# Top mass from asymptotic safety

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University of Heidelberg

**recent work with:**
**Aaron Held (1705.02342, 1707.01107 )**
**& Jan Pawlowski (1604.02041; Phys.Rev. D94 (2016) no.10, 104027)**

**Nic Christiansen (1702.07724; Phys.Lett. B770 (2017) 154-160)**

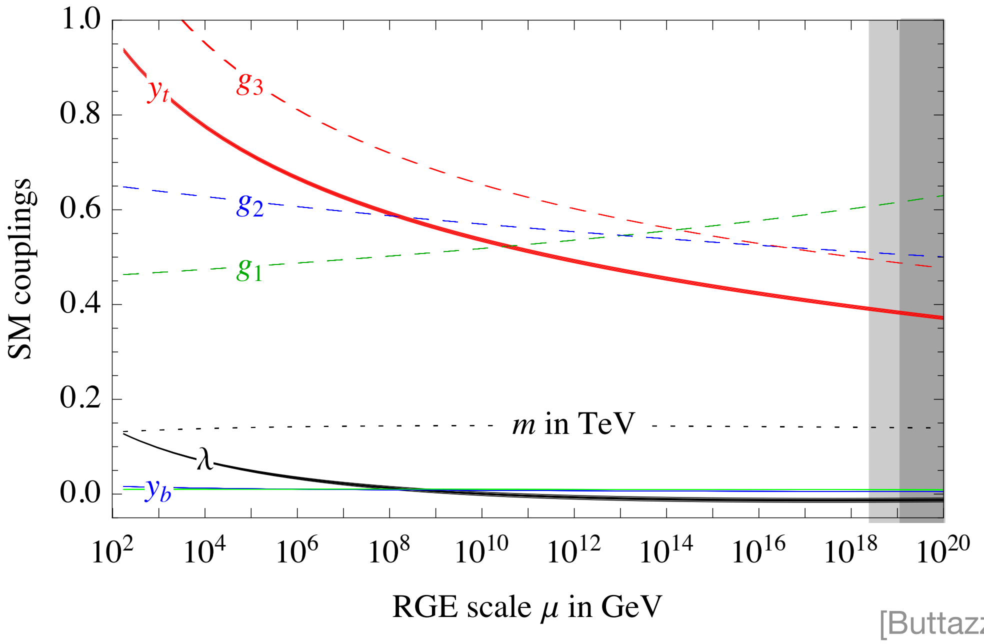
**Stefan Lippoldt (1611.05878; Phys.Lett. B767 (2017) 142-146)**

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

July 24, 2017
Particle and Astroparticle Theory Seminar
Max-Planck Institut für Kernphysik , Heidelberg

# What are the fundamental building blocks of our universe?

## Status of the Standard Model of particle physics

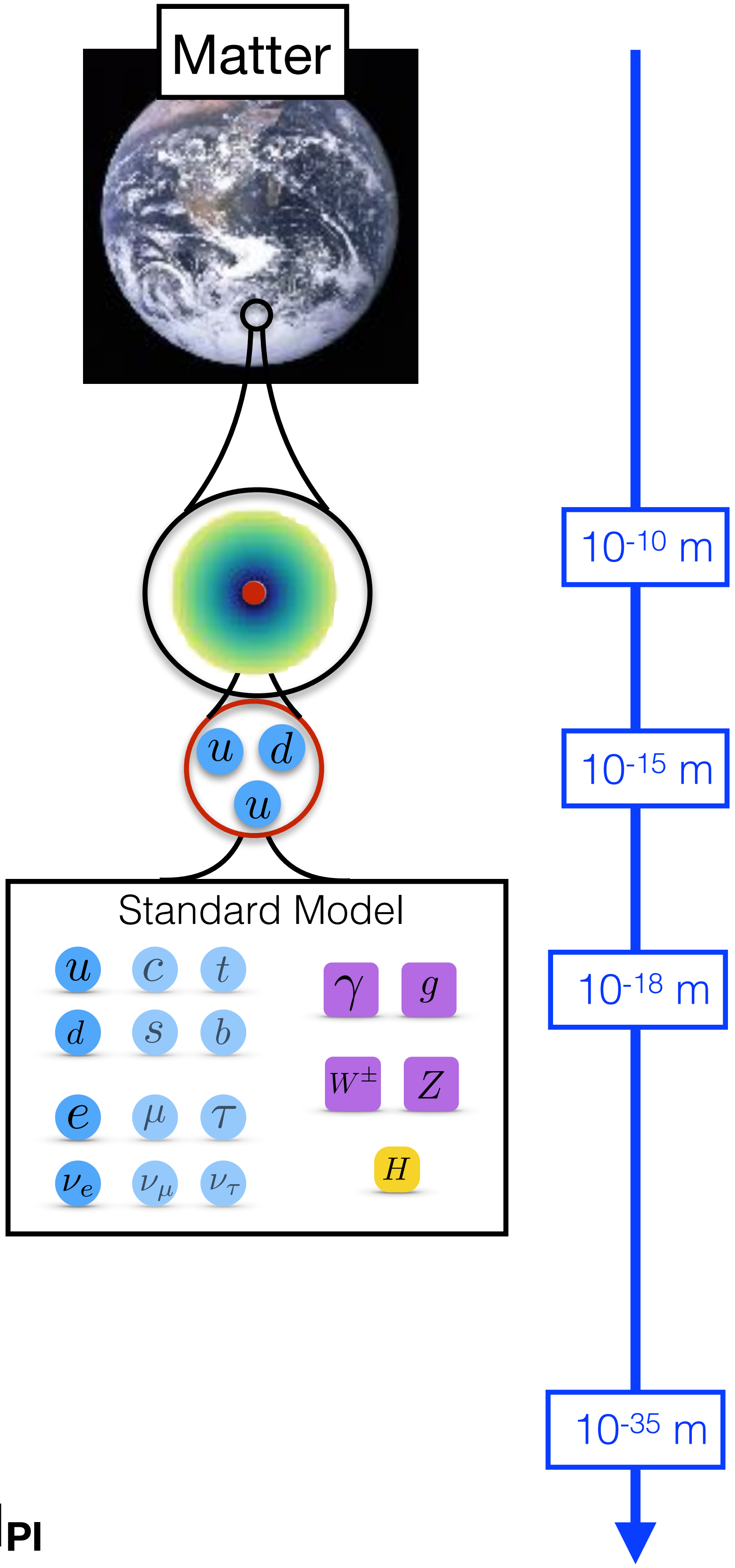



[Buttazzo et al. '13]

- can extend up to $M_{Pl}$ (@ $M_h$ ~ 125 GeV)
- breakdown in transplanckian regime (Landau pole/ triviality problem in Abelian hypercharge & Higgs-Yukawa)
- fails to include quantum gravity
- 19 free parameters

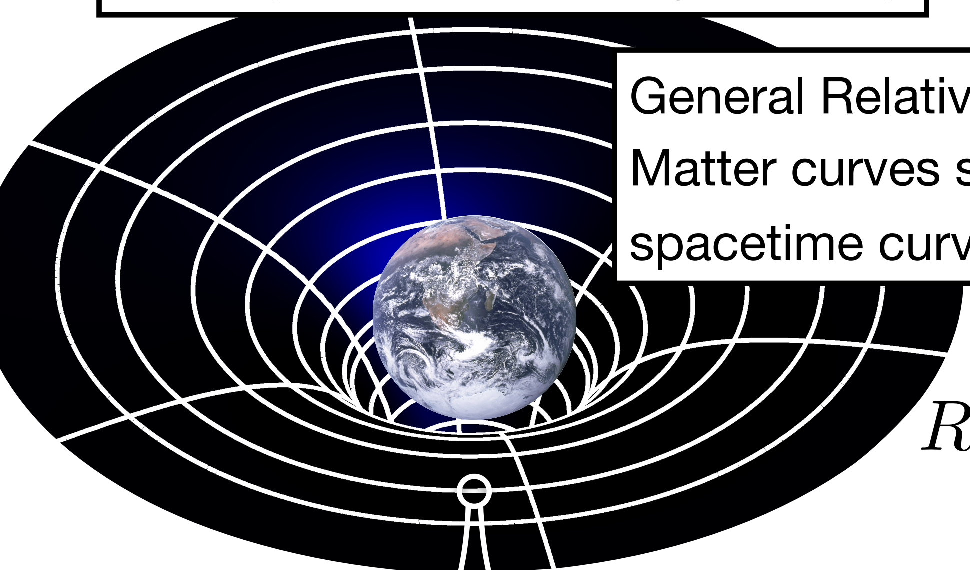
→ highly successful effective field theory

→ new physics (quantum gravity?!) required beyond $M_{Pl}$



# What are the fundamental building blocks of our universe?

Gravity = Spacetime geometry

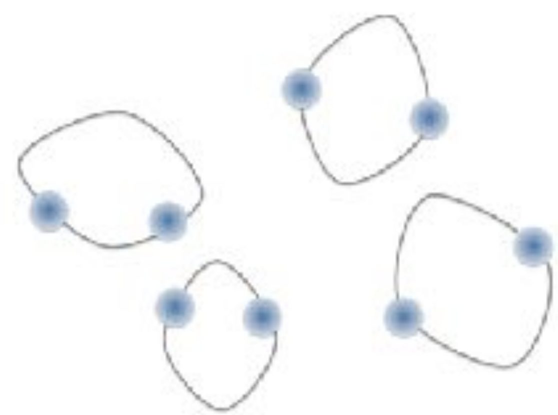


General Relativity:
Matter curves spacetime &
spacetime curvature tells matter how to move

