

erc

FNSNF

# Cosmological implications of gravitational waves



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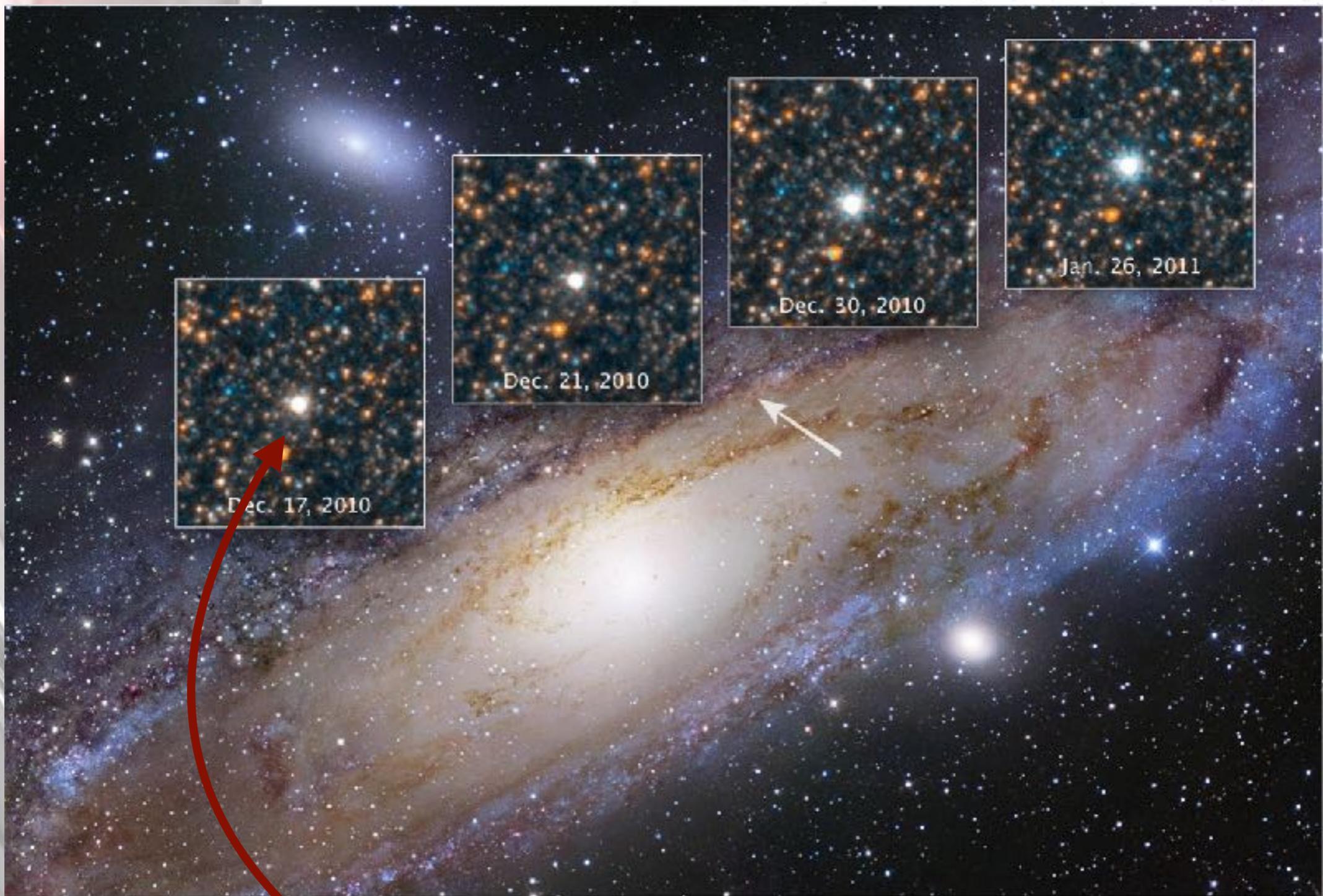
# The Universe until 1923



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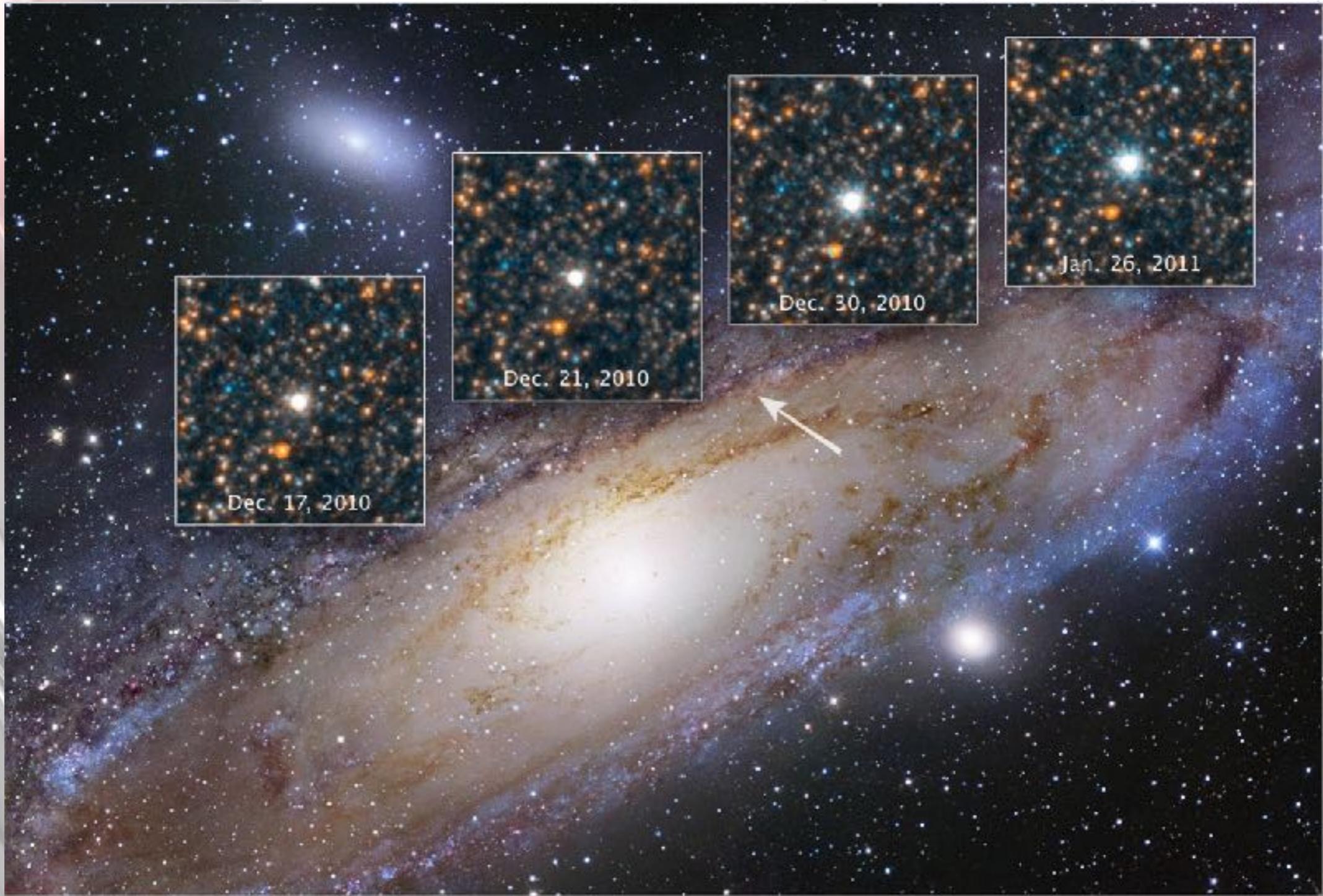


# The Universe until 1923



Cepheids

# The Universe until 1923



2.2 Mio light-year ( $\sim 10^{12}$  km)

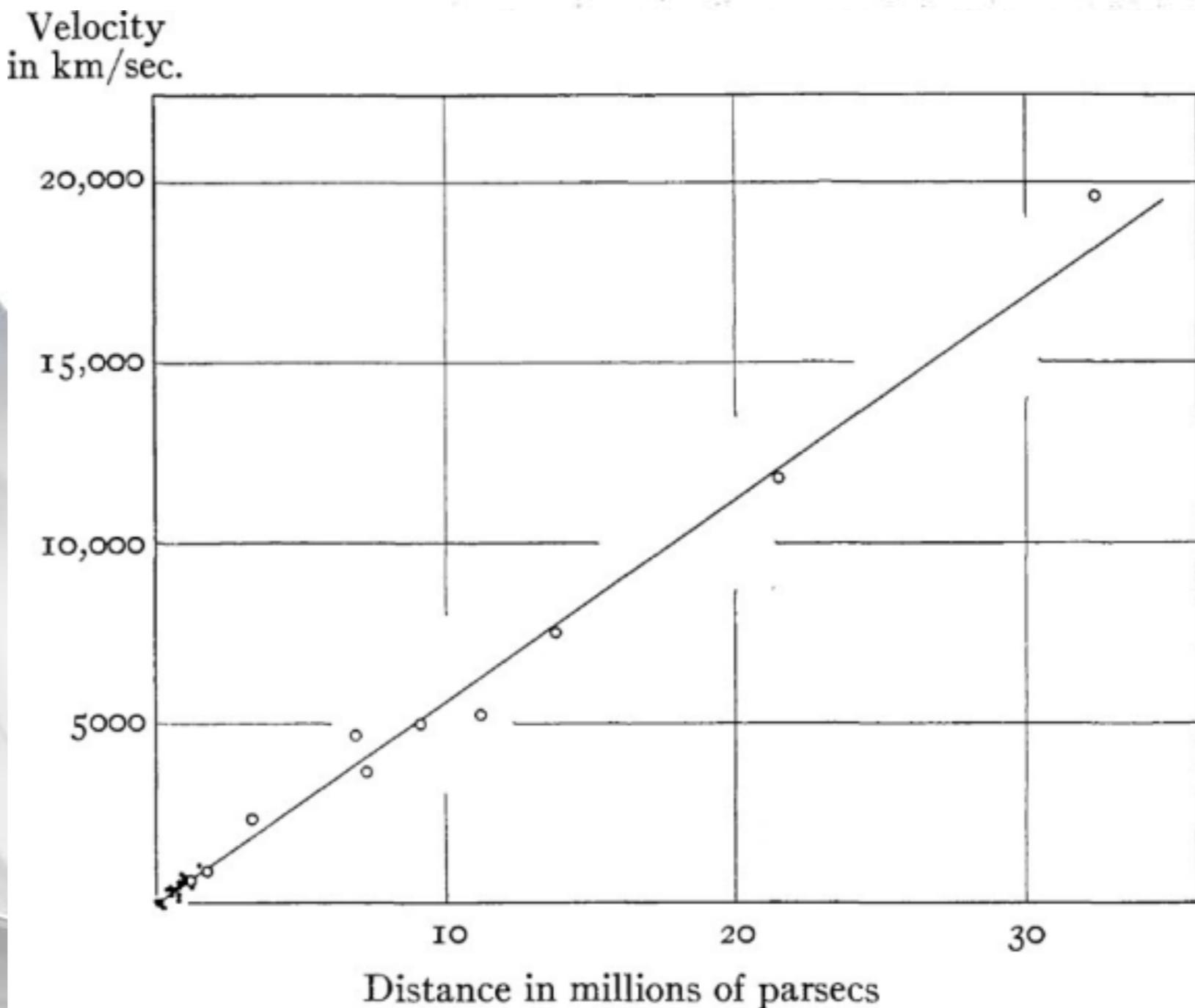
200 x larger than the diameter of MW

# The Universe until 1931



Hubble's law

$$v = H_0 D$$

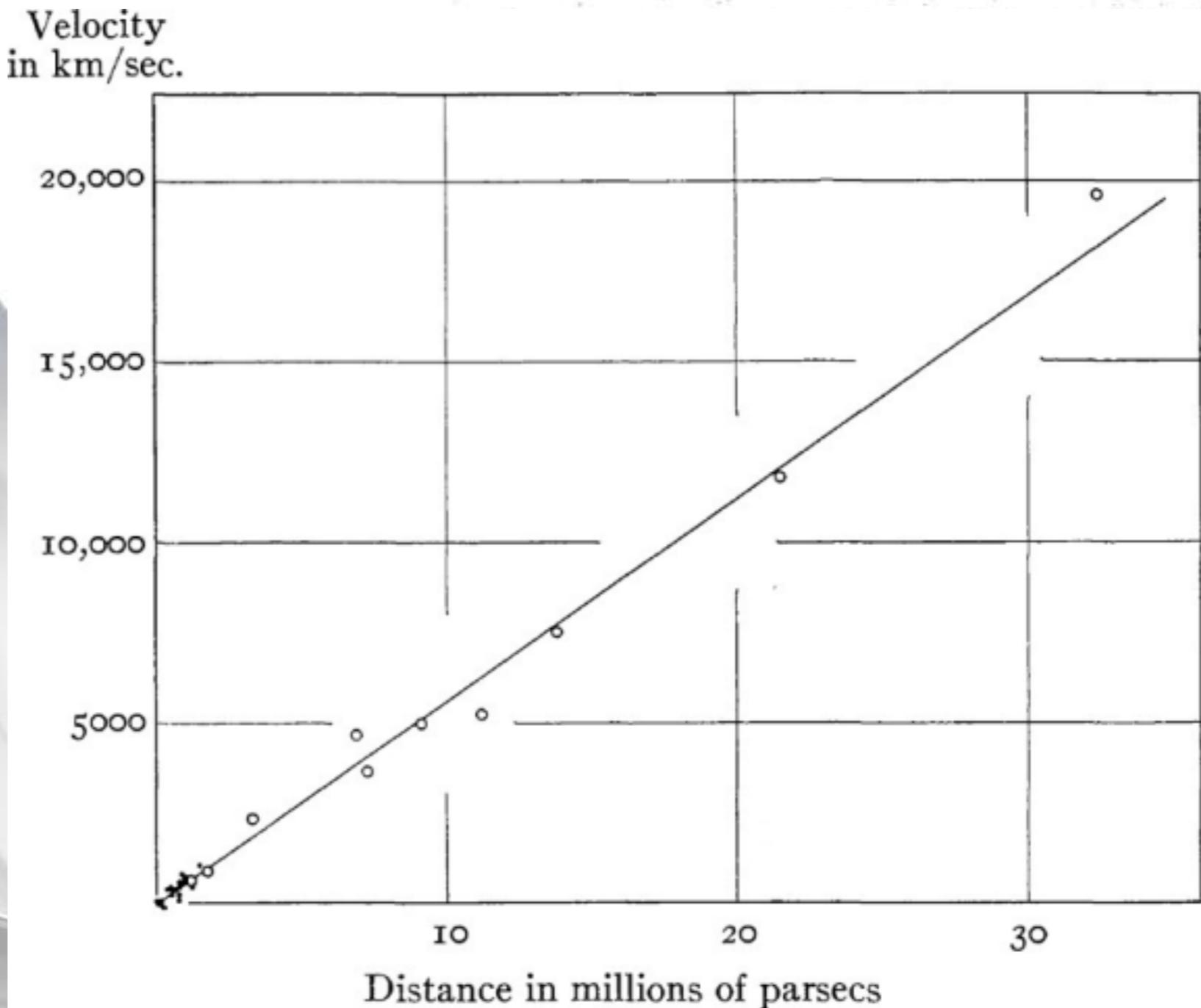


# The Universe until 1931



Hubble's law

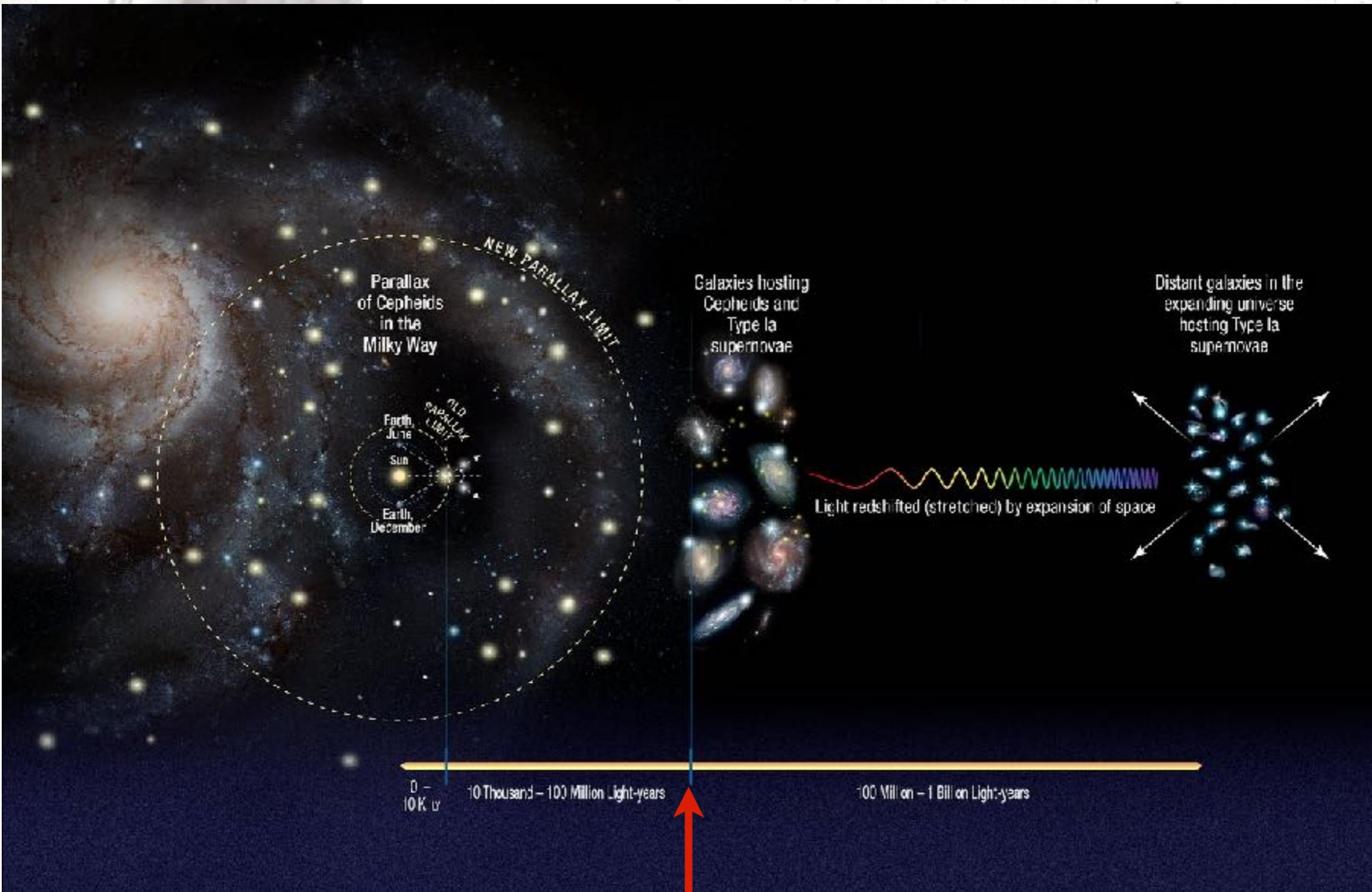
$$v = H_0 D$$



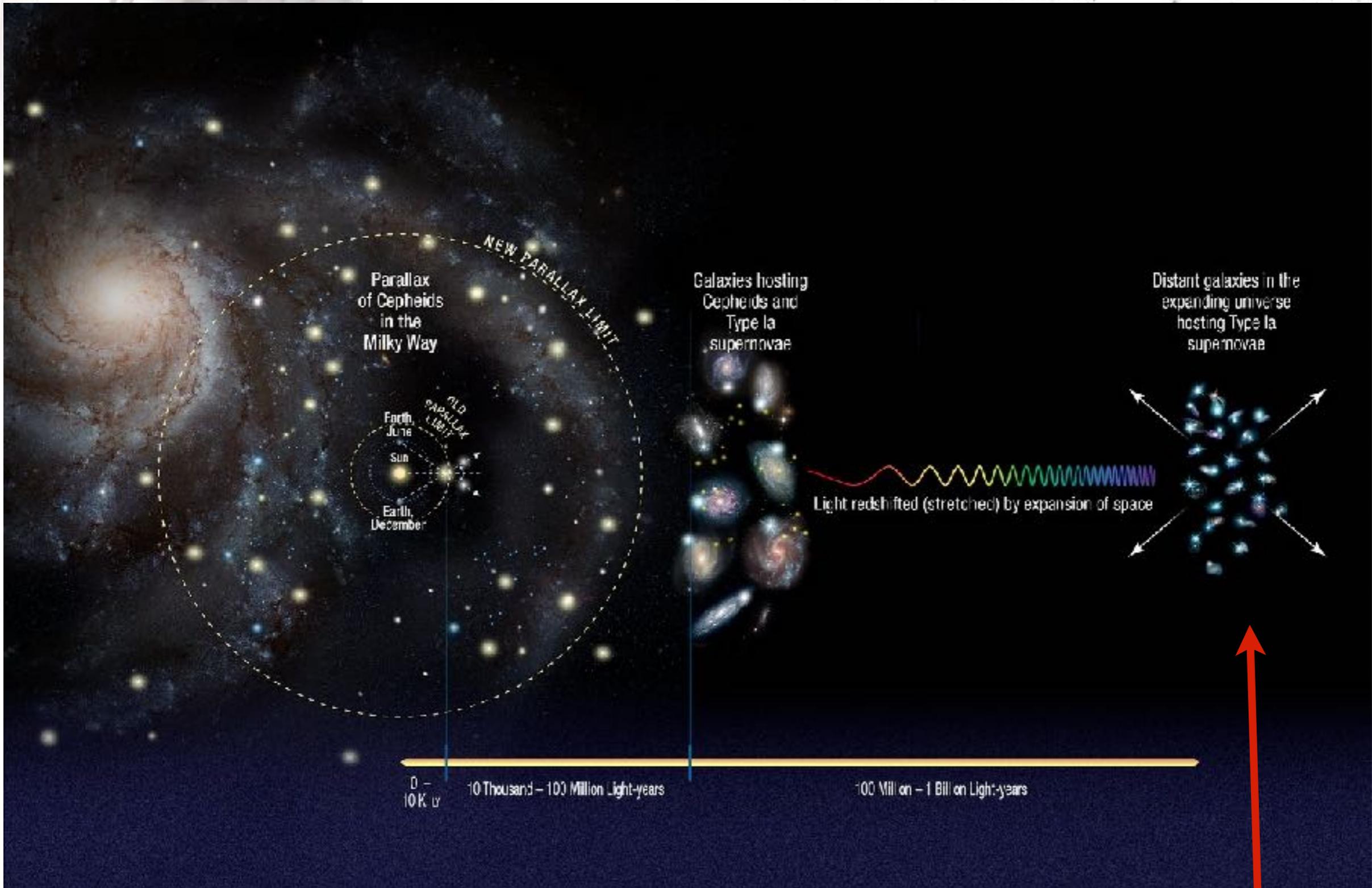
The universe is expanding!

