Collective neutrino oscillations
in the early Universe

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NBIA and DARK at the Niels Bohr Institute

UNIVERSITY OF COPENHAGEN

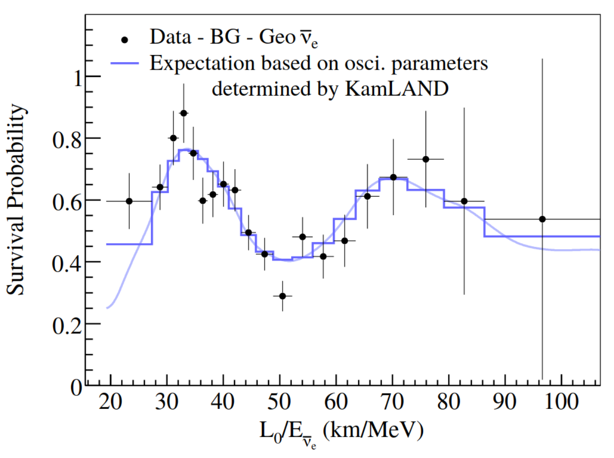
INTERACTIONS

Co-financed by the Connecting Europe Facility of the European Union

Sapere Aude

Neutrino oscillations

Vacuum oscillations



KamLAND: 0801.4589

Energy:

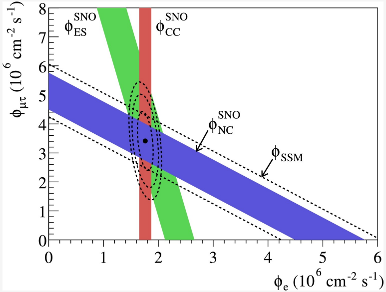
$$E = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p}.$$

Survival probability:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

Neutrino oscillations

Solar neutrinos



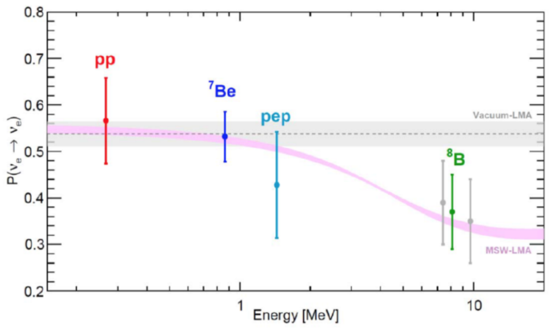
SNO: 1602.02469

Vacuum dominated at low energy.

Matter dominated at high energy.

MSW (Mikheev-Smirnov-Wolfenstein) potential:

$$V_e = \sqrt{2} G_F n_e$$



Borexino: 1810.12967

Density matrix

$a^\dagger_{i\mathbf{p}}$ - creation operator for neutrino with flavor i and momentum $\mathbf{p}$.
$a_{i\mathbf{p}}$ - annihilation operator for neutrino with flavor i and momentum $\mathbf{p}$.
The density matrices are

$$\rho_{ij} = \left\langle a^\dagger_i a_j \right\rangle_{\mathbf{p}} \, , \qquad \bar\rho_{ij} = \left\langle \bar a_{\mathbf{j}}{}^\dagger \bar a_{\mathbf{i}} \right\rangle_{\mathbf{p}} \, .$$

Neutrino oscillations described by

$$\frac{d\rho(\mathbf{p},\mathbf{x})}{dt} = -i\left[\mathcal{H}(\rho,\mathbf{p},\mathbf{x}),\rho(\mathbf{p},\mathbf{x})\right] \, , \qquad \frac{d\bar\rho(\mathbf{p},\mathbf{x})}{dt} = -i\left[\bar{\mathcal{H}}(\rho,\mathbf{p},\mathbf{x}),\bar\rho(\mathbf{p},\mathbf{x})\right] \, .$$

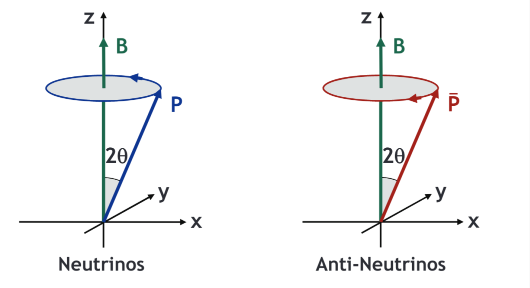
Hamiltonians for vacuum oscillations

$$\mathcal{H}(\rho,\mathbf{p},\mathbf{x}) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p} \, , \qquad \bar{\mathcal{H}}(\rho,\mathbf{p},\mathbf{x}) = -\frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p}$$

Polarization vector - two flavor oscillations

$$\rho = \frac{1}{2}\left(P_0 + \vec{P}\cdot\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}, \qquad \mathcal{H} - \frac{1}{2}\mathrm{Tr}(\mathcal{H}) = \frac{1}{2}\begin{pmatrix} V_z & V_x - iV_y \\ V_x + iV_y & -V_z \end{pmatrix}$$

$$\frac{d}{dt}\vec{P} = \vec{V}\times\vec{P}, \qquad \vec{V} = \frac{\Delta m^2}{2p}\vec{B}, \qquad \vec{\bar{V}} = -\frac{\Delta m^2}{2p}\vec{B}.$$