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \, T_{\mu\nu}$$

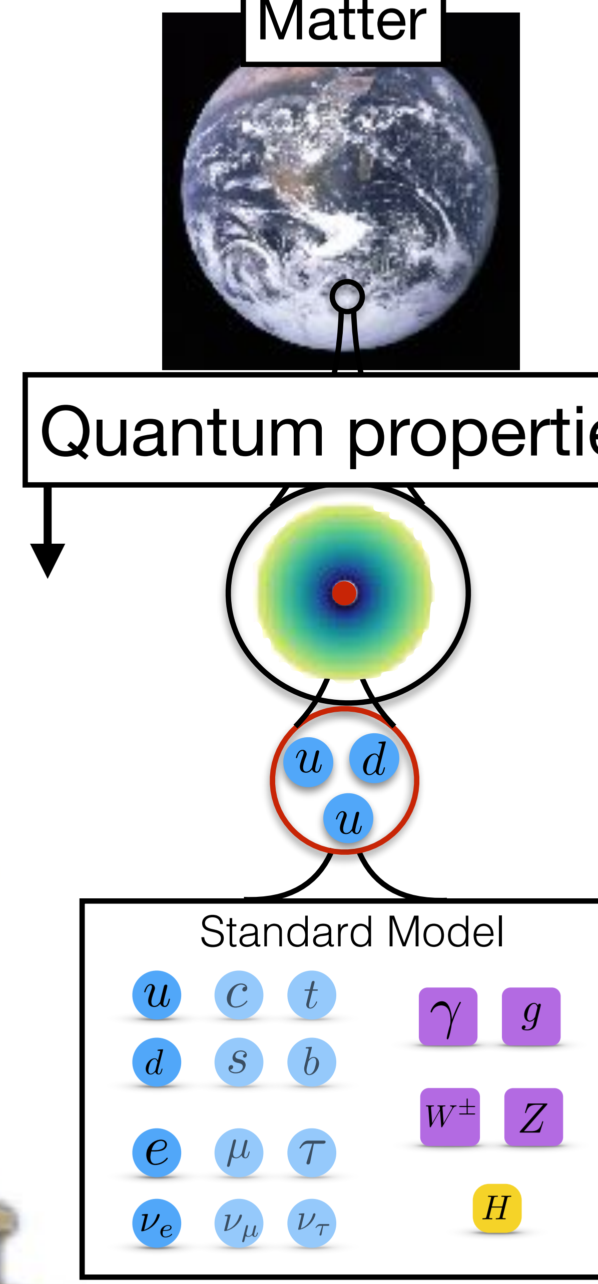
Quantum properties

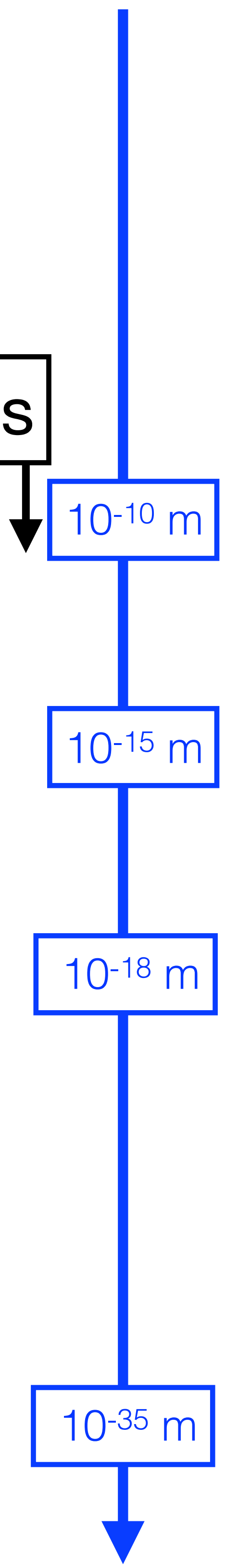
→ **string theory**
→ **supergravity**
→ **asymptotically safe gravity**
→ **loop quantum gravity**
→ **causal sets**
→ **non-commutative spacetime**




**Which way to quantum gravity?**

Matter



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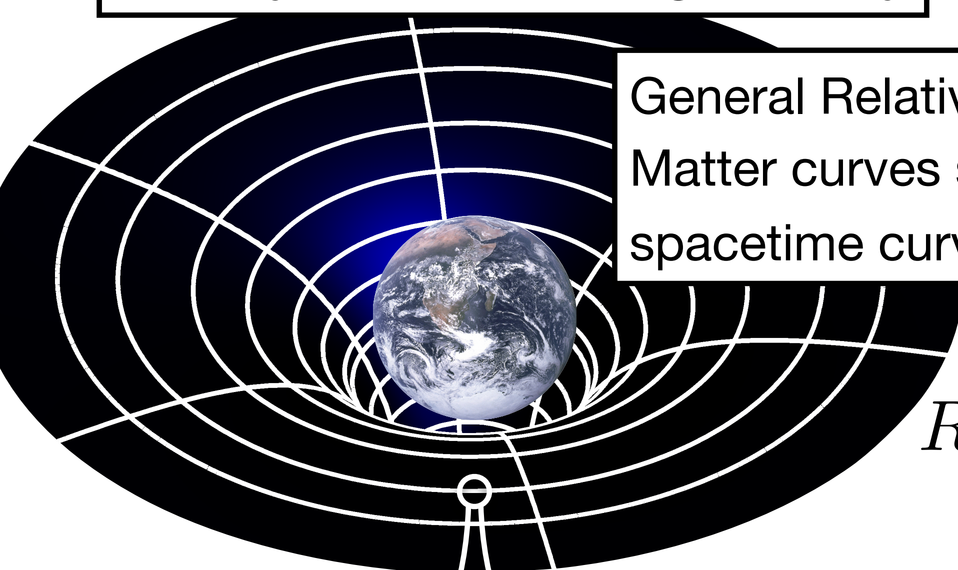

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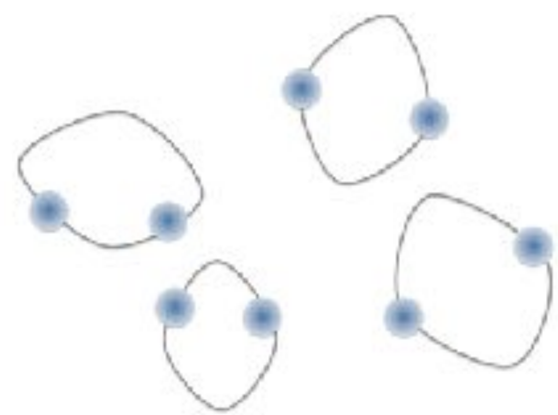