# Expansion of the Universe

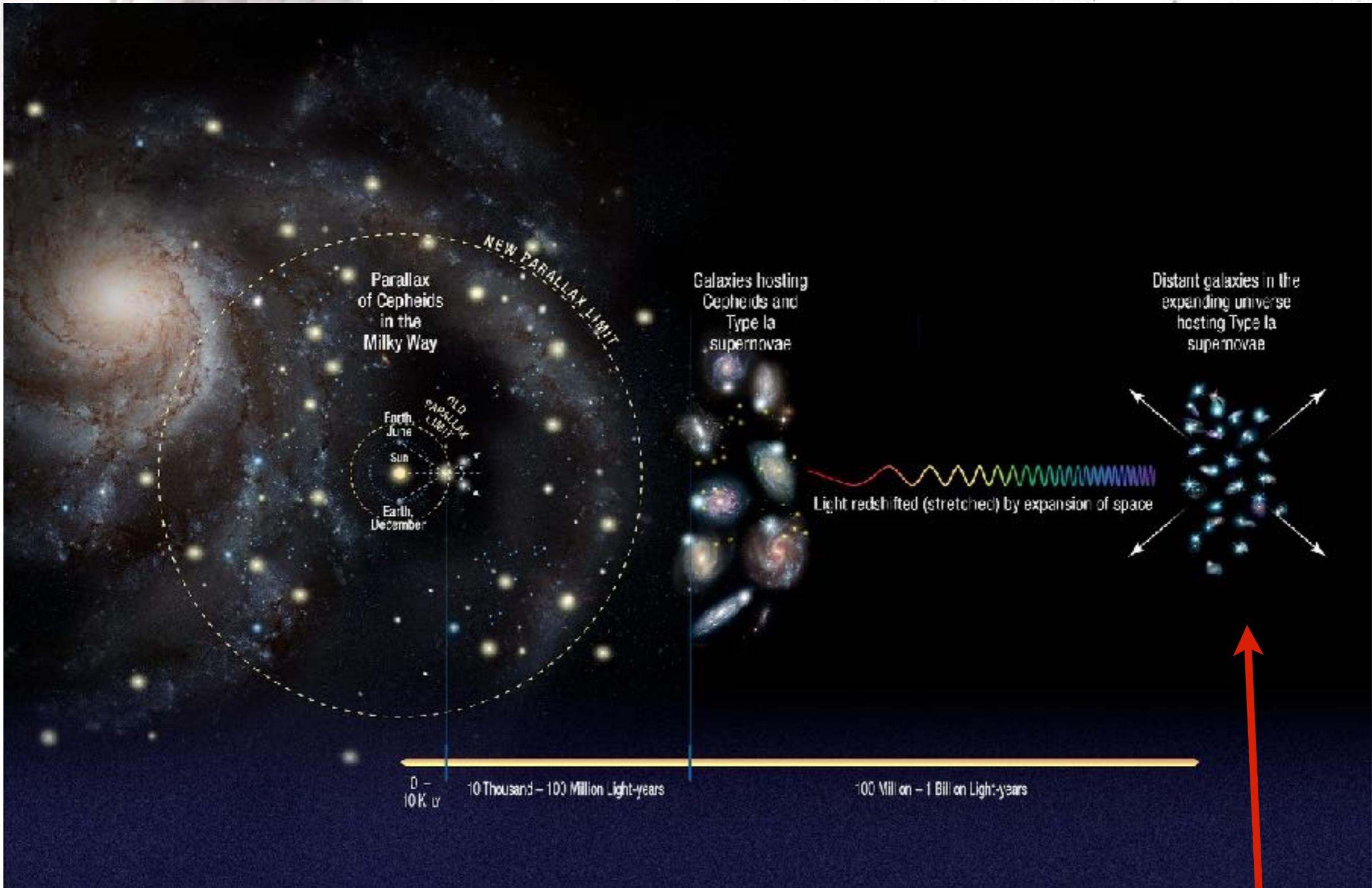




# Cepheids

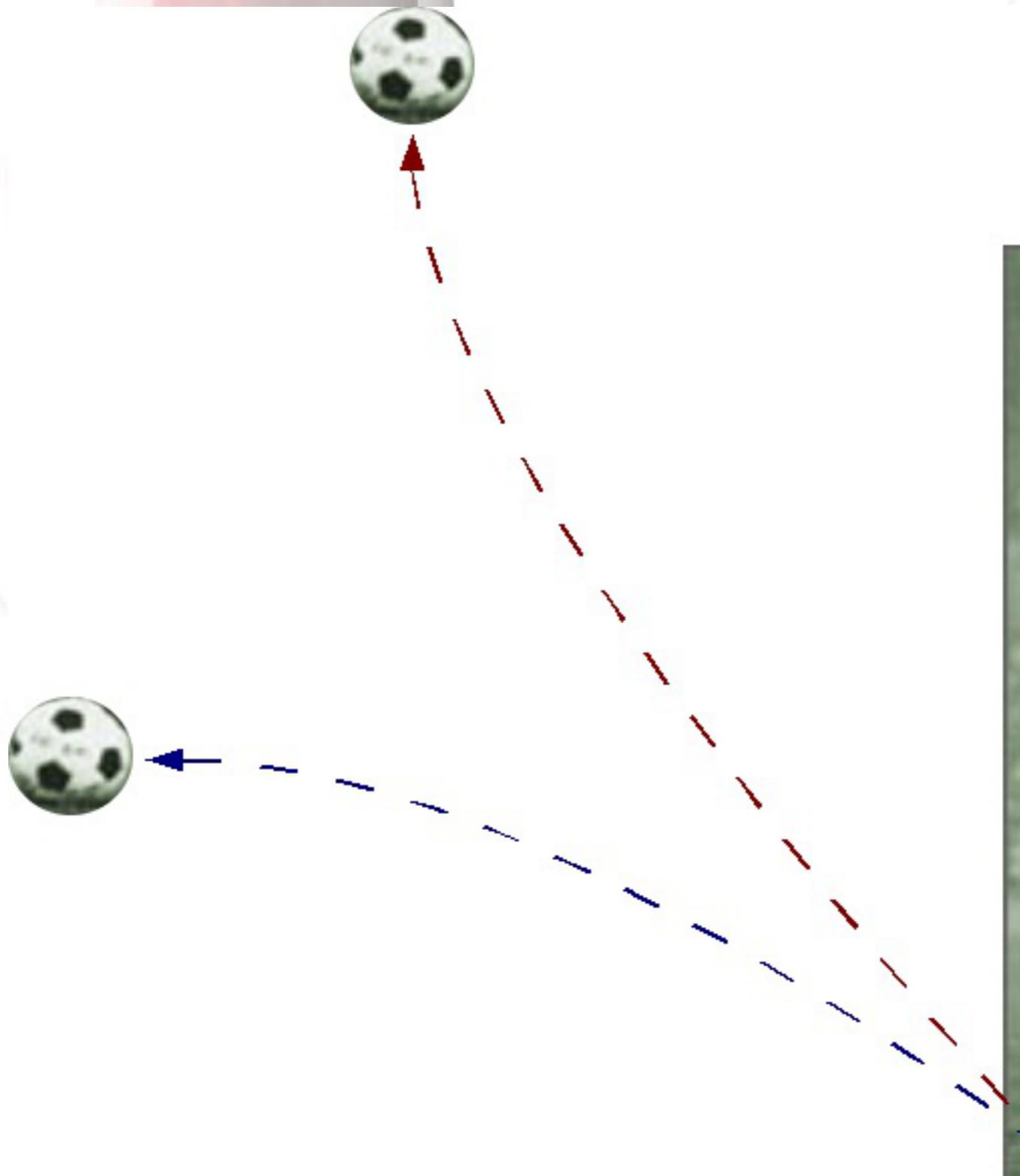


# Supernovae

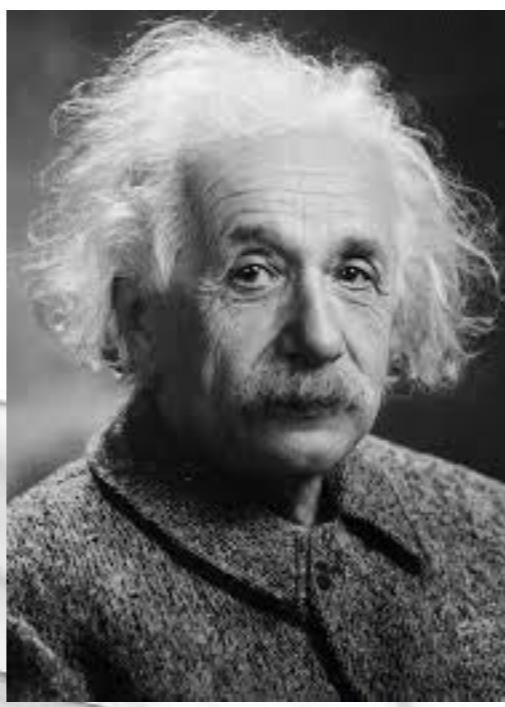


# Accelerated Expansion of the Universe!

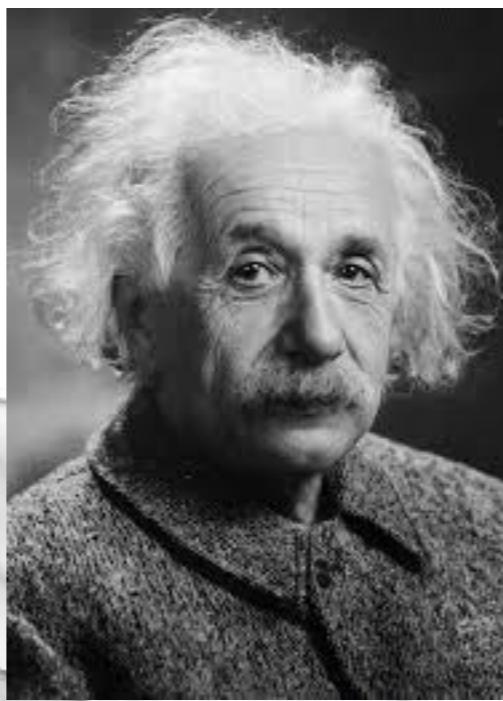
# Accelerated Expansion of the Universe



**How do we reconcile all these in a theory?**

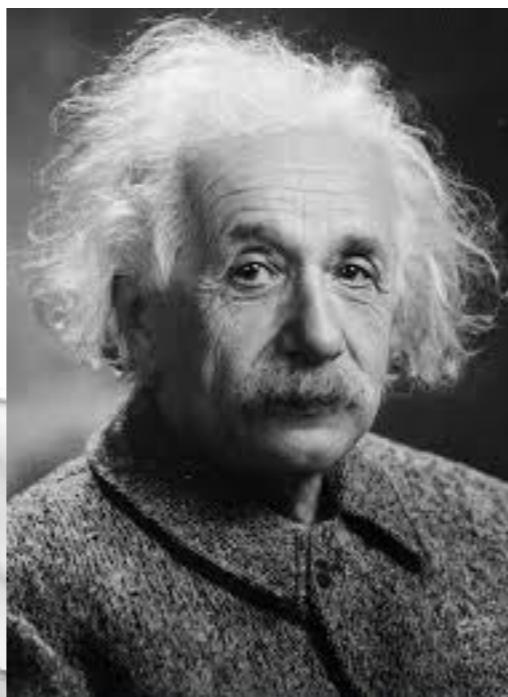


# General Relativity



# General Relativity

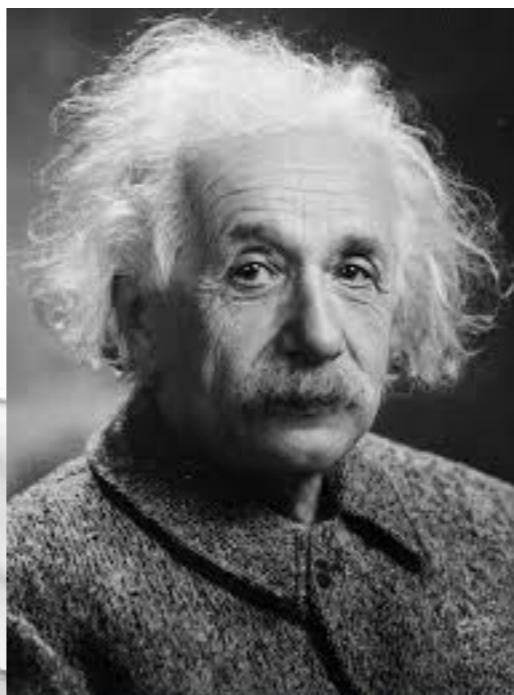
Gravity is



# General Relativity

Gravity is

property of  
space-time

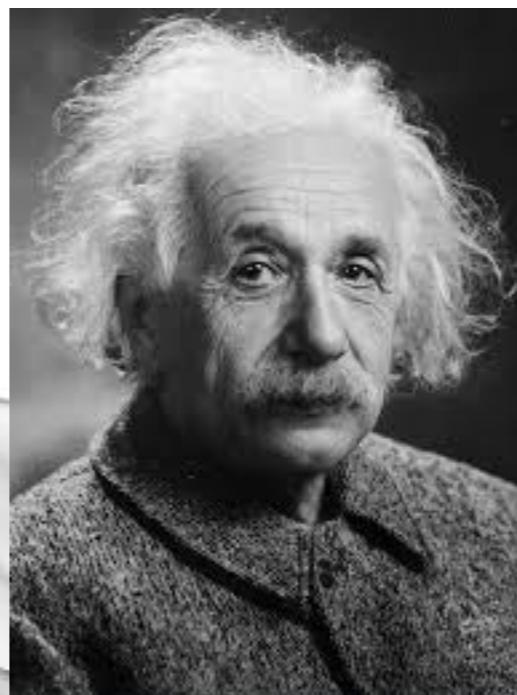


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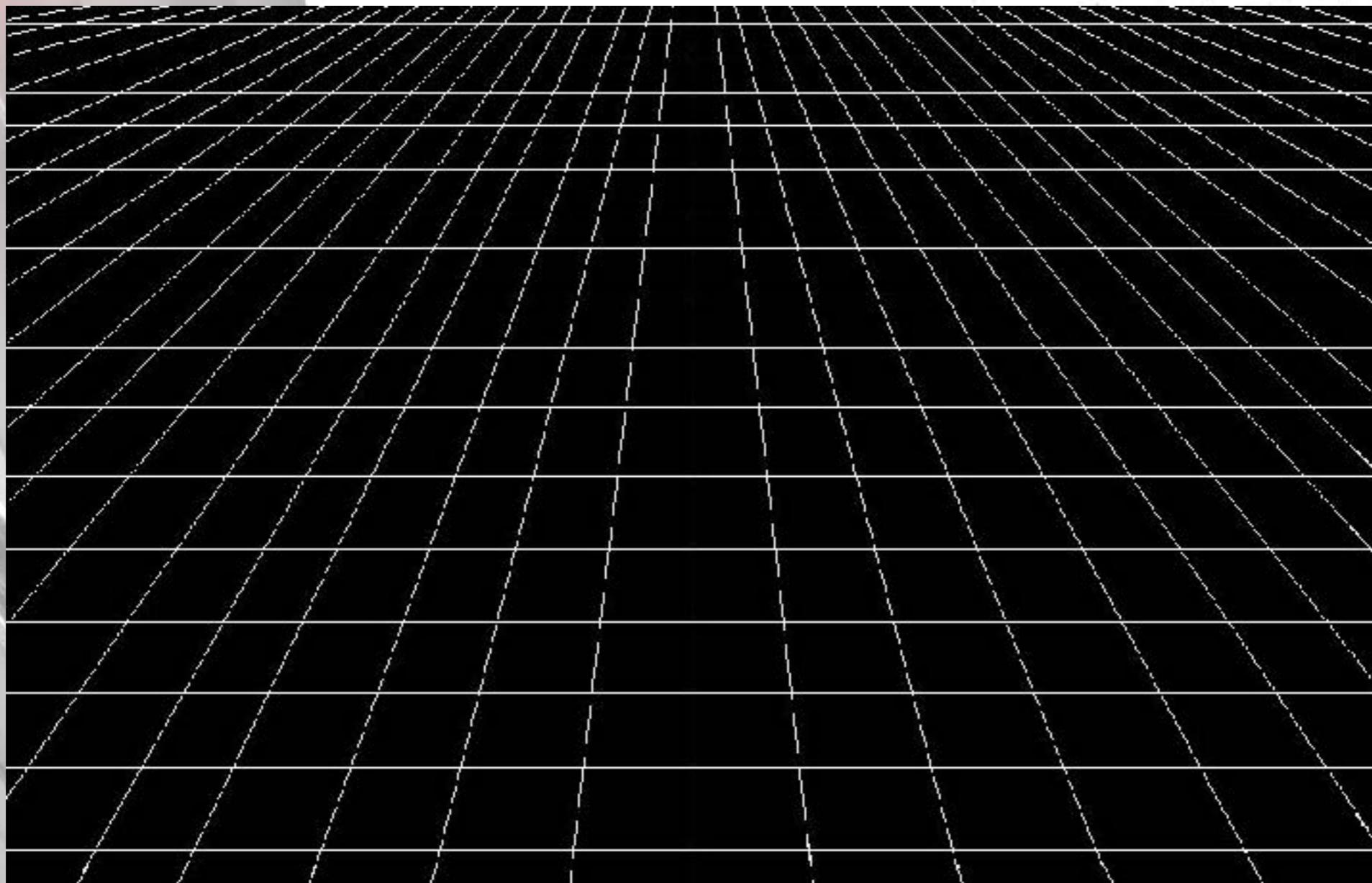
Gravity is

property of  
space-time

geometry



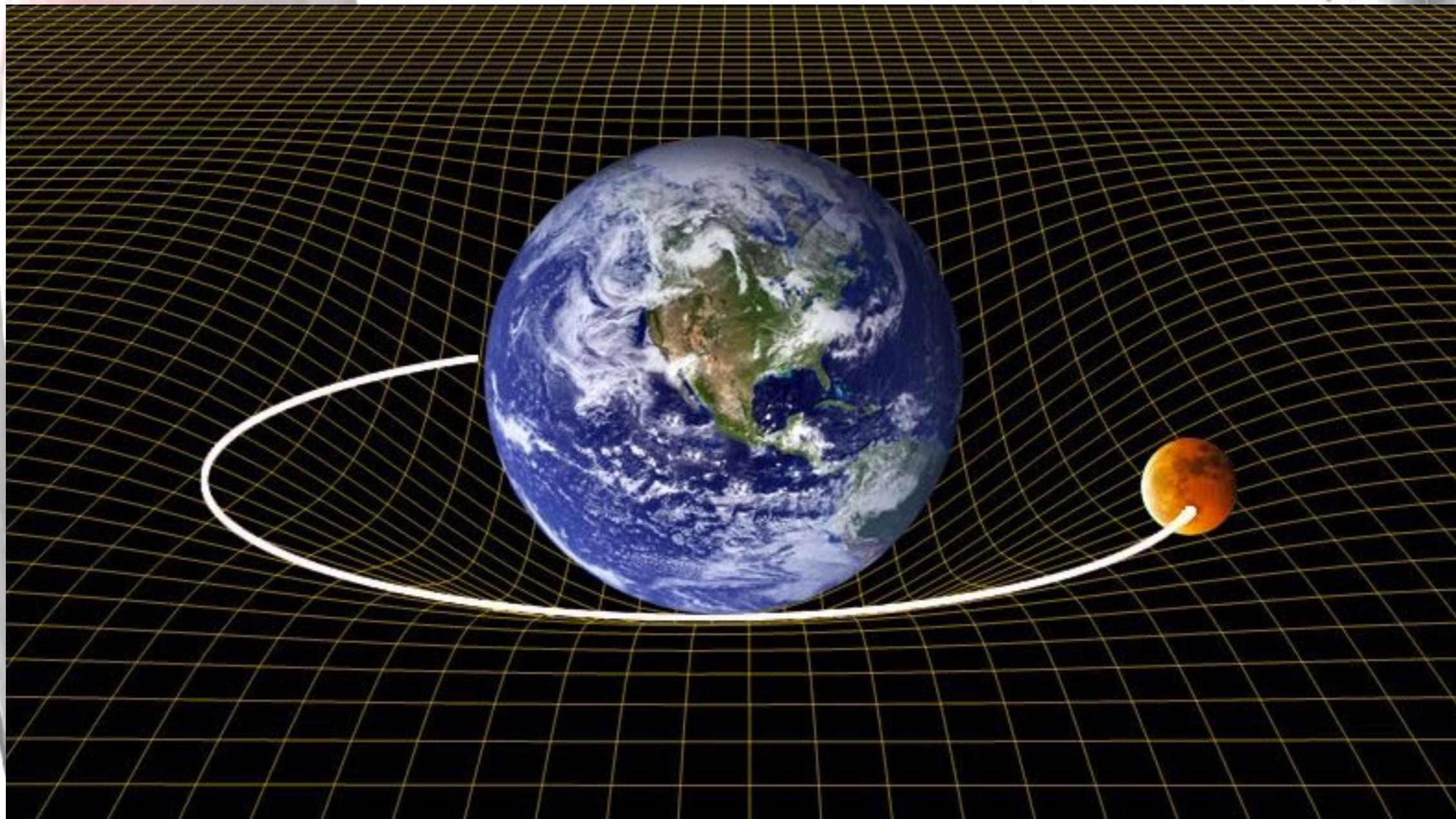
# General Relativity (Einstiens perspective)



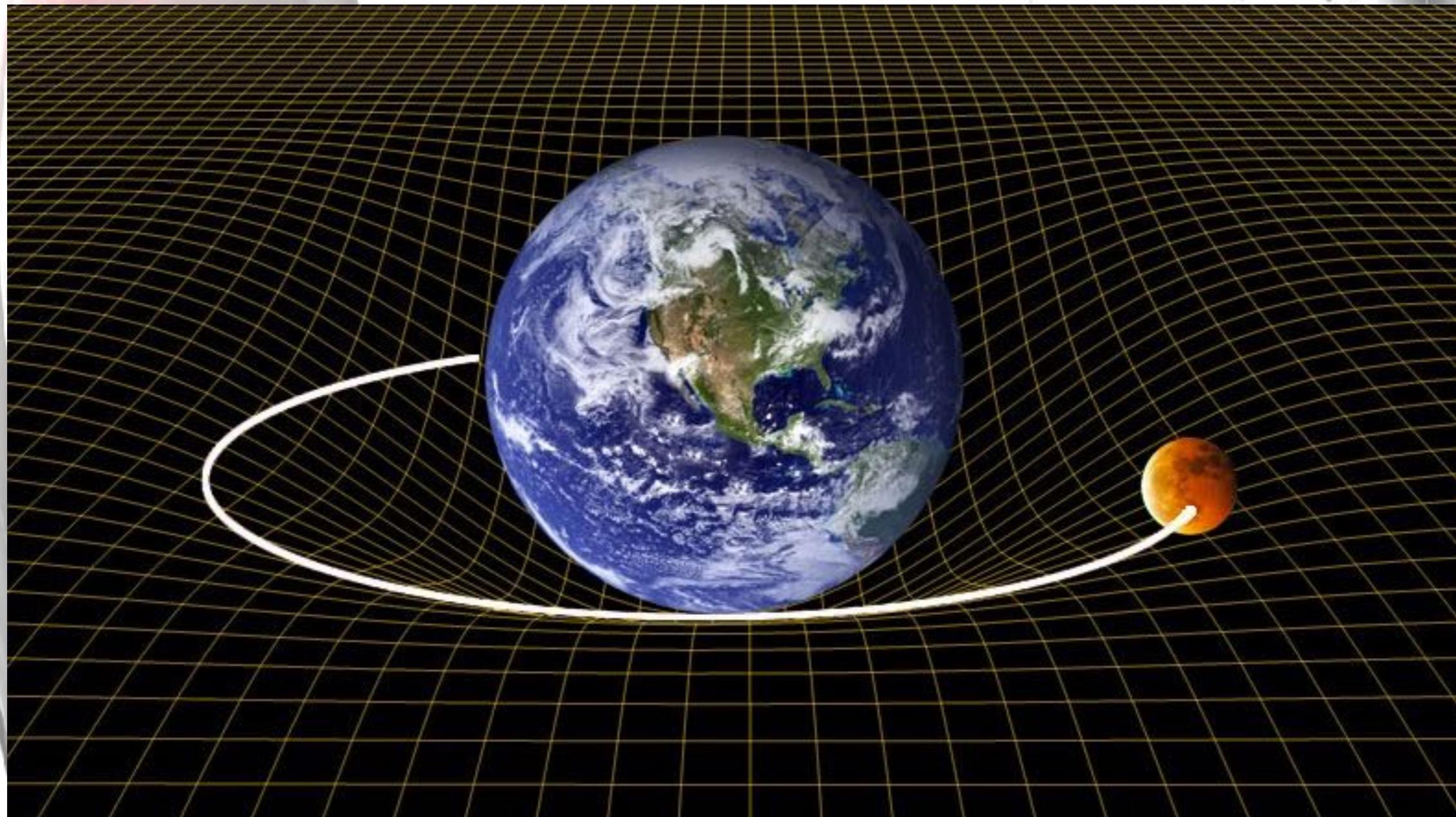
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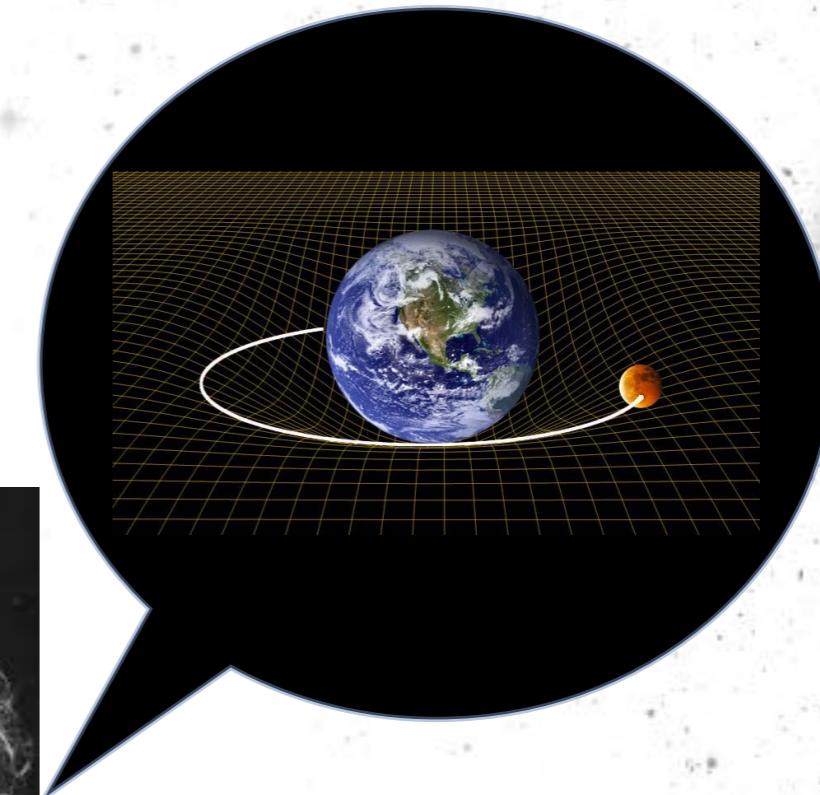
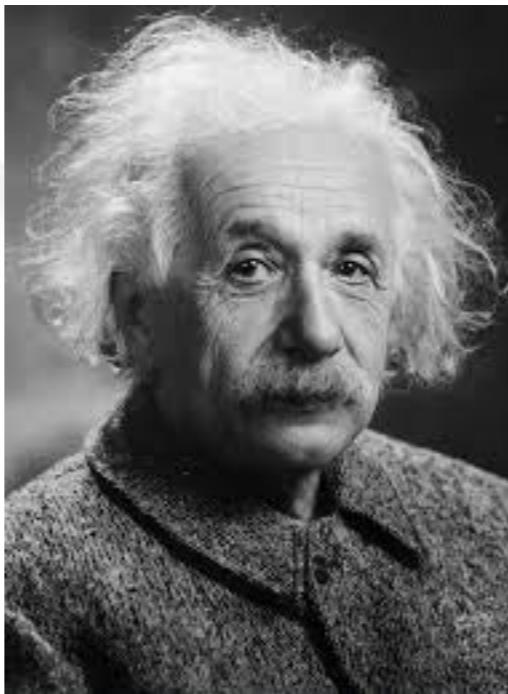


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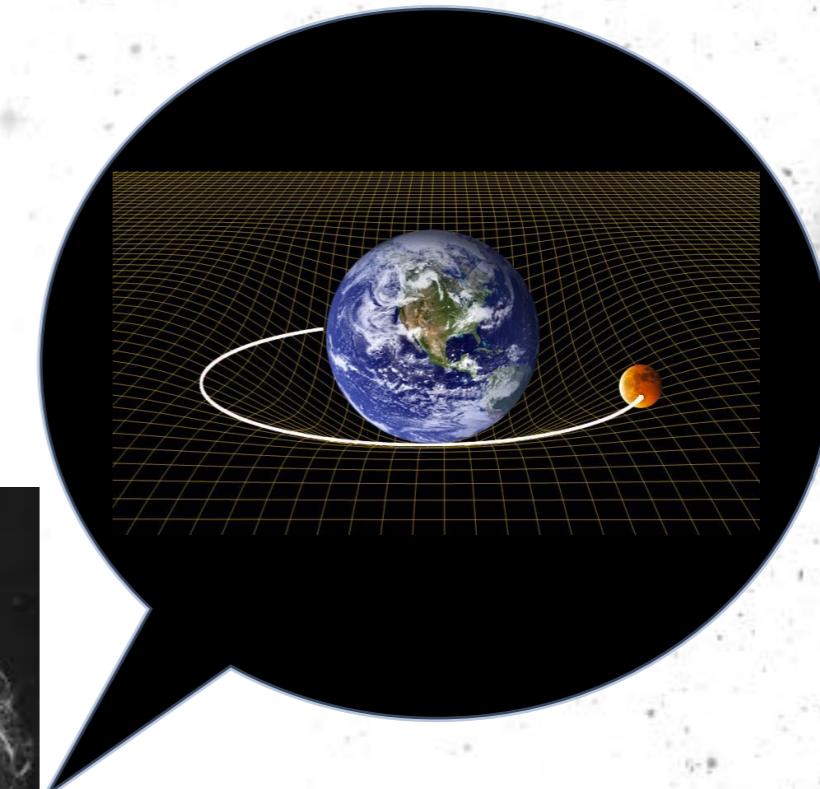
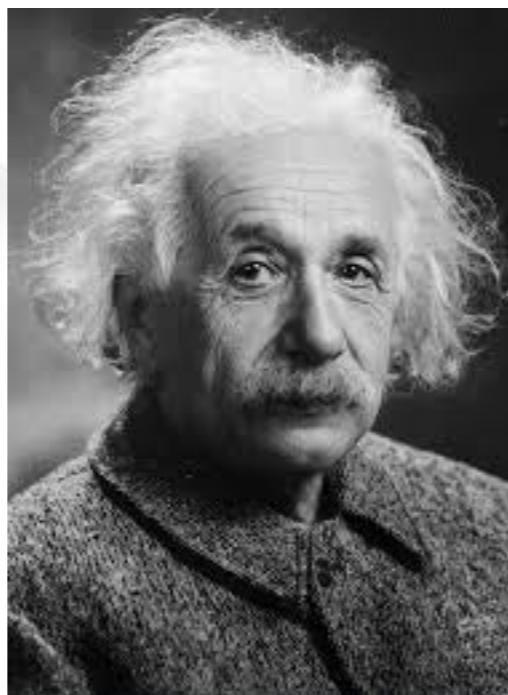
**GR = Curvature of space-time**

# General Relativity (Einstiens perspective)



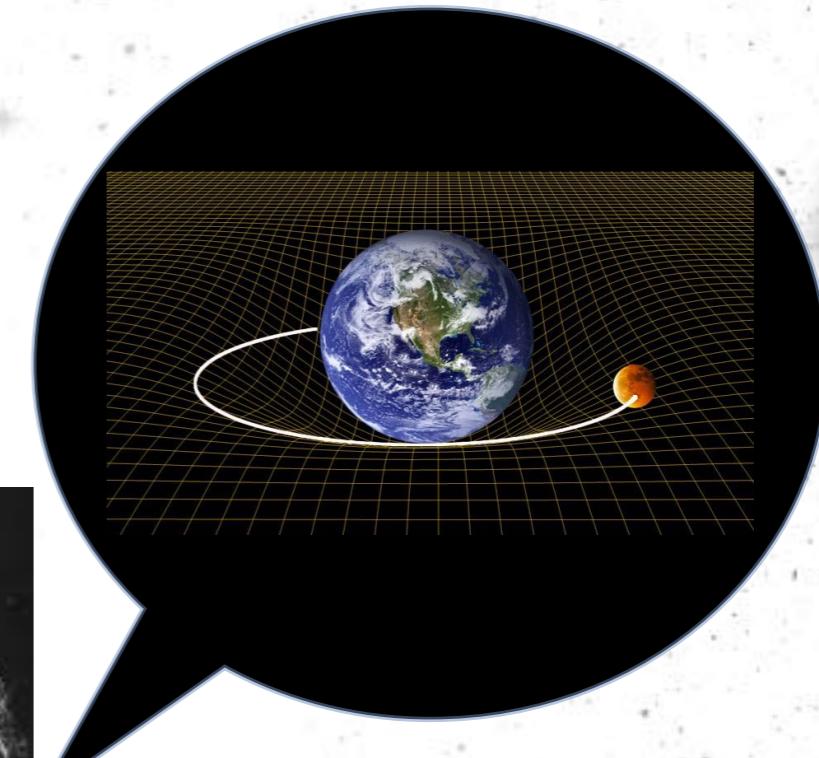
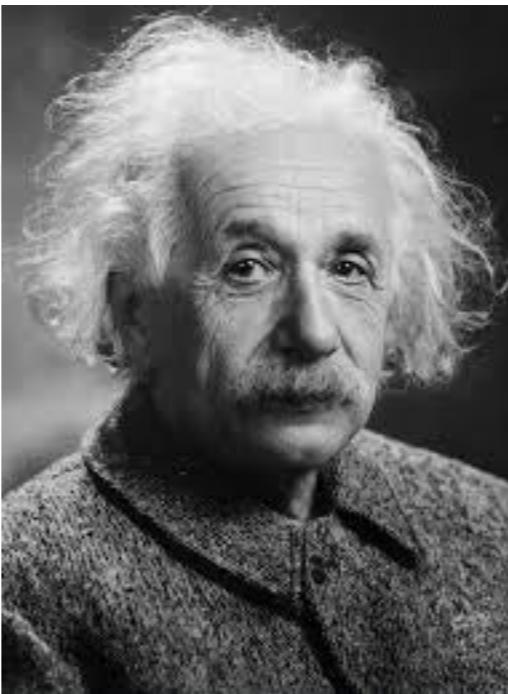
**Is this geometrical description unique?**

# General Relativity (Einstiens perspective)



**NO!**

# General Relativity (Einstiens perspective)



NO! → Trinity!

