

Nuclear structure factors for dark matter and CE ν NS

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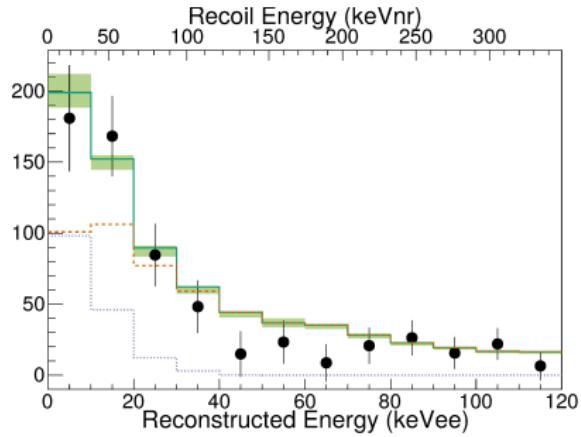
Seminar MPIK

Heidelberg, June 29, 2020

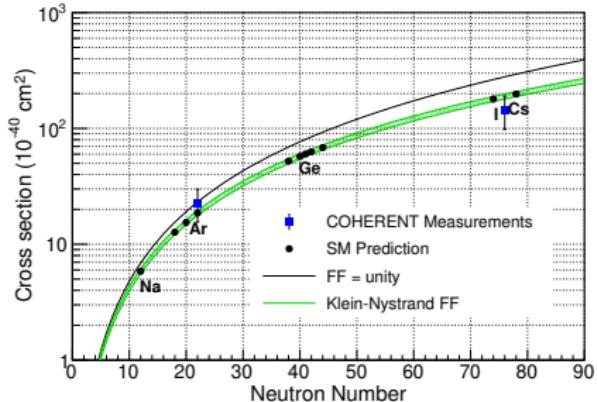
MH, Klos, Menéndez, Schwenk PRD 99 (2019) 055031

MH, Menéndez, Schwenk to appear

Recent observations of CE ν NS by COHERENT

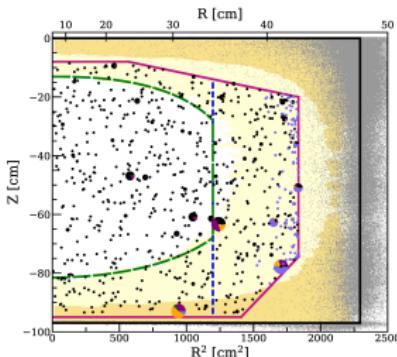
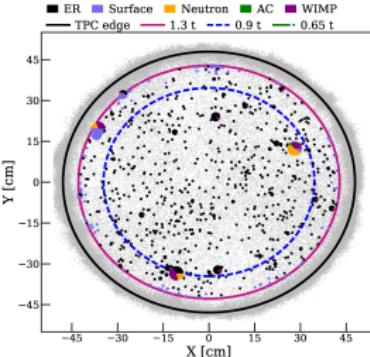
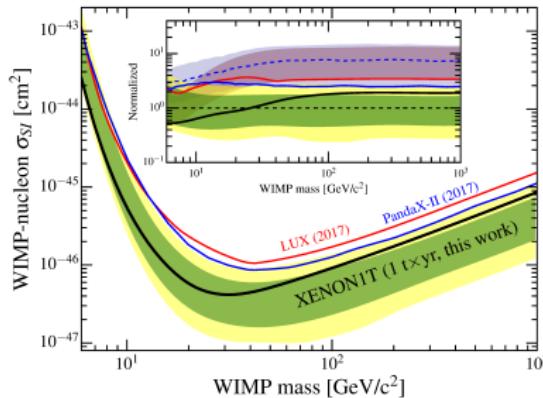


COHERENT 2020



- CE ν NS = coherent elastic neutrino–nucleus scattering Freedman 1974
- In SM: sensitive probe of the neutron distribution in nucleus
- Beyond SM: non-standard neutrino interactions?

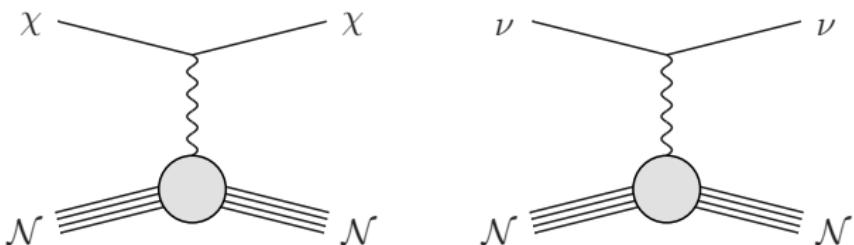
Direct detection of dark matter



XENON1T 2018

- Search for dark matter via nuclear recoils in large-scale liquid-noble-gas detectors
- Limits interpreted in terms of WIMP–nucleon cross sections

Weak Elastic Scattering with Nuclei



WIMP–nucleus scattering neutrino–nucleus scattering

kinematics	elastic, χ non-relativistic	elastic, ν relativistic
mediator	BSM	Z , BSM?
quantum numbers	unknown	$V - A$, others?
momentum transfer q	$\lesssim 200$ MeV	$\lesssim 50$ MeV

↪ need very similar nuclear responses (“**structure factors**”)

Weak Elastic Scattering with Nuclei: EFT approach

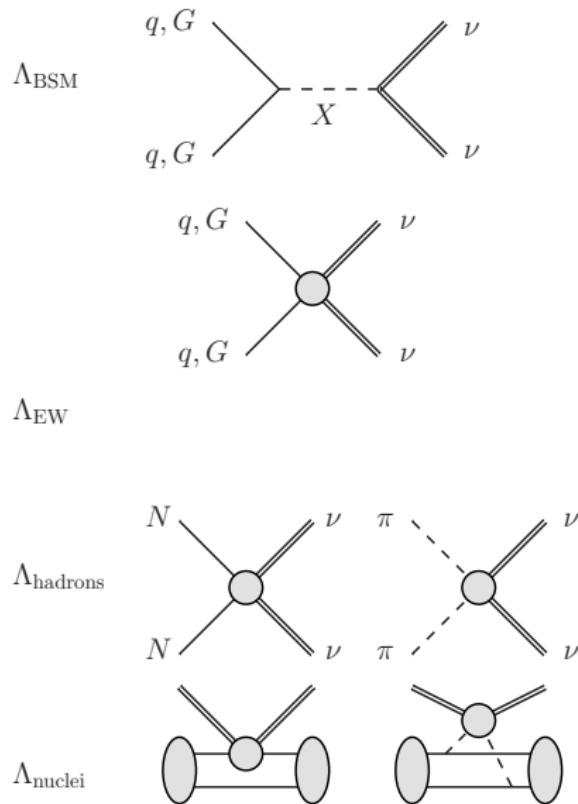
EFT approach to dark matter

Rate = BSM couplings \otimes hadronic matrix elements \otimes nuclear structure \otimes astrophysics