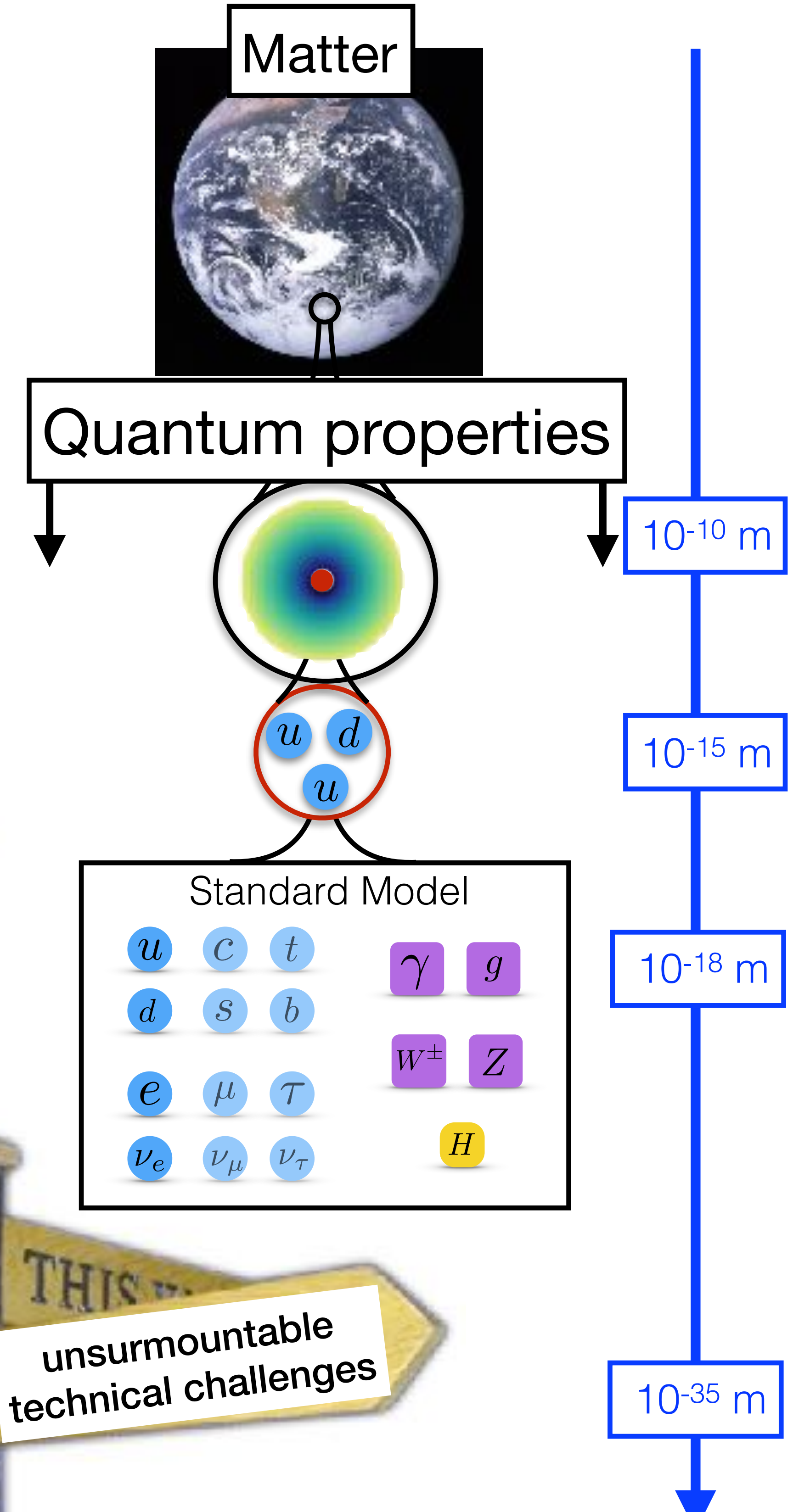


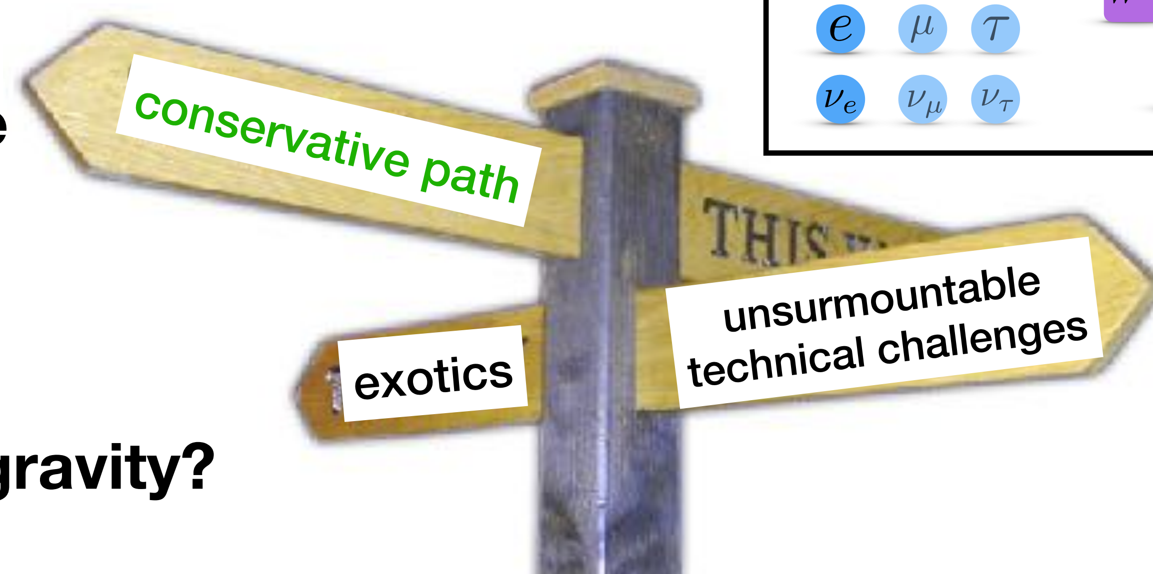


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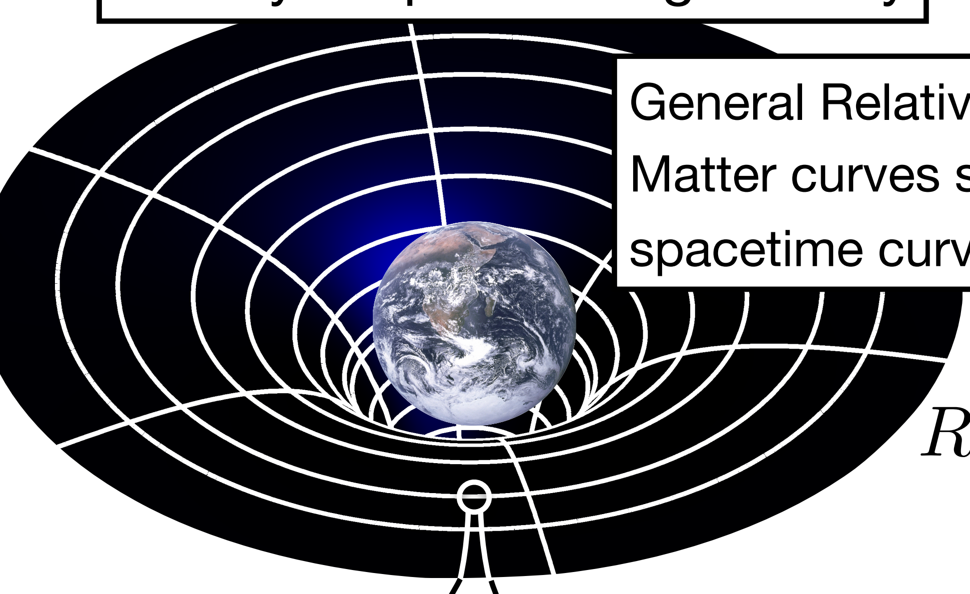

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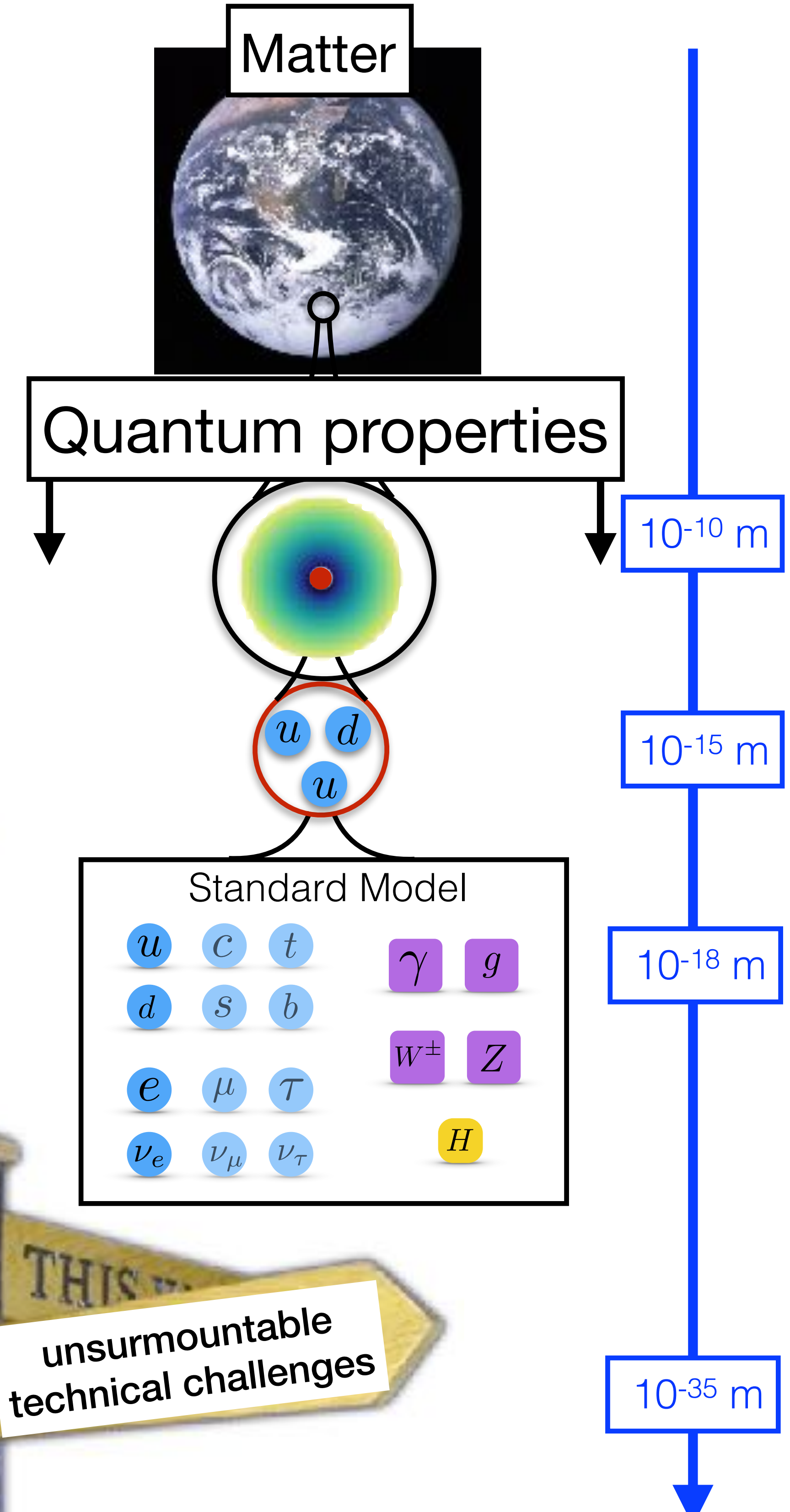
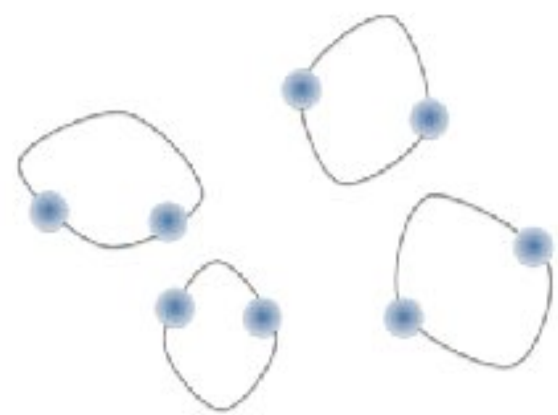
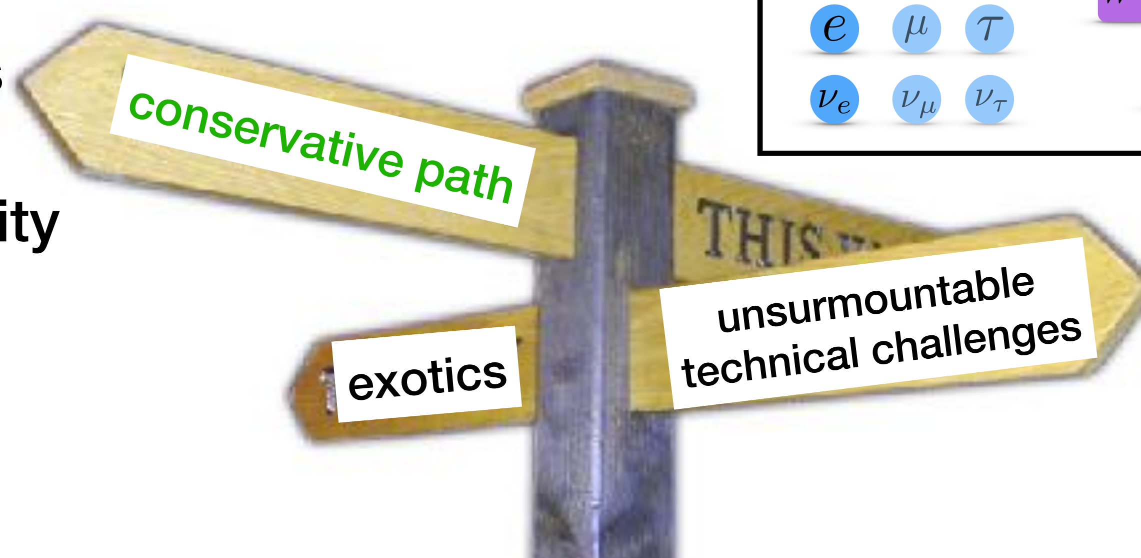


$\rightarrow$ **asymptotically safe gravity**

- local QFT framework
  - works for all other forces
- degrees of freedom: metric
  - good description of gravity at low energies

$\rightarrow$ could induce predictive UV completion of SM



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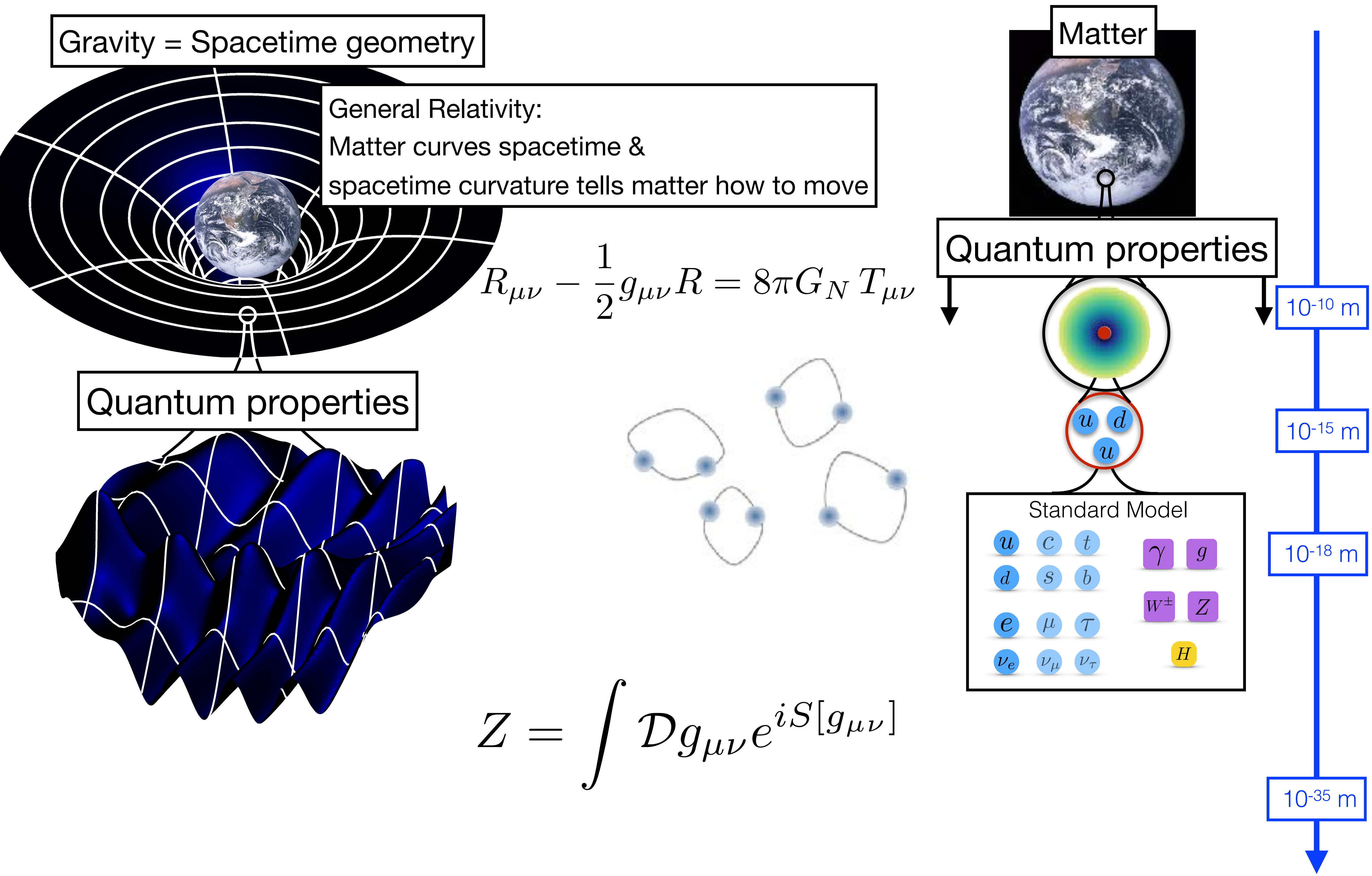