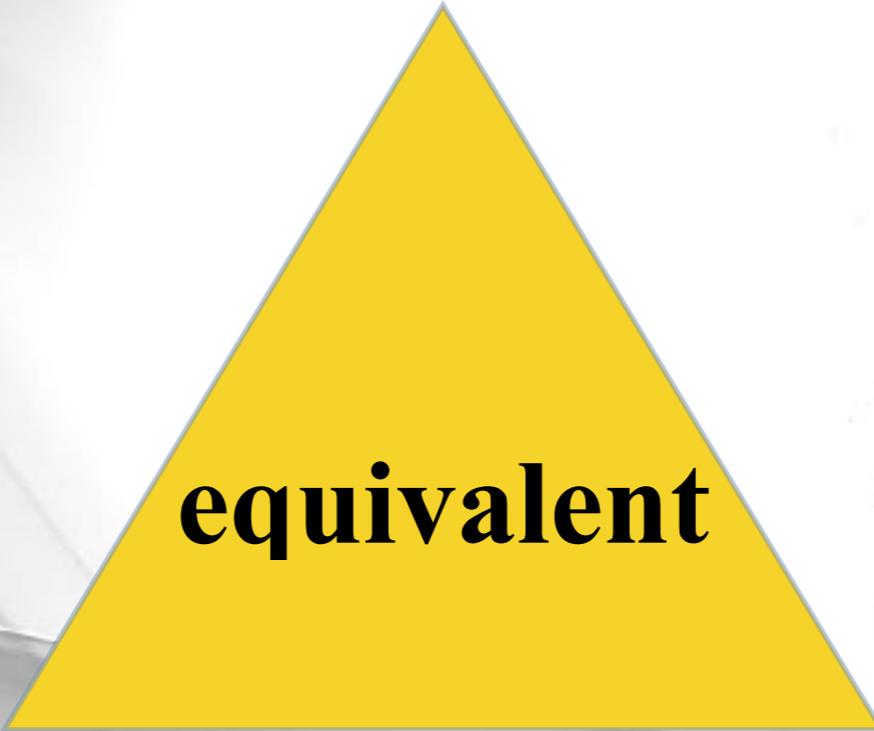
L.H & J.Beltran,  
T.Koivisto  
Universe 5 (2019) 7, 173,  
arXiv:1903.06830

# Geometrical Trinity of General Relativity

L.H & J.Beltran, T.Koivisto

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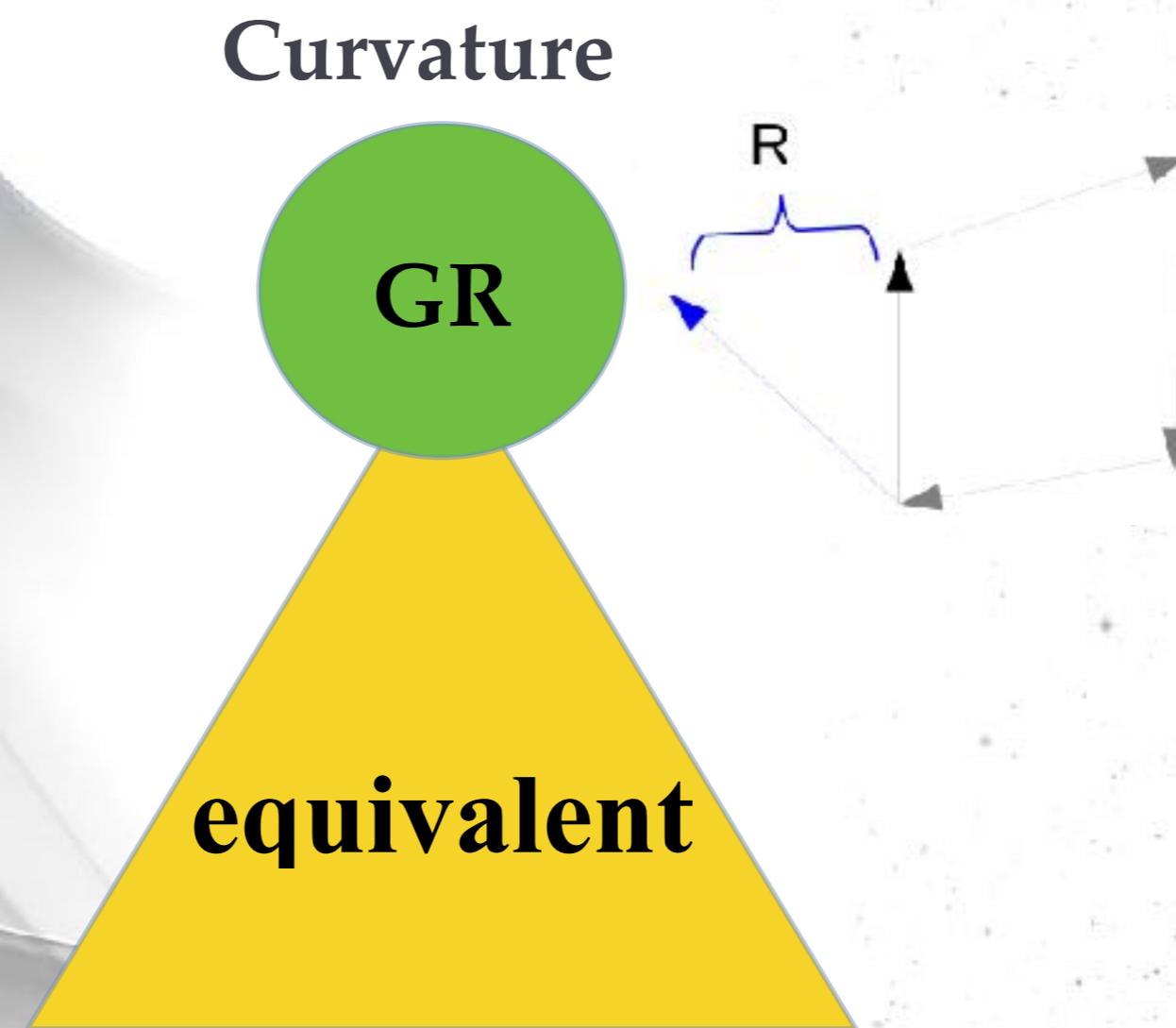
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equivalent

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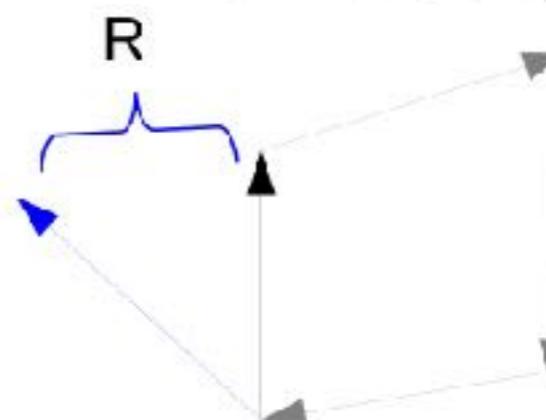
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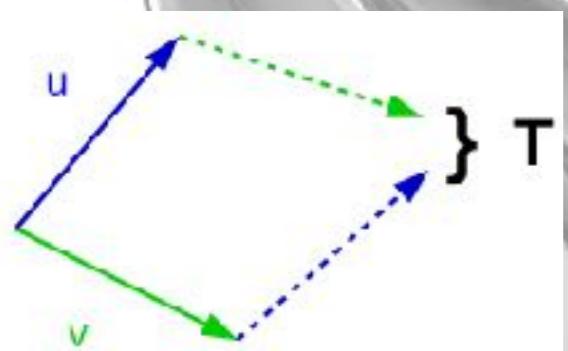
Curvature



equivalent

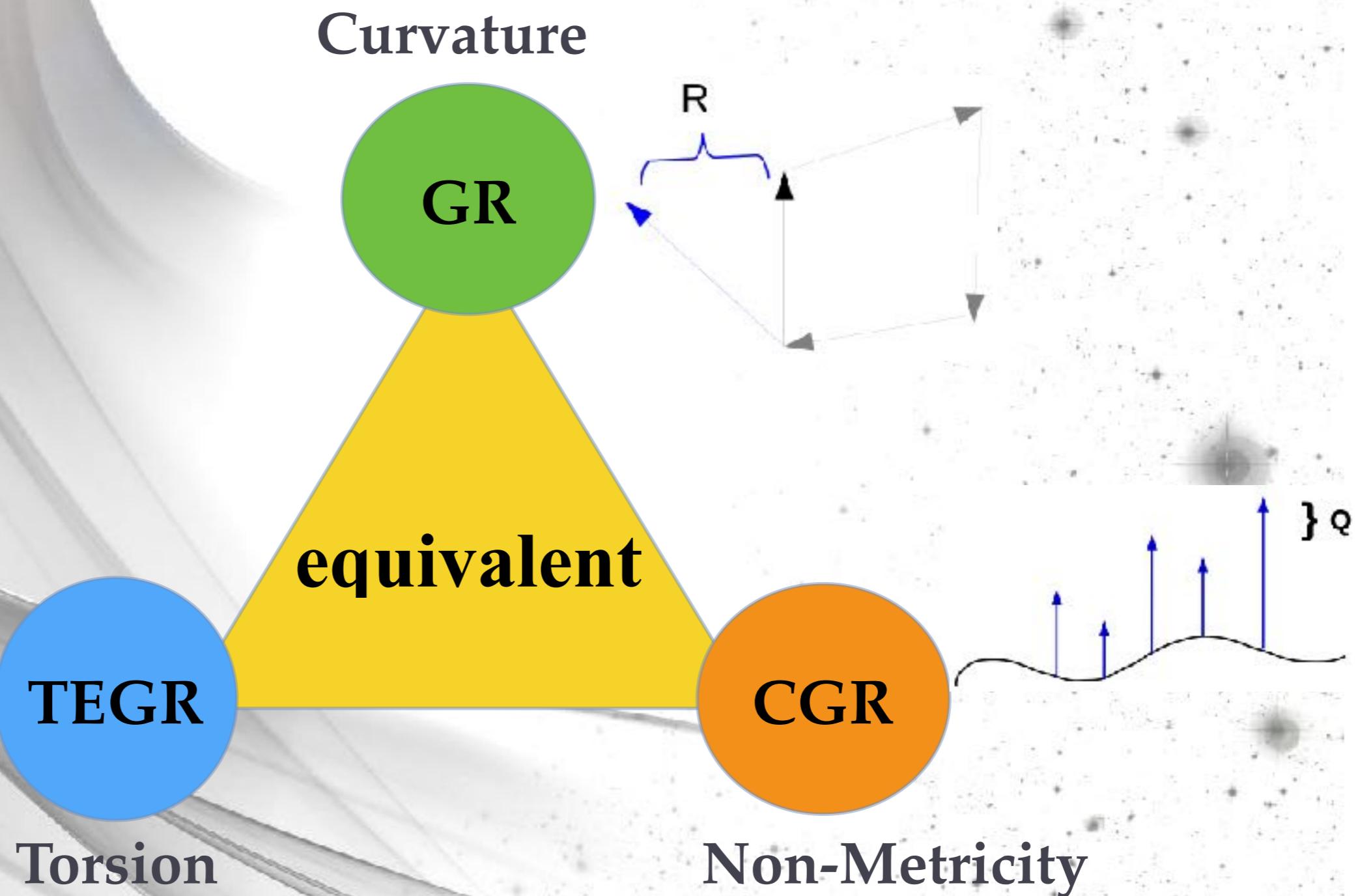
TEGR

Torsion

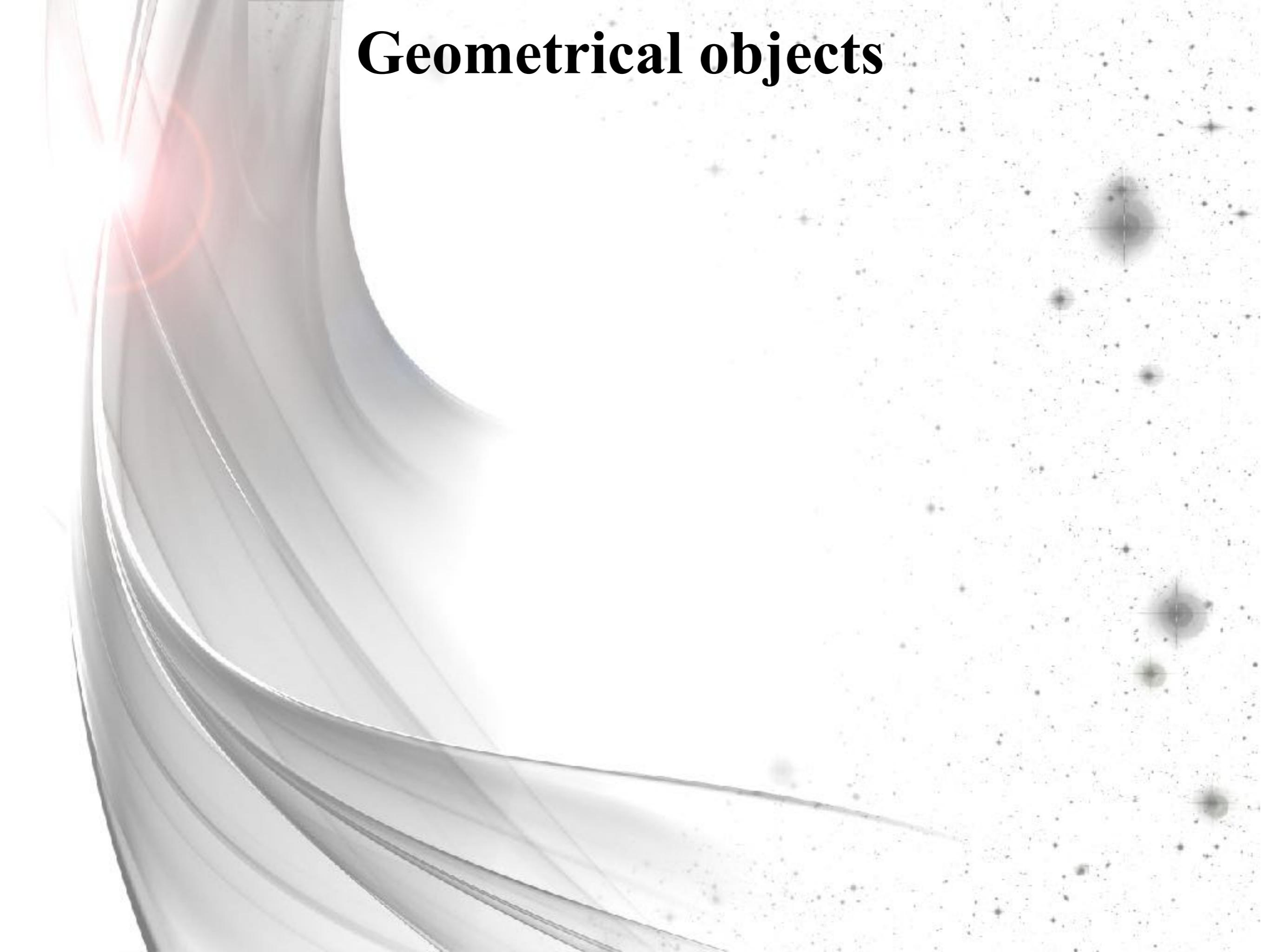


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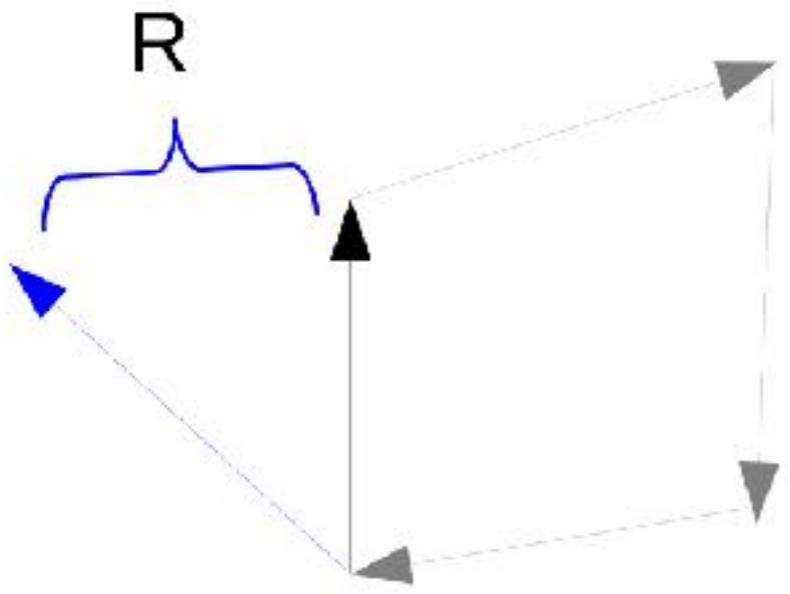
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# Geometrical objects

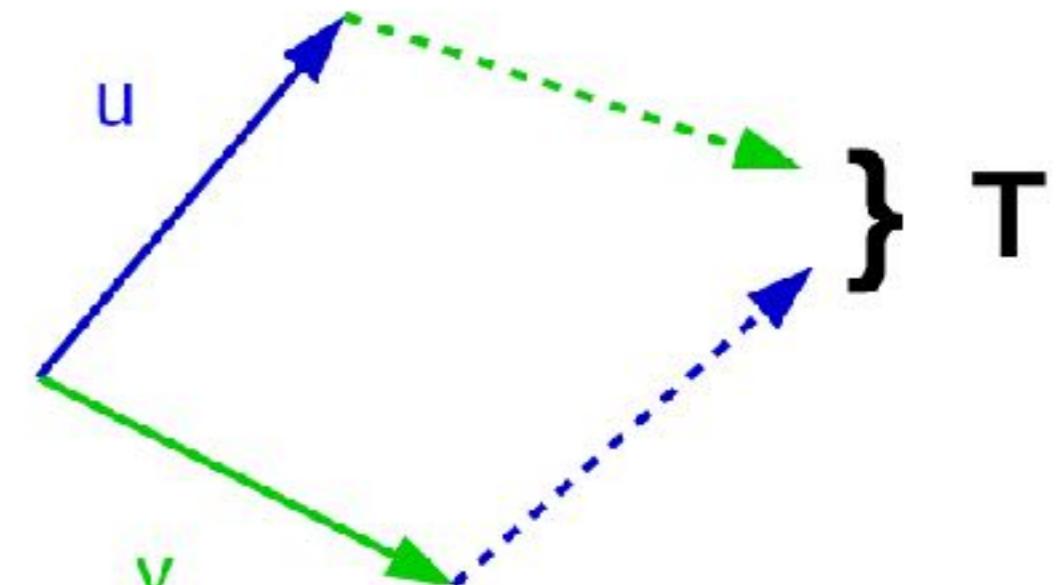
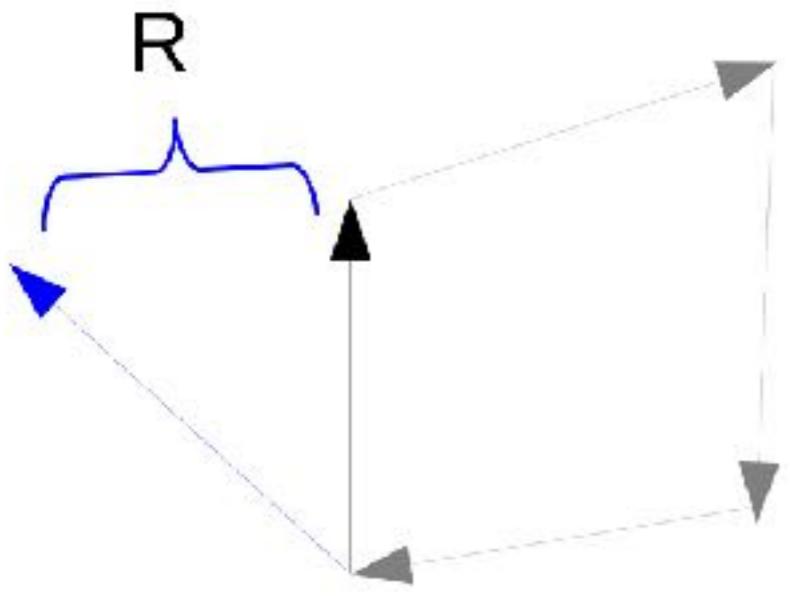


# Geometrical objects



- Curvature:  $R_{\beta\mu\nu}^\alpha \neq 0$

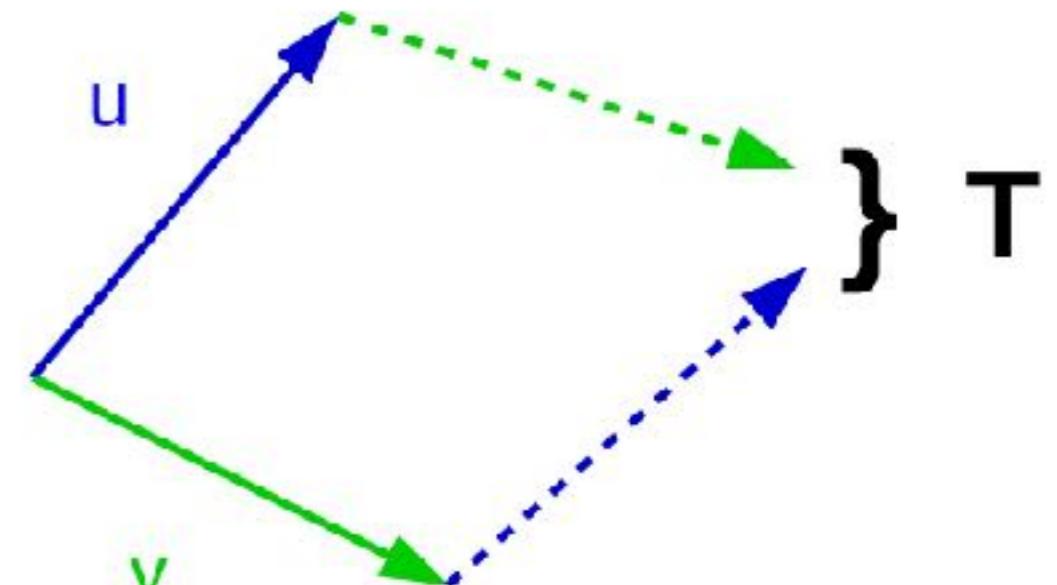
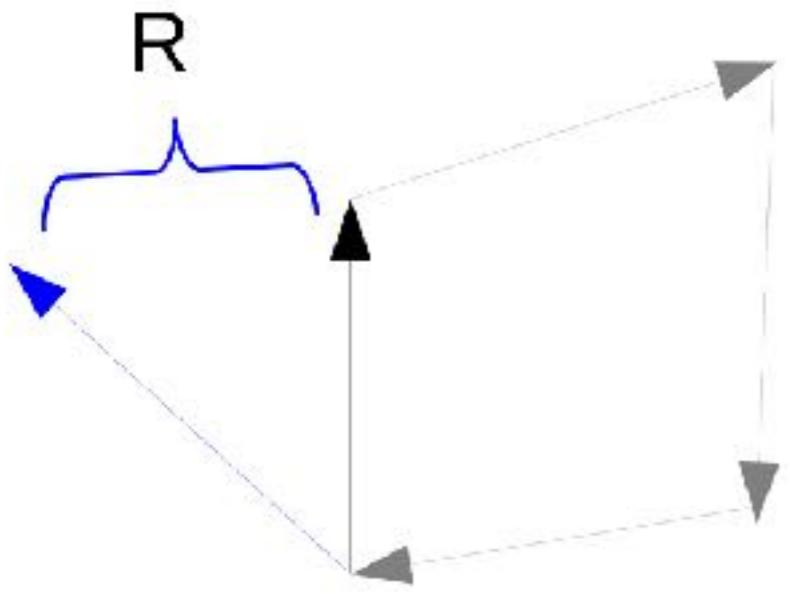
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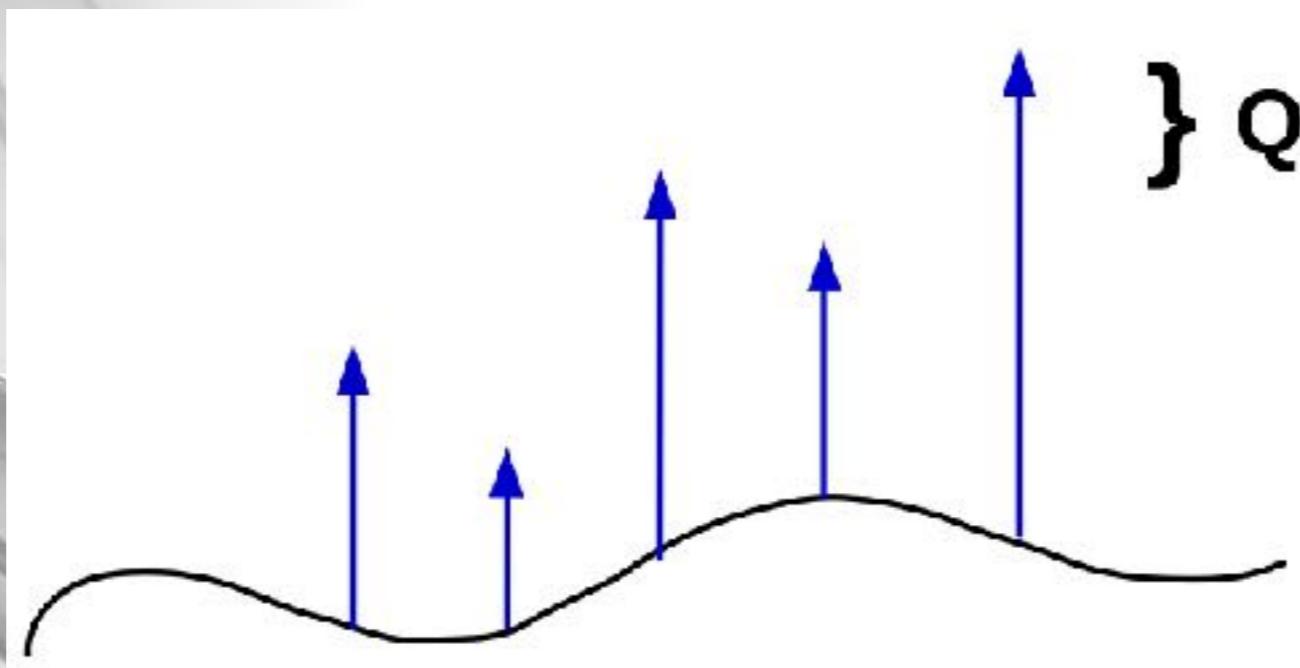
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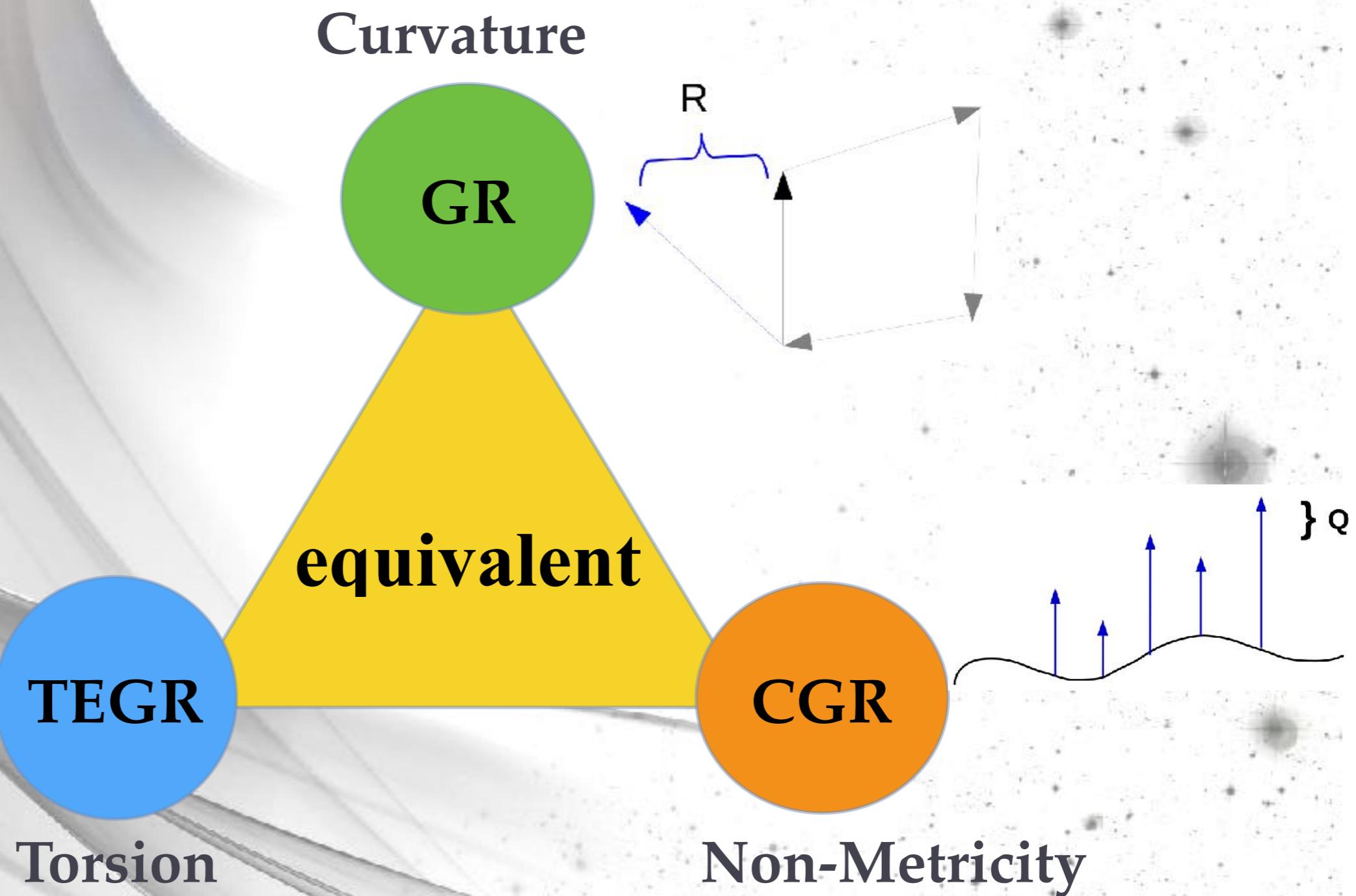
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- Non-metricity:  $Q_{\mu\nu}^{\alpha} \neq 0$

