{\zeta(3)^2} \quad (42)$$

$$\langle \sigma_3 v \rangle = \frac{64}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{\mathrm{Tr}\left(F_i^\dagger F_j\right) \mathrm{Tr}\left(K_j^\dagger K_i\right)}{M_i^2 M_j^2} \equiv \frac{64}{3\pi} \frac{T^2}{T_{\mathrm{reh}}^2} \frac{\alpha_3}{\zeta(3)^2} \quad (43)$$

Leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \to LQ}|^2 =$$



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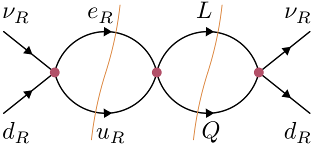
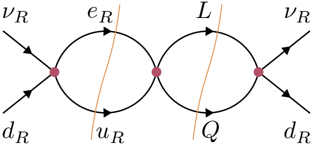


(44)

$$\Delta|T_{\nu_R d_R \to e_R u_R}|^2 =$$



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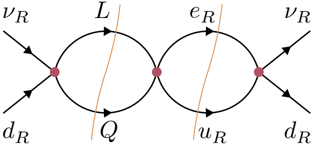
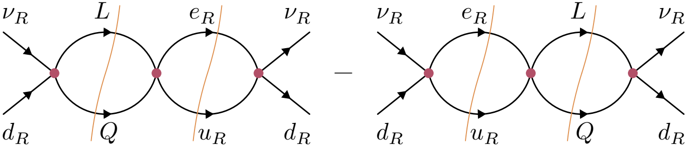


(45)

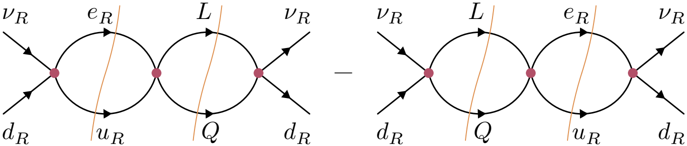
$$\Delta|T_{\nu_R d_R \to LQ}|^2 = -\Delta|T_{\nu_R d_R \to e_R u_R}|^2 \neq 0 \tag{46}$$

Leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \to LQ}|^2 = \quad (44)$$



$$\Delta|T_{\nu_R d_R \to e_R u_R}|^2 = \quad (45)$$



$$\Delta\langle\sigma_1 v\rangle \equiv \frac{512}{\pi^2}\frac{T^4}{T_{\rm reh}^6}\frac{\epsilon}{\zeta(3)^2} = -\Delta\langle\sigma_2 v\rangle \quad (47)$$

Freeze-in and wash-in

$$\left(\frac{\mathrm{d}\Delta_L}{\mathrm{d}x}\right)_{\text{source}} = -\left(\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x}\right)_{\text{source}} \quad \rightarrow \quad \left(\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x}\right)_{\text{source}} = 0 \tag{48}$$

Freeze-in and wash-in

$$\left(\frac{\mathrm{d}\Delta_L}{\mathrm{d}x}\right)_{\text{source}} = -\left(\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x}\right)_{\text{source}} \quad \rightarrow \quad \left(\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x}\right)_{\text{source}} = 0 \tag{48}$$

$$\left(\frac{\mathrm{d}\Delta_L}{\mathrm{d}x}\right)_{\text{wash-out}} \neq -\left(\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x}\right)_{\text{wash-out}} \quad \rightarrow \quad \left(\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x}\right)_{\text{wash-in}} \neq 0 \tag{49}$$

[Domcke *et al.* ’21]

Freeze-in and wash-in

$$\frac{\mathrm{d}\,Y_{\nu_R}}{\mathrm{d}x} = -\,\frac{1}{x^4}\,\frac{\Gamma}{H}\bigg|_{T_{\rm reh}}\left(Y_{\nu_R} - Y_{\nu_R}^{\rm eq}\right) \quad \leftarrow \quad \Gamma = \frac{5}{9}\,s\,Y_{\nu_R}^{\rm eq}\Big(\langle\sigma_1 v\rangle + \langle\sigma_2 v\rangle\Big) \tag{50}$$