# Quantum field theory for gravity

$$Z = \int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]}$$

**Which microscopic action?**

**Einstein-Hilbert action & perturbative quantisation:** $S = -\frac{1}{16\pi G_N} \int d^4x \sqrt{g}\, R$

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{16\pi G_N} h_{\mu\nu}$$

spin-2-field on flat background

**counterterms:**

1-loop: $R^2,\ R_{\mu\nu}R^{\mu\nu}$ — 't Hooft, Veltman '74; Deser, Nieuwenhuizen '74

2-loop: $C_{\mu\nu\kappa\lambda}C^{\kappa\lambda\rho\sigma}C_{\rho\sigma}{}^{\mu\nu}$ — Goroff, Sagnotti '86; Van de Ven '92

$\dots$

**breakdown of predictivity**

**consistent choice of S with finite number of free parameters?**

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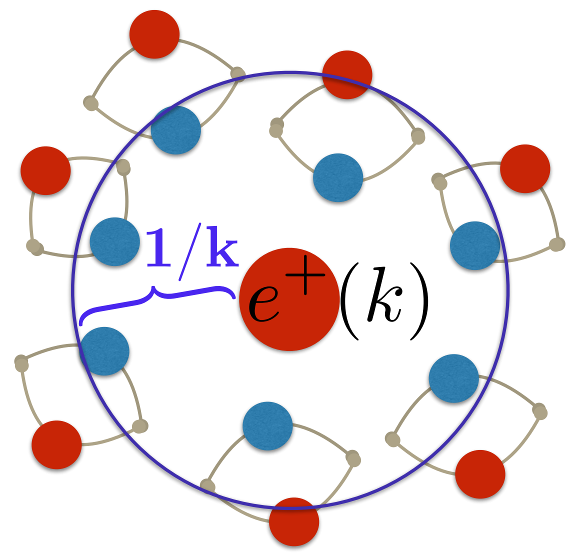
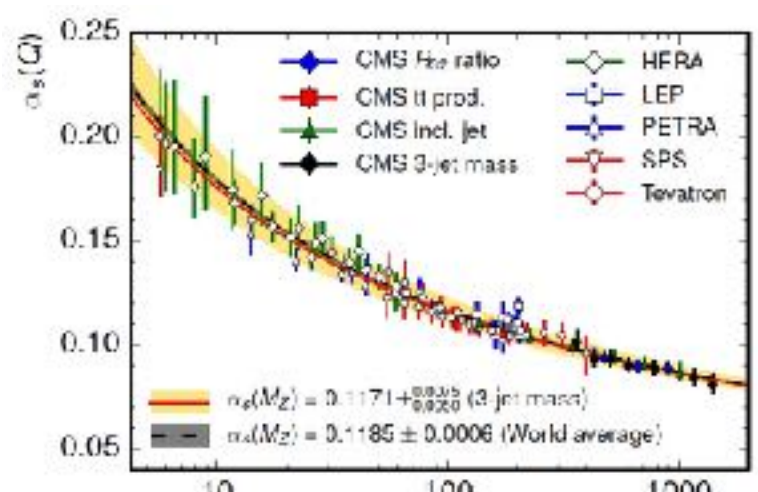



microscopic regime in fundamental theory (viable w/o ``new physics”): scale- invariance

Quantum fluctuations generate running (scale-dependent) couplings

Asymptotic freedom in non- Abelian gauge theories

[Gross, Wilczek ’73; Politzer ‘73]




CMS ‘14

$$\beta_{\alpha_s} = -\frac{7}{2\pi}\alpha_s^2 + \dots$$

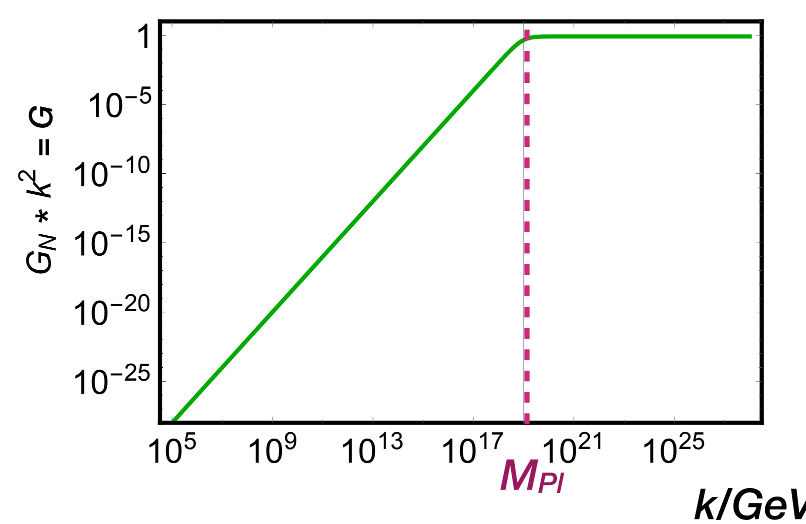
quark+ gluon fluctuations (universal @ 1loop)

$$\alpha_s^* = 0$$

Quantum gravity:
Newton coupling grows towards UV

asymptotically safe beyond $M_{Pl}$

[Weinberg ’76, ’79; Reuter ’96]




$$\beta_G = 2G - \frac{23\, G^2}{3\pi} + \dots$$

metric fluctuations

$$G^* = \frac{6\pi}{23}$$

[Codello, Percacci, Rahmede ’08]

# Asymptotic safety

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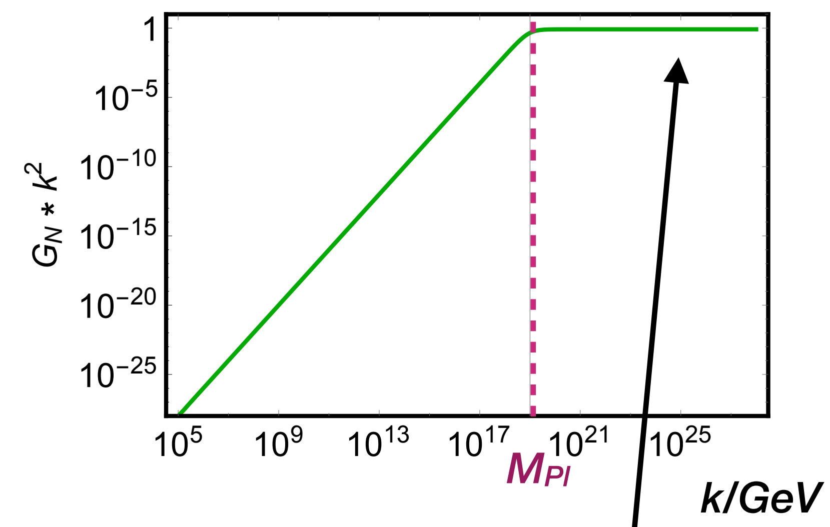
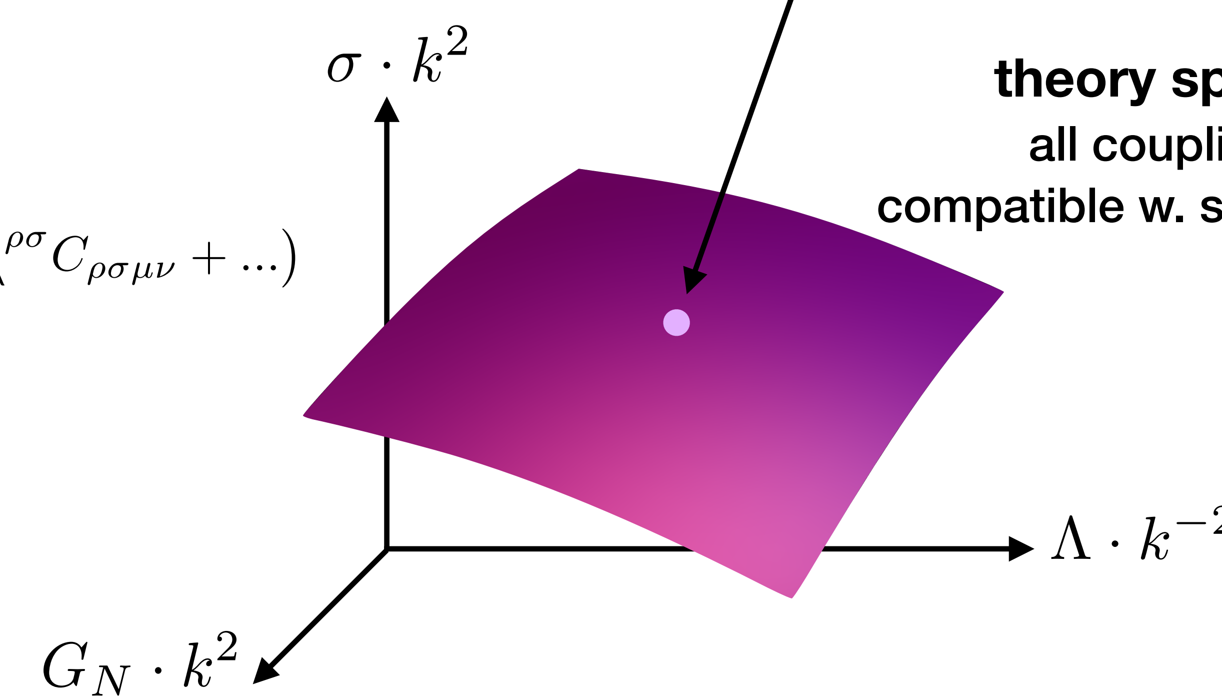



scale invariant
fixed-point regime
(microscopic dynamics @ $k \to \infty$)

higher-order interactions generated by quantum fluctuations:

$$\Gamma_k = -\frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,(R-2\Lambda)$$

$$+\int d^4x\,\sqrt{g}\,\left(a_1\,R^2 + a_2 R^{\mu\nu}R_{\mu\nu} + \sigma\, C^{\mu\nu\kappa\lambda}C_{\kappa\lambda}{}^{\rho\sigma}C_{\rho\sigma\mu\nu} + \ldots\right)$$




**theory space:**
all couplings
compatible w. symmetries

But at large scales our world
is clearly not scale-invariant?!