Raffelt and de Sousa Seixas: 1307.7625

Simple collective oscillations

Simple: homogeneous, isotropic

Neutrino-neutrino potential:

$$\vec{V}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \left(\vec{P}(\mathbf{p}') - \vec{\bar{P}}(\mathbf{p}')\right)(1 - \mathbf{v}' \cdot \mathbf{v}) \quad \Rightarrow \quad \frac{d}{dt}\vec{P} \propto (\vec{P} - \vec{\bar{P}}) \times \vec{P}\,.$$

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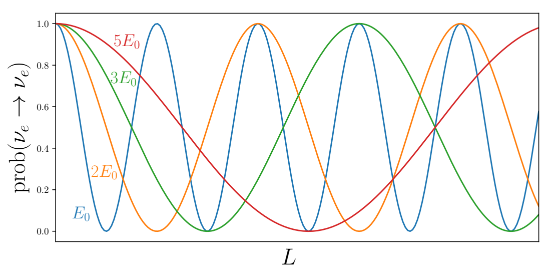
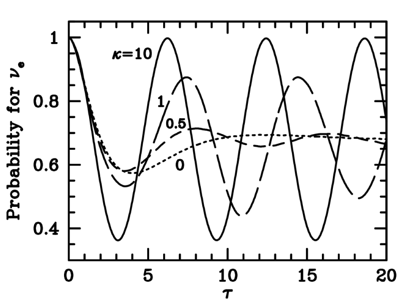
Vacuum oscillations:
$\mathrm{prop}(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$
mixing angle $\theta = \pi/4 \approx 0.79$.



Synchronized oscillations:
mixing angle $\theta = 0.46$.



Pastor, Raffelt, Semikoz: hep-ph/0109035

Simple collective oscillations

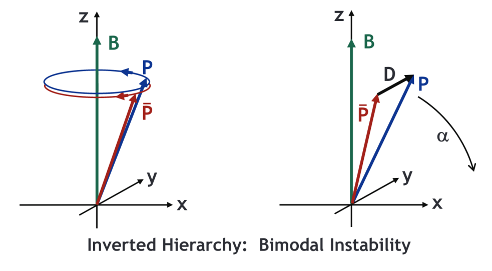
Simple: homogeneous, isotropic

Polarization vector:

$$\rho = \frac{1}{2}\begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}, \qquad \frac{d}{dt}\vec{P} = \vec{V} \times \vec{P},$$

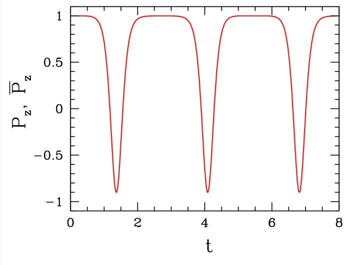
$$\vec{V}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3}\left(\vec{P}(\mathbf{p}') - \vec{\bar{P}}(\mathbf{p}')\right)(1 - \mathbf{v}' \cdot \mathbf{v}) \quad \Rightarrow \quad \frac{d}{dt}\vec{P} \propto \vec{D} \times \vec{P}\,.$$

Bipolar oscillations:



Raffelt and de Sousa Seixas: 1307.7625

mixing angle $\theta = 0.01$



Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695

Collective oscillations can:

- Synchronize oscillations.

- Lead to large conversion for small θ.

Treating neutrino oscillations in the early Universe

Quantum kinetic equations

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)\rho(\mathbf{p},\mathbf{x}) = -i\left[\mathcal{H}(\rho,\mathbf{p},\mathbf{x}),\rho(\mathbf{p},\mathbf{x})\right] + \mathcal{C}(\rho,\mathbf{p},\mathbf{x})\,,$$

Treating neutrino oscillations in the early Universe

Quantum kinetic equations

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Hamiltonian:

$$\mathcal{H}(\rho,\mathbf{p},\mathbf{x}) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p} + \sqrt{2}G_F\int\frac{d^3\mathbf{p}'}{(2\pi)^3}\left(\rho(\mathbf{p}',\mathbf{x}) - \bar{\rho}(\mathbf{p}',\mathbf{x})\right)(1-\mathbf{v}'\cdot\mathbf{v})$$

vacuum term   asymmetric neutrino — neutrino term

$$-\frac{8\sqrt{2}G_F p}{4}\,\frac{\mathcal{E}_l + \frac{1}{3}\mathcal{P}_l}{m_W^2}$$

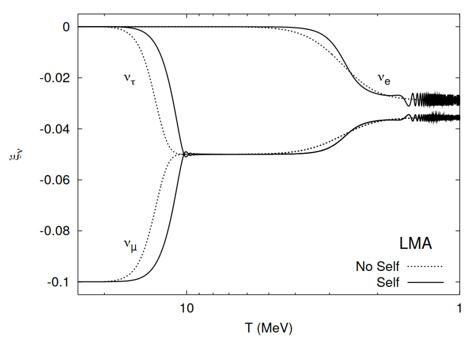
symmetric matter term

Neutrino oscillations in the early Universe

Large lepton asymmetry

Dolgov, Hansen, Pastor, Petcov, Raffelt and Semikoz: hep-ph/0201287

Synchronized oscillations and large chemical potentials:



Fermi-Dirac distribution:

$$f(p) = \frac{1}{\exp(p/T - \xi) + 1} \,.$$

In equilibrium, $\bar{\xi} = -\xi$.