↪ most efficiently addressed in **effective field theory**

- Same approach to CE ν NS (with or without SM prediction for the Wilson coefficients)
- Will assume a **heavy mediator**, e.g. Z in SM
- Light BSM physics can be added to set of BSM operators, e.g. light Z'
 - ↪ requires same hadronic/nuclear input

Scales



➊ **BSM scale** Λ_{BSM} : \mathcal{L}_{BSM}

➋ **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda_{\text{BSM}}^i} \mathcal{O}_{i,k}$

➌ Integrate out **EW physics**
(start here if only SM)

➍ **Hadronic scale**: nucleons and pions
→ effective interaction Hamiltonian H_i

➎ **Nuclear scale**: $\langle \mathcal{N} | H_i | \mathcal{N} \rangle$
→ nuclear wave function

First step: effective operators

- **Vector and axial-vector operators**

$$\mathcal{L}_q^{\text{SM}} = -\frac{G_F}{\sqrt{2}} \sum_{q=u,d} \left(C_q^V \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \bar{q} \gamma_\mu q + C_q^A \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \bar{q} \gamma_\mu \gamma_5 q \right)$$

↪ can add more operators in BSM case

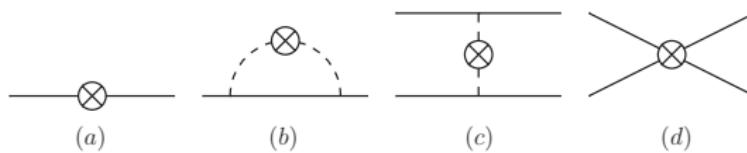
- In SM:

$$C_u^V = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad C_d^V = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \quad C_u^A = -\frac{1}{2} \quad C_d^A = \frac{1}{2}$$

- Related to typical CE ν NS notation by

$$C_q^V - C_q^V|_{\text{SM}} = \epsilon_{ee}^{qV} = \epsilon_{ee}^{qL} + \epsilon_{ee}^{qR} \quad C_q^A - C_q^A|_{\text{SM}} = -\epsilon_{ee}^{qA} = \epsilon_{ee}^{qR} - \epsilon_{ee}^{qL}$$

Second step: hadronic matrix elements



- **Single-nucleon level:** vector and axial-vector form factors
→ charge radii, magnetic moments, ...
- **Beyond single-nucleon level:** pion-exchange diagrams \leftrightarrow two-body currents
→ can all be addressed systematically in **chiral effective field theory**

Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
 - Based on **chiral symmetry** of QCD
 - **Power counting**
 - **Low-energy constants**
 - Hierarchy of multi-nucleon forces
 - Consistency of NN and $3N$
- modern theory of nuclear forces
- Long-range part related to **pion–nucleon scattering**

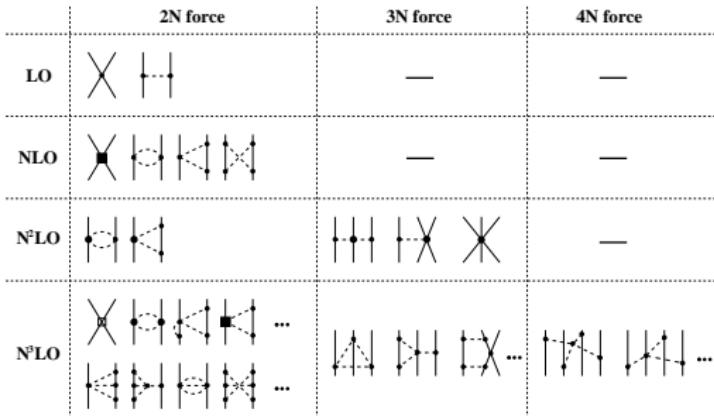
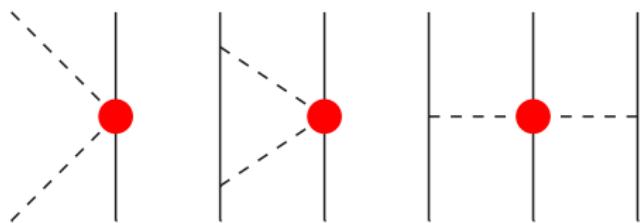
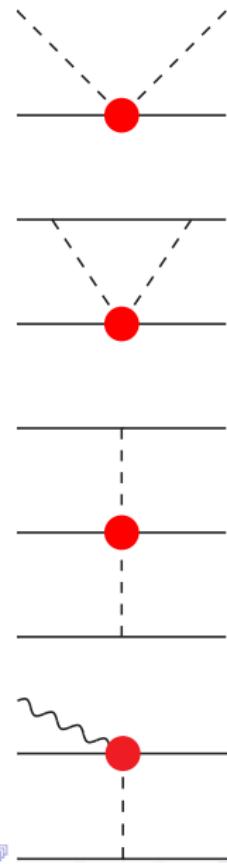


Figure taken from 1011.1343



Chiral EFT: currents

- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial-vector current**
- Two-body effects well established in β decay, magnetic moments, ...
- Formalism can be applied to other than vector and axial-vector currents
↪ sizable two-body effect for a **scalar current**



Third step: nuclear structure factors

- **Nuclear matrix element** of the single-nucleon and few-nucleon amplitudes
- Ideally: based on nuclear interactions from chiral EFT
 - “ab-initio” calculation Coupled-cluster calculation for ^{40}Ar , Payne et al. 2019
- Not yet available for heavier nuclei, challenge to get charge radii right
- Therefore: here us **nuclear shell model** with phenomenological interactions
- We can do better than simple parameterizations in terms of proton/neutron radii!

CE_νNS cross section

$$\begin{aligned} \frac{d\sigma_{\nu N}}{dT} = & \frac{G_F^2 m_A}{2\pi} \left[\left(2 - \frac{m_A T}{E_\nu^2} \right) \left| \left(g_V^0 - \dot{g}_{V,1}^0 q^2 - \frac{g_V^0 + 2g_{V,2}^0}{8m_N^2} q^2 \right) \mathcal{F}_+^M(q^2) + \left(g_V^1 - \dot{g}_{V,1}^1 q^2 - \frac{g_V^1 + 2g_{V,2}^1}{8m_N^2} q^2 \right) \mathcal{F}_-^M(q^2) \right. \right. \\ & + \frac{g_V^0 + 2g_{V,2}^0}{4m_N^2} q^2 \mathcal{F}_+^{\Phi''}(q^2) + \frac{g_V^1 + 2g_{V,2}^1}{4m_N^2} q^2 \mathcal{F}_-^{\Phi''}(q^2) \Big|^2 \\ & \left. \left. + \left(2 + \frac{m_A T}{E_\nu^2} \right) \frac{2\pi}{2J+1} \left((g_A^0)^2 S_{00}^T(q^2) + g_A^0 g_A^1 S_{01}^T(q^2) + (g_A^1)^2 S_{11}^T(q^2) \right) \right] \right] \end{aligned}$$

Notation:

- Momentum transfer $q^2 = 2m_A T$ with target mass m_A , $T \in [0, 2E_\nu^2/(m_A + 2E_\nu)]$
- $g_V^0, \dot{g}_V^0, g_A^0$ etc.: combination of Wilson coefficients and hadronic matrix elements
↪ “**matching relations**”
- $\mathcal{F}_\pm^M(q^2)$: isoscalar/isovector charge distribution
- $S_{ij}^T(q^2)$: only transverse part T contributes in spin-dependent response
- Interference term neglected: $T/E_\nu \lesssim 2E_\nu/m_A$ tiny

CE_νNS cross section

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• Subleading contributions

- Radius and relativistic corrections included
- Quasi-coherent $\mathcal{F}_I^{\Phi''}(q^2)$ spin-orbit response Serot 1978
- Two-body corrections kinematically suppressed for vector response, but relevant for axial vector

Charge radii and neutron skin

	^{23}Na	^{40}Ar	^{74}Ge	^{132}Xe	^{127}I	^{133}Cs
$\sqrt{\langle r_{\text{ch}}^2 \rangle}$ [fm] (th)	3.00	3.43	4.08	4.77	4.73	4.79
(exp)	2.9936(21)	3.427(3)	4.0742(12)	4.7808(49)	4.7500(81)	4.8041(46)
$\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ [fm] shell-model interaction	0.04 SDPF.SM	0.11 RG	0.17 GCN	0.28 USDB	0.26 GCN	0.27 GCN

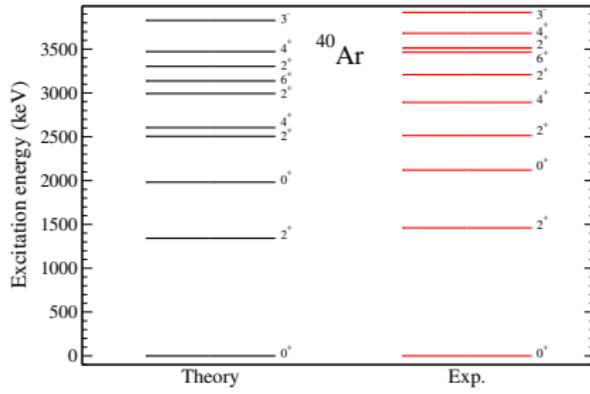
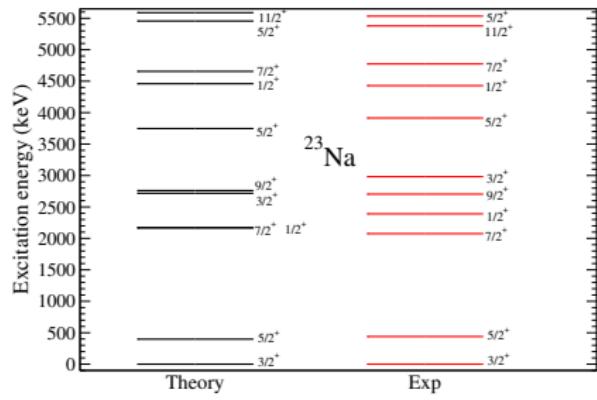
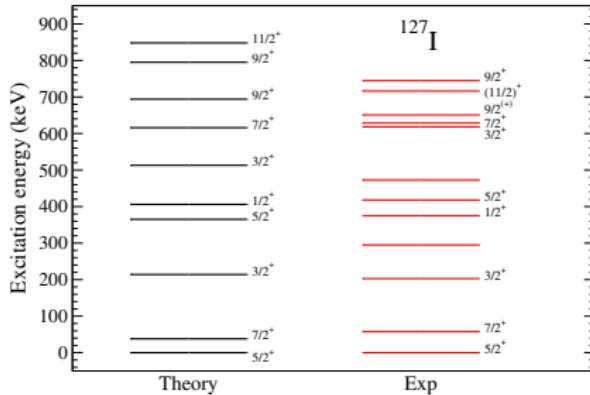
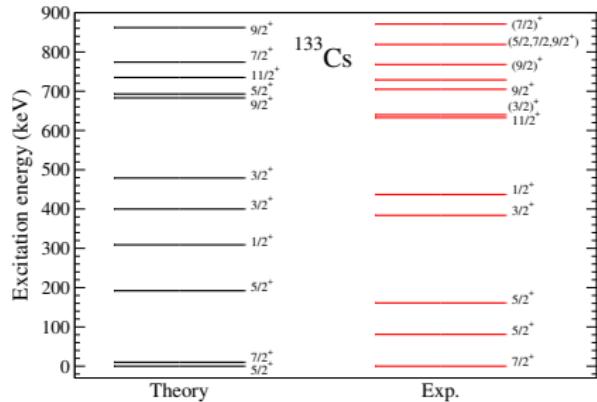
- Excellent agreement for charge radii

$$\langle r_{\text{ch}}^2 \rangle = \langle r_p^2 \rangle + \langle r_{E,p}^2 \rangle + \frac{N}{Z} \langle r_{E,n}^2 \rangle + \langle r_{\text{rel}}^2 \rangle + \langle r_{\text{spin-orbit}}^2 \rangle$$

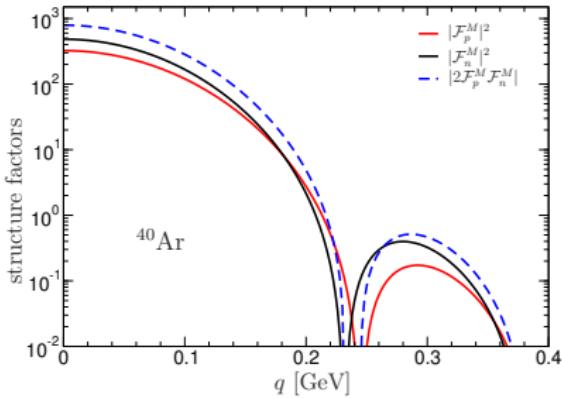
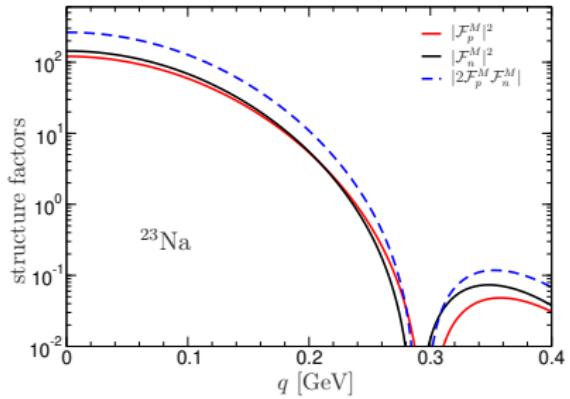
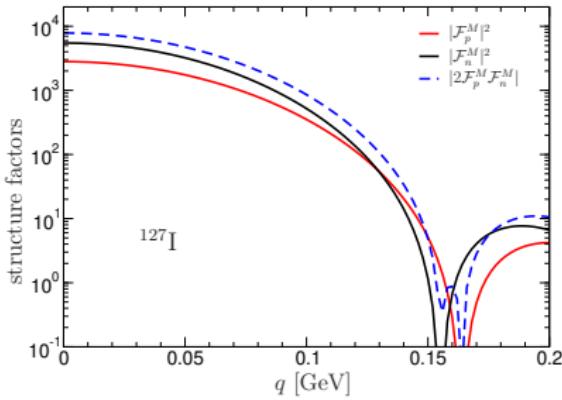
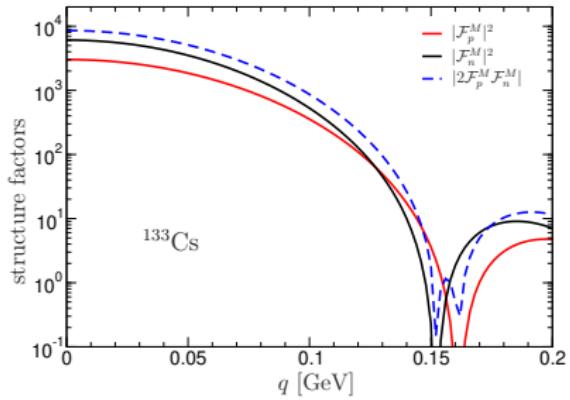
- But: charge radii used to constrain parameters
- Point-neutron radii more uncertain, e.g. from coupled-cluster Payne et al. 2019
- $\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} |_{^{40}\text{Ar}} = 0.035 \dots 0.09 \text{ fm}$
- Related to structure factors by