Freeze-in and wash-in

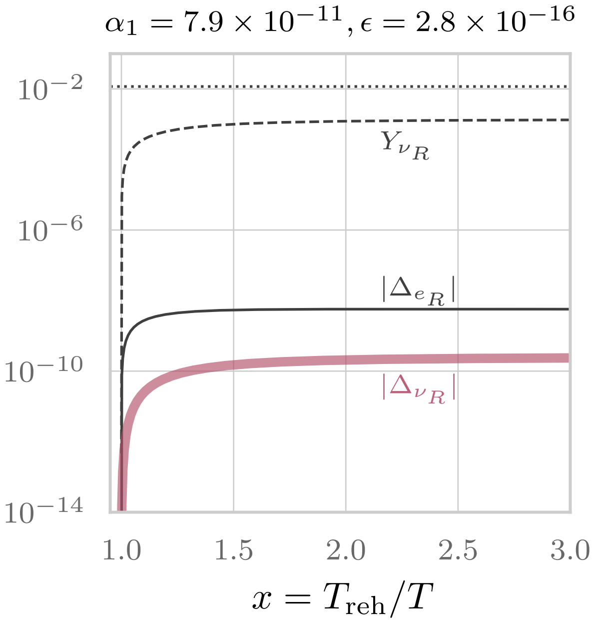
$$\frac{\mathrm{d}\,Y_{\nu_R}}{\mathrm{d}x} = -\frac{1}{x^4}\frac{\Gamma}{H}\bigg|_{T_{\rm reh}}\left(Y_{\nu_R} - Y_{\nu_R}^{\rm eq}\right) \quad \leftarrow \quad \Gamma = \frac{5}{9}\,s\,Y_{\nu_R}^{\rm eq}\Big(\langle\sigma_1 v\rangle + \langle\sigma_2 v\rangle\Big) \tag{50}$$

$$\begin{aligned}\frac{\mathrm{d}\Delta_L}{\mathrm{d}x} =& \frac{Y_{\nu_R}^{\rm eq}}{9H}\frac{\mathrm{d}s}{\mathrm{d}x}\bigg\{\Delta\langle\sigma_1 v\rangle\left(Y_{\nu_R}^{\rm eq} - Y_{\nu_R}\right) + \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) \\ &+ \frac{8}{9}\langle\sigma_1 v\rangle\left[\Delta_L - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\rm eq}}\left(\Delta_L - \frac{3}{2}\Delta_{\nu_R}\right)\right]\bigg\}\end{aligned} \tag{51}$$

$$\begin{aligned}\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x} =& \frac{Y_{\nu_R}^{\rm eq}}{9H}\frac{\mathrm{d}s}{\mathrm{d}x}\bigg\{\Delta\langle\sigma_2 v\rangle\left(Y_{\nu_R}^{\rm eq} - Y_{\nu_R}\right) - \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) \\ &+ \frac{8}{9}\langle\sigma_2 v\rangle\left[2\Delta_{e_R} - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\rm eq}}\left(2\Delta_{e_R} - \frac{3}{2}\Delta_{\nu_R}\right)\right]\bigg\}\end{aligned} \tag{52}$$

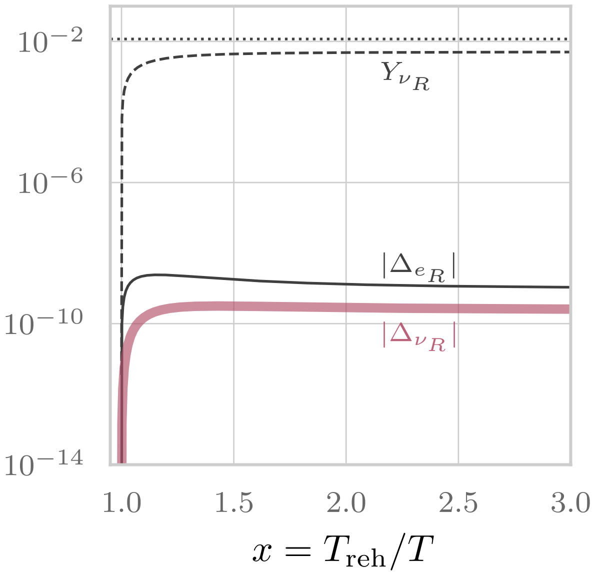
[Blažek *et al.* '24]