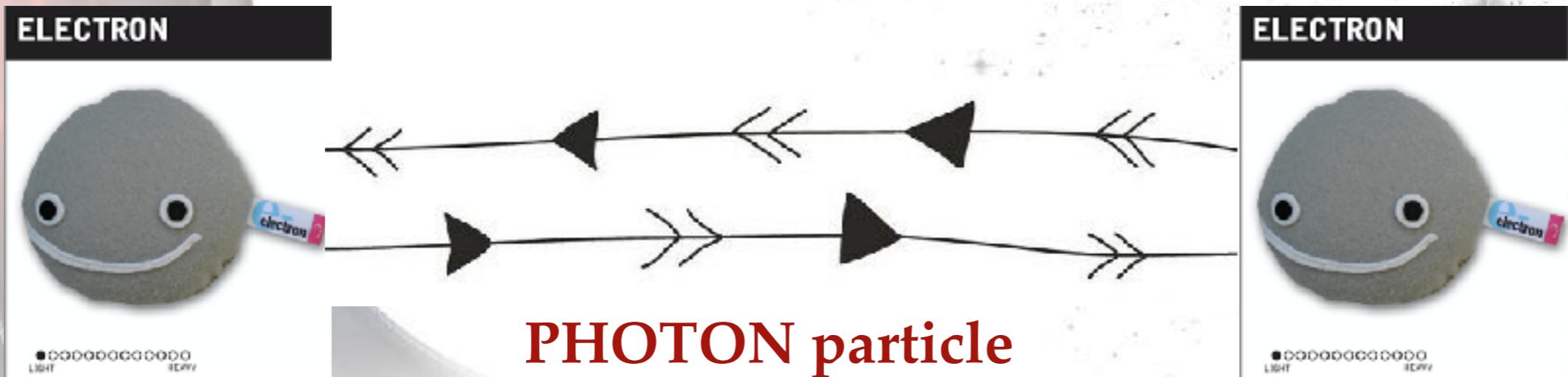
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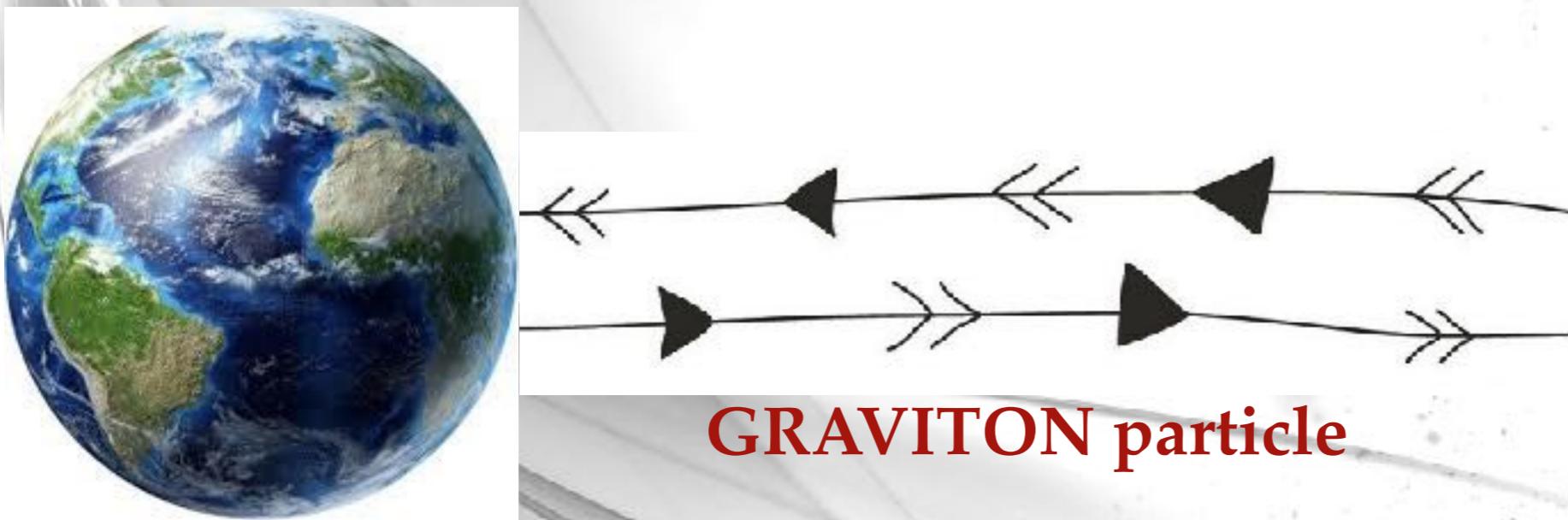
# Particle Physics Perspective

- Electromagnetic interactions



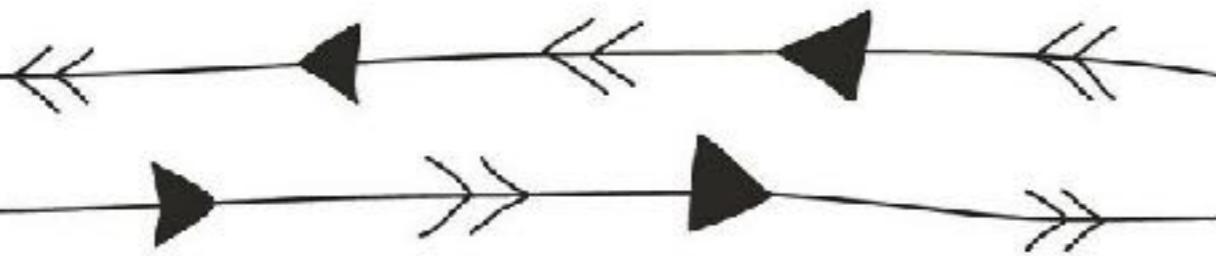
**PHOTON particle**

- Gravitational interactions



**GRAVITON particle**

# Particle Physics Perspective



**GRAVITON particle**



**What is the most natural candidate for  
the GRAVITON particle?**

## Facts that we know about gravity:

### Gravity

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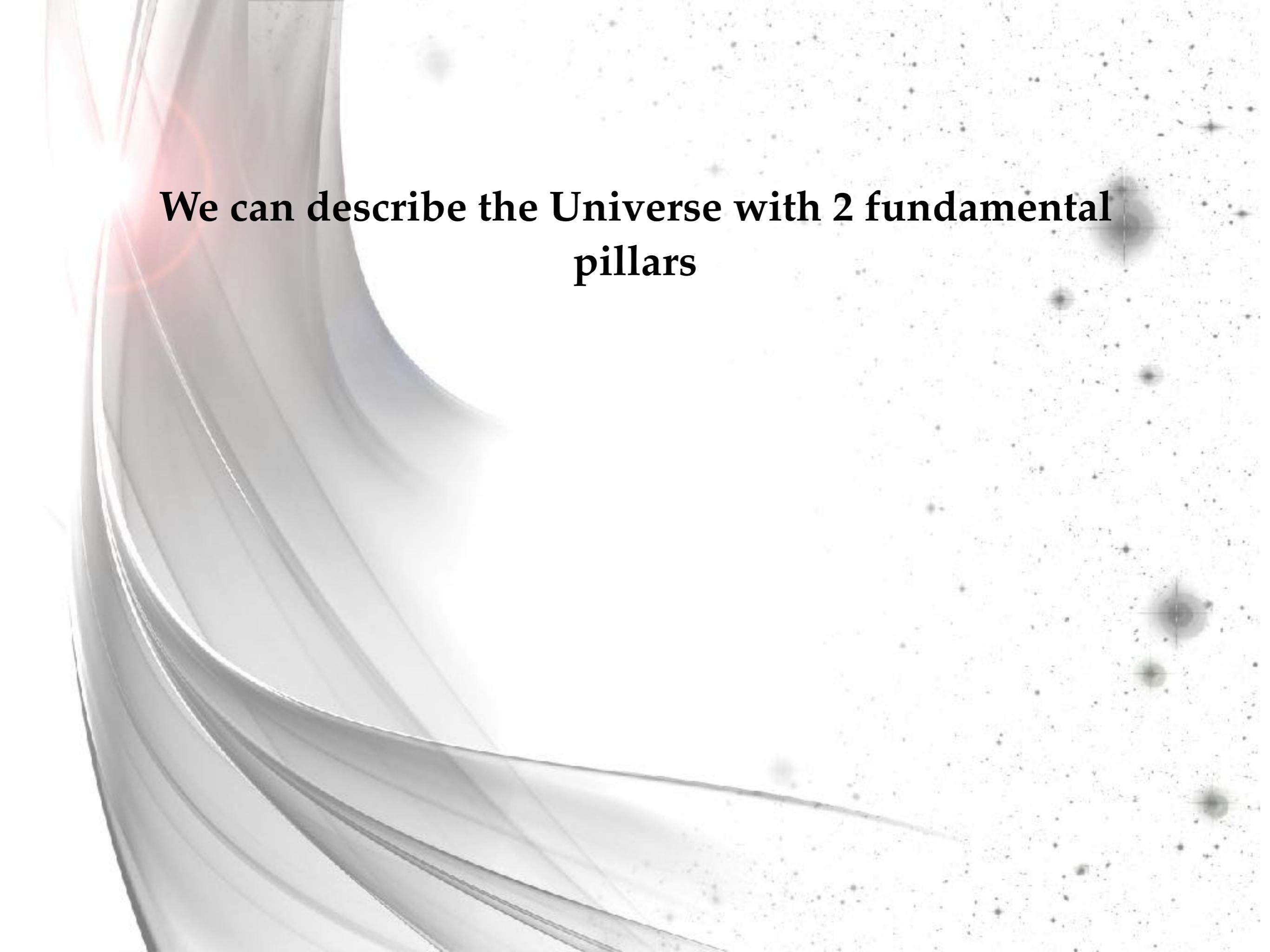
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This leaves massless spin-2 particle as a natural candidate!

→ General Relativity



**We can describe the Universe with 2 fundamental pillars**

We can describe the Universe with 2 fundamental pillars

GR

General Relativity

We can describe the Universe with 2 fundamental pillars

GR

General Relativity

CP

Cosmological Principle

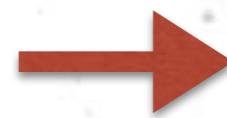
We can describe the Universe with 2 fundamental pillars

GR

General Relativity

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Cosmological Principle



Homogeneity  
& Isotropy

We can describe the Universe with 2 fundamental pillars

GR

General Relativity

CP

Cosmological Principle



Homogeneity  
& Isotropy

spherically symmetric  
for every observer

(FLRW model)

# Budget of the Universe

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

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Observations indicate

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

DM

- $\sim 27\%$  with  $p \sim 0$

- Cold Dark Matter: non-relativistic weakly interacting matter

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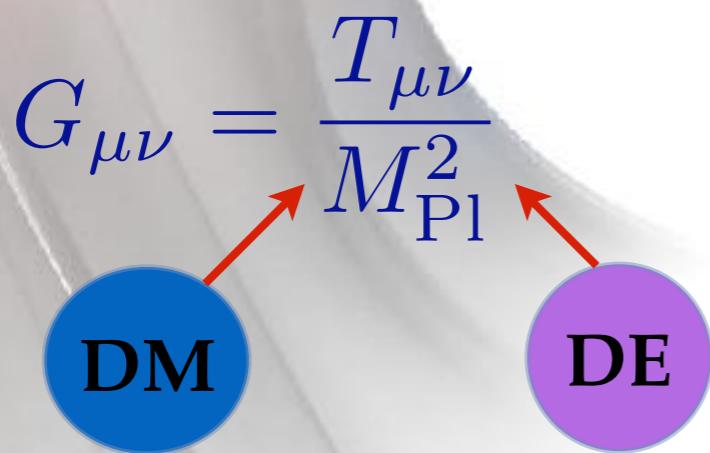
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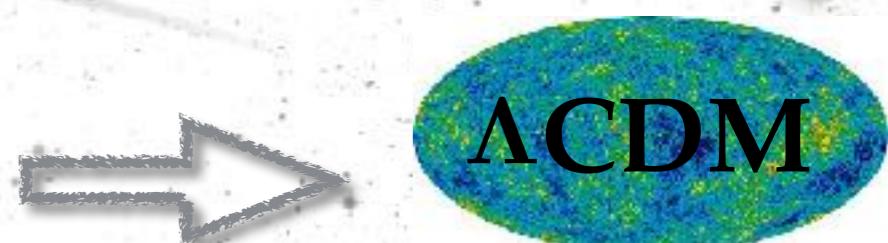
$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$


- $\sim 4.9\%$  baryons
- $\sim 27\%$  with  $p \sim 0$
- $\sim 68\%$  with  $p < -\frac{1}{3}\rho$

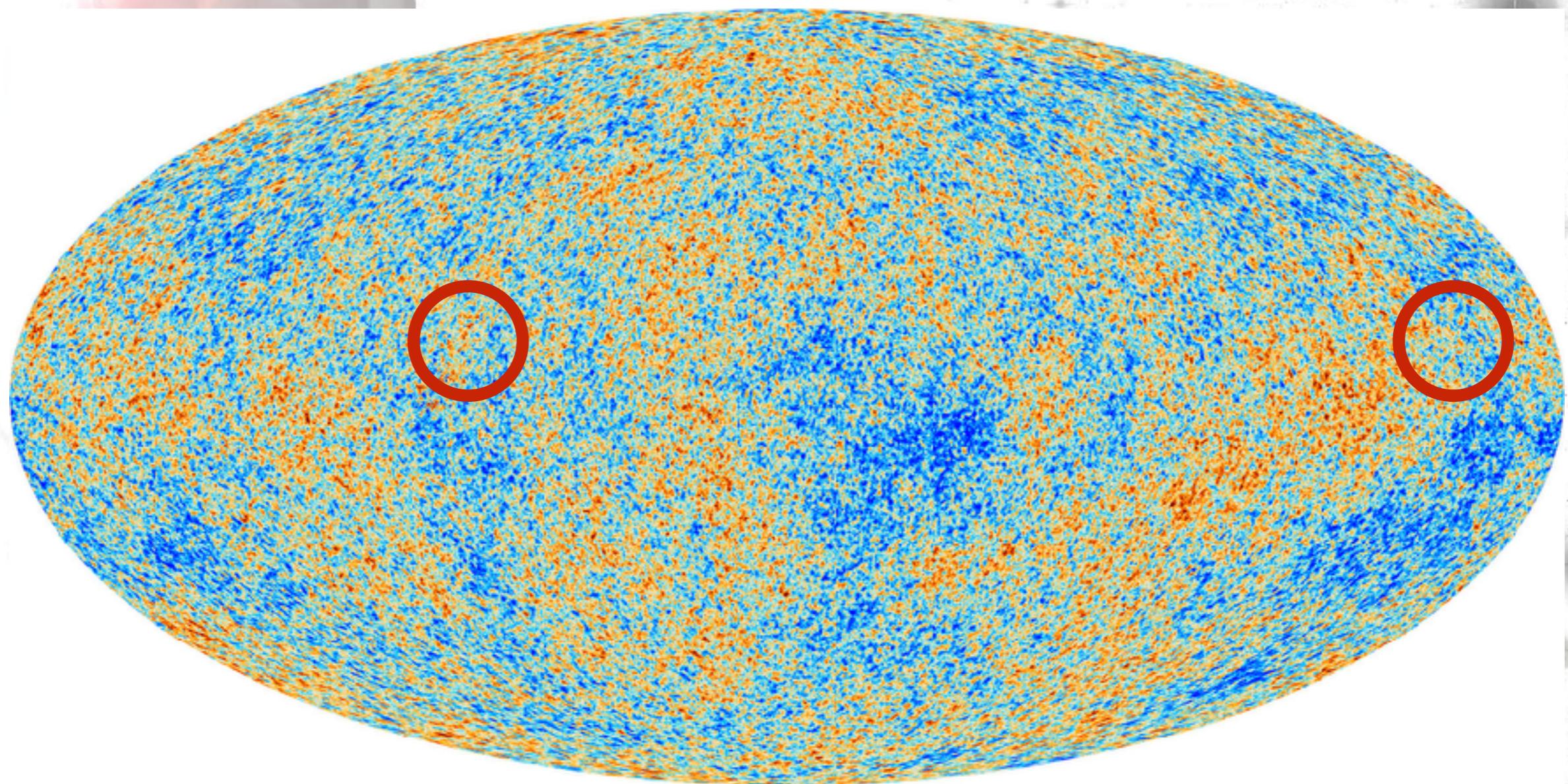
- Cold Dark Matter: non-relativistic weakly interacting matter
- Dark Energy: energy with **negative pressure** → accelerating universe

Cosmological Constant  
 $\Lambda$

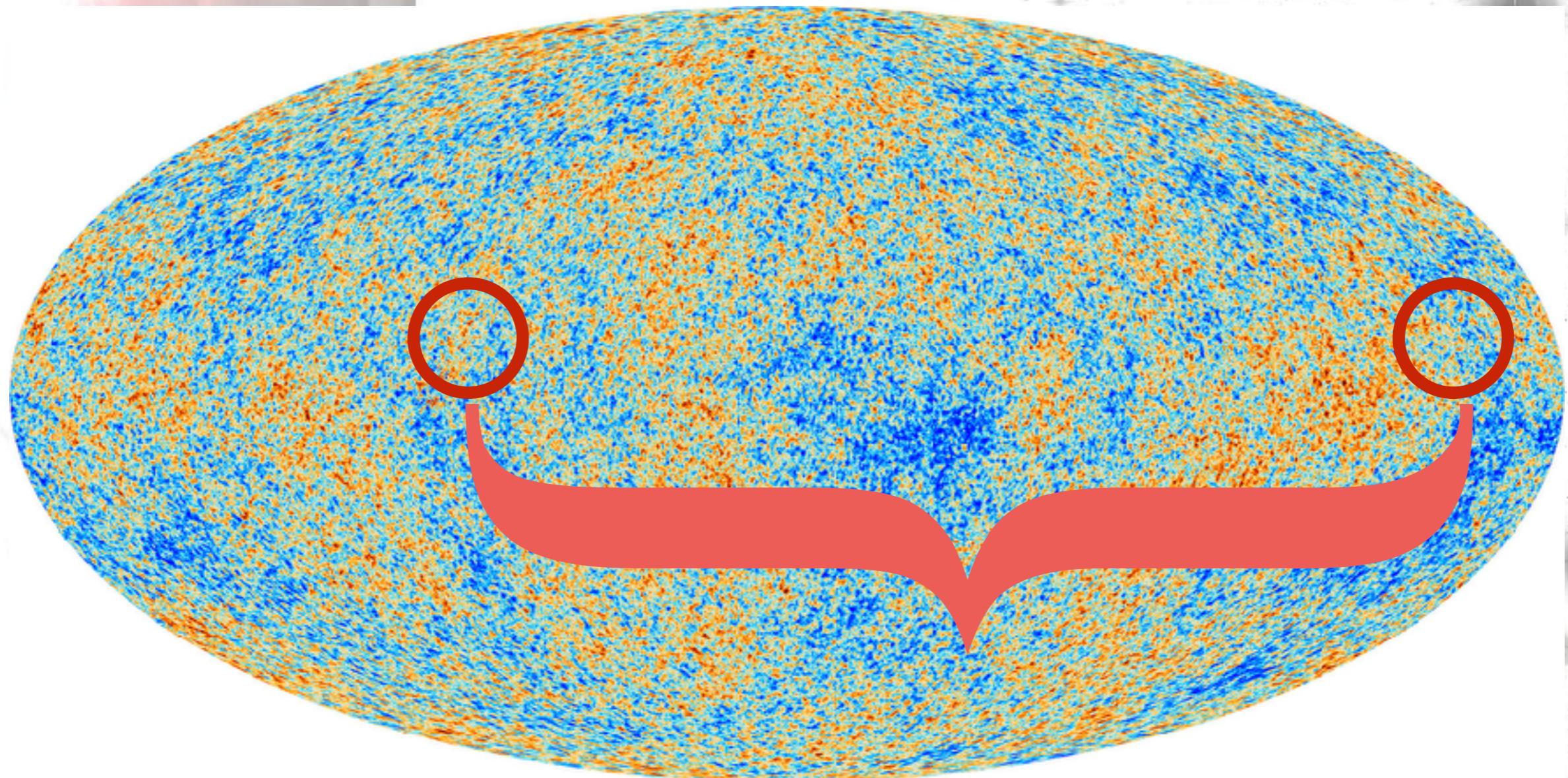
$$p = -\rho$$



CMB

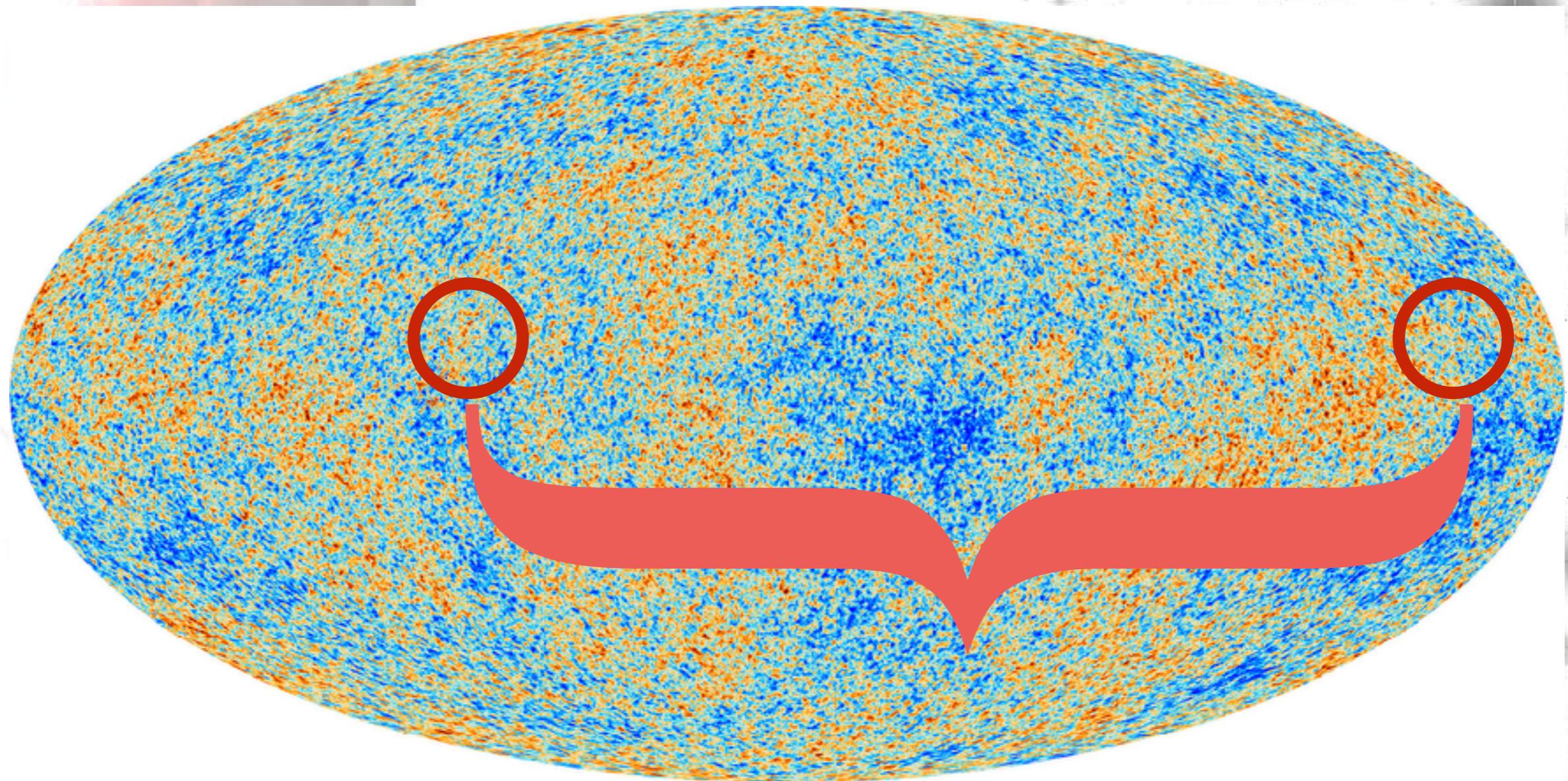


CMB

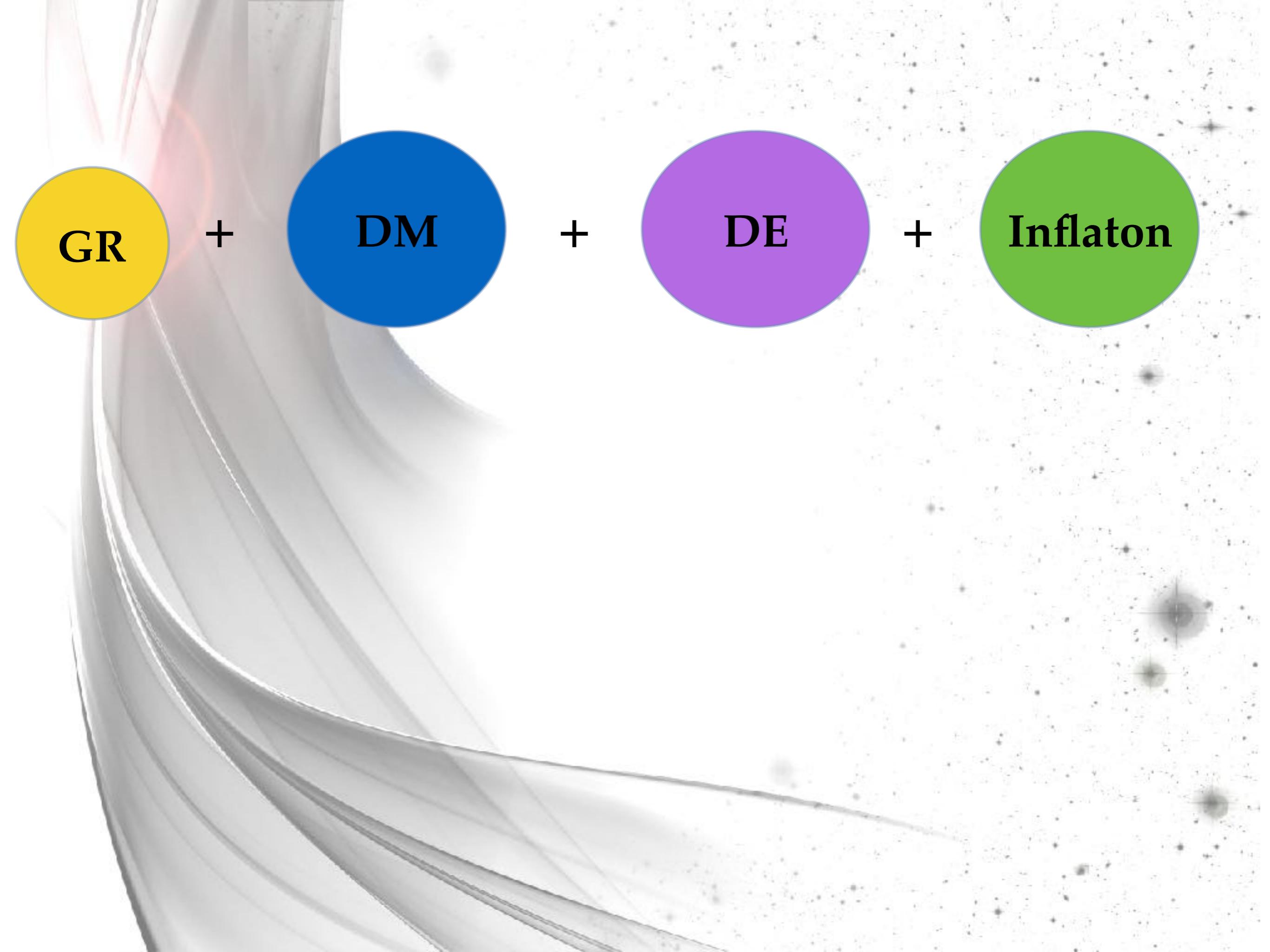


How can causally not connected regions have the  
same fluctuations?