$$\langle r_p^2 \rangle = -\frac{3}{Z} \frac{d}{dq^2} (\mathcal{F}_+^M(q^2) + \mathcal{F}_-^M(q^2))|_{q^2=0} \quad \langle r_n^2 \rangle = -\frac{3}{N} \frac{d}{dq^2} (\mathcal{F}_+^M(q^2) - \mathcal{F}_-^M(q^2))|_{q^2=0}$$

Spectra



Structure factors

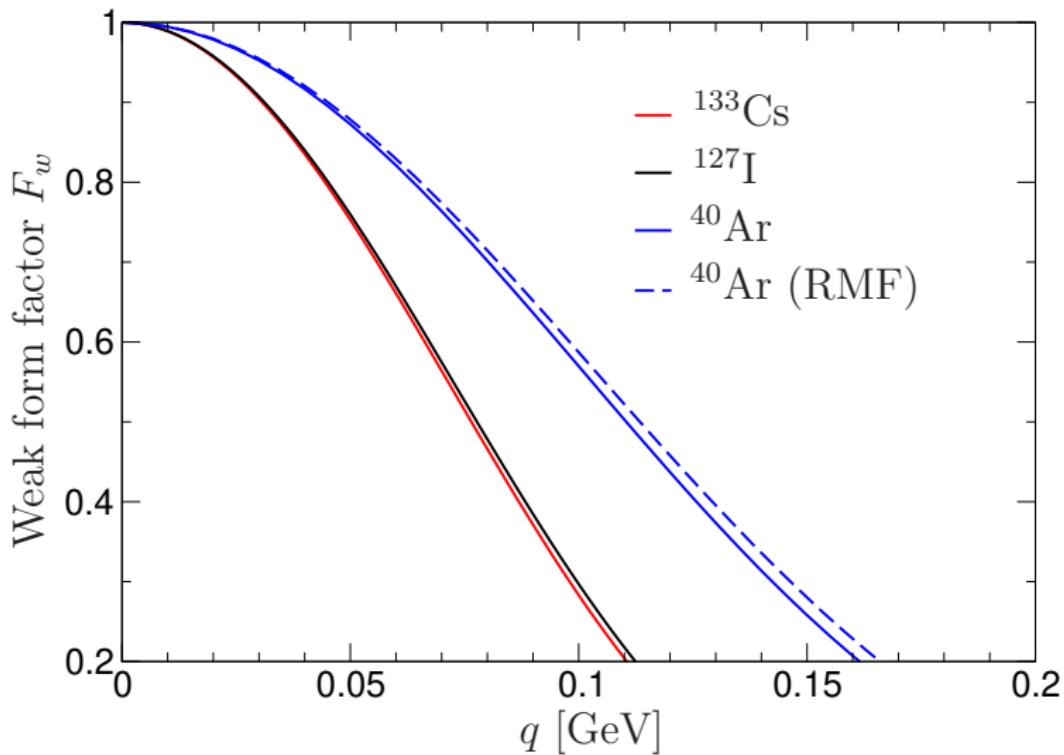


The weak form factor

$$\frac{d\sigma_{\nu N}}{dT} = \frac{G_F^2 m_A}{4\pi} \left(1 - \frac{m_A T}{2E_\nu^2}\right) Q_w^2 |F_w(q^2)|^2 \quad Q_w = N - (1 - 4 \sin^2 \theta_W) Z$$

- In the SM define so-called **weak form factor** $F_w(q^2)$
- Q_w does not actually factorize
- $F_w(q^2)$ depends on \mathcal{F}_\pm^M , $\mathcal{F}_+^{\Phi''}(q^2)$, with coefficients determined by the Wilson coefficients and hadronic matrix elements
- Once Wilson coefficients are changed away from the SM, **the form factor will change!**
- EFT decomposition allows one to keep track of these changes

The weak form factor



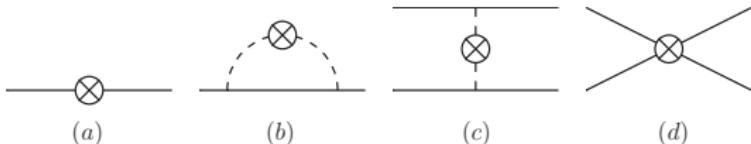
A question to experiment

- Current list of isotopes MH, Menéndez, Schwenk to appear:

- Cesium: ^{133}Cs
- Iodine: ^{127}I
- Argon: ^{40}Ar
- Sodium: ^{23}Na
- Germanium: $^{70,72,73,74,76}\text{Ge}$
- Xenon: $^{128,129,130,131,132,134,136}\text{Xe}$

- Should we consider anything else?