Conclusion: Bounds on ξ_{ν_e} apply to all flavors.

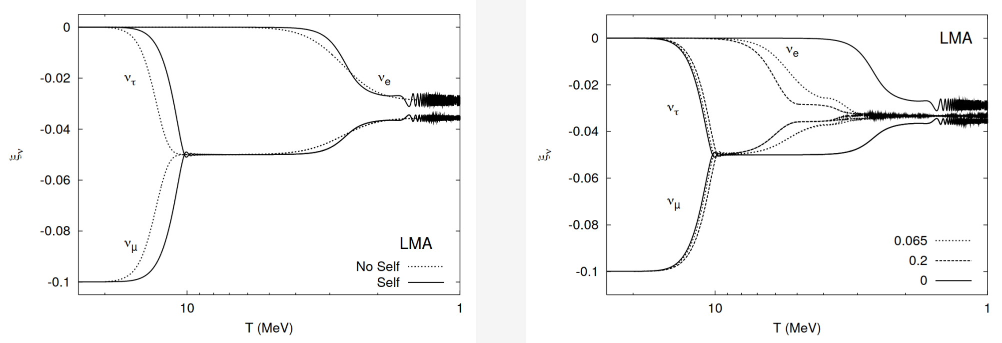
see also Wong: hep-ph/0203180 and Abazajian, Beacom and Bell: astro-ph/0203442

Neutrino oscillations in the early Universe

Large lepton asymmetry

Dolgov, Hansen, Pastor, Petcov, Raffelt and Semikoz: hep-ph/0201287

Synchronized oscillations and large chemical potentials: (measured $\tan(\theta_{13})^2 = 0.02$)



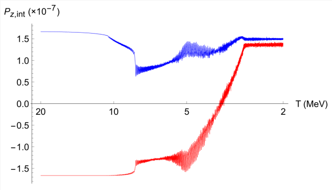
Conclusion: Bounds on ξ_{ν_e} apply to all flavors.

see also Wong: hep-ph/0203180 and Abazajian, Beacom and Bell: astro-ph/0203442

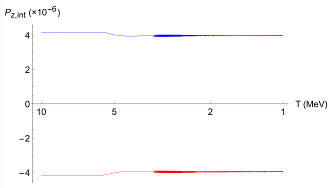
Neutrino oscillations in the early Universe

Diversity

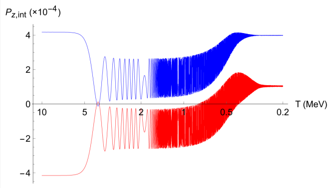
Asymmetric MSW



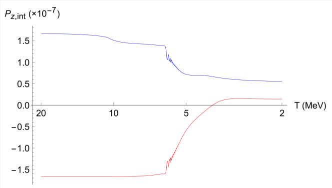
Minimal transformation



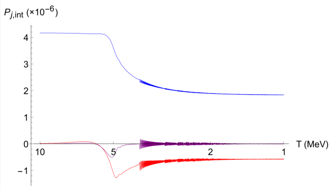
Large synch. oscillations



Damped



Damped



$$P_{z,\mathrm{int}} = \frac{n_{\nu_e} - n_{\nu_x}}{T^3}$$

Johns, Mina, Cirigliano, Paris and Fuller: 1608.01336

Neutrino oscillations equilibrate flavors in general.

Neutrino oscillations equilibrate flavors in general.

Details depend on initial conditions.

Breaking isotropy

$$\rho = \frac{1}{2}\begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}$$

Neutrino-neutrino term

$$\vec{V}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3} (\vec{P}(\mathbf{p}') - \vec{\bar{P}}(\mathbf{p}'))(1 - \mathbf{v}' \cdot \mathbf{v})$$

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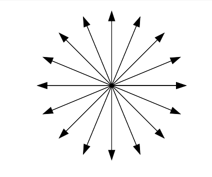
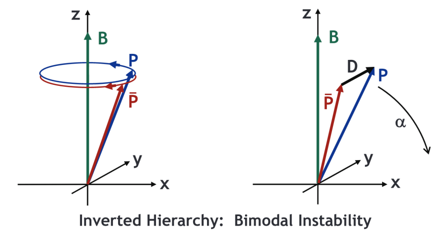


$$\frac{d}{dt}\vec{P} \propto \vec{D} \times \vec{P}$$

Neutrino-neutrino term

$$\vec{V}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3}(\vec{P}(\mathbf{p}') - \vec{\bar{P}}(\mathbf{p}'))(1 - \mathbf{v}' \cdot \mathbf{v})$$

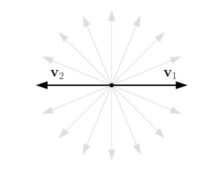
Bipolar oscillations - Inverted mass ordering



Raffelt and de Sousa Seixas: 1307.7625

Breaking isotropy

$$\rho = \frac{1}{2}\begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}$$

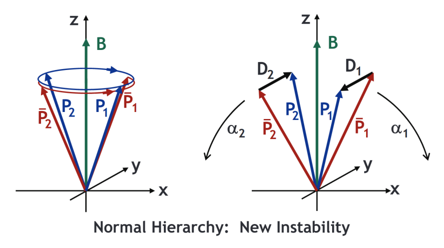


$$\frac{d}{dt}\vec{P}_1 \propto \vec{D}_2 \times \vec{P}_1$$
$$\frac{d}{dt}\vec{P}_2 \propto \vec{D}_1 \times \vec{P}_2$$