CMB



**INFLATION: early phase of accelerated expansion!**



GR

+

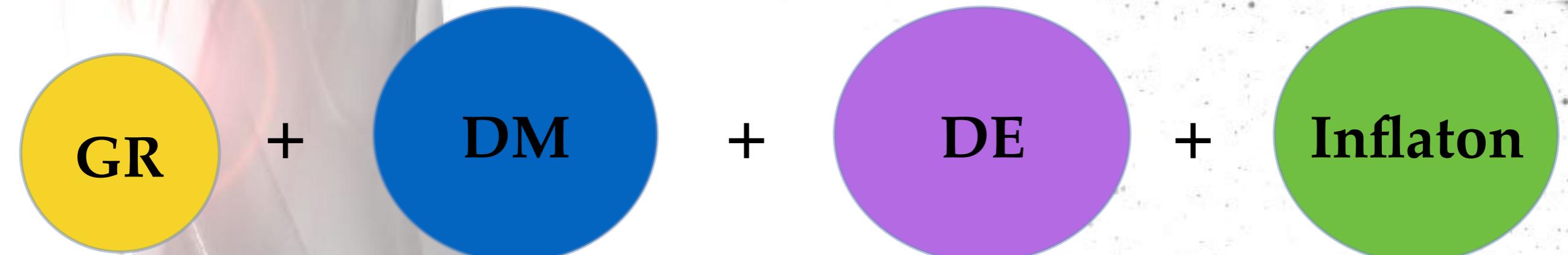
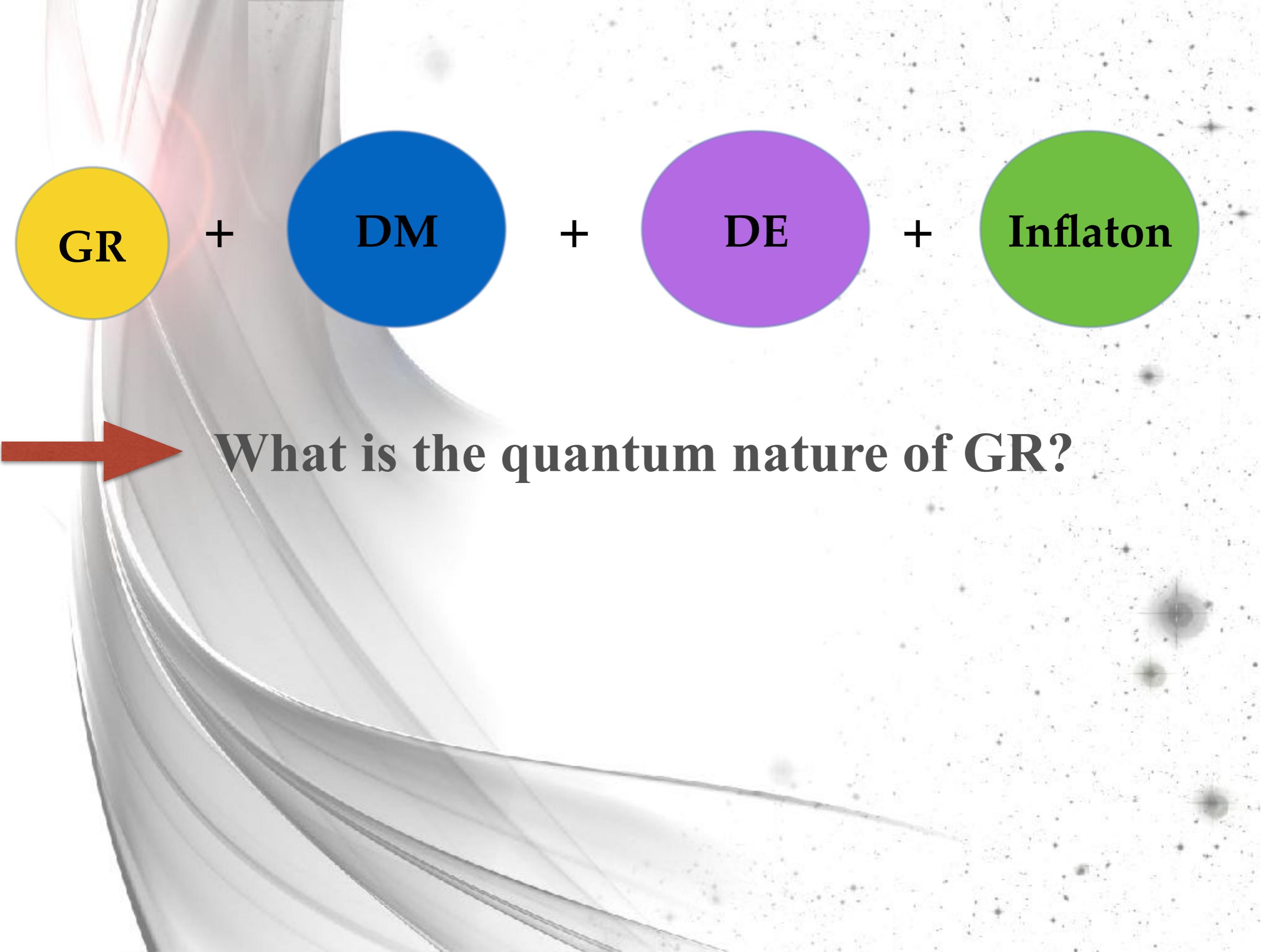
DM

+

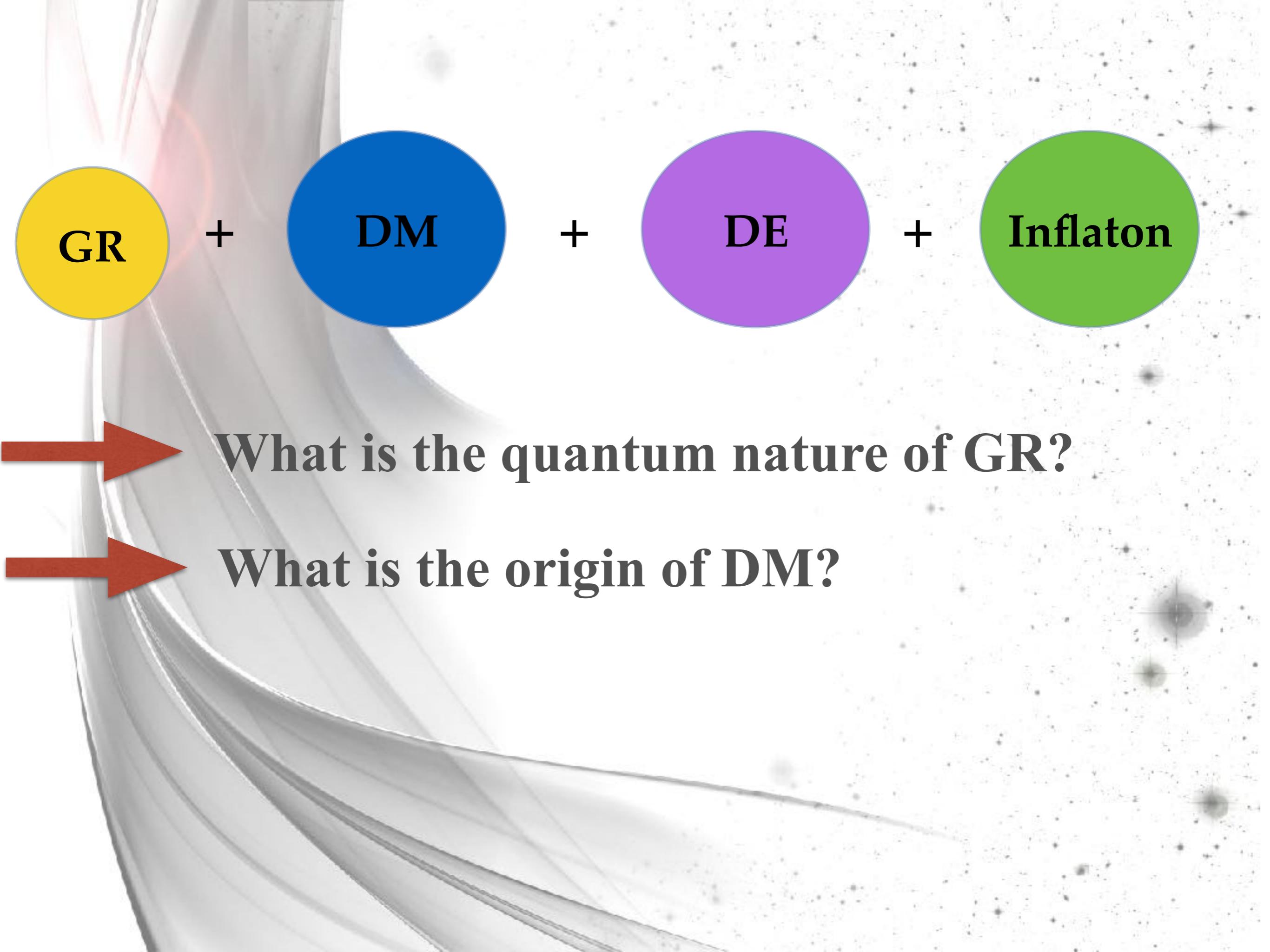
DE

+

Inflaton



What is the quantum nature of GR?



GR

+

DM

+

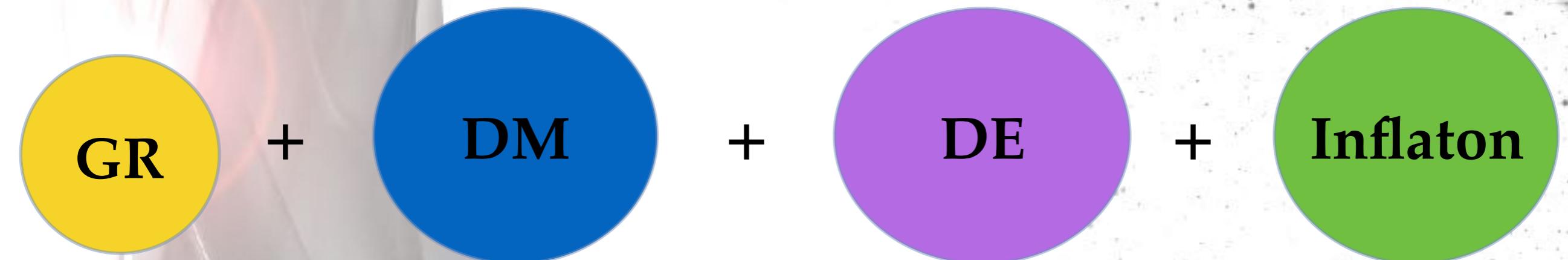
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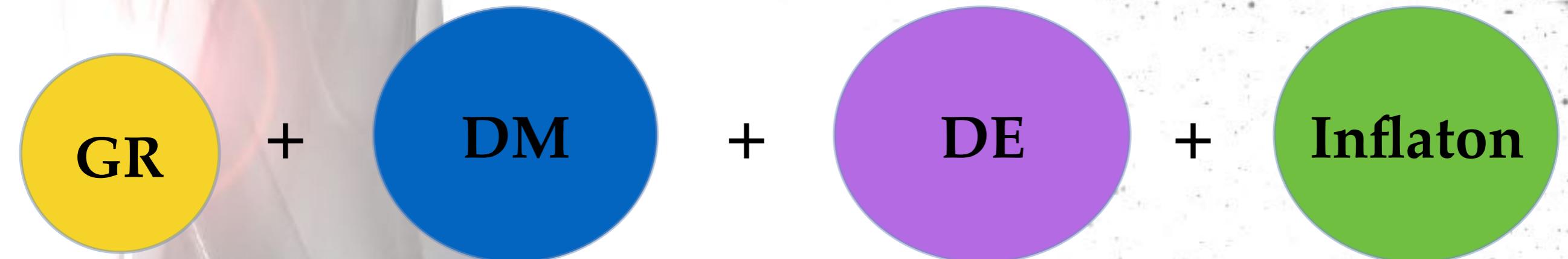
What is the origin of DM?



What is the quantum nature of GR?

What is the origin of DM?

What causes these two accelerated expansion phases of the universe?

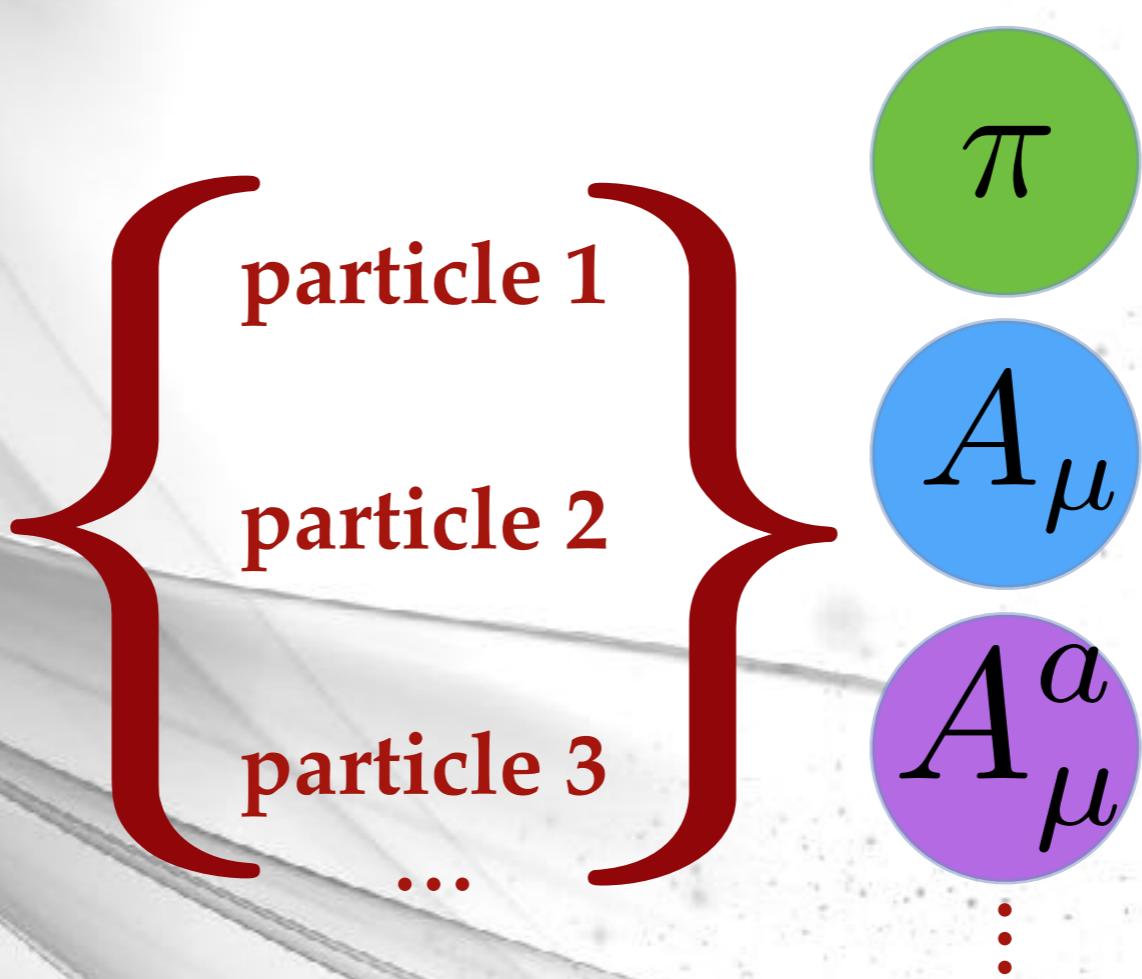


What is the quantum nature of GR?

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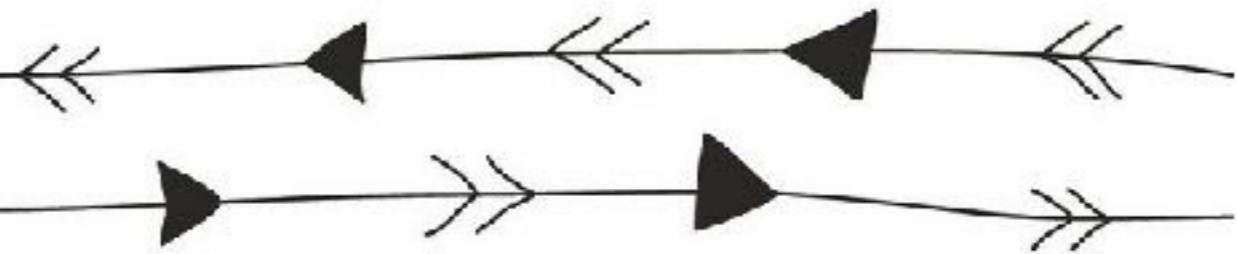
What causes these two accelerated expansion phases of the universe?

Is DE just a cosmological constant or a field evolving in time like the inflaton?





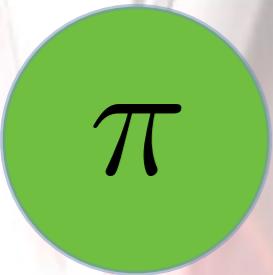
DM      DE      Infl.



particle 1  
particle 2  
particle 3  
...

$\pi$   
 $A_\mu$   
 $A_\mu^a$

⋮

 $\pi$ 

# Scalar Field (Galileons)



$\pi$

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- second order equations of motion
- Lorentz invariant and local
- shift and galileon symmetry  $\pi \rightarrow \pi + c + x_\mu b^\mu$

$\pi$

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- second order equations of motion
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## Galileon interactions

$$\mathcal{L}_2 = (\partial\pi)^2$$

$$\mathcal{L}_3 = (\partial\pi)^2[\Pi]$$

$$\mathcal{L}_4 = (\partial\pi)^2([\Pi]^2 - [\Pi^2])$$

$$\mathcal{L}_5 = (\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3])$$

Nicolis, Rattazzi, Trincherini  
Phys.Rev.D79 064036, 2009

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

$$[\Pi] = \square \pi$$

# Scalar-Tensor-interactions

The generalisation to curved space-time yield the rediscovery of Horndeski interactions

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T. Kobayashi, M. Yamaguchi, J. Yokoyama,  
Prog. Theor. Phys. 126 (2011),511-529

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4,X}([\Pi]^2 - [\Pi^2])$$

$$\mathcal{L}_5 = G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu} - \frac{G_{5,X}}{6}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^2])$$

$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

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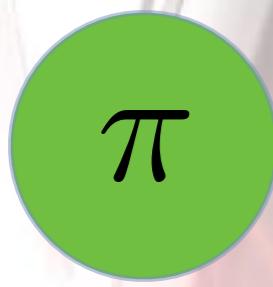
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Non-minimal couplings modifying the gravity sector

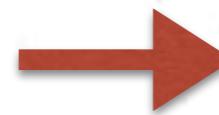
 $\pi$ 

# Scalar Field (Galileons)

Realization of the

CP

Cosmological Principle



Homogeneity  
& Isotropy

is very simple:

$$\pi = \pi(t)$$

$A_\mu$ 

# Vector Field (Generalized Proca)



$A_\mu$ 

# Vector Field (Generalized Proca)

- second order equations of motion
- Lorentz invariant and local
- 3 propagating degrees of freedom

$A_\mu$ 

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- Lorentz invariant and local
- 3 propagating degrees of freedom

L. H., JCAP 1405, 015 (2014),  
arXiv:1402.7026

G.Tasinato JHEP 1404 (2014)067  
arXiv:1402.6450

Allys, Peter, Rodriguez, JCAP  
1602 (2016) 02, 004

L.H & J.Beltran,  
Phys.Lett.B757 (2016) 405-411,  
arXiv:1602.03410

$$\mathcal{L}_2 = f_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = f_3(A^2) \partial \cdot A$$

$$\mathcal{L}_4 = f_4(A^2) [(\partial \cdot A)^2 - \partial_\rho A_\sigma \partial^\sigma A^\rho]$$

$$\mathcal{L}_5 = f_5(A^2) [(\partial \cdot A)^3 - 3(\partial \cdot A) \partial_\rho A_\sigma \partial^\sigma A^\rho$$

$$+ 2\partial_\rho A_\sigma \partial^\gamma A^\rho \partial^\sigma A_\gamma] + \tilde{f}_5(A^2) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \partial_\alpha A_\beta$$

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# Vector-Tensor Theories

L. H., JCAP 1405, 015 (2014),  
arXiv:1402.7026

L.H & J.Beltran,  
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$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

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$$\begin{aligned} \mathcal{L}_5 = & G_5(Y) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,Y} \left[ (\nabla \cdot A)^3 \right. \\ & \left. + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3 (\nabla \cdot A) \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right] \\ & - \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \nabla_\alpha A_\beta \end{aligned}$$

$$\mathcal{L}_6 = G_6(Y) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$

$$Y = -\frac{1}{2} A_\mu A^\mu$$

# Vector-Tensor Theories

Non-minimal couplings  
modifying the gravity  
sector

L. H., JCAP 1405, 015 (2014),  
arXiv:1402.7026

L.H & J.Beltran,  
Phys.Lett.B757 (2016) 405-411,  
arXiv:1602.03410

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# Vector-Tensor Theories

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$$- \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \nabla_\alpha A_\beta$$

$$\mathcal{L}_6 = G_6(Y) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$

$$Y = -\frac{1}{2} A_\mu A^\mu$$

$A_\mu$ 

# Vector Field (Generalized Proca)

**It was believed that a single vector field is in tension with**

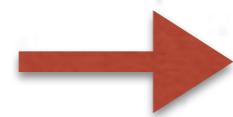
$A_\mu$ 

# Vector Field (Generalized Proca)

It was believed that a single vector field is in tension with

CP

Cosmological Principle



Homogeneity  
& Isotropy

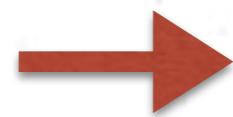
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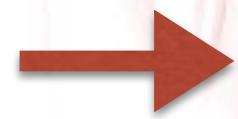
Cosmological Principle



Homogeneity  
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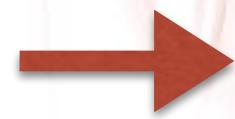
and therefore not appropriate for dark energy applications.

# Vector Field (Generalized Proca)



**Generalized Proca initiated a radiacal change of view!**

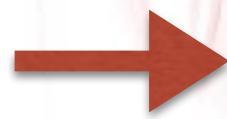
# Vector Field (Generalized Proca)



**Generalized Proca initiated a radiacal change of view!**

$$A_\mu = (A_0(t), 0, 0, 0)$$

# Vector Field (Generalized Proca)

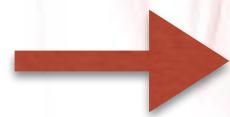


Generalized Proca initiated a radiacal change of view!

GR

$$+ \boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + G_2\left(-\frac{1}{2}A_\mu A^\mu\right) + G_3\left(-\frac{1}{2}A_\mu A^\mu\right)\nabla_\alpha A^\alpha}$$

# Vector Field (Generalized Proca)



Generalized Proca initiated a radiacal change of view!

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

- Dark Energy fixed point

G.Tasinato JHEP 1404 (2014)067

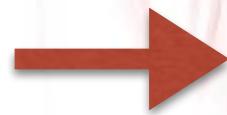
arXiv:1402.6450

L.H. & de Felice,Kase,Mukohyama,

Tsujikawa,Zhang, JCAP

1606,2016,06,048, arXiv:1603.05806

# Vector Field (Generalized Proca)



Generalized Proca initiated a radiacal change of view!

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

- Dark Energy fixed point

G.Tasinato JHEP 1404 (2014)067

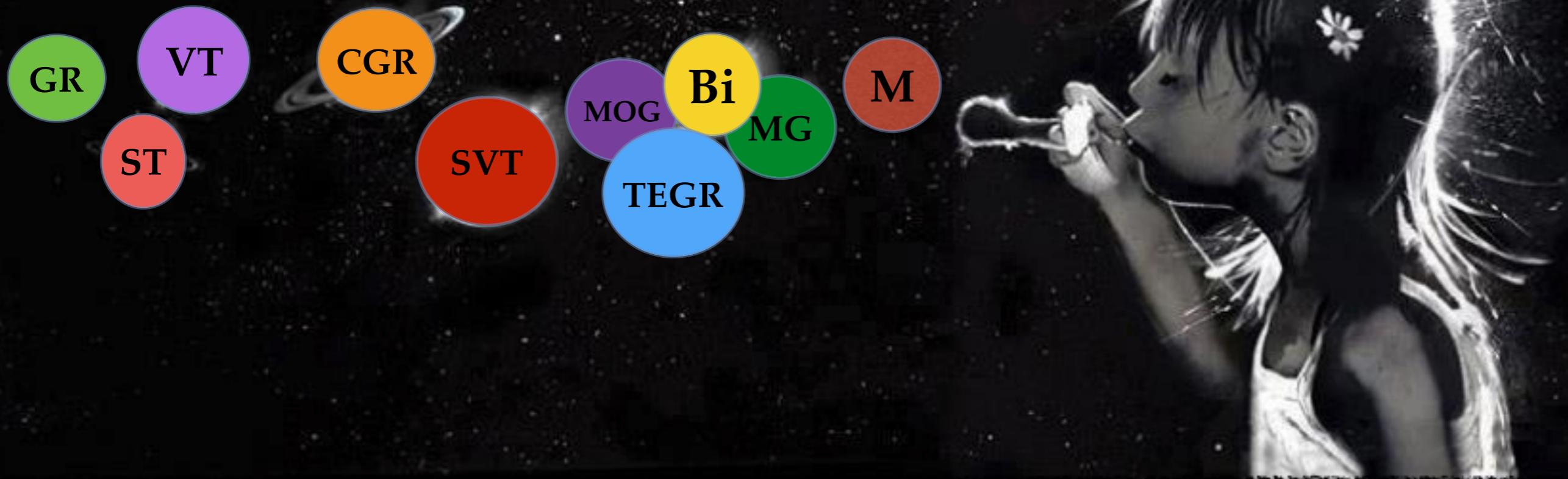
arXiv:1402.6450

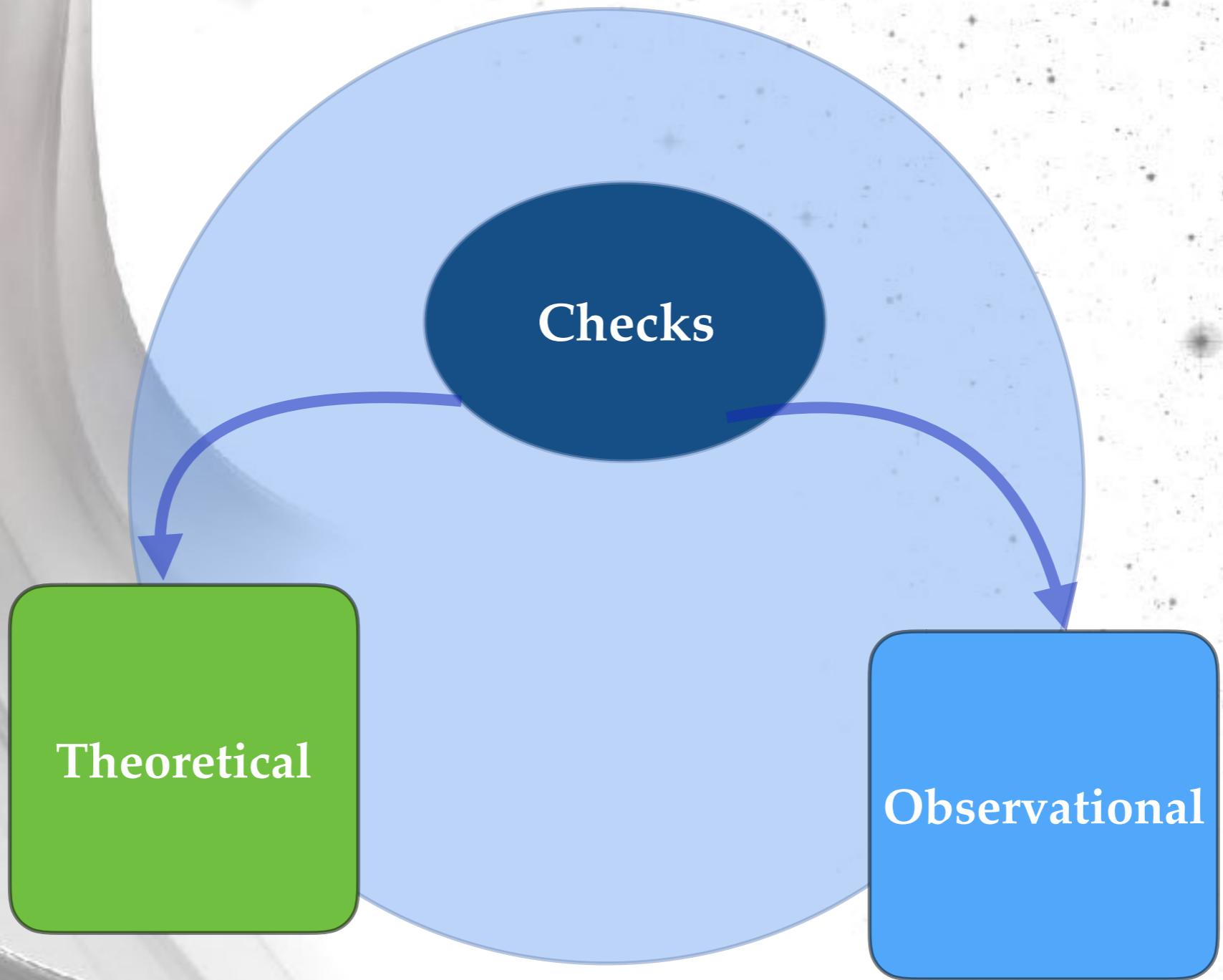
L.H. & de Felice,Kase,Mukohyama,  
Tsujikawa,Zhang, JCAP  
1606,2016,06,048, arXiv:1603.05806