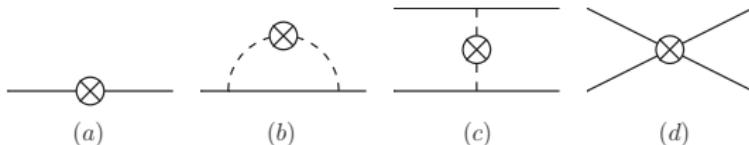
Cross section and nuclear structure factors



$$\begin{aligned}\frac{d\sigma_{XN}}{dq^2} = & \frac{1}{4\pi v^2} \left| \sum_{l=\pm} \left(c_l^M - \frac{q^2}{m_N^2} \dot{c}_l^M \right) \mathcal{F}_l^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2} \sum_{l=\pm} c_l^{\Phi''} \mathcal{F}_l^{\Phi''}(q^2) \right|^2 \\ & + \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{l=\pm} \xi_i(q, v_T^\perp) c_l^{M,i} \mathcal{F}_l^M(q^2) \right|^2 \\ & + \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right)\end{aligned}$$

- Decomposition into **nuclear structure factors** \mathcal{F} , S_{ij} and coefficients c , a
- Three classes of contributions:
 - (Sub-) Leading 1b responses** (a): $c_l^M \mathcal{F}_l^M(q^2)$, $c_l^{\Phi''} \mathcal{F}_l^{\Phi''}(q^2)$, $|a_\pm|^2 S_{ij}(q^2)$
 - Radius corrections** (b): $\dot{c}_l^M \mathcal{F}_l^M(q^2)$
 - Two-body currents** (c), (d): $c_\pi \mathcal{F}_\pi(q^2)$, $c_b \mathcal{F}_b(q^2)$
- (a)+(b) essentially **nucleon form factors**, but (c)+(d) genuinely new effects

Cross section and nuclear structure factors



$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{1}{4\pi v^2} \left| \sum_{I=\pm} \left(c_I^M - \frac{q^2}{m_N^2} \dot{c}_I^M \right) \mathcal{F}_I^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_B \mathcal{F}_B(q^2) + \frac{q^2}{2m_N^2} \sum_{I=\pm} c_I^{\Phi''} \mathcal{F}_I^{\Phi''}(q^2) \right|^2 \\ + \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{I=\pm} \xi_i(q, v_T^\perp) c_I^{M,i} \mathcal{F}_I^M(q^2) \right|^2 \\ + \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right)$$

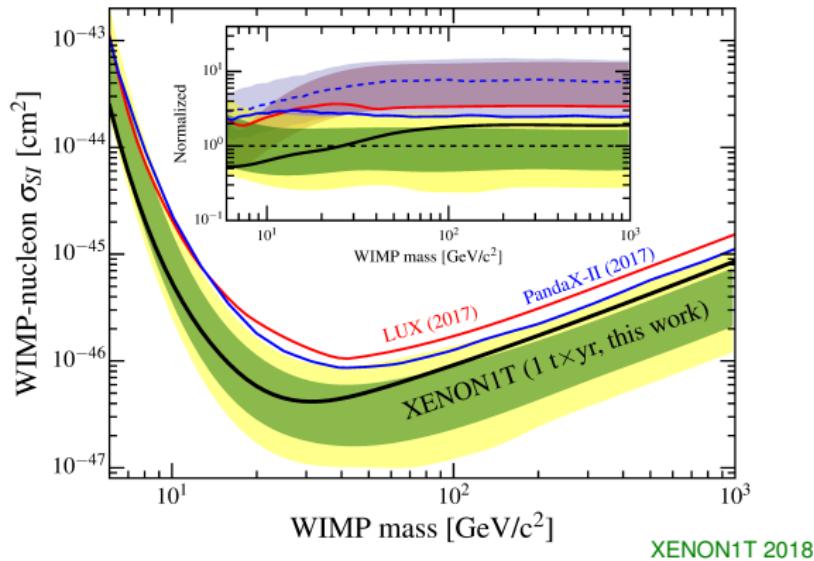
- Nuclear structure interpretation:

- $\mathcal{F}_I^M(q^2)$: $\mathbb{1} \Rightarrow$ charge distribution (coherent), $\mathcal{F}_\pm^M(0) = Z \pm N$
- $\mathcal{F}_I^{\Phi''}(q^2)$: $i \mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \Rightarrow$ spin-orbit interaction (quasi-coherent)
- $S_{ij}(q^2)$: $\mathbf{S}_x \cdot \mathbf{S}_N, \mathbf{S}_x \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q} \Rightarrow$ spin average (not coherent),
 $S_{00}(0) \pm S_{01}(0) + S_{11}(0) = \frac{(2J+1)(J+1)}{4\pi J} |\langle \mathbf{S}_{p/n} \rangle|^2$

- Coefficients: convolution of **Wilson coefficients** and **nucleon matrix elements**

$$c_\pm^M = \frac{\zeta}{2} (f_p \pm f_n) + \dots \quad f_N = \frac{m_N}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^N + \dots \quad \langle N | m_q \bar{q} q | N \rangle = m_N f_q^N$$

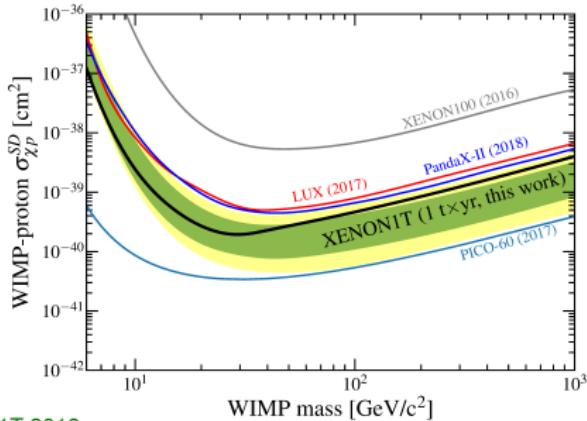
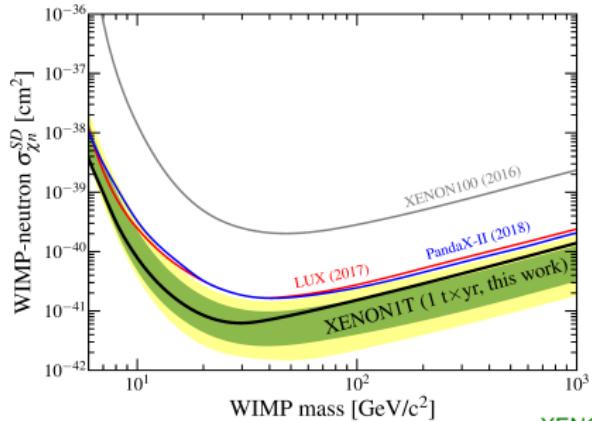
Case 1: spin-independent scattering



- All $c = 0$, $a = 0$ except for c_+^M : **spin-independent scattering**

$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{\sigma_{\chi N}^{SI}}{4\mu_N^2 v^2} |\mathcal{F}_+^M(q^2)|^2 \quad \sigma_{\chi N}^{SI} = \frac{\mu_N^2}{\pi} |c_+^M|^2 \quad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

Case 2: spin-dependent scattering

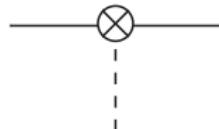


- All $c = 0, a_+ = \pm a_-$: **spin-dependent scattering**

$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{\sigma_{\chi N}^{\text{SD}}}{3\mu_N^2 v^2} \frac{\pi}{2J+1} S_N(q^2) \quad \sigma_{\chi N}^{\text{SD}} = \frac{3\mu_N^2}{\pi} |a_+|^2$$