Neutrino-neutrino term

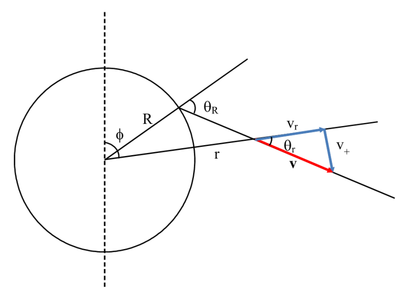
$$\vec{V}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3}(\vec{P}(\mathbf{p}') - \vec{\bar{P}}(\mathbf{p}'))(1 - \mathbf{v}'\cdot\mathbf{v})$$

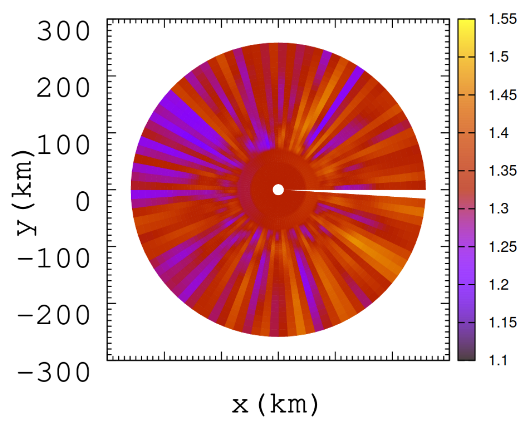
Two bin model - Normal mass ordering



Raffelt and de Sousa Seixas: 1307.7625

Breaking isotropy



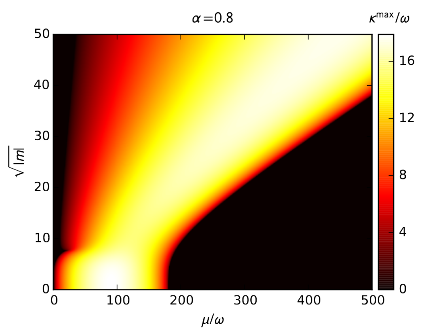


Mirizzi: 1506.06805

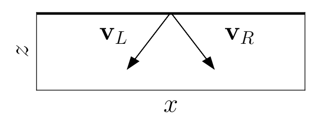
Breaking homogeneity

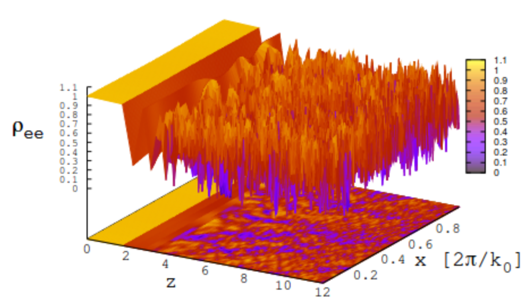
A simple line model

Linear stability analysis: Growthrate κ



Duan and Shalgar: 1412.7097





Mirizzi, Mangano and Saviano: 1503.03485

Collective oscillations can break isotropy and homogeneity.

Collective oscillations can break isotropy and homogeneity.

The breaking can make conversion in NO possible.

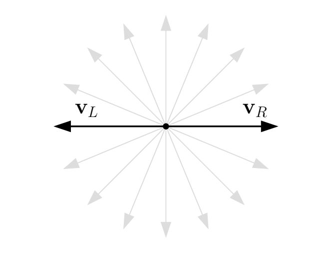
Collective oscillations in the early Universe

Homogeneous universe model with two angle bins

RSLH, Shalgar and Tamborra: 2012.03948

Assumptions:

- Universe is homogeneous.
- Angular dependence approximated with two angle bins. (left moving L and right moving R not to be confused with chirality of the particles)
- Two neutrino oscillation framework.
- Relaxation-time-like approximation for the collision term.



Expansion of the Universe

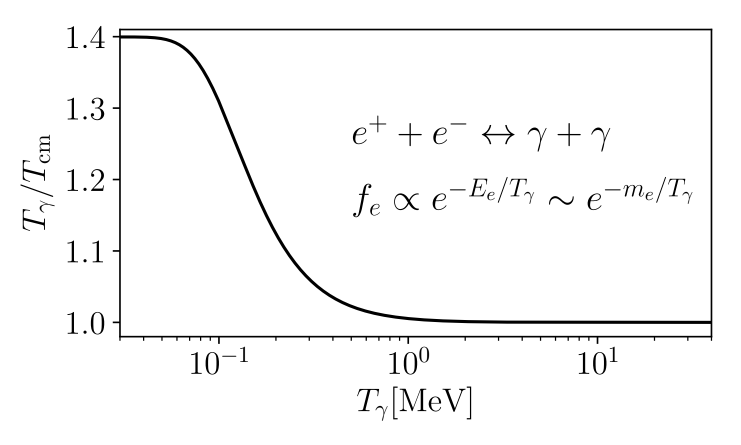
2012.03948

Friedmann equation:

$$H = \sqrt{\frac{8\pi\rho}{3}}\frac{1}{m_{\rm Pl}},$$

Continuity equation:

$$\dot{\rho} = -3H(\rho + P),$$



Collision term - approximations

2012.03948

Divide into four different types of reactions each with a rate:

1. Scattering with electrons and positrons, $\Gamma_{s,\alpha}$
2. Annihilations to electrons and positrons, $\Gamma_{a,\alpha}$
3. Neutrino-neutrino scatterings, $\Gamma_{\nu\nu}$
4. Neutrino-antineutrino collisions, $\Gamma_{\nu\bar{\nu}}$

Equilibrium distributions are assumed when calculating the rates.