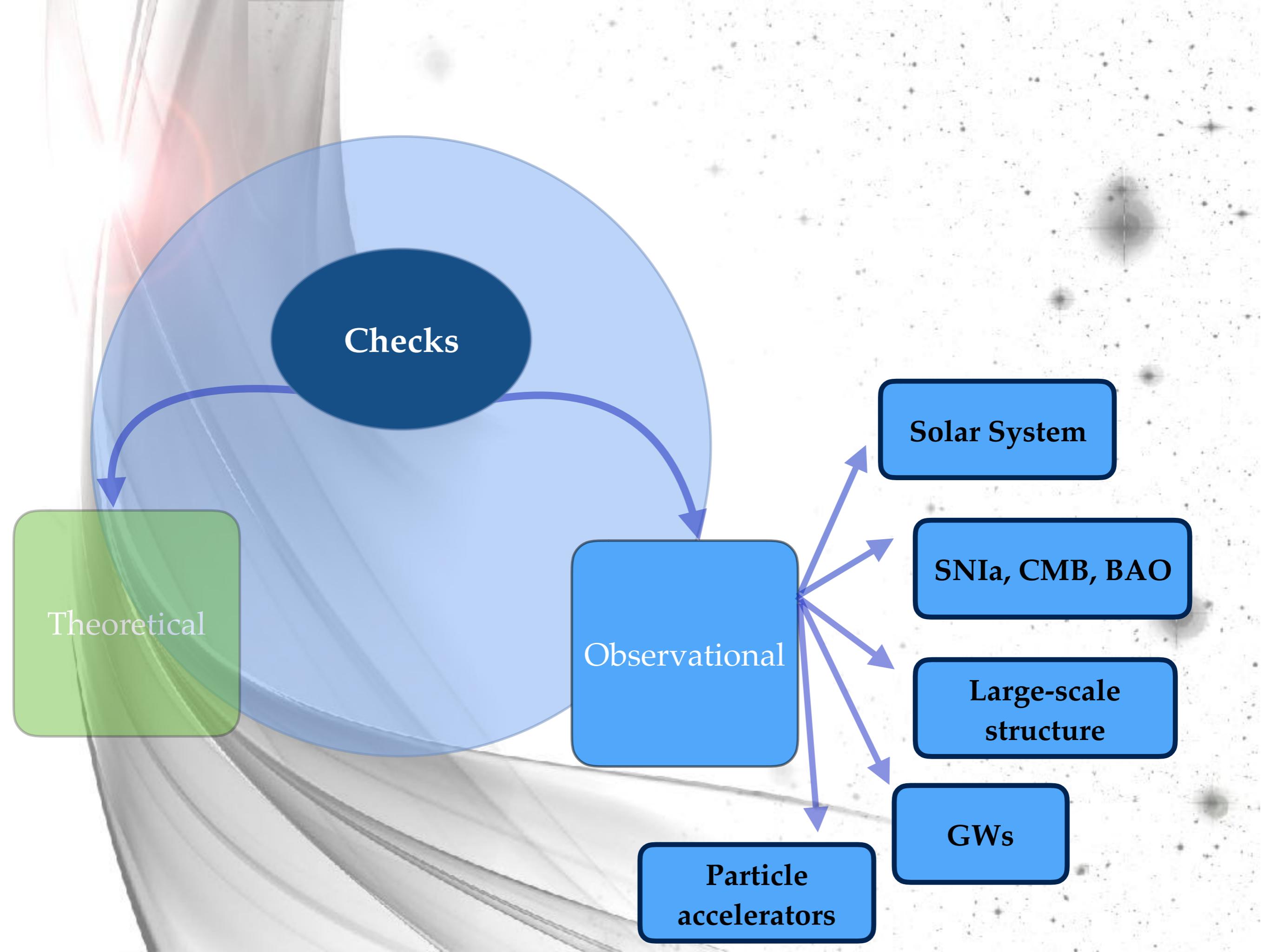
- Reduces the  $H_0$  tension and delivers a better fit to data

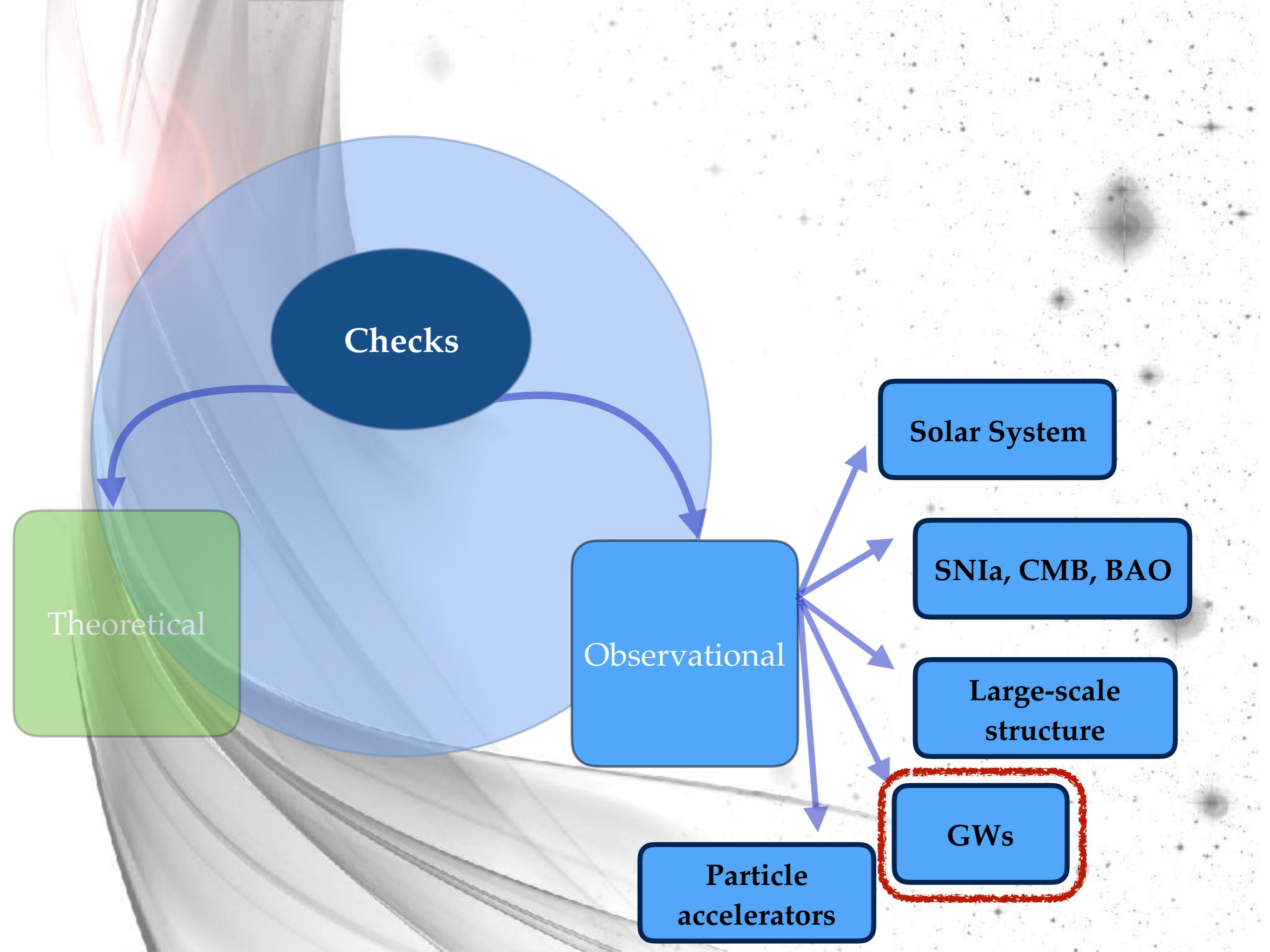
L.H. & de Felice,Tsujikawa, PRD95  
(2017)12, 123540, arXiv:1703.09573

# Cosmological Models









GWs

# What is a Gravitational Wave?

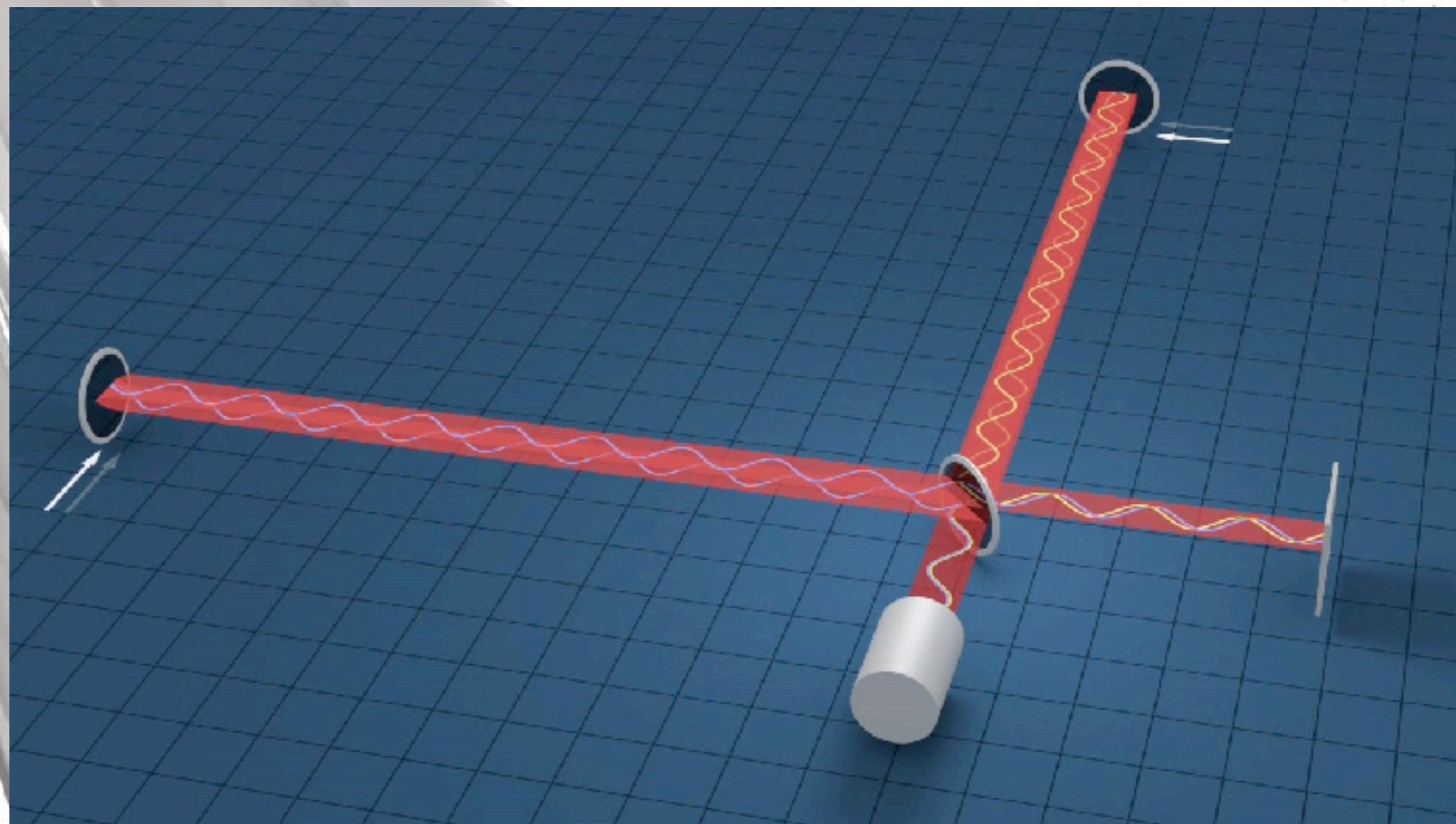
**GWs**

**GWs are ripples of space-time**

**Giving rise to changes in the lengths and shapes**

GWs are ripples of space-time

Giving rise to changes in the lengths and shapes



**GWs are ripples of space-time**

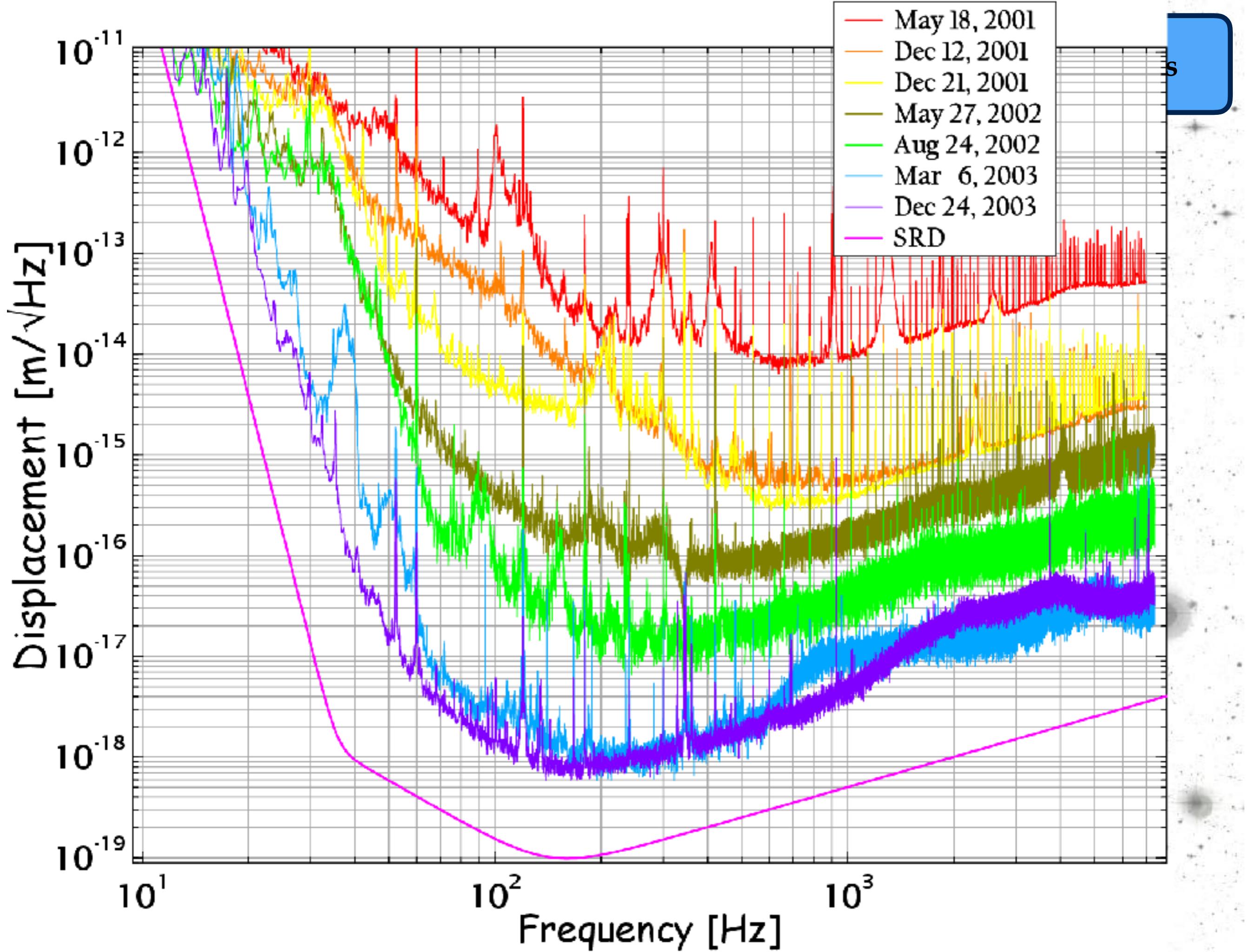
**Giving rise to changes in the lengths and shapes**

**The measurement challenge**

$$h = \frac{\Delta L}{L} \leq 10^{-21}$$

$$L = 4\text{km}$$

$$\Delta L = 4 \times 10^{-18} \text{ meters}$$

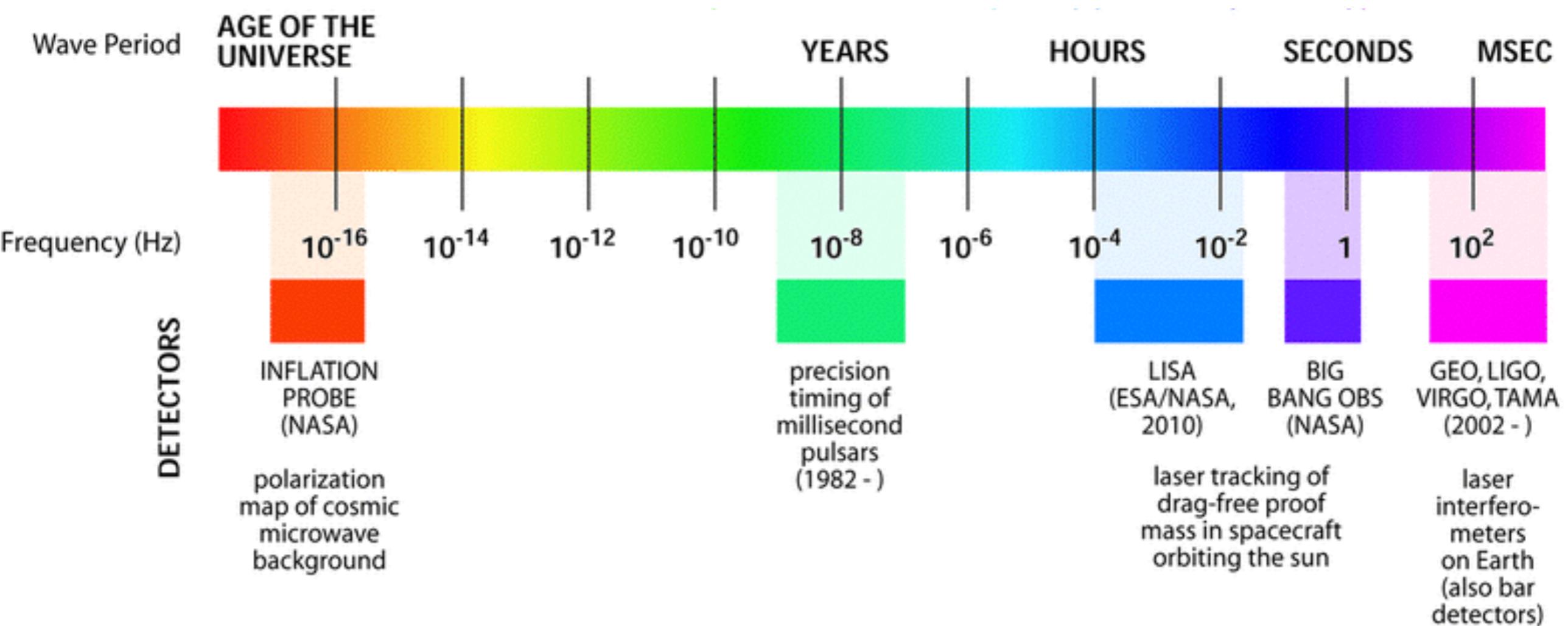


**GWs are ripples of space-time**

**Giving rise to changes in the lengths and shapes**

**Like the electromagnetic spectrum there is  
the GW spectrum**

# THE GRAVITATIONAL WAVE SPECTRUM



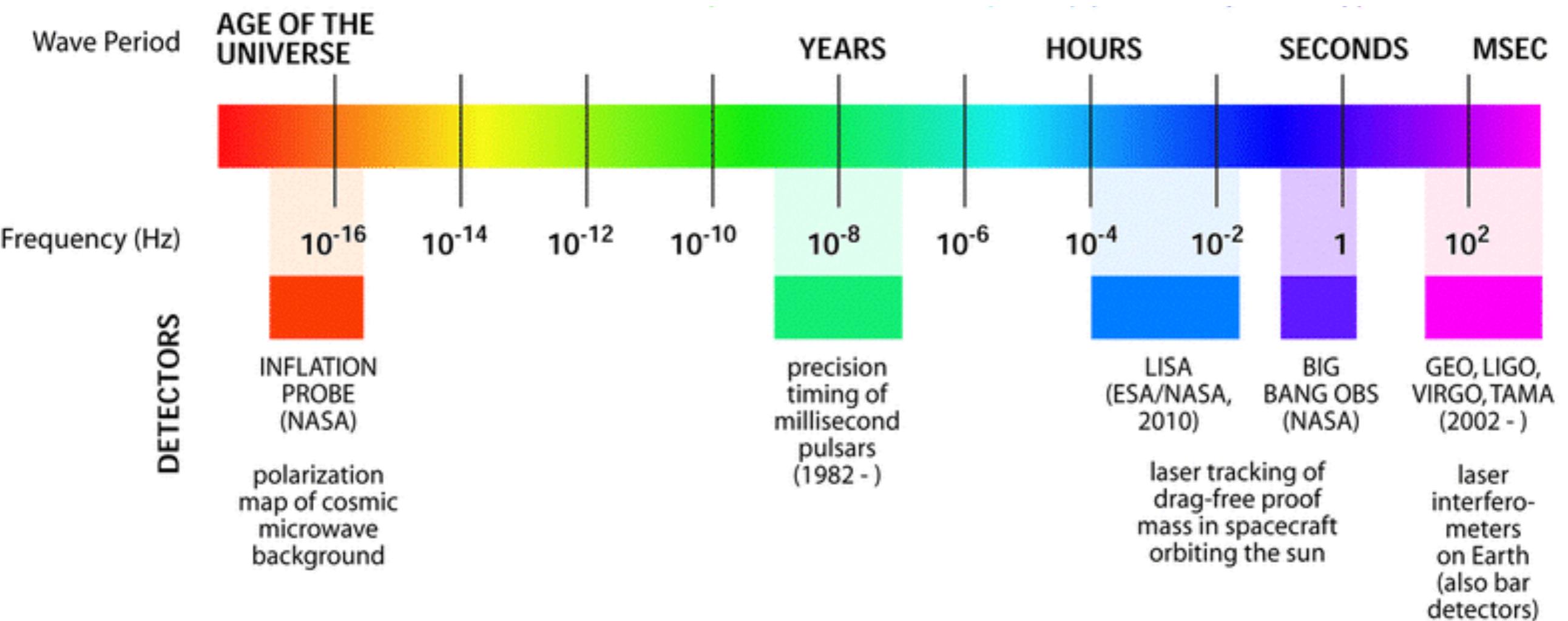
# Pulsar Timing Array

a set of pulsars is analysed to search for correlated signatures in the pulse arrival times

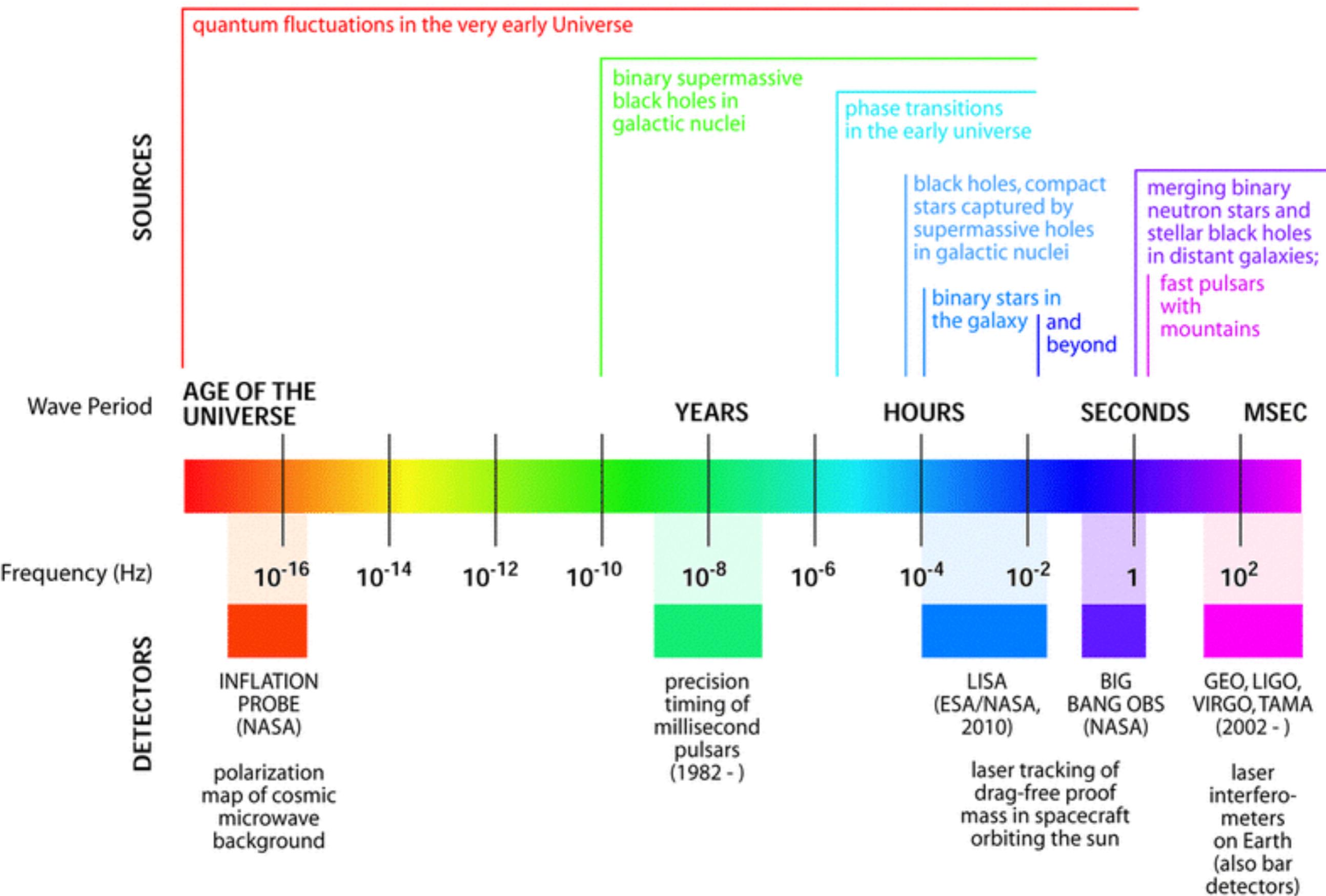


# THE GRAVITATIONAL WAVE SPECTRUM

Different wavelengths probe  
BHs of different sizes  
Neutron stars and BHs at  
different separation and lifetimes