Numerical solution

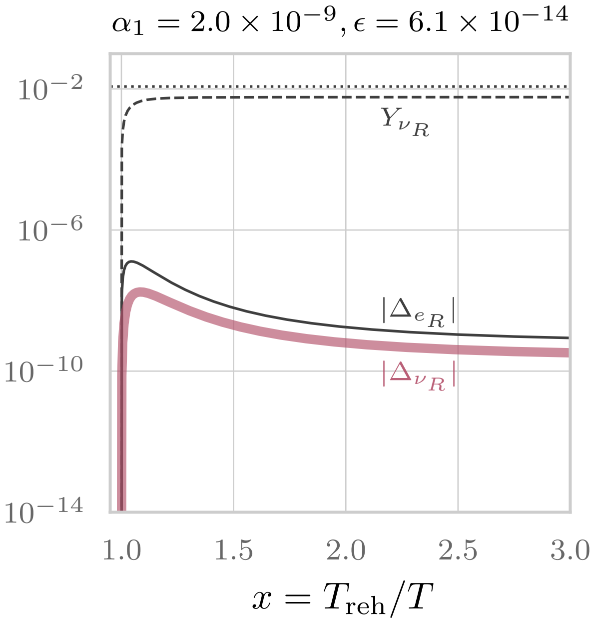


Numerical solution

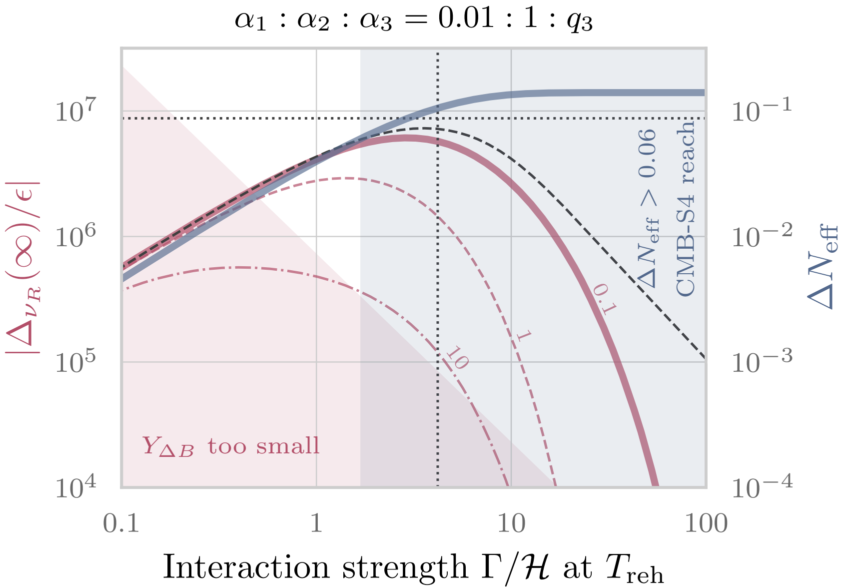
$\alpha_1 = 5.9 \times 10^{-10}, \epsilon = 3.9 \times 10^{-16}$



Numerical solution

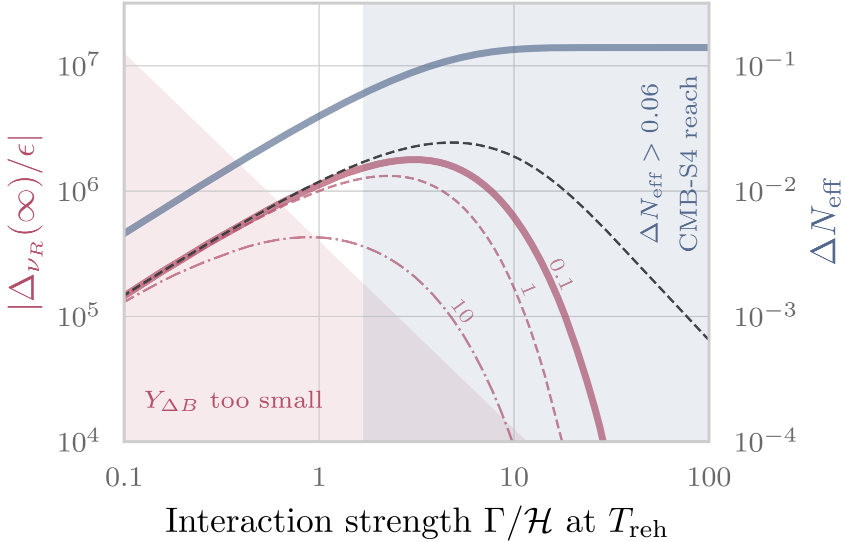


Numerical solution



Numerical solution

$\alpha_1 : \alpha_2 : \alpha_3 = 1 : 2 : q_3$



Summary II.

- Dirac leptogenesis through scatterings of massless particles works with sufficiently complex model.

- Right-handed neutrino source-term vanishes.

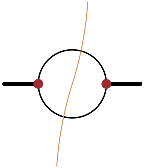
- Asymmetry is washed in from small differences in standard-model particle asymmetries.

Part III.

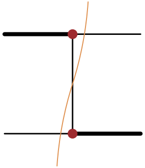
Further consequences of unitarity

Unitarity and the Boltzmann equation

$$\gamma_{fi}^{\mathrm{eq}} = \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i^{\mathrm{eq}}(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} \Big(-\mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \dots \Big) \tag{53}$$



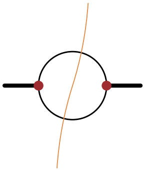
$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2](2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$



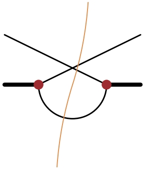
?

Unitarity and the Boltzmann equation

$$\gamma_{fi}^{\mathrm{eq}} = \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i^{\mathrm{eq}}(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} \Big(- \mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \dots \Big) \tag{53}$$

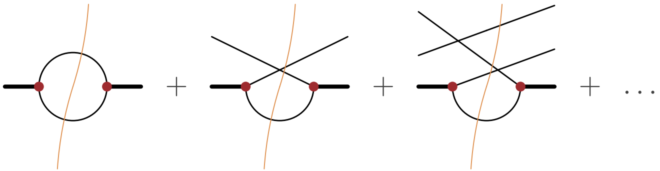


$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2](2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \mathrm{e}^{-E_{k_1}/T} (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$

Unitarity and the Boltzmann equation



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \left[1 + \frac{1}{\mathrm{e}^{E_{k_1}/T} - 1}\right] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2) \tag{54}$$

[Blažek, Maták '21b]