Functional form: One equilibrium distribution in each gain term.

$$f(T_{\text{eq}}, \mu) = \frac{1}{\exp(p/T_{\text{eq}} - \mu/T_{\text{eq}}) + 1},$$

All other distribution functions are represented by normalized energy densities, $u_{\alpha\beta}$.

Collision term - approximations

2012.03948

Divide into four different types of reactions each with a rate:

1. Scattering with electrons and positrons, $\Gamma_{s,\alpha}$, $T_{\rm eq} = T_\gamma$, $\mu = \pi_\alpha$
2. Annihilations to electrons and positrons, $\Gamma_{a,\alpha}$, $T_{\rm eq} = T_\gamma$, $\mu = \mu_\alpha$
3. Neutrino-neutrino scatterings, $\Gamma_{\nu\nu}$, $T_{\rm eq} = T_{\nu_\alpha}$, $\mu = \pi_{\nu_\alpha}$
4. Neutrino-antineutrino collisions, $\Gamma_{\nu\bar{\nu}}$, $T_{\rm eq} = T_{\nu_\alpha}$, $\mu = \pi_{\nu_\alpha}$

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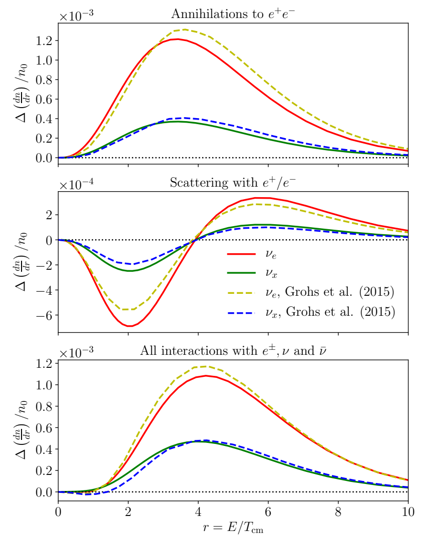
Collision term

2012.03948

For electron neutrinos:

$$\begin{aligned}\mathcal{C}_{ee} = {} & \Gamma_{a,e}\left[\left(\frac{T_\gamma}{T_{\rm cm}}\right)^4 f(T_\gamma,\mu_e) - \rho_{ee}\right] \\ & + \Gamma_{s,e}\left[f(T_\gamma,\pi_e) - \rho_{ee}\right] \\ & - \Gamma_G \mathrm{Re}\left(\bar{u}_{ex}\rho^*_{ex}\right) \\ & + \Gamma_{\nu\nu}\left(2u_{ee} + u_{xx}\right)\left(f_{\nu_e} - \rho_{ee}\right) \\ & + \Gamma_{\nu\bar\nu}\left(4\bar{u}_{ee} + \bar{u}_{xx}\right)\left(f_{\nu_e} - \rho_{ee}\right) \\ & + \Gamma_{\nu\bar\nu}\left(\bar{u}_{xx} f_{\nu_x} - \bar{u}_{ee}\rho_{ee}\right) \\ & + \mathrm{Re}\left[\left(\Gamma_{\nu\nu}u^*_{ex} + 4\Gamma_{\nu\bar\nu}\bar{u}^*_{ex}\right)\left(f_{ex} - \rho_{ex}\right)\right],\end{aligned}$$



Linear stability analysis

2012.03948

Write density matrix:

$$\rho_R = \frac{1}{2}\mathrm{Tr}(\rho_R) + \frac{1}{2}\begin{pmatrix} s_R & \epsilon_R \\ \epsilon_R^* & -s_R \end{pmatrix} .$$

- Assume $\boldsymbol{\epsilon} = (\epsilon_R, \bar{\epsilon}_R, \epsilon_L, \bar{\epsilon}_L)^T = (0, 0, 0, 0)^T$ is a fixed point (zero time derivative).
- Linearize the equations for small ϵ:

$$\frac{d}{dt}\boldsymbol{\epsilon} = i\mathsf{M}\boldsymbol{\epsilon}$$

- Assume collective solution $\boldsymbol{\epsilon} = \exp(-i\Omega t)\mathbf{Q}$.
- Determine Ω such that $(\mathsf{M} - \mathsf{I})\mathbf{Q} = 0$ has solutions (determine eigenvalues).