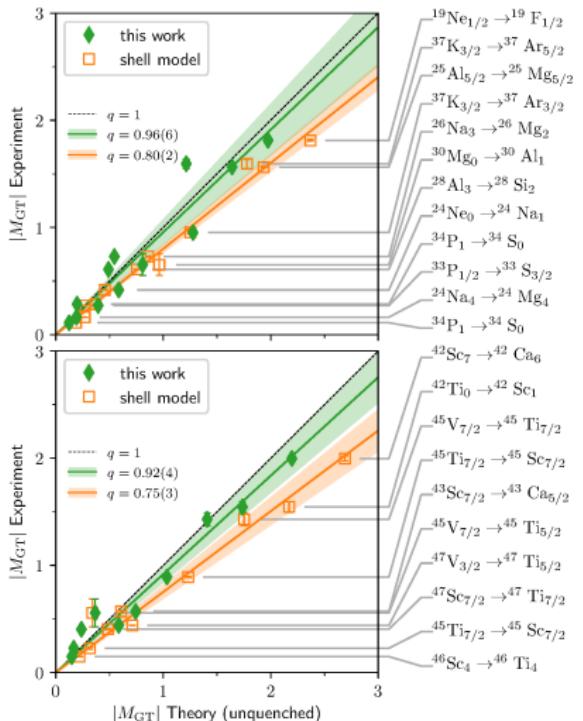
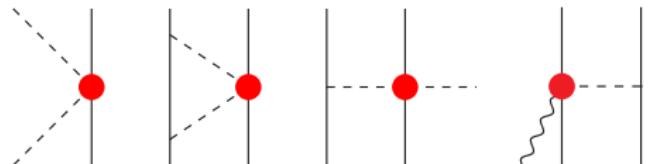
- Xe sensitive to proton spin due to **two-body currents**

Klos, Menéndez, Gazit, Schwenk 2013



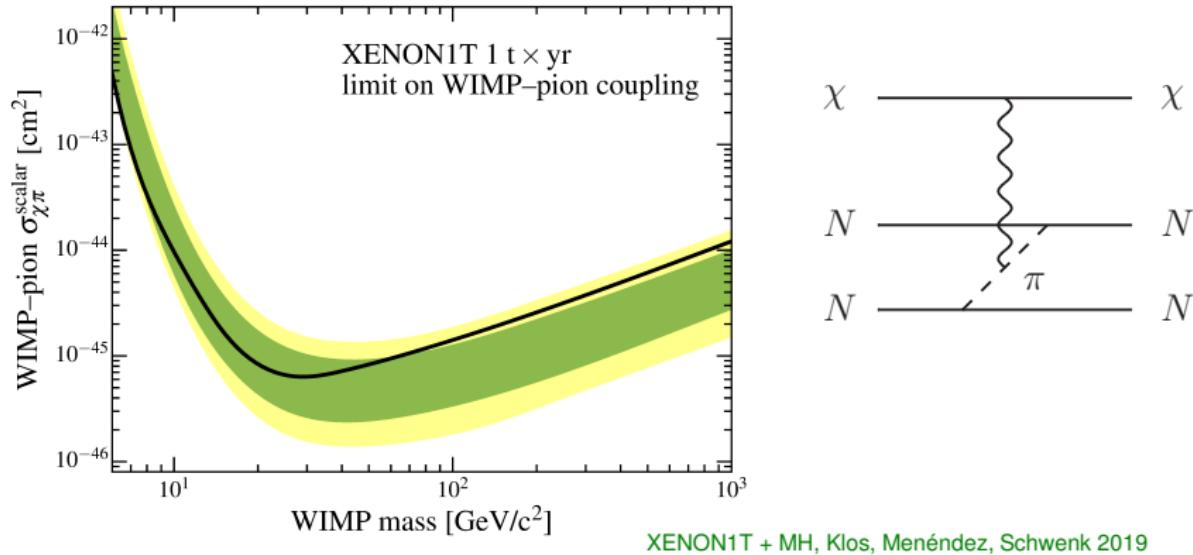
Case 2: spin-dependent scattering

- Recent EFT developments:
 - Improved understanding of **two-body currents** in light and medium-size nuclei from ab-initio methods
 - Improved determination of **low-energy constants** MH, Ruiz de Elvira, Kubis, Meißner 2015
- Permits improved SD responses both for dark matter and CE ν NS



Gysbers et al. 2019

Case 3: WIMP–pion scattering



- Only c_π nonzero: **WIMP–pion scattering**

$$\frac{d\sigma_{\chi N}}{dq^2} = \frac{\sigma_{\chi\pi}^{\text{scalar}}}{\mu_\pi^2 v^2} |\mathcal{F}_\pi(q^2)|^2 \quad \sigma_{\chi\pi}^{\text{scalar}} = \frac{\mu_\pi^2}{4\pi} |c_\pi|^2 \quad \mu_\pi = \frac{m_\chi M_\pi}{m_\chi + M_\pi}$$

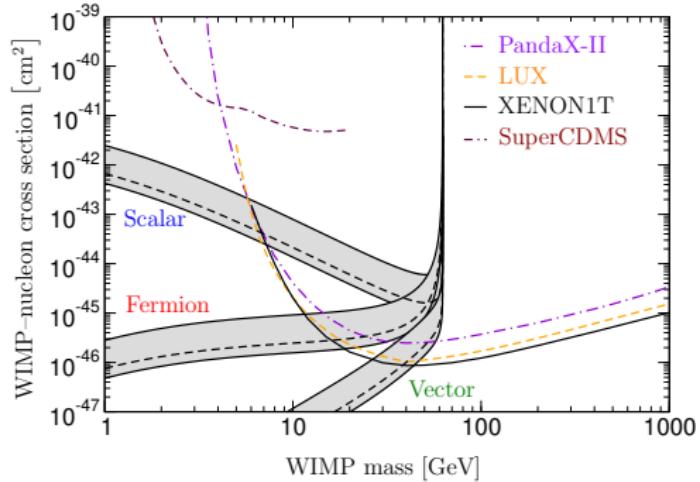
- Expression in terms of cross section depends on underlying operator, here for a scalar $\bar{\chi}\chi\bar{q}q$

Higgs Portal dark matter

- **Higgs Portal:** WIMP interacts with SM via the Higgs

- **Scalar:** $H^\dagger H S^2$
- **Vector:** $H^\dagger H V_\mu V^\mu$
- **Fermion:** $H^\dagger H \bar{f} f$

- If $m_h > 2m_\chi$, should happen at the LHC
↪ limits on **invisible Higgs decays**