Vacuum diagrams and complex phases

- Used to represent the weak-basis invariants. [Botella, Nebot, Vives '06; Nilles, Ratz, Trautner, Vaudrevange '18]

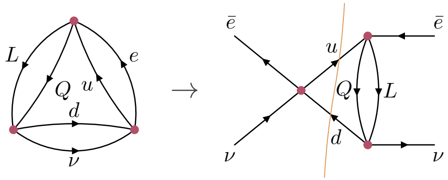
- Reversing the arrows on charged-particle propagators must lead to a topologically inequivalent vacuum diagram.

$$\Delta|T_{fi}|^2 = \sum_n \left(\mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nf}\,\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\,\mathrm{i}\,T_{fn}\,\mathrm{i}\,T_{ni} \right) - \dots \tag{55}$$

$$\mathcal{L} = \frac{1}{2}\bar{L}^c F_i L \bar{X}_i + \bar{e}^c_R G_i \nu_R \bar{X}_i + \text{H.c.}$$

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}^c_R G_i \nu_R \bar{X}_i + \bar{u}^c_R K_i e_R \bar{X}_i + \text{H.c.}$$

Vacuum diagrams and the Boltzmann equation



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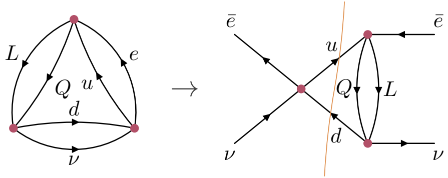
Vacuum diagrams and the Boltzmann equation



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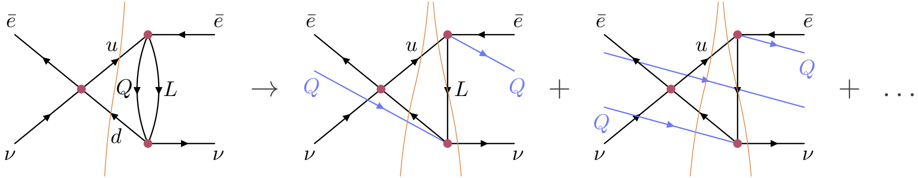
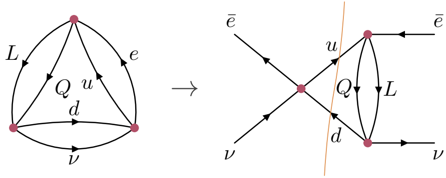


(57)

$$\frac{\mathrm{i}}{p^2 + \mathrm{i}\epsilon} \quad \to \quad 2\pi \sum_{w=1}^{\infty} \left(-1\right)^w \left[f^{\mathrm{eq}}(|p^0|)\right]^w \delta(p^2) = -2\pi f_{\mathrm{FD}}(|p^0|)\delta(p^2) \tag{58}$$

Vacuum diagrams and the Boltzmann equation



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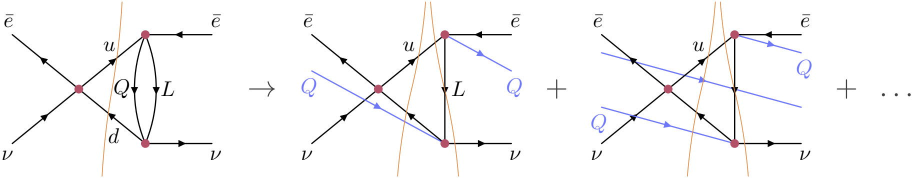

(57)

$$\Delta\gamma^{\rm eq}_{\nu\bar{e}(Q)\to u\bar{d}(Q)} + \Delta\gamma^{\rm eq}_{(\nu)\bar{e}Q\to(\nu)\bar{L}u} = 0 \tag{59}$$

The assumptions we made

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ - \sum_p F_p a_p^\dagger a_p \right\}, Z = \prod_p Z_p \tag{60}$$

$$e^{-E_p/T} \to e^{-F_p} = \frac{f_p}{1 \pm f_p} \qquad f_p = \mathrm{Tr}\left[a_p^\dagger a_p \rho\right] \tag{61}$$

$$\rho' = S\rho S^\dagger \quad \to \quad (1 + \mathrm{i}\,T)\rho(1 - \mathrm{i}\,T + \mathrm{i}\,T\mathrm{i}\,T - \ldots) \tag{62}$$

The collision term for the Boltzmann equation is obtained as $\mathrm{Tr}\left[a_p^\dagger a_p\big(\rho - \rho'\big)\right]/V_4$.

[McKellar, Thomson ’94; Blažek, Maták ’21b]

What else can be done?