Linear stability analysis

2012.03948

μ - Neutrino term.
ω_λ - Matter and vacuum term.
$s = \rho_{ee} - \rho_{xx}$ - Difference between ν_e and ν_x densities.
Eigenvalues: ($\epsilon = \exp(-i\Omega t)\mathbf{Q}$)

$$\Omega_1^\pm = \frac{1}{2}\left(3\mu(s-\bar{s}) \pm \sqrt{-4\omega_\lambda\mu(s+\bar{s}) + 4\omega_\lambda^2 + \mu^2(s-\bar{s})^2}\right),$$
$$\Omega_2^\pm = \frac{1}{2}\left(\mu(s-\bar{s}) \pm \sqrt{4\omega_\lambda\mu(s+\bar{s}) + 4\omega_\lambda^2 + \mu^2(s-\bar{s})^2}\right).$$

Eigenvectors:

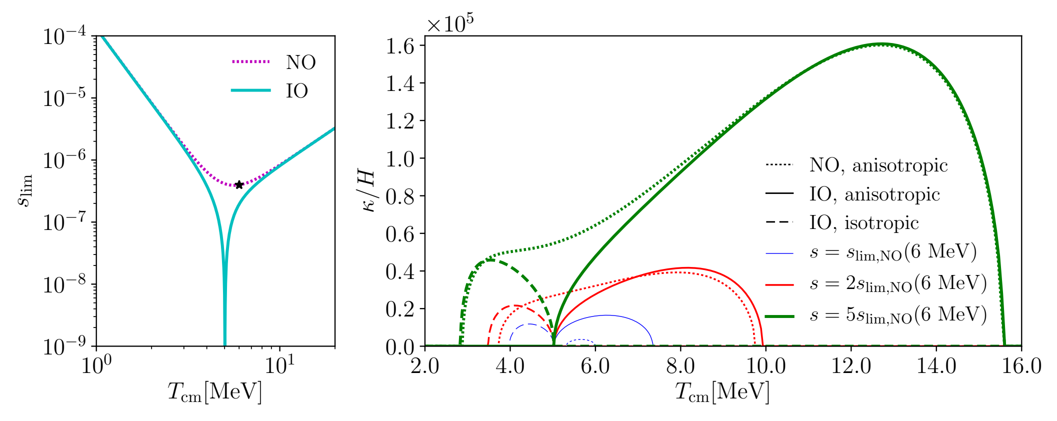
$$\mathbf{w}_1^{\pm T} = \left(\frac{\mu(s-2\bar{s})+\omega_\lambda-\Omega_1^\pm}{\mu\bar{s}}, \quad -1, \quad -\frac{\mu(s-2\bar{s})+\omega_\lambda-\Omega_1^\pm}{\mu\bar{s}}, \quad 1\right)^T, \quad \text{(asymmetric in } R \leftrightarrow L)$$
$$\mathbf{w}_2^{\pm T} = \left(\frac{\mu s+\omega_\lambda-\Omega_2^\pm}{\mu\bar{s}}, \quad 1, \quad \frac{\mu s+\omega_\lambda-\Omega_2^\pm}{\mu\bar{s}}, \quad 1\right)^T. \quad \text{(symmetric in } R \leftrightarrow L)$$

Recall: $\epsilon = (\epsilon_R, \bar{\epsilon}_R, \epsilon_L, \bar{\epsilon}_L)^T$.

Linear stability analysis

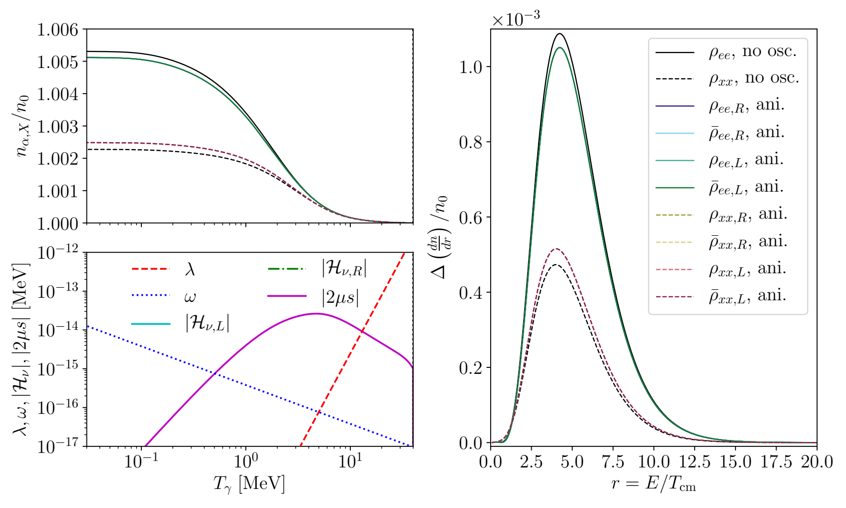
2012.03948

$$s = \rho_{ee} - \rho_{xx}, \qquad s_{\rm lim} = \left| \frac{56\pi^4}{270\zeta(3) m_W^2} \langle p \rangle T_{\rm cm} + \frac{\sqrt{2}\pi^2 \Delta m^2}{6\zeta(3) G_F} \frac{1}{\langle p \rangle T_{\rm cm}^3} \right| .$$