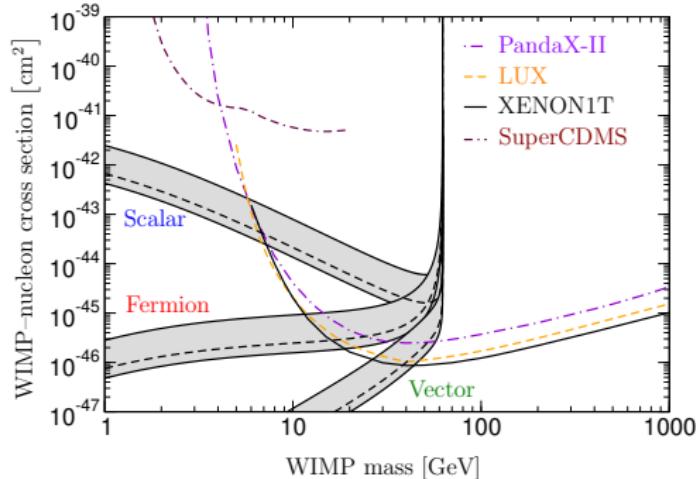


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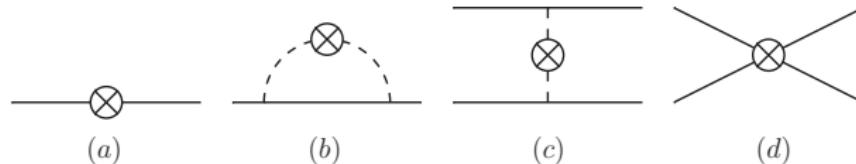


- Translation requires input for **Higgs–nucleon coupling**

$$f_N = \sum_{q=u,d,s,c,b,t} f_q^N = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q^N + \mathcal{O}(\alpha_s) \quad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle$$

- Issues: input for $f_N = 0.260 \dots 0.629$ outdated, 2b currents missing

Higgs–nucleon coupling



- **One-body contribution**

$$f_N^{1b} = 0.307(9)_{ud}(15)_s(5)_{\text{pert}} = 0.307(18)$$

- Limits on WIMP–nucleon cross section subsume **2b effects**

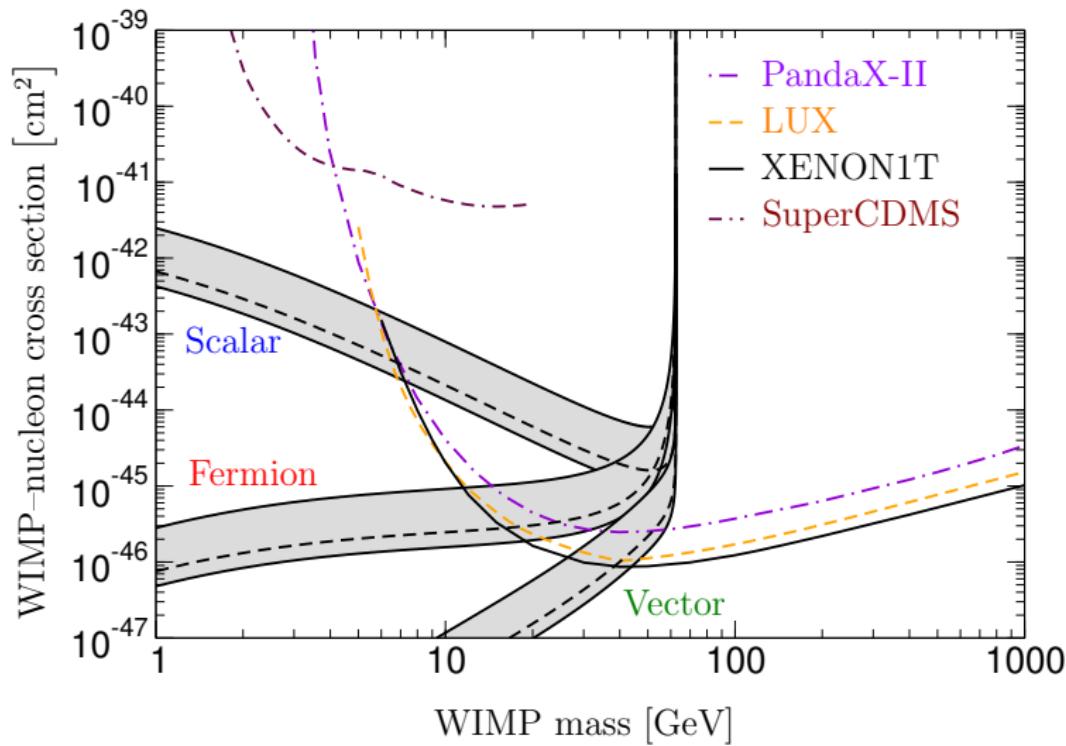
→ have to be included for meaningful comparison

- **Two-body contribution**

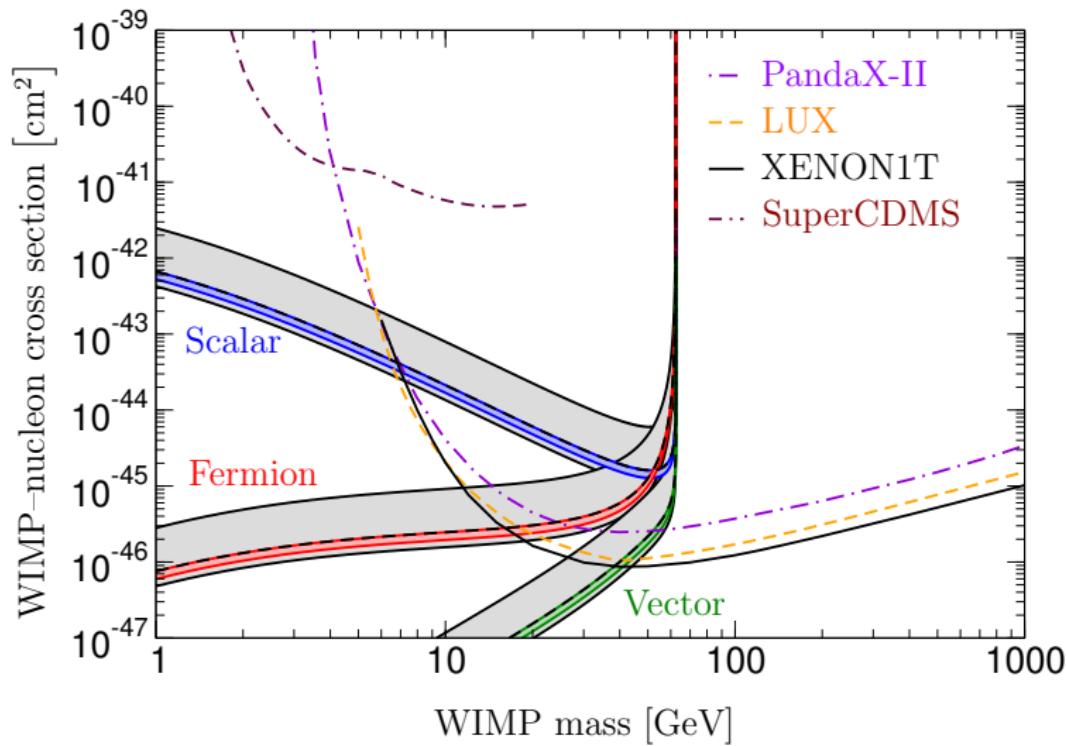
- Need s and θ_μ^μ currents
- Treatment of θ_μ^μ tricky: several ill-defined terms combine to $\langle \Psi | T + V_{NN} | \Psi \rangle = E_b$
- A cancellation makes the final result anomalously small

$$f_N^{2b} = [-3.2(0.2)_A(2.1)_{\text{ChEFT}} + 5.0(0.4)_A] \times 10^{-3} = 1.8(2.1) \times 10^{-3}$$