- Thermal masses from anomalous thresholds. [Blažek, Maták '22]

- CPT and unitarity constraints generalized to thermal-corrected asymmetries. In the seesaw type-I leptogenesis at $\mathcal{O}(Y^4 Y_t^2)$ they look like

$$\Delta\gamma^{\text{eq}}_{N_i Q \to lt} + \Delta\gamma^{\text{eq}}_{N_i (Q) \to lH(Q)} + \Delta\gamma^{\text{eq}}_{N_i (Q) \to \bar{l}\bar{H}(Q)} + \Delta\gamma^{\text{eq}}_{N_i Q \to \bar{l}QQ\bar{t}} = 0, \tag{63}$$

$$m^2_{H,Y_t}(T) \frac{\partial}{\partial m_H^2}\Big|_0 \left(\Delta\gamma^{\text{eq}}_{N_i \to lH} + \Delta\gamma^{\text{eq}}_{N_i \to \bar{l}\bar{H}}\right) = 0. \tag{64}$$

[Blažek, Maták, Zaujec '22]

- No double-counting from intermediate states. Resonant dark matter annihilation at fixed order. [Maták '24; see also Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

Summary III.

- Vacuum diagrams must not be invariant under the arrow reversal to come with a complex phase.

- Completing the Boltzmann equations by all possible unitary cuts represents thermal corrections (even when you do not notice that).

Thank you for your attention!

Backup

Description of multiparticle states

Single-particle state: $|p\rangle \quad \leftarrow \quad$ particle species, momentum, spin, ...

$$\rho_p = \frac{1}{Z_p} \exp\left\{ -F_p a_p^\dagger a_p \right\}, \quad Z_p = \mathrm{Tr} \exp\left\{ -F_p a_p^\dagger a_p \right\} = \frac{\exp F_p}{\exp F_p - 1} \tag{65}$$

[Wagner '91]

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[Wagner '91]

$$f_p = \mathrm{Tr}\, \rho_p a_p^\dagger a_p = \frac{1}{\exp F_p - 1} \quad \rightarrow \quad \mathring{f}_p \stackrel{\text{def.}}{=} \exp\left\{ - F_p \right\} \tag{66}$$

Boltzmann equation from density matrix evolution

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ -\sum_p F_p a_p^\dagger a_p \right\}, \quad Z = \prod_p Z_p \tag{67}$$

$$\rho' = S\rho S^\dagger \quad \to \quad \rho' - \rho = \mathrm{i}\, T\rho - \mathrm{i}\rho T^\dagger + T\rho T^\dagger \tag{68}$$

$$\rho' - \rho = T\rho T^\dagger - \frac{1}{2} TT^\dagger \rho - \frac{1}{2}\rho TT^\dagger + \frac{\mathrm{i}}{2}\left[T + T^\dagger, \rho\right]$$

[McKellar, Thomson '94]

$$\mathrm{i}\, T^\dagger = \frac{\mathrm{i}}{2}(T + T^\dagger) + \frac{1}{2} TT^\dagger, \quad \mathrm{i}\, T = \frac{\mathrm{i}}{2}(T + T^\dagger) - \frac{1}{2} TT^\dagger \tag{69}$$

Boltzmann equation from density matrix evolution

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ -\sum_p F_p a_p^\dagger a_p \right\}, \quad Z = \prod_p Z_p \tag{67}$$

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$$\rho' - \rho = T\rho T^\dagger - \frac{1}{2} T T^\dagger \rho - \frac{1}{2}\rho T T^\dagger + \frac{\mathrm{i}}{2}\left[T + T^\dagger, \rho\right]$$

[McKellar, Thomson '94]

$$f_p' - f_p = \mathrm{Tr}\left[a_p^\dagger a_p (\rho' - \rho)\right] \tag{70}$$

$$\left[a_p^\dagger a_p, \rho\right] = 0 \quad \to \quad f_p' - f_p = \mathrm{Tr}\left[a_p^\dagger a_p \left(\rho\, \mathrm{i}\, T \mathrm{i}\, T^\dagger - \mathrm{i}\, T\rho\, \mathrm{i}\, T^\dagger\right)\right] \tag{71}$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \mathrm{Tr}\left[a_p^\dagger a_p\left(\rho\,\mathrm{i}T\mathrm{i}T^\dagger - \mathrm{i}T\rho\,\mathrm{i}T^\dagger\right)\right] \tag{72}$$

$$= \sum_{k=1}^{\infty}(-1)^k \mathrm{Tr}\left[a_p^\dagger a_p\left(\mathrm{i}T\rho(\mathrm{i}T)^k - \rho(\mathrm{i}T)^{k+1}\right)\right]$$

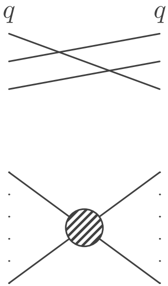
$$= \sum_{k=1}^{\infty}(-1)^k \frac{1}{Z_1 Z_2 \ldots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \ldots (\mathrm{i}T)^k_{in} \mathrm{i}T_{ni}$$

[Blažek, Maták '21b]

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \dots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]



$$\propto \quad \mathring{f}_q^{\,3} \quad \to \quad Z_q = \sum_{i_q=0}^{\infty} \mathring{f}_q^{\,i_q} \tag{73}$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \ldots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \ldots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]

$|q\rangle \neq |p\rangle$

1. i_q goes up by one in some $\mathrm{i}\,T$, down in another.
2. The same in a reversed order.
3. i_q is lowered and increased in the same $\mathrm{i}\,T$.