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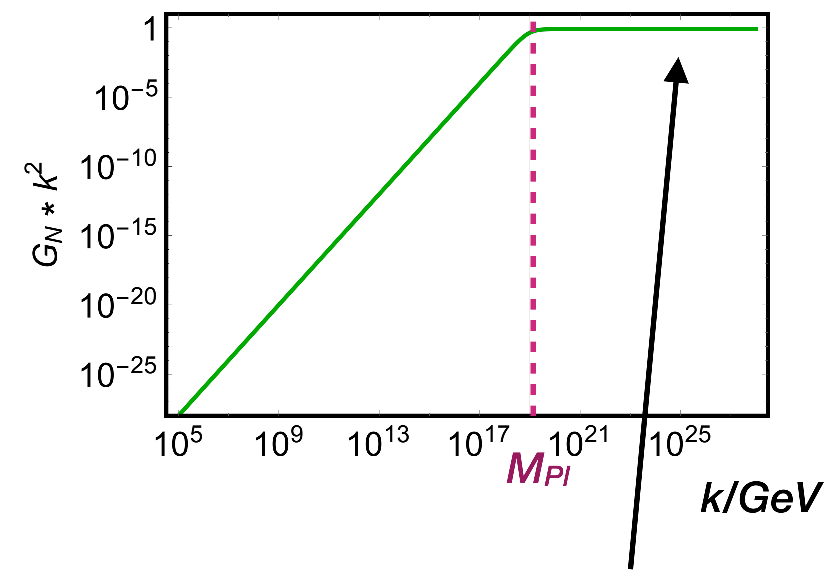


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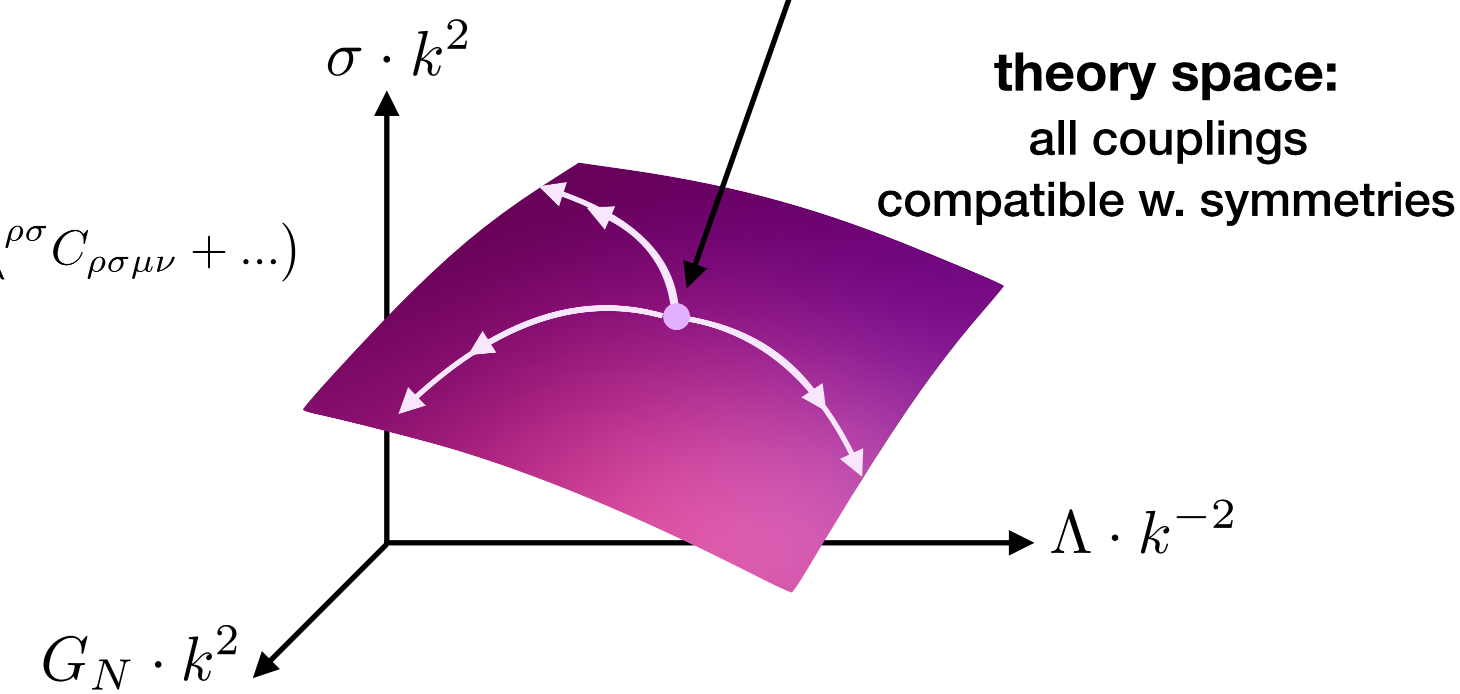


But at large scales our world
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$\longrightarrow$ Flow away from fixed point

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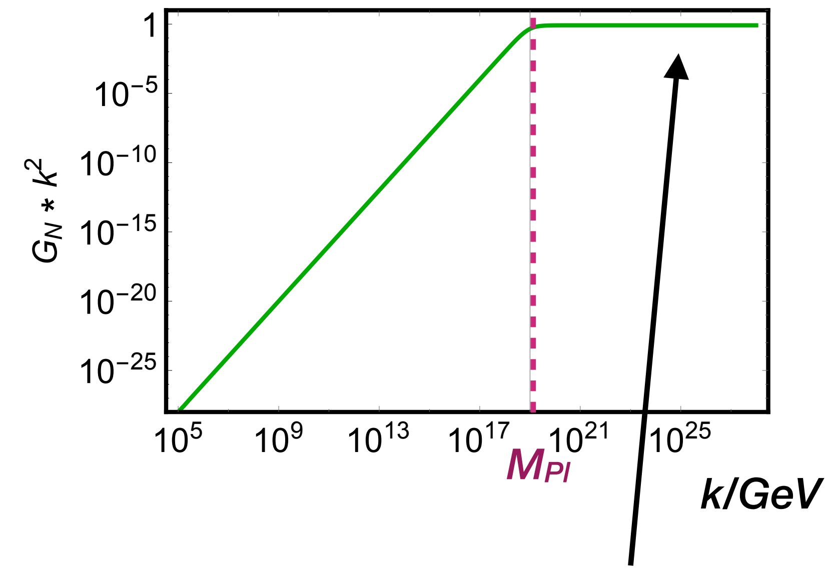


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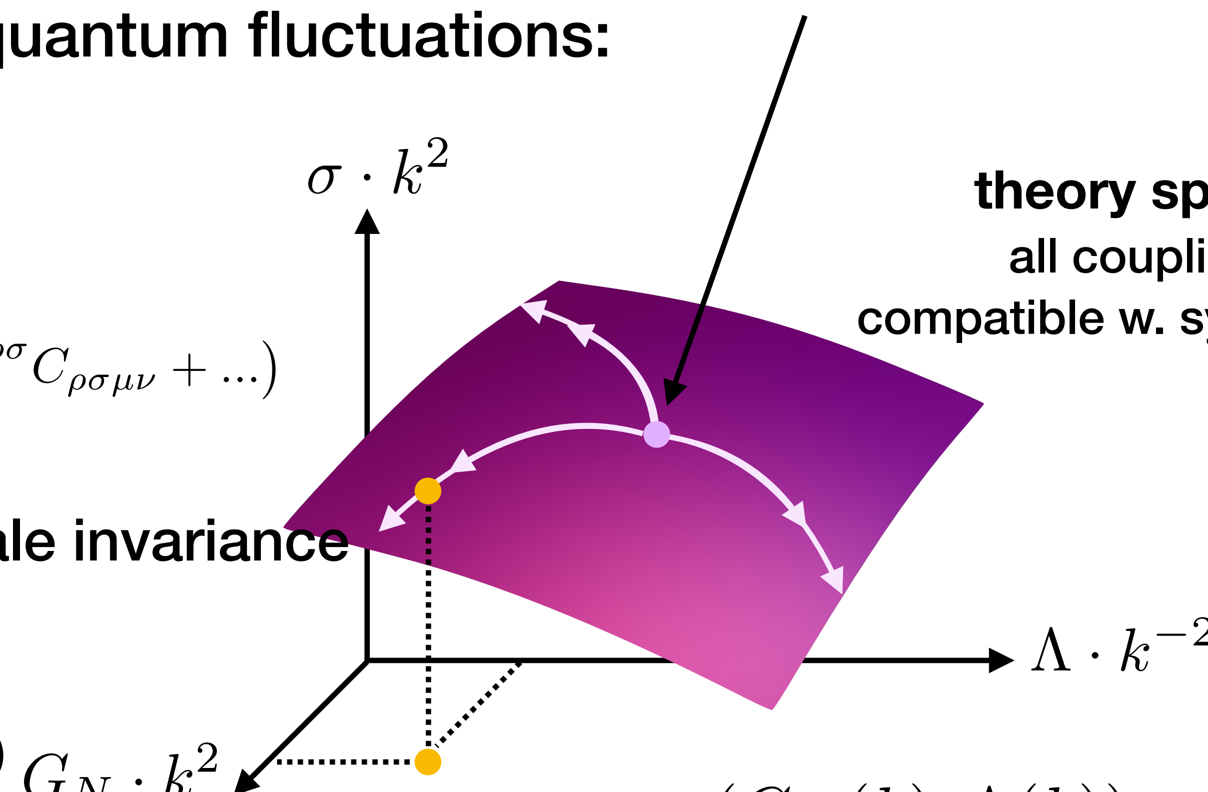
low-energy dynamics:

- free parameters encode deviation from scale invariance (relevant couplings)
- irrelevant couplings ***predicted***
  (qm fluc's force flow to stick to critical hypersurface)