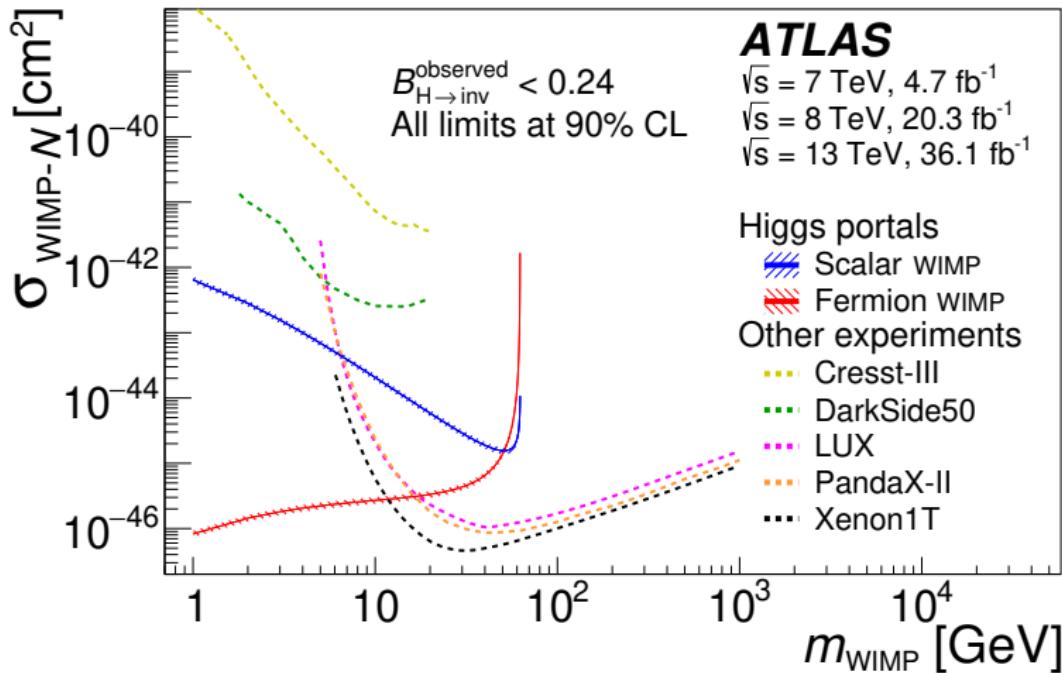
Improved limits for Higgs Portal dark matter



Improved limits for Higgs Portal dark matter



Improved limits for Higgs Portal dark matter



ATLAS 2019

ChiralEFT4DM: a PYTHON package for dark matter

- ChiralEFT4DM: PYTHON package for dark matter structure factors
<https://theorie.ikp.physik.tu-darmstadt.de/strongint/ChiralEFT4DM.html>
- Includes:
 - (Quasi-) Coherent structure factors for F, Si, Ar, Ge, Xe
 - Nucleon matrix elements and matching relations for spin-1/2 and spin-0 WIMP
 - 1b and 2b responses up to third chiral order
 - $S, P, V, A, T, \theta_\mu^\mu$, and spin-2 effective operators that can lead to a coherent response
 - Convolution with Standard Halo Model

Conclusions

- **EFT approach** to dark matter and CE ν NS
- Separation into
 - **Wilson coefficients**
 - **Hadronic matrix elements**
 - **Nuclear structure factors**
- Two-body corrections kinematically suppressed for leading, coherent response
- Axial-vector part similar, but not identical to spin-dependent dark matter
- Cannot just use the weak form factor for when constraining BSM (weak charge does not factorize)
- Can systematically include non-standard interactions in **chiral EFT**

Chiral counting for neutrino–nucleus scattering

	Nucleon	<i>V</i>		<i>A</i>			Nucleon	<i>S</i>	<i>P</i>
WIMP		<i>t</i>	x	<i>t</i>	x	WIMP			
<i>V</i>	1b	0	1 + 2	2	0 + 2	<i>S</i>	1b	2	1
	2b	4	2 + 2	2	4 + 2		2b	3	5
	2b NLO	—	—	5	3 + 2		2b NLO	—	4
<i>A</i>	1b	0 + 2	1	2 + 2	0	<i>P</i>	1b	2 + 2	1 + 2
	2b	4 + 2	2	2 + 2	4		2b	3 + 2	5 + 2
	2b NLO	—	—	5 + 2	3		2b NLO	—	4 + 2

Chiral counting for neutrino–nucleus scattering

Nucleon		V		A		Nucleon		S	P
ν	t	x	t	x	ν	$1b$	2	1	
V	1b	0	1	2	0	S	2b	3	5
	2b	4	2	2	4		2b NLO	—	4
	2b NLO	—	—	5	3		1b	2	1
A	1b	0	1	2	0	P	2b	3	5
	2b	4	2	2	4		2b NLO	—	4
	2b NLO	—	—	5	3		1b	2	1

Chiral counting for neutrino–nucleus scattering

Nucleon		V		A		Nucleon		S	P
ν	t	x	t	x	ν	$1b$	2	1	
V	1b	0	1	2	0	S	2b	3	5
	2b	4	2	2	4		2b NLO	—	4
	2b NLO	—	—	5	3		1b	2	1
A	1b	0	1	2	0	P	2b	3	5
	2b	4	2	2	4		2b NLO	—	4
	2b NLO	—	—	5	3		1b	2	1

- **Standard interactions:** AA as for dark matter, vector 2b
- **Non-standard interactions:** potentially large corrections for scalar 2b currents

Coherence effects

- Six distinct nuclear responses

Fitzpatrick et al. 2012, Anand et al. 2013

- $M \leftrightarrow O_1 \leftrightarrow SI$
- $\Sigma', \Sigma'' \leftrightarrow O_4, O_6 \leftrightarrow SD$
- $\Phi'' \leftrightarrow O_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
- $\Delta, \tilde{\Phi}'$: not coherent

- **Quasi-coherence** of Φ''

- Spin-orbit splitting
- Coherence until mid-shell
- About 20 coherent nucleons in Xe
- Interference $M-\Phi'' \leftrightarrow O_1-O_3$

- Coherent 2b currents:

- Scalar $\propto N + Z$
- Vector $\propto N - Z$

↪ concentrate on scalar case