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \ldots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \ldots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]

$\underline{|q\rangle \neq |p\rangle}$

1. i_q goes up by one in some $\mathrm{i}\,T$, down in another.

$$\left\langle \frac{1}{\sqrt{(i_q+1)!}} a_q^{i_q+1} a_q^{\dagger} \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \quad \rightarrow \quad \sqrt{1+i_q} \tag{74}$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \ldots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \ldots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]

$\underline{|q\rangle \neq |p\rangle}$

1. i_q goes up by one in some $\mathrm{i}\,T$, down in another.

$$\left\langle \frac{1}{\sqrt{(i_q+1)!}} a_q^{i_q+1} a_q^\dagger \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \quad \to \quad \sqrt{1+i_q} \tag{74}$$

$$\sum_{i_q=1}^{\infty} (1+i_q) \mathring{f}_q^{\,i_q} = Z_q (1+f_q) \tag{75}$$

Boltzmann equation from density matrix evolution

$$f_p' - f_p = \sum_{k=1}^{\infty} (-1)^k \frac{1}{Z_1 Z_2 \dots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \dots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]

$|q\rangle \neq |p\rangle$

2. and 3.

$$\left\langle \frac{1}{\sqrt{(i_q - 1)!}} a_q^{i_q - 1} a_q \frac{1}{\sqrt{i_q!}} a_q^{\dagger i_q} \right\rangle \quad \to \quad \sqrt{i_q} \tag{76}$$

$$\sum_{i_q=1}^{\infty} i_q \mathring{f}_q^{\,i_q} = Z_q f_q \tag{77}$$

Boltzmann equation from density matrix evolution

$$f'_p - f_p = \sum_{k=1}^{\infty}(-1)^k \frac{1}{Z_1 Z_2 \ldots} \sum_{\{i\}} \sum_{\{n\}} \left(n_p - i_p\right) \mathring{f}_1^{\,i_1} \mathring{f}_2^{\,i_2} \ldots (\mathrm{i}\,T)^k_{in} \mathrm{i}\,T_{ni} \tag{72}$$

[Blažek, Maták '21b]

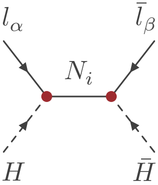
$\underline{|q\rangle = |p\rangle}$

$$n_p - i_p = +1 \quad \rightarrow \quad Z_p(1 + f_p) \tag{78}$$

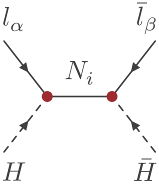
$$n_p - i_p = -1 \quad \rightarrow \quad -Z_p f_p \tag{79}$$

Unstable-particle intermediate states

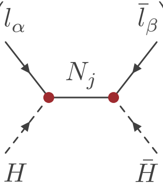
$$\left|T_{lH\to\bar{l}\bar{H}}\right|^2 \leftarrow \left|\sum_i \right.$$



$$\left.\right|^2 = \sum_{i,j}$$



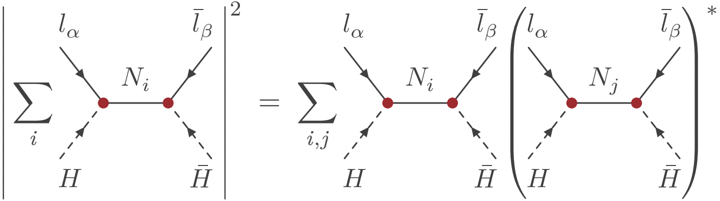
$$\left(\right.$$



$$\left.\right)^* \qquad (80)$$

Unstable-particle intermediate states

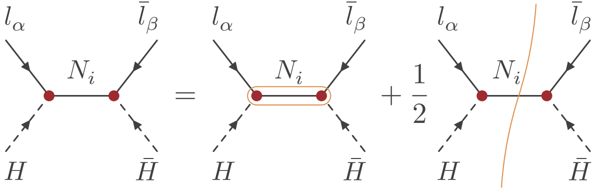
$$|T_{lH\to\bar{l}\bar{H}}|^2 \leftarrow$$



(80)

Terms with $i \neq j$

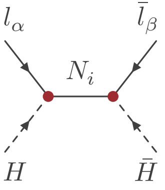
$$\frac{1}{s - M^2 + \mathrm{i}\epsilon} = \mathcal{P}\frac{1}{s - M^2} + \mathrm{i}\pi\delta(s - M^2) \tag{81}$$



(82)

Unstable-particle intermediate states

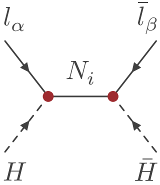
Terms with $i = j$



$$\left| \; \cdots \; \right|^2 \propto \left| \frac{1}{s - M^2 + \mathrm{i}\epsilon} \right|^2 \quad \to \quad \left| \frac{1}{s - M^2 + \mathrm{i}M\Gamma} \right|^2 \tag{83}$$