**theory space:**
all couplings
compatible w. symmetries




$\to \sigma = \sigma(G_N(k), \Lambda(k))$

# Asymptotic safety in a nutshell

**theory space:**
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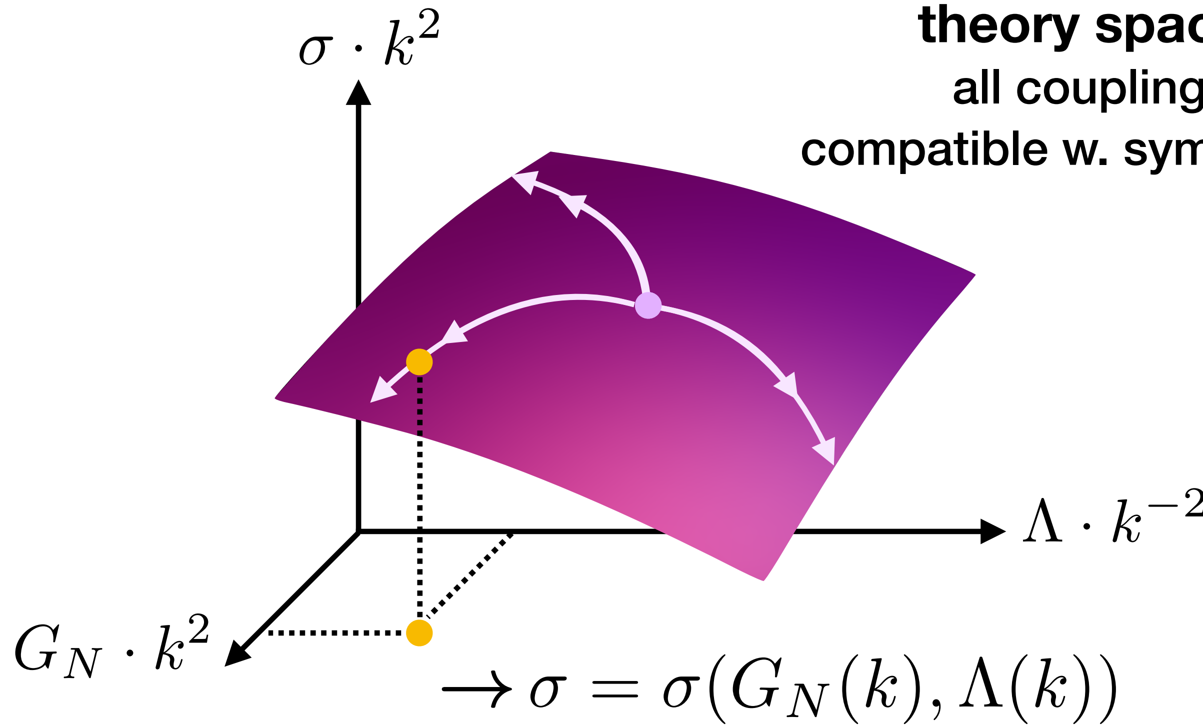



**Theory space features an interacting fixed point with a finite number of relevant directions. (At least) one trajectory emanating from the fixed point reaches a phenomenologically viable IR regime.**

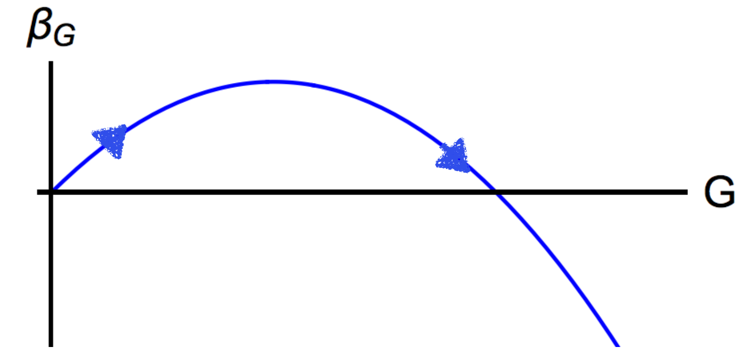
$\rightarrow$ UV complete

$\rightarrow$ predictive
(finite # free parameters)

$\rightarrow$ predictions for irrelevant couplings match observations

# Hints for asymptotic safety of gravity?

- **$\epsilon$ expansion in d=2+$\epsilon$** $\beta_G = \epsilon G - \frac{38}{3} G^2$




Weinberg '76; Christensen, Duff '78
Gastmans, Kallosh, Truffin '78

- **lattice simulations (Euclidean/Causal Dynamical Triangulations)**
continuum limit not conclusively established

Ambjorn, Jurkiewicz, Loll '01, '04…
Coumbe, Laiho '11, '17

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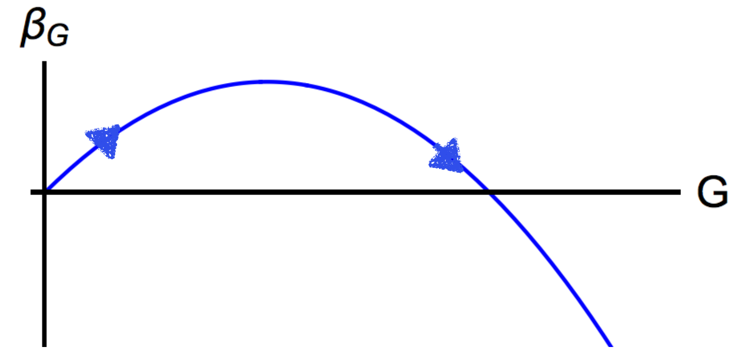


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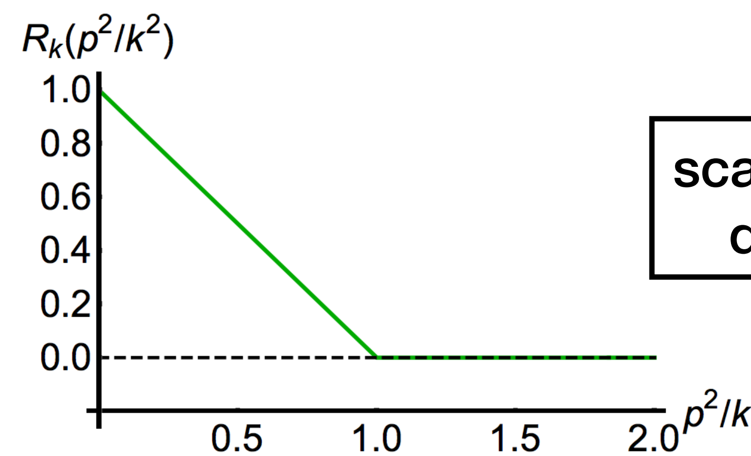
- **Functional Renormalization Group**

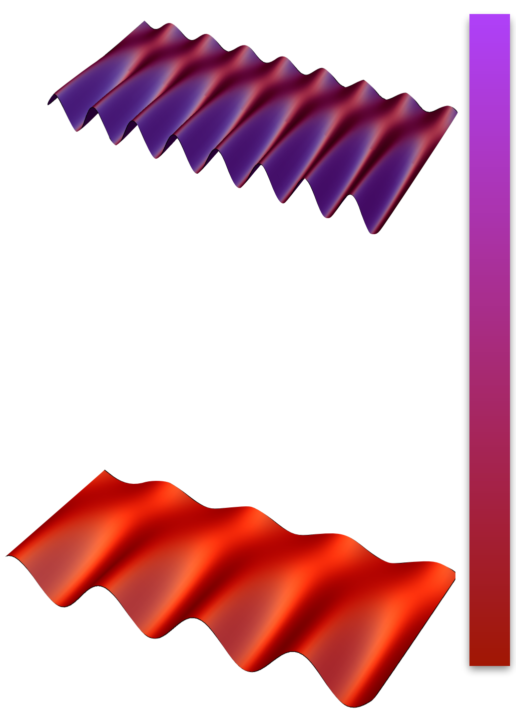
$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \frac{1}{2}\int \varphi(-p) R_k(p) \varphi(p)}$$

probe scale dependence of QFT

Wetterich ’93, Reuter ‘96




scale- and momentum-dependent ``mass”



$S$

$\Gamma_k$ contains effect of quantum fluctuations above k

$$\Gamma_k = \sum_i g_i(k) \int d^d x \, \mathcal{O}^i \;\; \rightarrow k\partial_k \Gamma_k = \sum_i \beta_{g_i} \int d^d x \mathcal{O}^i$$

$\Gamma_{k\rightarrow 0}$

Wetterich equation: $\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k =$ 

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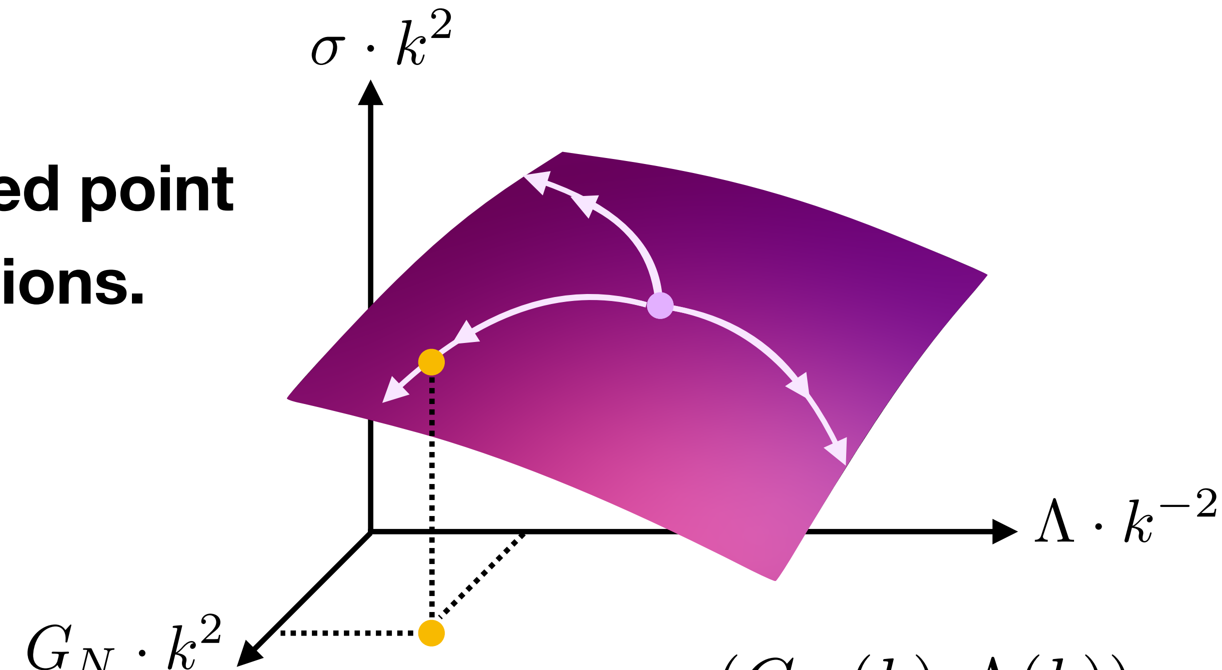
$\rightarrow$ UV complete