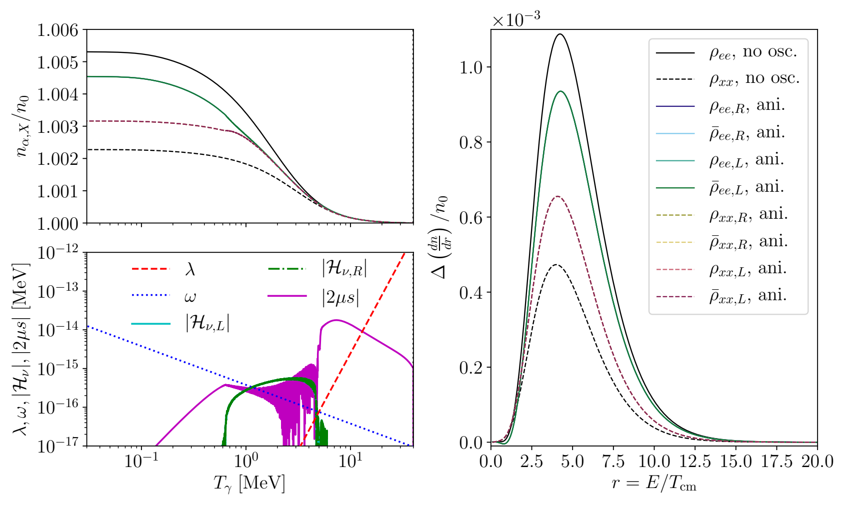
Isotropic initial conditions, NO

2012.03948



Isotropic initial conditions, IO

2012.03948



Anisotropic initial conditions, IO

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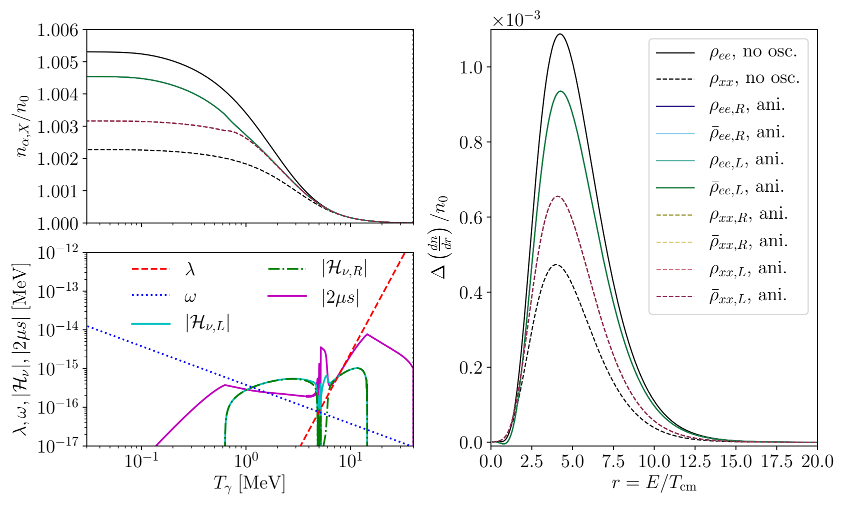
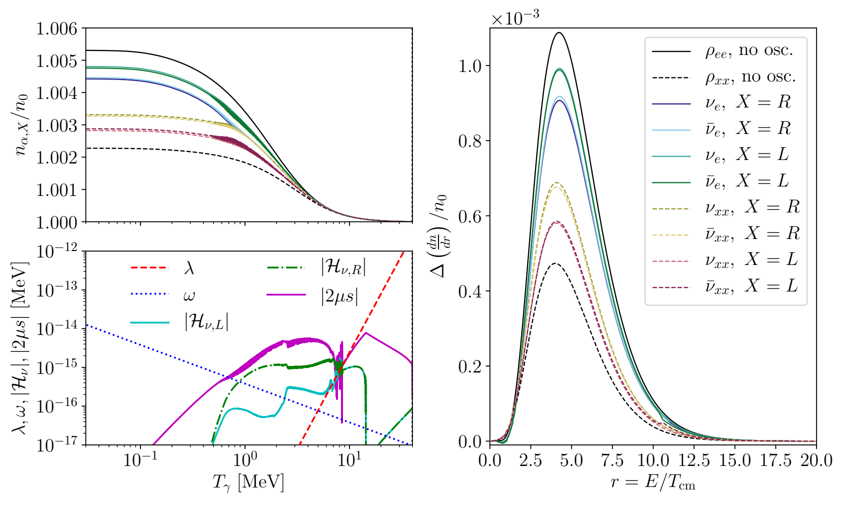


Anisotropic initial condition, NO

2012.03948



$\nu - \bar{\nu}$ asymmetry

Homogeneous, isotropic and single energy

Linearize:

$$\rho \sim \begin{bmatrix} s - \delta & \epsilon \\ \epsilon^* & -s + \delta \end{bmatrix}$$

$$\frac{d}{dt} \begin{pmatrix} \epsilon \\ \bar{\epsilon} \end{pmatrix} = i\mathsf{M} \begin{pmatrix} \epsilon \\ \bar{\epsilon} \end{pmatrix}$$

$$\frac{d(\delta - \bar{\delta})}{dt} = \omega \sin 2\theta (\mathrm{Im}(\epsilon) + \mathrm{Im}(\bar{\epsilon}))$$