Unstable-particle intermediate states

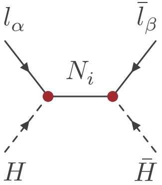
Terms with $i = j$



$$\left| \text{diagram} \right|^2 \propto \left| \frac{1}{s - M^2 + \mathrm{i}\epsilon} \right|^2 \quad \rightarrow \quad \frac{\epsilon}{M\Gamma} \left| \frac{1}{s - M^2 + \mathrm{i}\epsilon} \right|^2 \tag{83}$$

Unstable-particle intermediate states

Terms with $i = j$



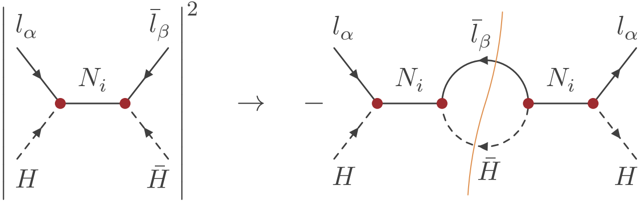
$$\left| \; \cdot \; \right|^2 \propto \left| \frac{1}{s - M^2 + \mathrm{i}\epsilon} \right|^2 \quad \to \quad \frac{\pi}{M_i \Gamma} \delta(s - M^2) \tag{83}$$

$$|T_{lH \to \bar{l}\bar{H}}|^2 \approx |T_{lH \to N}|^2 \times \mathrm{Br}(N \to \bar{l}\bar{H}) \tag{84}$$

double-counting the inverse decays

Unstable-particle intermediate states

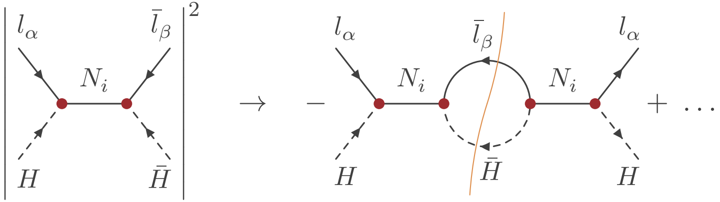
Terms with $i = j$ from holomorphic cuts



(85)

Unstable-particle intermediate states

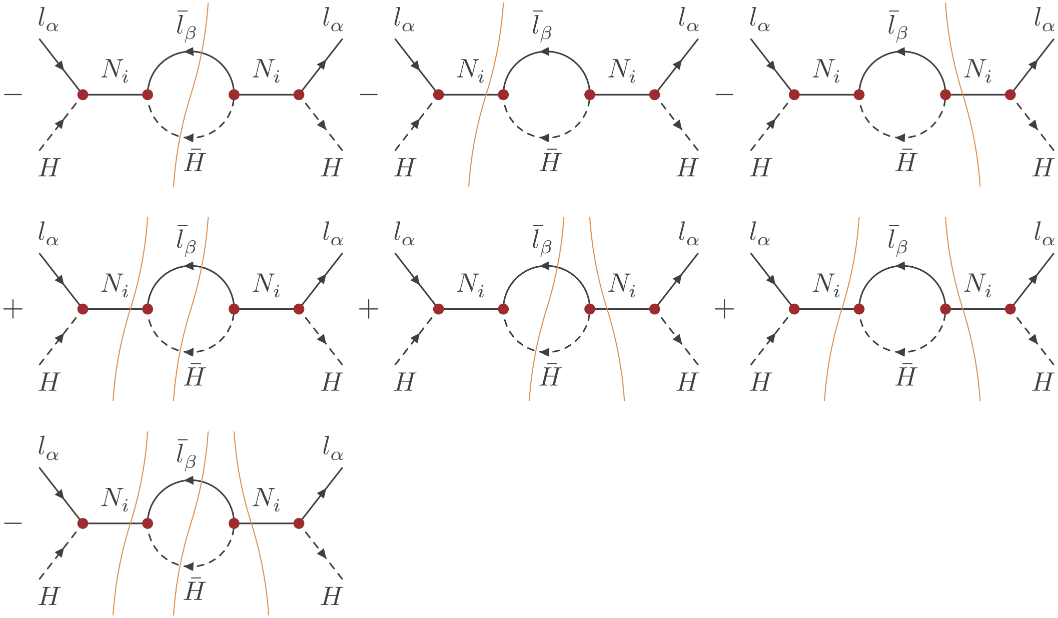
Terms with $i = j$ from holomorphic cuts



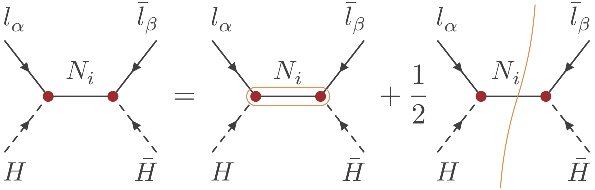
(85)

$$|T_{fi}|^2 = -\mathrm{i}\,T_{if}\,\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nf}\,\mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nk}\,\mathrm{i}\,T_{kf}\,\mathrm{i}\,T_{fi} + \ldots \tag{86}$$