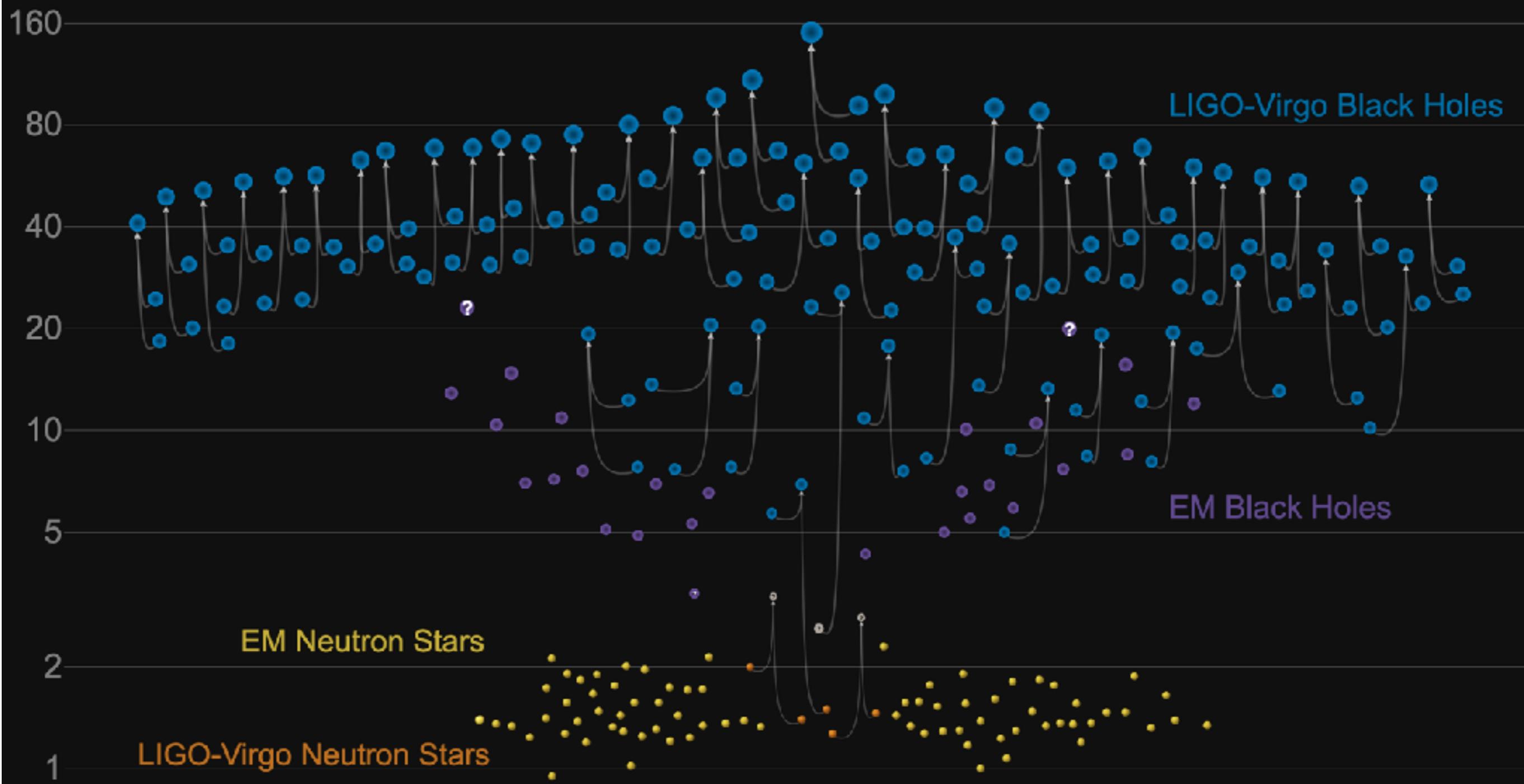


# THE GRAVITATIONAL WAVE SPECTRUM



# Masses in the Stellar Graveyard

*in Solar Masses*



GWTC-2 plot v1.0  
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

# Gravitational Waveform

We test gravity by the shapes of  
the gravitational waveform

# Gravitational Waveform

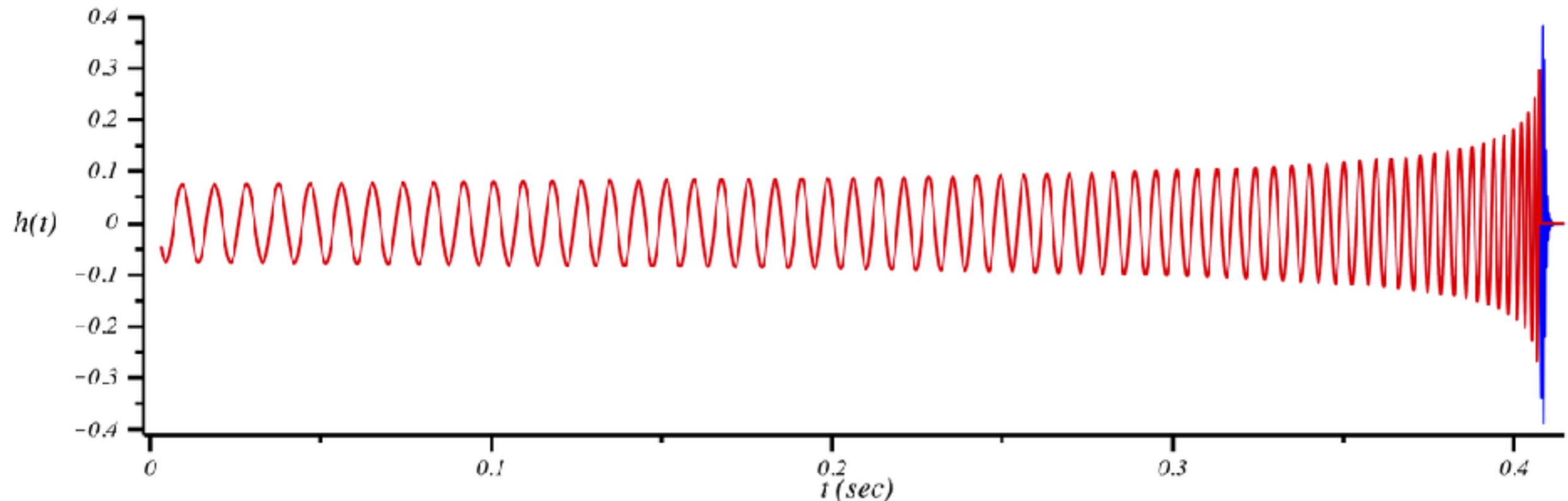
Different wavelengths probe  
BHs of different sizes  
Neutron stars and BHs at  
different separation and lifetimes

$$w = \sqrt{\frac{GM}{r^3}}$$

$$f_{\text{GW}} = \frac{w}{\pi}$$

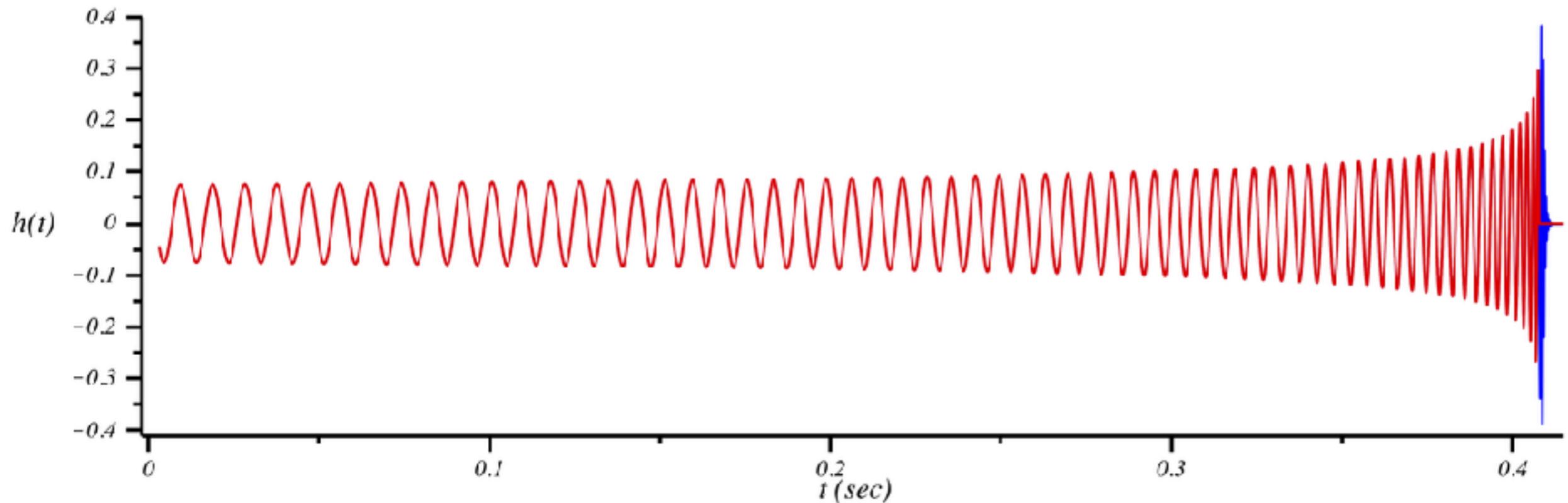
# Gravitational Waveform

From frequency evolution one can infer masses



# Gravitational Waveform

From frequency evolution one can infer masses

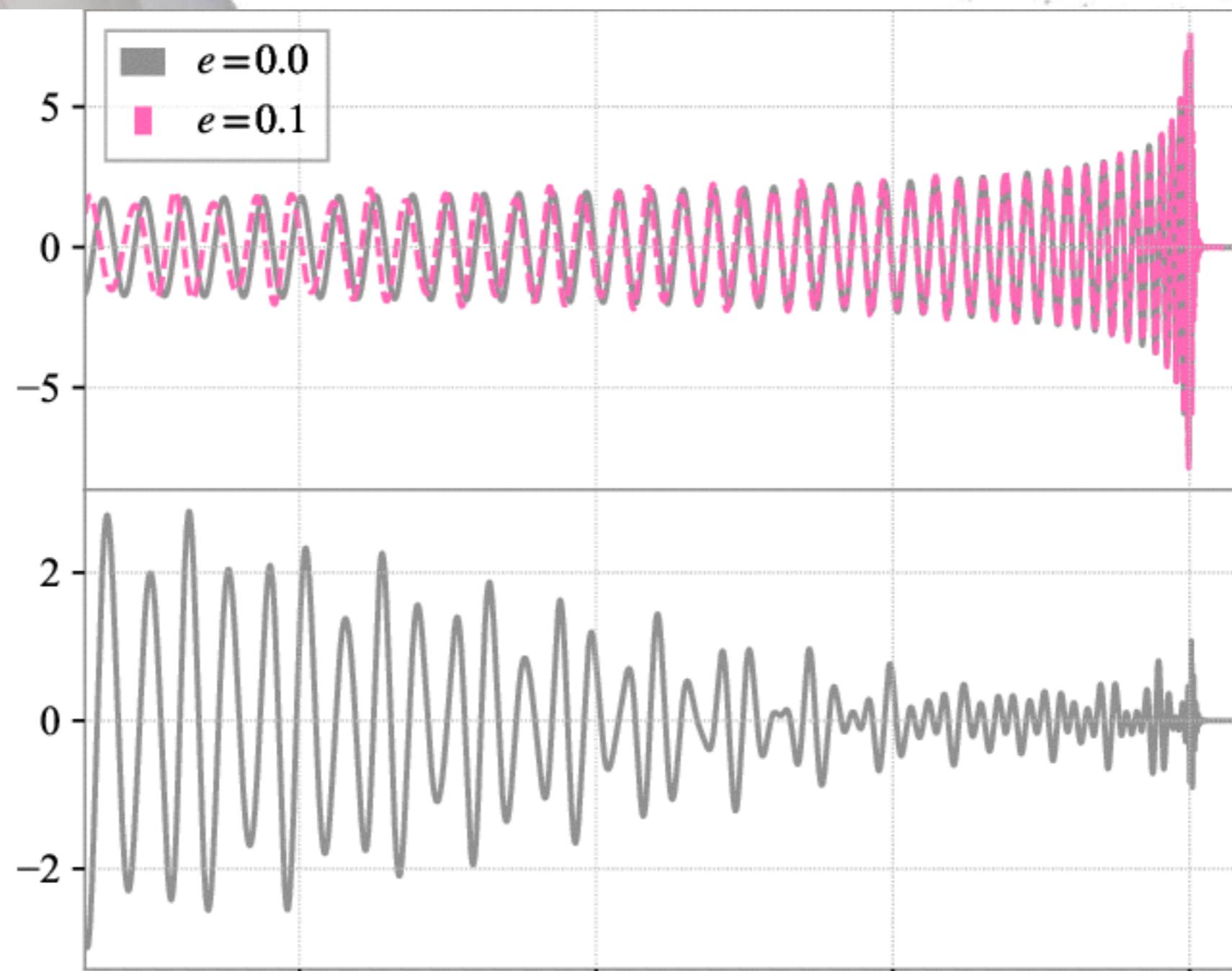


From amplitude and masses one infers distance

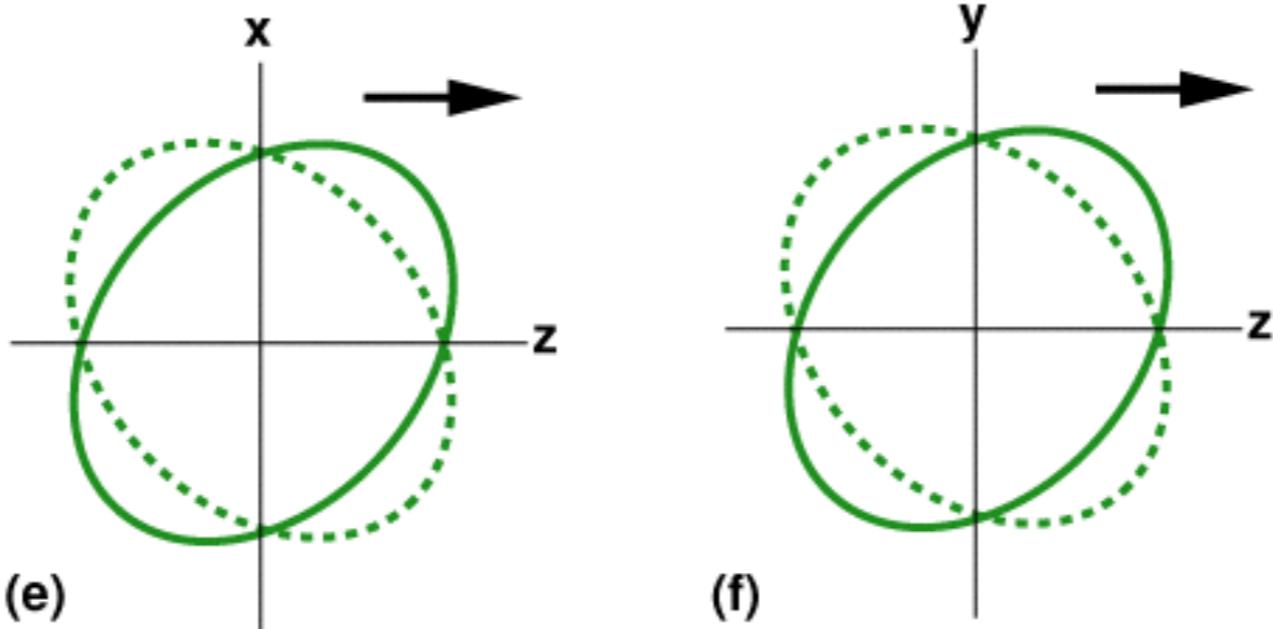
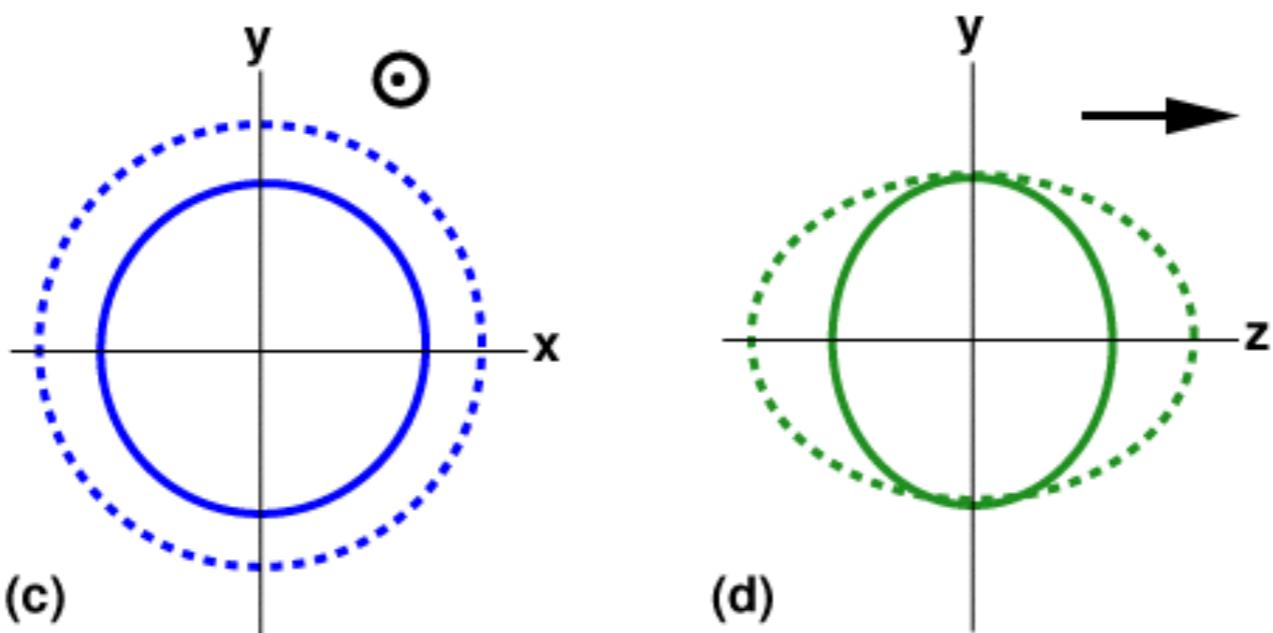
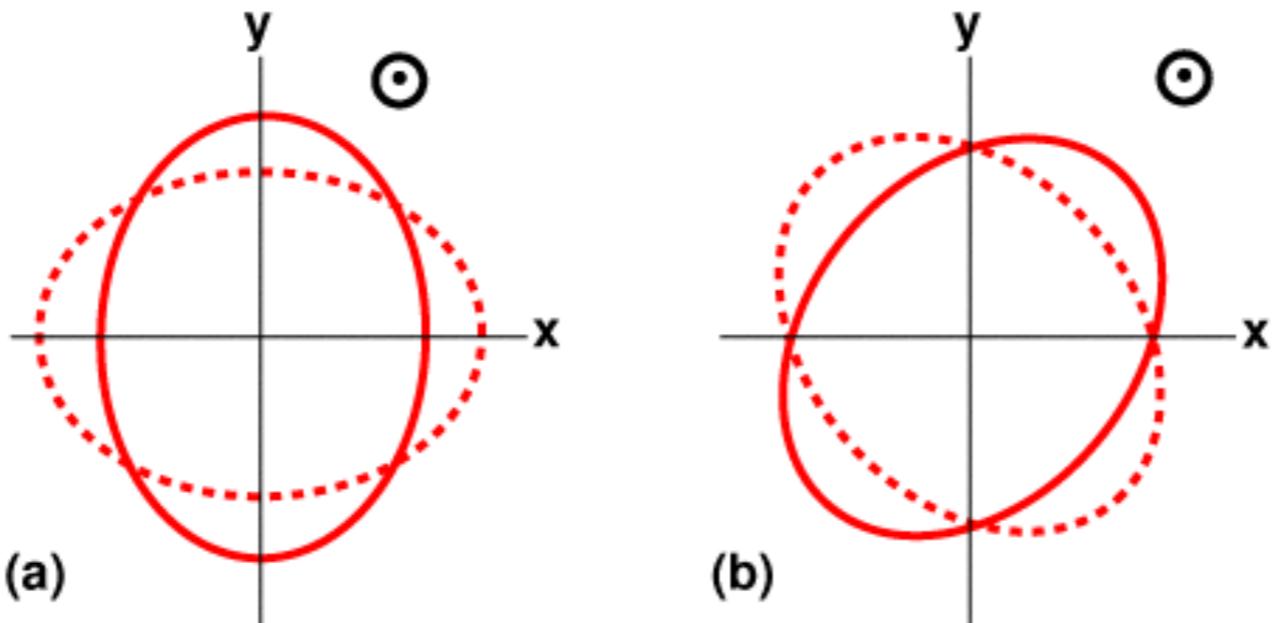
From time of arrival, amplitude and phase at detectors  
one infers sky location.

# Gravitational Waveform

From modulations of amplitude and phase one infers spins and eccentricity



# Gravitational-Wave Polarization

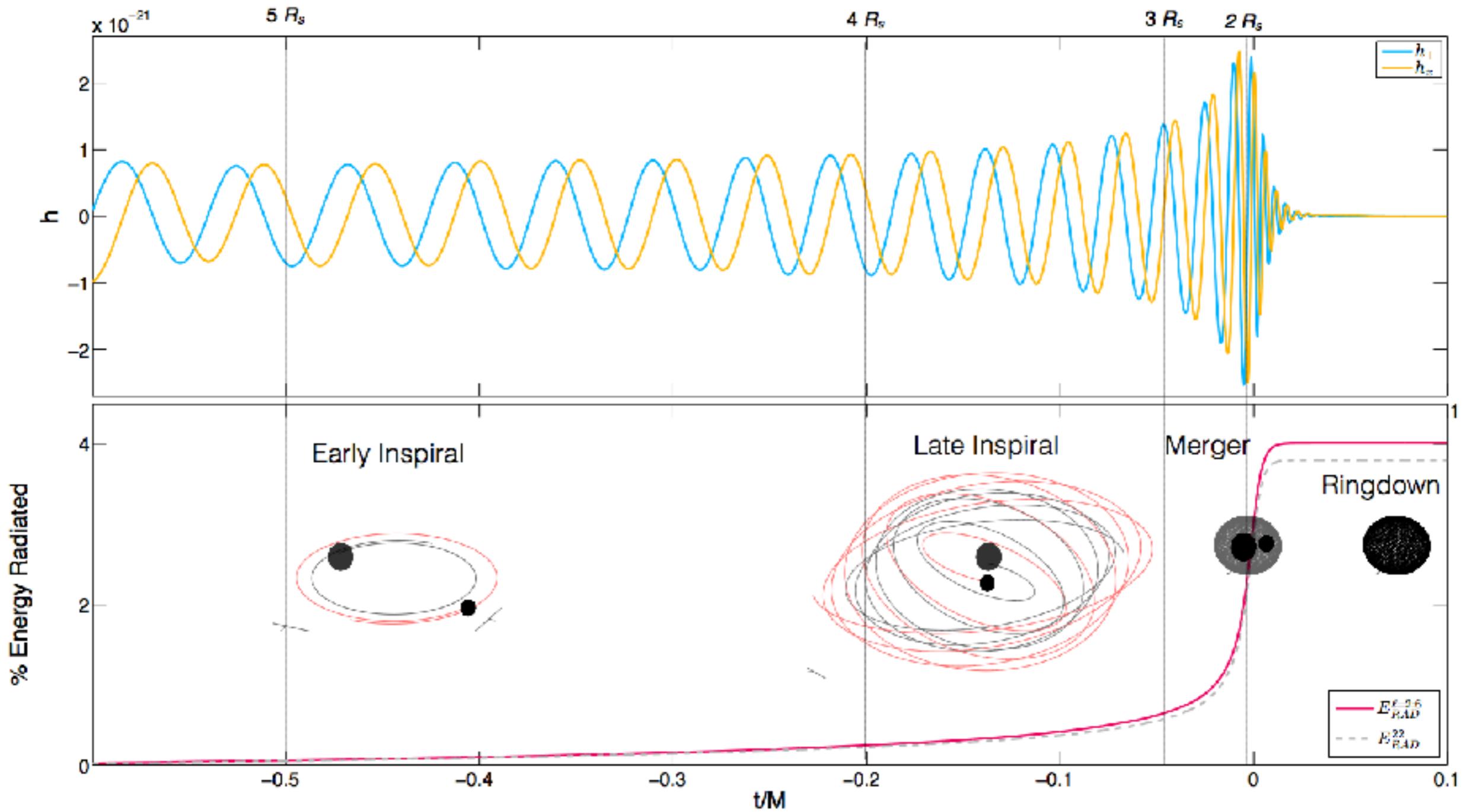


# Gravitational Waveform

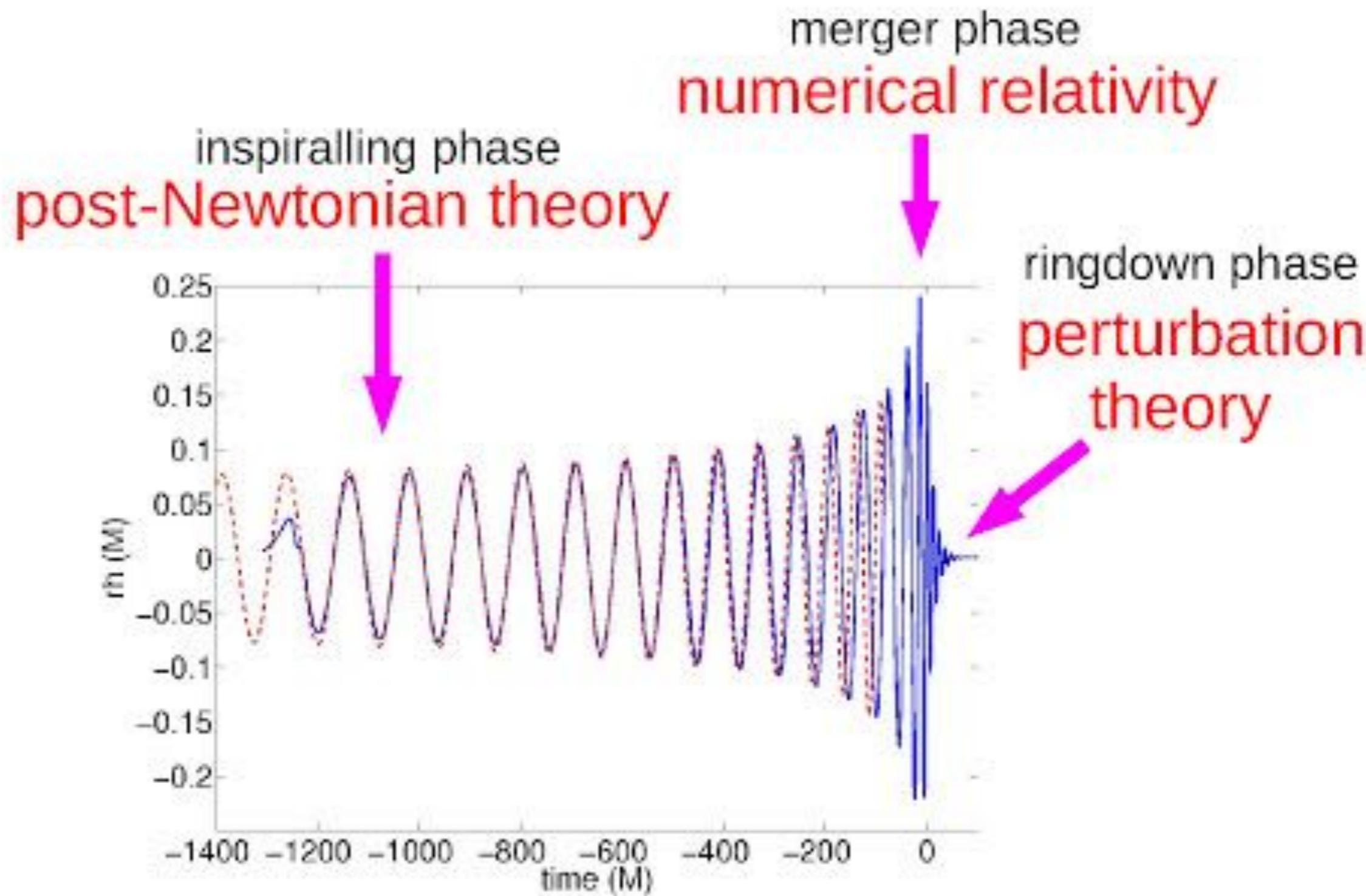
We have to solve Einstein's field equations in order to generate the GW waveform

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

# Gravitational Waveform



# Gravitational Waveform



# Gravitational Waveform

Linear regime

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu}$$

# Gravitational Waveform

Linear regime

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu}$$

$$S_T^{(2)} = \sum_{\lambda} \int d^4x a^3 M_{\text{Pl}}^2 \left\{ \dot{h}_{\lambda}^2 - \frac{c_T^2}{a^2} (\partial h_{\lambda})^2 \right\}$$