$\rightarrow$ predictive



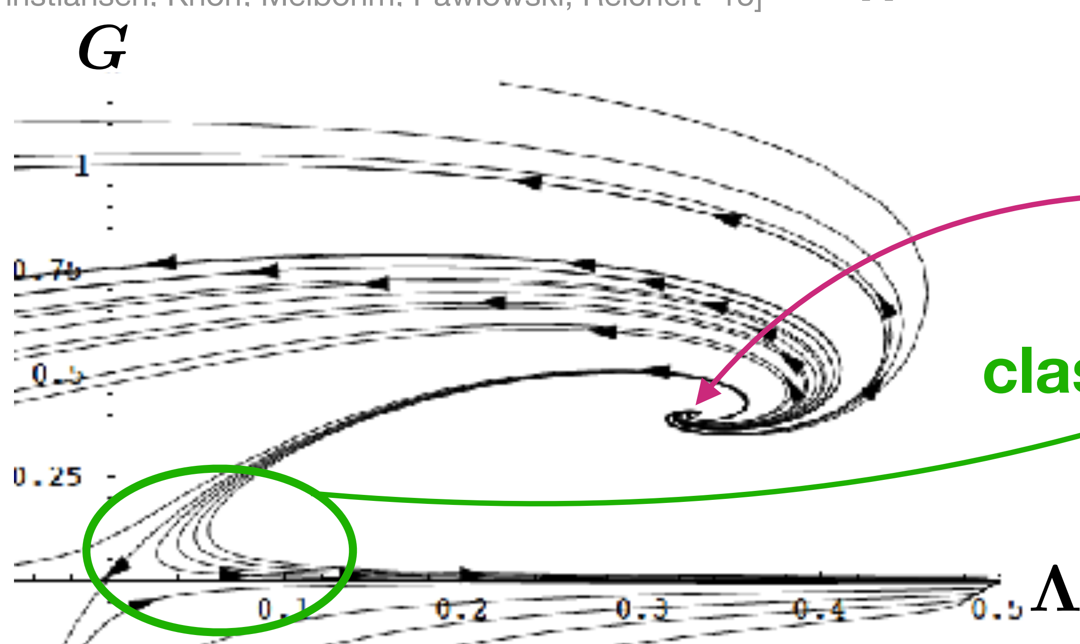

$\rightarrow \sigma = \sigma(G_N(k), \Lambda(k))$

| fixed point | operators | relevant | irrelevant |
|---|---|---|---|
| ✓ | $\sqrt{g}$ | x | |
| ✓ | $\sqrt{g}R$ | x | |

[Reuter '96, Lauscher, Reuter '01; Reuter, Saueressig '02; Becker, Reuter '14; Christiansen, Knorr, Meibohm, Pawlowski, Reichert '15]

**tool: functional RG equation in truncations of theory space**
**(well-tested for interacting fixed points)**

[Wetterich '93, Reuter '96]




**UV fixed point**

**classical gravity regime**

[Reuter, Saueressig '01]

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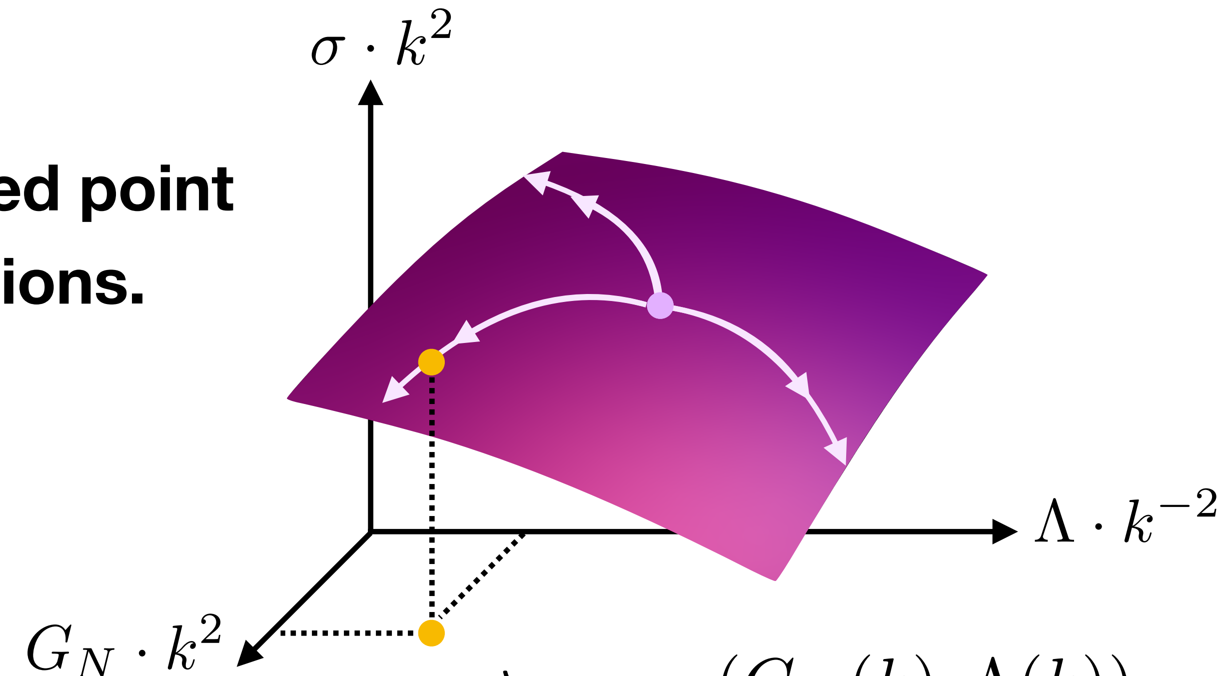


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| | . . . | | | . . . |
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[Wetterich '93, Reuter '96]

**ordering principle:**
**canonical mass dimension of couplings**

→ **control over approximations with finitely many couplings**

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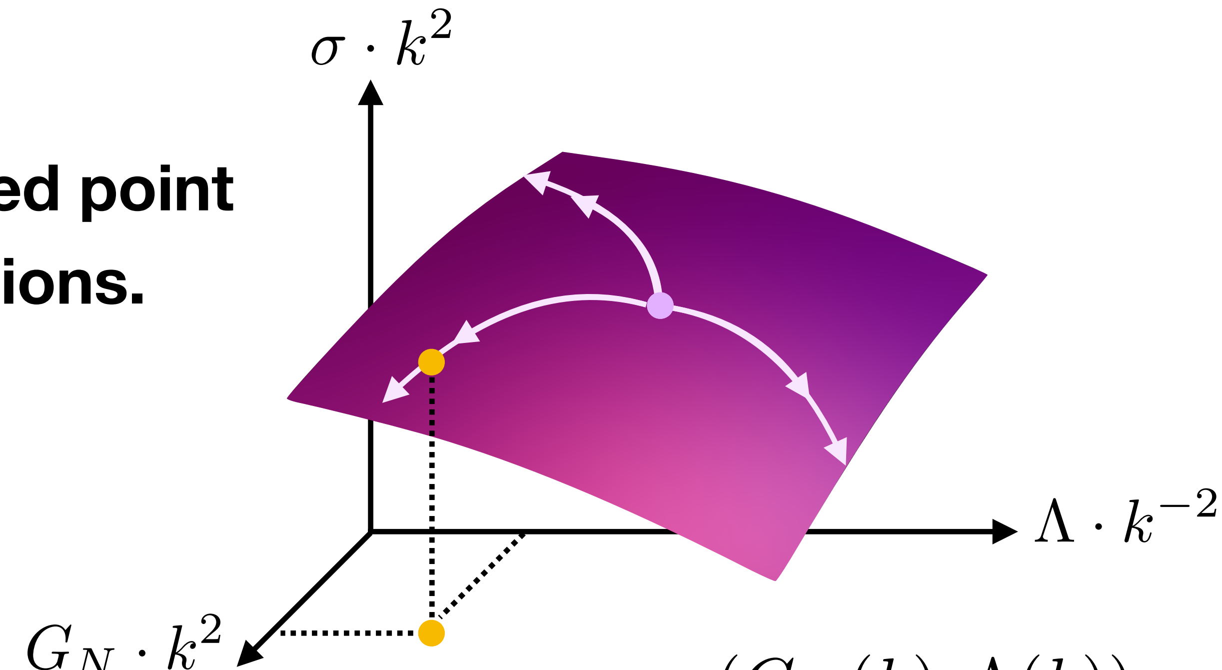


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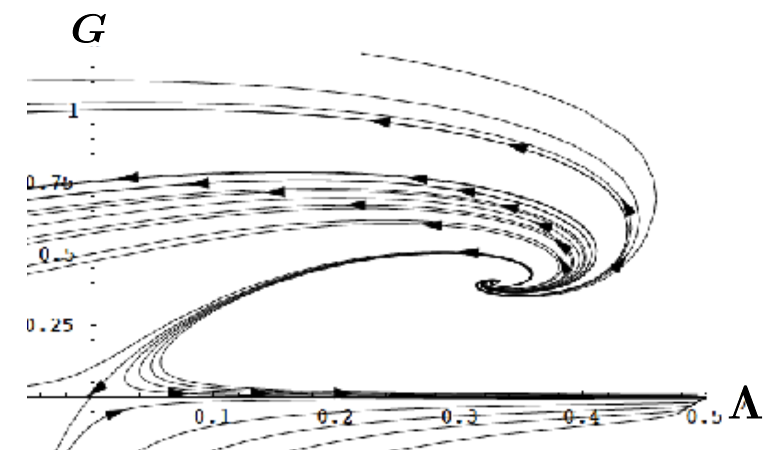
**status: compelling hints for asymptotic safety in pure gravity**

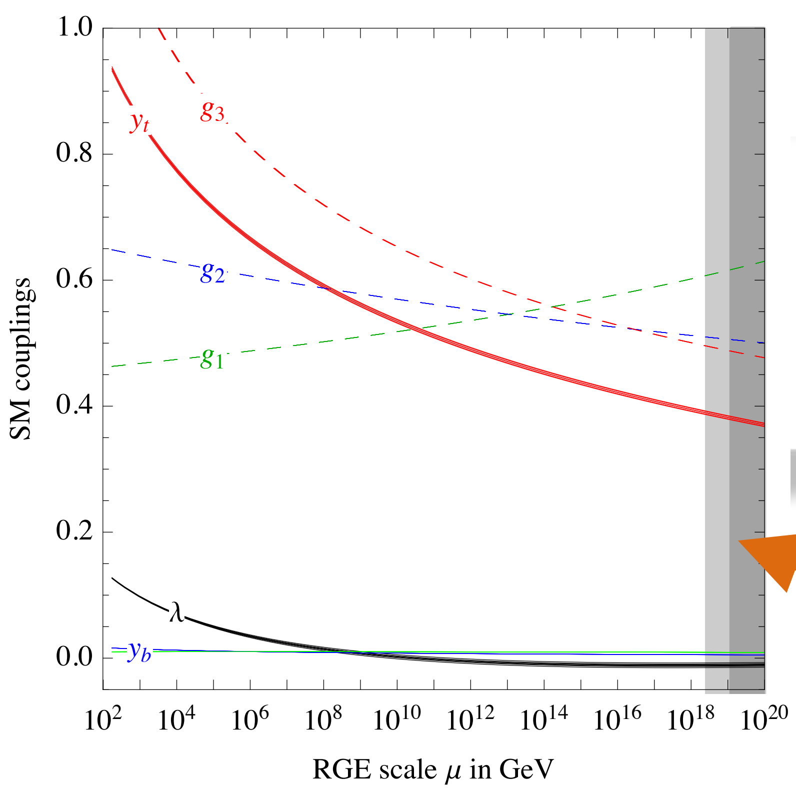
$\rightarrow$ **what about matter?**

# Asymptotic safety for gravity & matter

Can quantum fluctuations
of matter destroy a
consistent quantum gravity model?