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Symmetric initial conditions ($\bar{\epsilon}_0 = \epsilon_0$) gives

$$\mathrm{Im}(\bar{\epsilon}) = -\mathrm{Im}(\epsilon).$$

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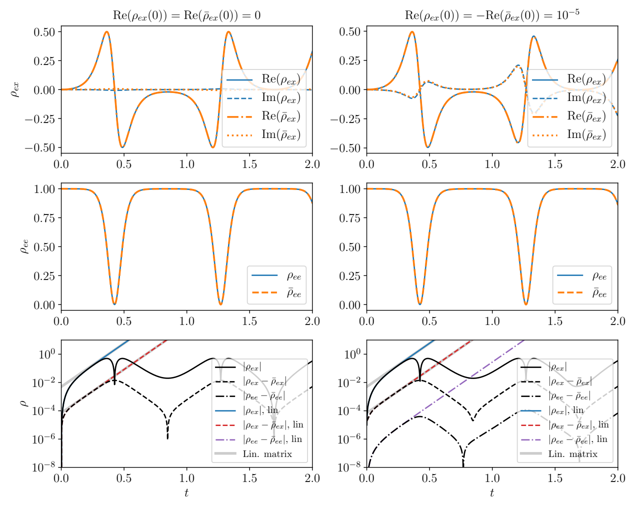

Enhancement of $\nu - \bar{\nu}$ asymmetry:

- Found numerically for early Universe conditions.

- Confirmed in simple model.

- Similar effect known from active-sterile oscillations.

Effect on Neff

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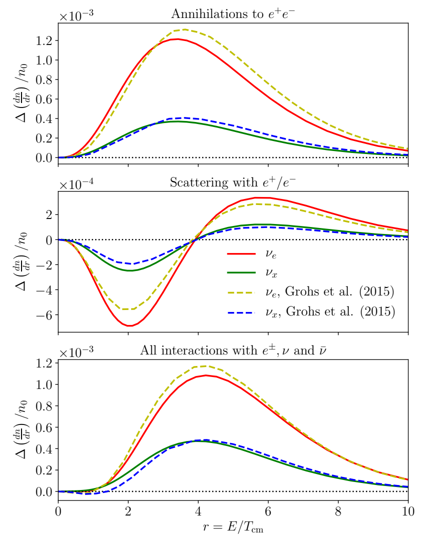
Collective oscillations enhance ν_x and supress ν_e.

Collision term from e^+e^- annihilation:

$$\mathcal{C}_{a,ee} = \Gamma_{a,e}\left[\left(\frac{T_\gamma}{T_{\rm cm}}\right)^4 f(T_\gamma,\mu_e) - \rho_{ee}\right] ,$$
$$\mathcal{C}_{a,xx} = \Gamma_{a,x}\left[\left(\frac{T_\gamma}{T_{\rm cm}}\right)^4 f(T_\gamma,\mu_x) - \rho_{xx}\right] .$$

$$\Gamma_{a,e} > \Gamma_{a,x} .$$

Oscillations are expected to enhance $N_{\rm eff}$.



Change in $N_{\rm eff}$ in the two angle bin model

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$$N_{\rm eff} \equiv \frac{\rho_\nu}{7/8\rho_\gamma}\left(\frac{11}{7}\right)^3$$
$$\approx \left(\frac{\int dr\, r^3(\rho_{ee} + \bar\rho_{ee} + \rho_{xx} + \bar\rho_{xx})}{2\int dr r^3 f_0} + 1\right)\frac{(11/7)^3}{(T_\gamma/T_{\rm cm})^4}\,.$$

For no neutrino oscillations, $N_{\rm eff} = 3.04596$ (not very accurate).

The cases with oscillations give:

	NO, isotropic	NO, anisotropic	NO, anisotropic ($\mu_{\rm ini} = 10^{-9}$)	IO, both
$\Delta N_{\rm eff}$	0.9×10^{-4}	5.0×10^{-4}	4.9×10^{-4}	5.8×10^{-4}

These are only indications from a simple model with an approximated collision term!

Summary

- Collective oscillations in the early Universe can enhance Neff.
- They can break isotropy and we expect them to break homogeneity.
- They might amplify a small neutrino-antineutrino asymmetry by several orders of magnitude.

Future perspectives

- Full angular distributions.
- Breaking homogeneity.
- Full collision term.
- Three neutrino effects.
- Beyond Standard Model physics.
  - Large lepton asymmetry.
  - Low temperature reheating.
  - Sterile neutrinos.
- Going beyond meanfield.

Thank you for your attention!

Homogeneous universe model with two angle bins

$$\frac{\partial \rho_R(p)}{\partial t} - Hp\frac{\partial \rho_R(p)}{\partial p} = -i[\mathcal{H}_R(\rho_R, \rho_L, p), \rho_R] + \mathcal{C}_R(\rho_R, \rho_L, p) \,,$$
$$\frac{\partial \rho_L(p)}{\partial t} - Hp\frac{\partial \rho_L(p)}{\partial p} = -i[\mathcal{H}_L(\rho_R, \rho_L, p), \rho_L] + \mathcal{C}_L(\rho_R, \rho_L, p) \,,$$

Hamiltonian:

$$\mathcal{H}_R(\rho_R, \rho_L, p) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p} + \sqrt{2}G_F \int \frac{dp'}{2\pi^2}(\rho_L(p') - \bar{\rho}_L^*(p'))$$
$$- \frac{8\sqrt{2}G_F p}{3}\frac{\mathcal{E}_l}{m_W^2} \,,$$

Effects of collisions and potentials

$$P_{z,\text{int},X} = \int \frac{dr}{4\pi^2} P_{z,X}(r) \qquad \text{for} \quad X \in \{R, L\}$$