Unstable-particle intermediate states

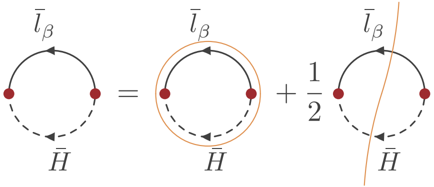


Unstable-particle intermediate states



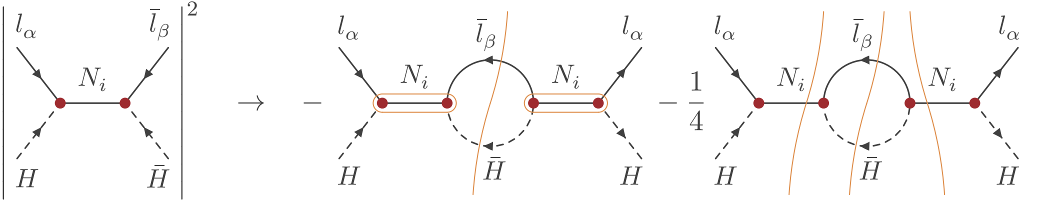
(87)

$$-\mathrm{i}\Sigma = -\mathrm{i}\,\mathrm{Re}\,\Sigma + \mathrm{Im}\,\Sigma \quad \rightarrow$$



(88)

Unstable-particle intermediate states

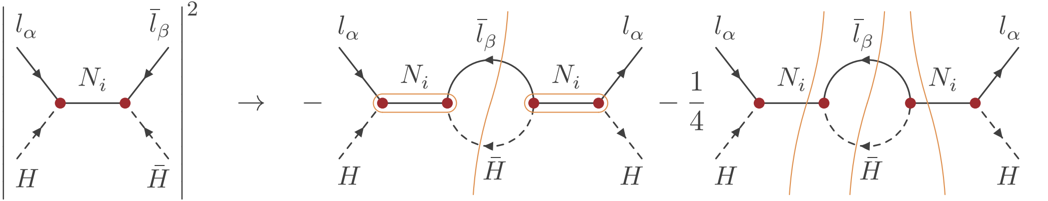


(89)

$$\propto \operatorname{Im}\Sigma(s) \times \frac{(s - M_i^2)^2 - \epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2} = \operatorname{Im}\Sigma(s) \times -\frac{\partial}{\partial s}\mathcal{P}\frac{1}{s - M_i^2}$$

[Hannessdottir, Mizera ’22; Maták ’23]

Unstable-particle intermediate states



$$\propto \ \mathrm{Im}\,\Sigma(s) \times \frac{(s - M_i^2)^2 - \epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2} = \mathrm{Im}\,\Sigma(s) \times -\frac{\partial}{\partial s}\mathcal{P}\frac{1}{s - M_i^2} \tag{89}$$

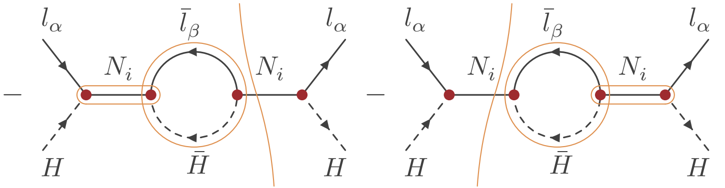
[Hannessdottir, Mizera ’22; Maták ’23]

No double-counting

$$\frac{\epsilon}{M_i \Gamma_i}\left\{\frac{1}{(s - M_i^2)^2 + \epsilon^2} - \frac{2\epsilon^2}{[(s - M_i^2)^2 + \epsilon^2]^2}\right\} \quad \to \quad 0 \tag{90}$$

[Maták ’22]

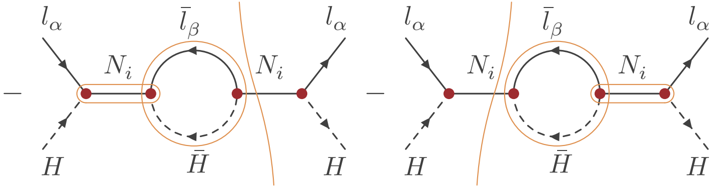
. . . the rest



(91)

$$\propto \; 2\,\mathrm{Re}\,\Sigma(s) \times \delta(s - M_i^2)\mathcal{P}\frac{1}{s - M_i^2} = \mathrm{Re}\,\Sigma(s) \times -\frac{\partial}{\partial s}\delta(s - M_i^2) \tag{92}$$

... the rest



(91)

$$\propto \; 2\,\mathrm{Re}\,\Sigma(s) \times \delta(s - M_i^2)\mathcal{P}\frac{1}{s - M_i^2} = \mathrm{Re}\,\Sigma(s) \times -\frac{\partial}{\partial s}\delta(s - M_i^2) \tag{92}$$

$$\to \quad \left\{ \mathrm{Re}\,\Sigma(s)\frac{\partial}{\partial s} + \frac{\partial\,\mathrm{Re}\,\Sigma(s)}{\partial s} \right\}\Bigg|_{s=M_i^2} \left|T_{l_\alpha H \to N_i}\right|^2 \tag{93}$$

[Maták '23]