# Gravitational Waveform

**Linear regime**

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu}$$

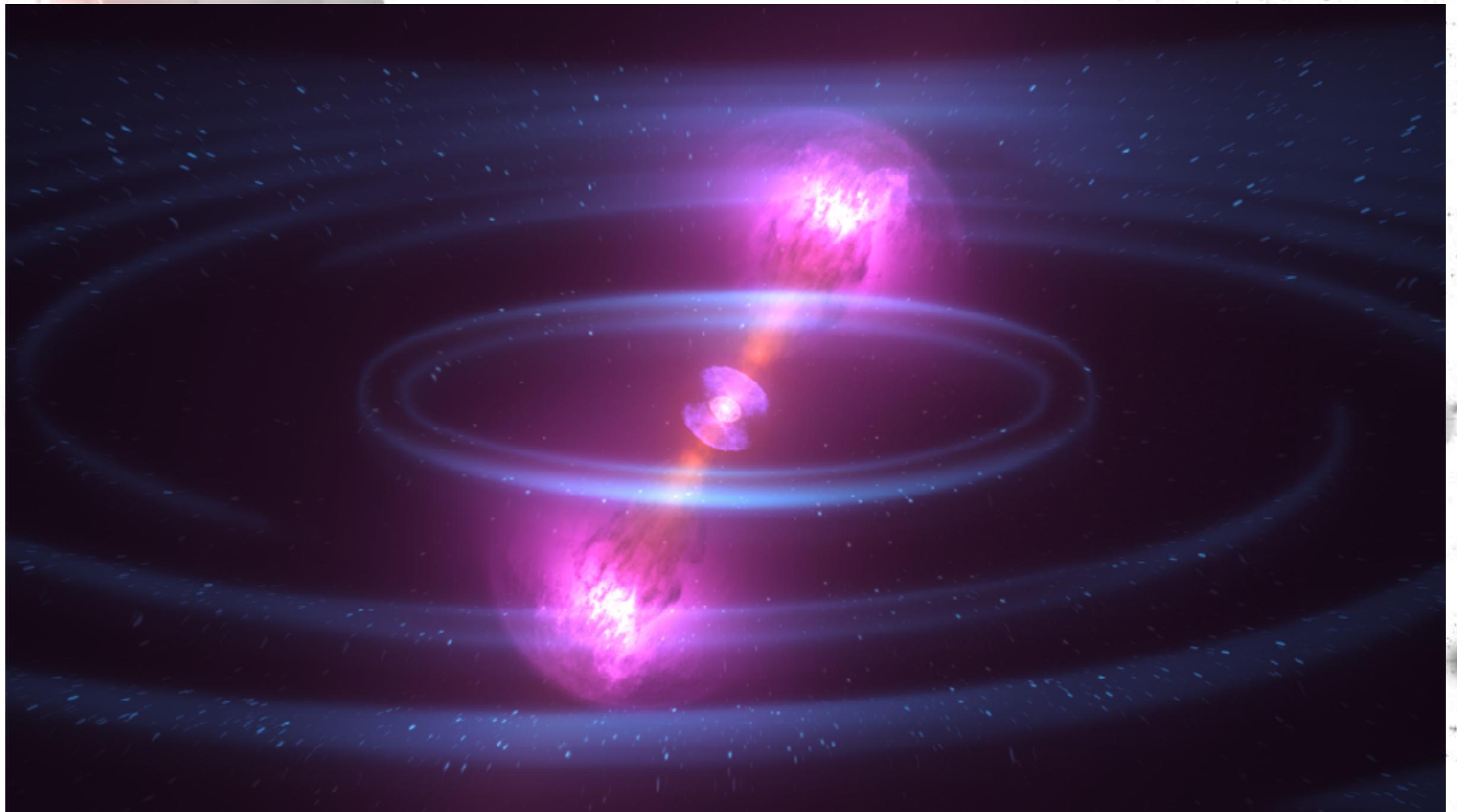
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**GW strain**

$$h_{\text{GW}}(f) = A_{\text{GW}}(f) e^{i\psi_{\text{GW}}(f)}$$

**GWs**

# Multi-messenger: GWs & EM



GWs

# Multi-messenger: GWs & EM

Gravitational Waves propagate  
with the speed of light!

$$c_{\text{gw}}^2 = 1$$

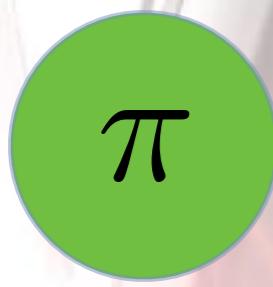
# Gravitational Waveform

Linear regime

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$c \pm 10^{-15}$



$\pi$

# Scalar Field (Horndeski)



Luminal GWs propagation

$\pi$

# Scalar Field (Horndeski)

Luminal GWs propagation

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + \cancel{G_{4,X}([\Pi]^2 - [\Pi^2])}$$

$$\mathcal{L}_5 = \cancel{G_5(\pi, X)} G_{\mu\nu} \Pi^{\mu\nu} - \frac{\cancel{G_{5,X}}}{6} (\cancel{[\Pi]^3} - 3\Pi[\Pi^2] + 2\Pi^2)$$

$$c_{\text{gw}}^2 = 1$$

P. Creminelli, F. Vernizzi, PRL119  
(2017) 251302

J. M. Ezquiaga, M. Zumalacarregui,  
PRL119 (2017) 251304

$A_\mu$ 

# Vector Field (Generalized Proca)



## Luminal GWs propagation

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

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$$\left. + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3 (\nabla \cdot A) \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right]$$

$$- \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \nabla_\alpha A_\beta$$

$$\mathcal{L}_6 = G_6(Y) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$

$$c_{\text{gw}}^2 = 1$$

$A_\mu$ 

# Vector Field (Generalized Proca)

Luminal GWs propagation

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$$c_{\text{gw}}^2 = 1$$

$A_\mu$ 

# Vector Field (Generalized Proca)

No effects on GWs, but on scalar perturbations!

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

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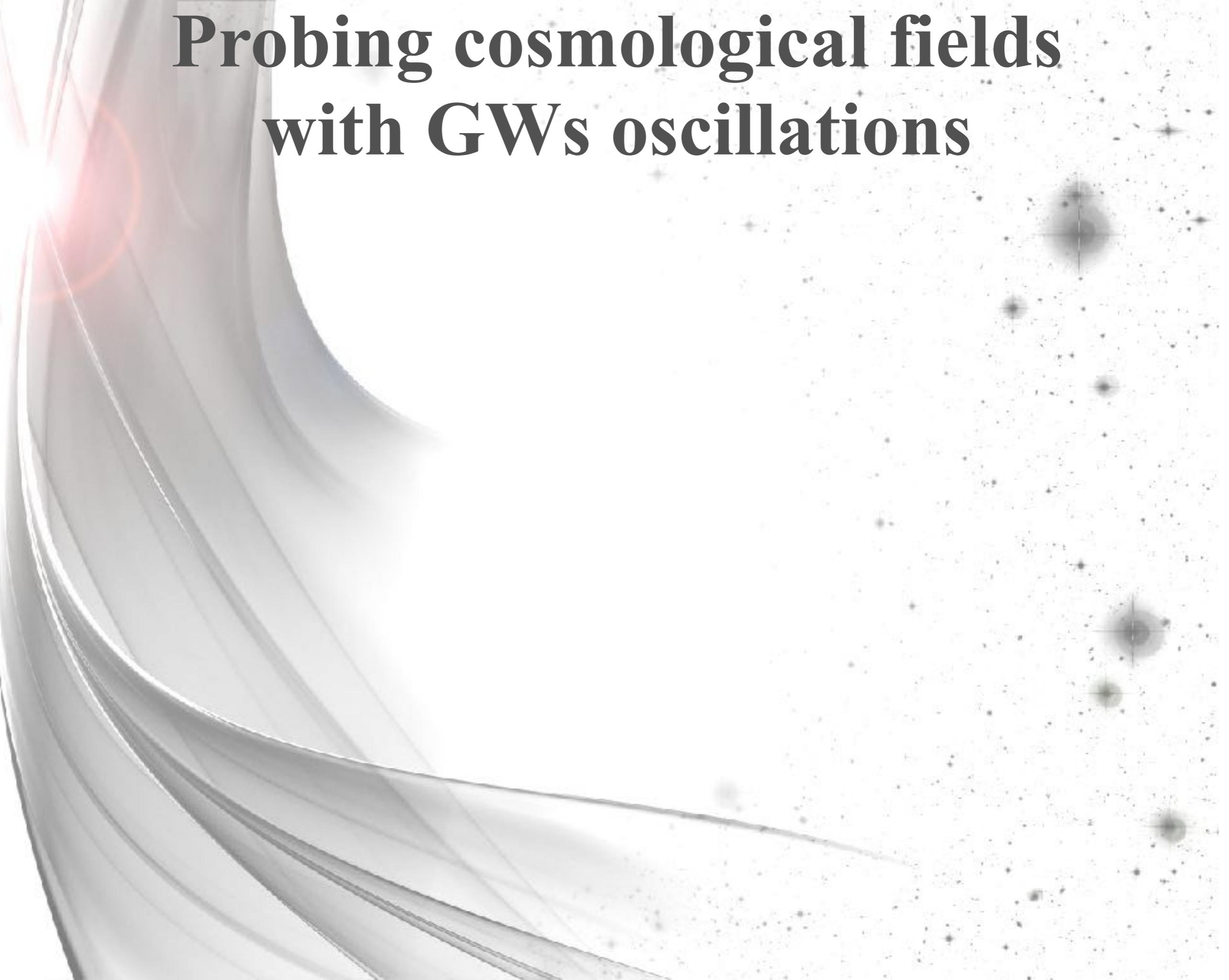
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$$- \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \nabla_\alpha A_\beta$$

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$$c_{\text{gw}}^2 = 1$$

# Probing cosmological fields with GWs oscillations



# Probing cosmological fields with GWs oscillations

GR

+

YM

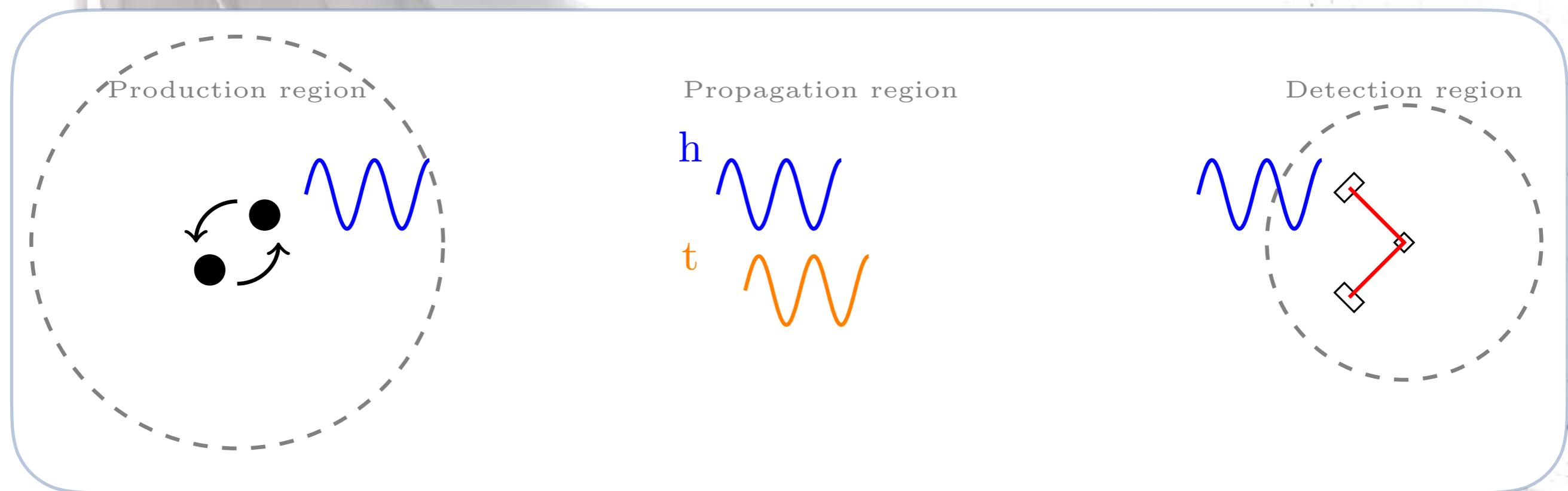
$$g_{\mu\nu}$$

$$A_\mu^a$$

# Probing cosmological fields with GWs oscillations

GR + YM

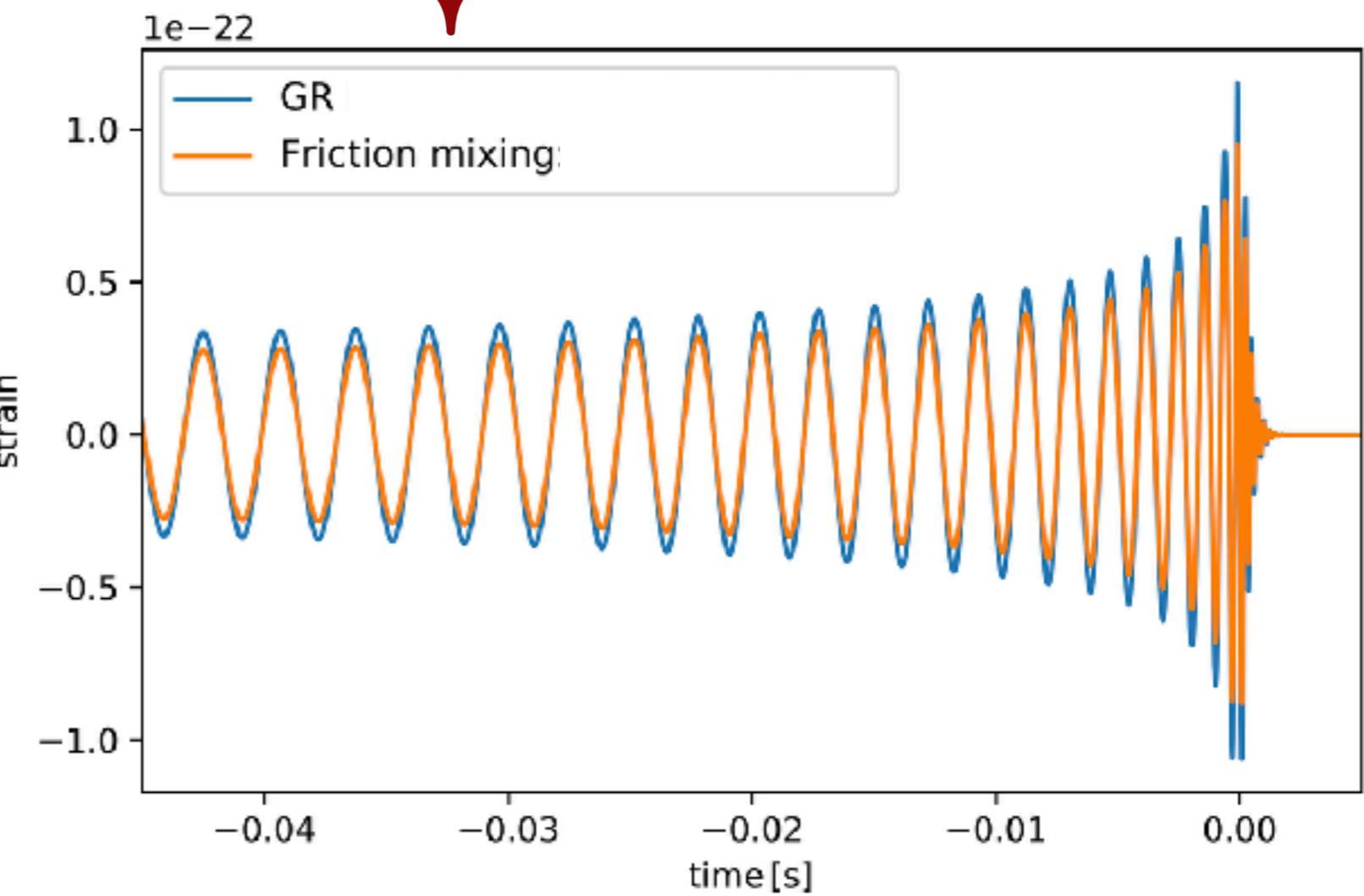
$$g_{\mu\nu} \quad A_{\mu}^a$$



# Probing cosmological fields with GWs oscillations

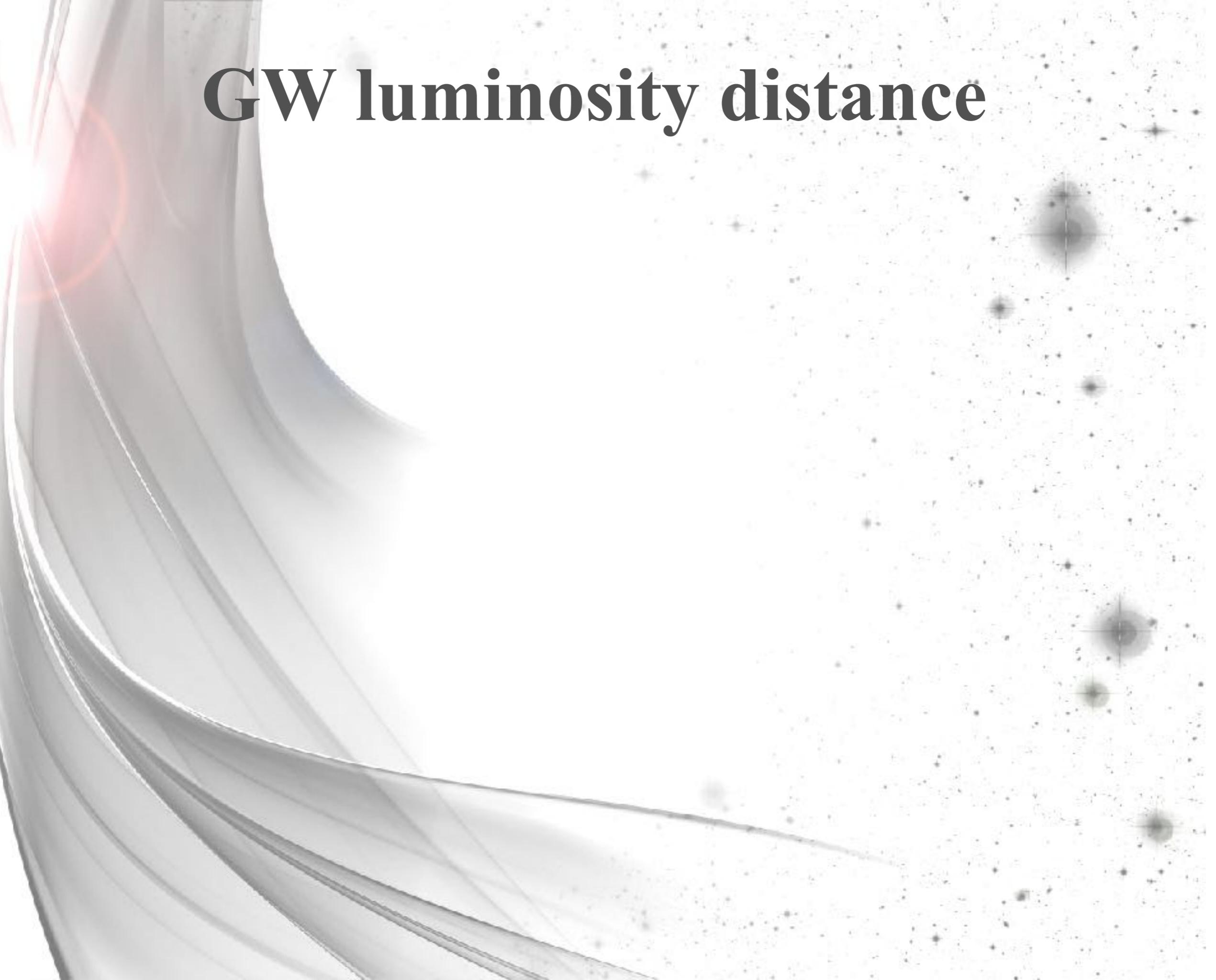


$$\left[ \frac{d^2}{d\eta^2} + \begin{pmatrix} 0 & -2\alpha \\ 2\alpha & 4\Delta\nu \end{pmatrix} \frac{d}{d\eta} + \begin{pmatrix} c_h^2 & 0 \\ 0 & c_t^2 \end{pmatrix} k^2 \right] \begin{pmatrix} h \\ t \end{pmatrix} = 0$$



L.H & J.Beltran, J. Ezquiaga,  
arXiv:1912.06104

# GW luminosity distance



# GW luminosity distance

$$d_{\text{L}}^{\text{GW}} = \frac{M_c^{5/3} f^{2/3}}{|h_{+,\times}|} F_{+,\times}$$

# GW luminosity distance

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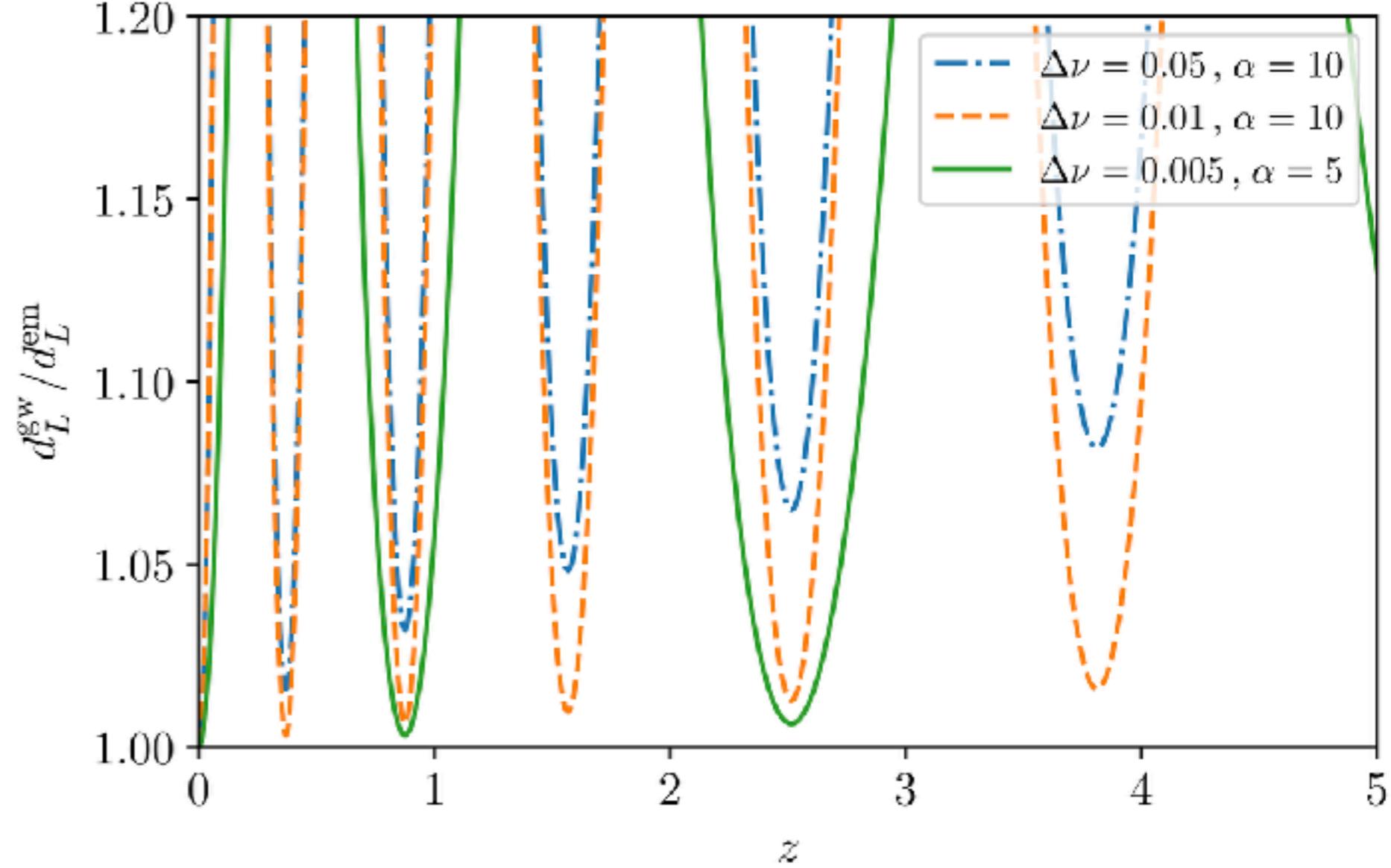
GWs as standard sirens!

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{\text{DE}}(z)}$$

# GW luminosity distance

GR + YM

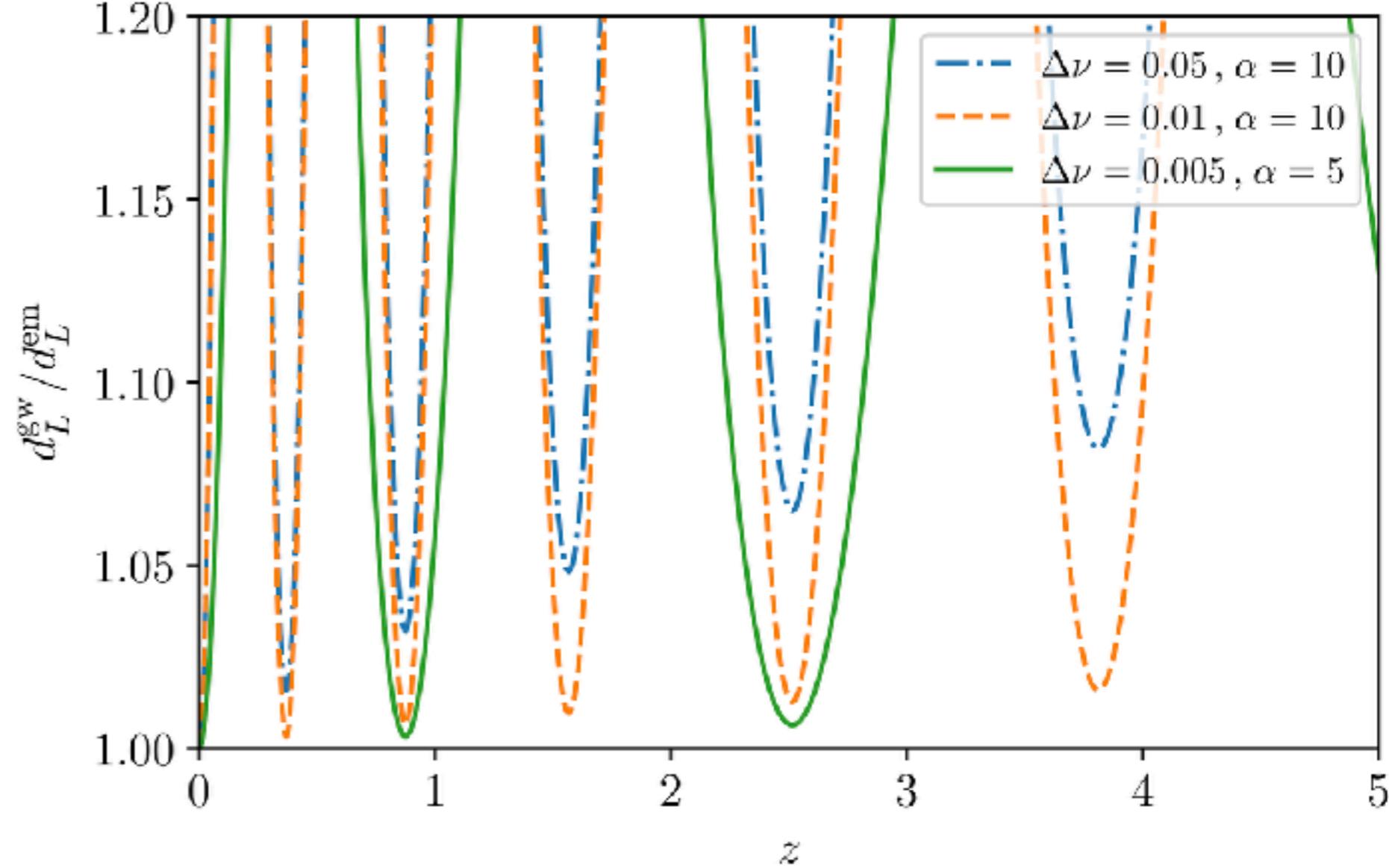
$$g_{\mu\nu} \quad A_{\mu}^a$$



# GW luminosity distance

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# A systematic approach to generalisations of General Relativity and their cosmological implications

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## Abstract

A century ago, Einstein formulated his elegant and elaborate theory of General Relativity, which has so far withstood a multitude of empirical tests with remarkable success. Notwithstanding the triumphs of Einstein's theory, the tenacious challenges of modern cosmology and of particle physics have motivated the exploration of further generalised theories of spacetime. Even though Einstein's interpretation of gravity in terms of the curvature of spacetime is commonly adopted, the assignment of geometrical concepts to gravity is ambiguous because General Relativity allows three entirely different, but equivalent approaches of which Einstein's interpretation is only one. From a field-theoretical perspective, however, the construction of a consistent theory for a Lorentz-invariant massless spin-2 particle uniquely leads to General Relativity. Keeping Lorentz invariance then implies that any modification of General Relativity will inevitably introduce additional propagating degrees of freedom into the gravity sector. Adopting this perspective, we will review the recent progress in constructing consistent field theories of gravity based on additional scalar, vector and tensor fields. Within this conceptual framework, we will discuss theories with Galileons, with Lagrange densities as constructed by Horndeski and beyond, extended to DHOST interactions, or containing generalized Proca fields and extensions thereof, or several Proca fields, as well as bigravity theories and scalar-vector-tensor theories. We will review the motivation of their inception, different formulations, and essential results obtained within these classes of theories together with their empirical viability.

**Keywords:** Modified Gravity, Massive Gravity, Scalar-Tensor theories, Generalized Proca, Multi-Proca, Scalar-Vector-Tensor theories, Cosmology

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