**quantum gravity dynamics** **matter dynamics**




Buttazzo et al. '13

Can quantum fluctuations
of gravity generate
a viable UV completion for matter?

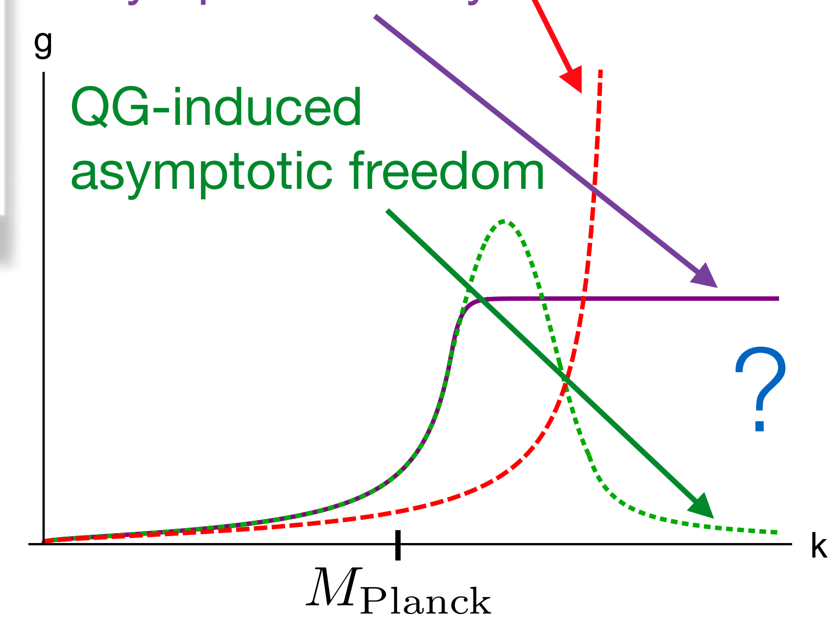
match onto SM
at Planck scale

QG generated
fixed point

triviality in Higgs-Yukawa & U(1)

QG-induced
asymptotic safety

QG-induced
asymptotic freedom



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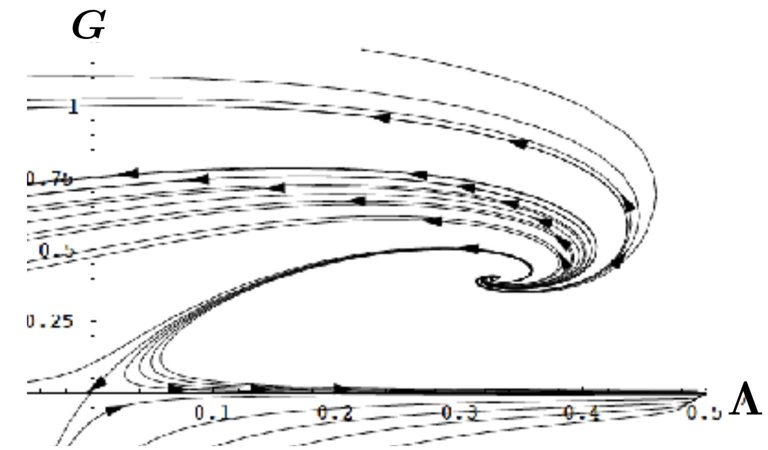
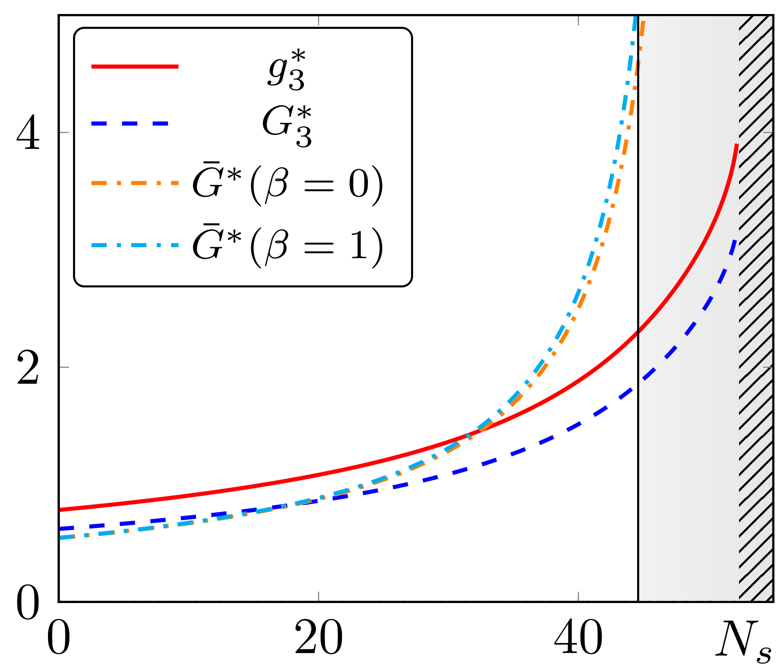
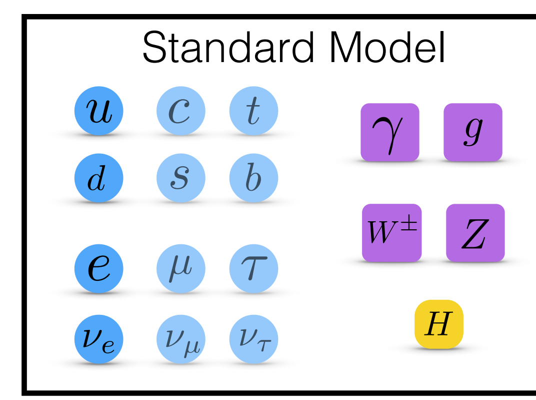



**matter matters:**




[AE, Labus, Pawlowski, Reichert to appear]

**minimally coupled SM fields**




**matter content of SM (& small extensions)**
**admits a gravity fixed point in Einstein-Hilbert truncation**

[Dona, AE, Percacci '13, '14]

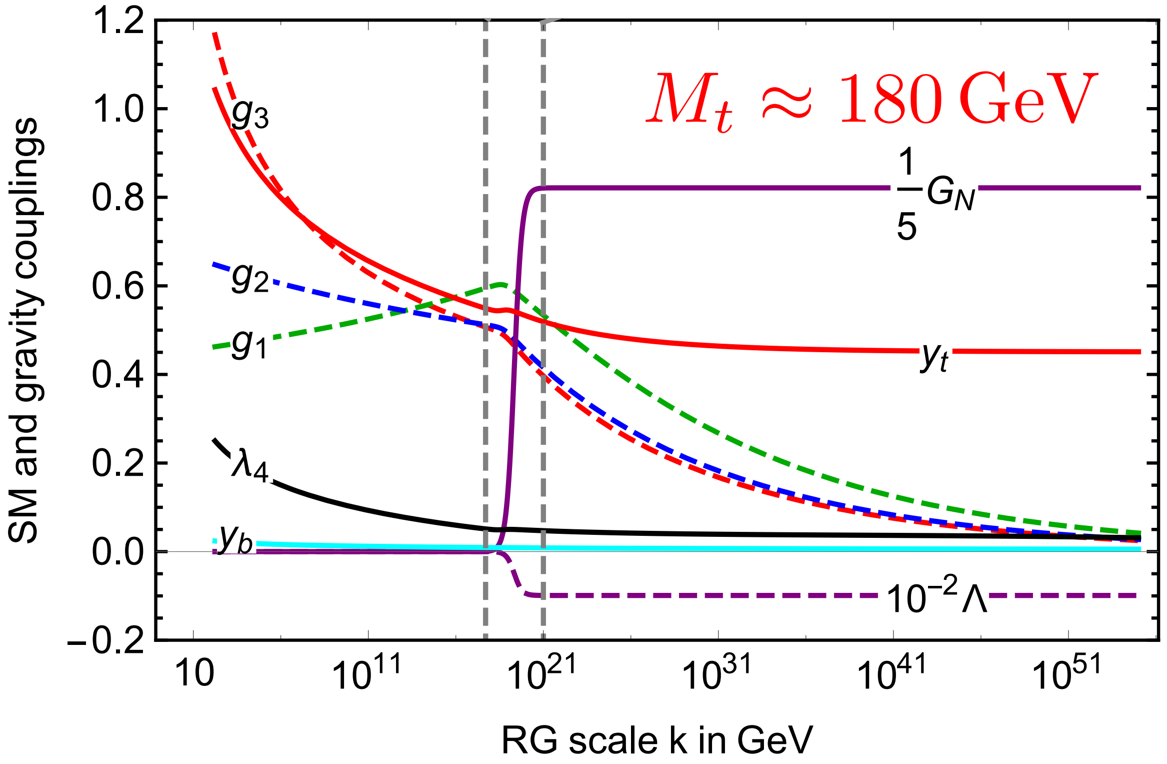
[Meibohm, Pawlowski, Reichert '15
Dona, AE, Labus, Percacci '15]

# Quantum-gravity induced UV completion for the SM


**results within simple truncations**

$\rightarrow$ **convergence of results in extended truncations: stay tuned…**

# Quantum-gravity induced UV completion for the SM




**within simple truncations:**

- **asymptotic freedom in all gauge couplings (incl. Abelian hypercharge)**

[Daum, Harst, Reuter ’10; Folkerts, Litim, Pawlowski ’11; Harst, Reuter ’11, Christiansen, AE ’17]

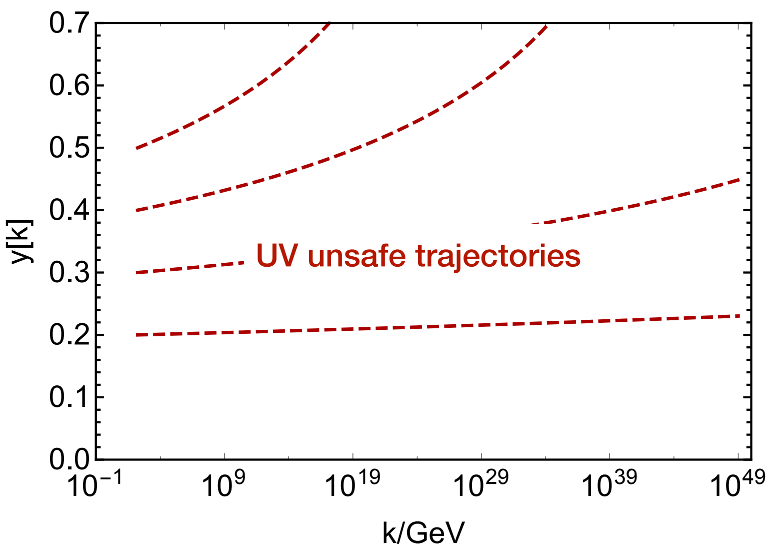
- **asymptotic safety in top Yukawa coupling with $M_t$ >> $M_b$ fixed uniquely**

[AE, Held, Pawlowski ’16; AE, Held 05/17, 07/17]

# Top mass from asymptotic safety

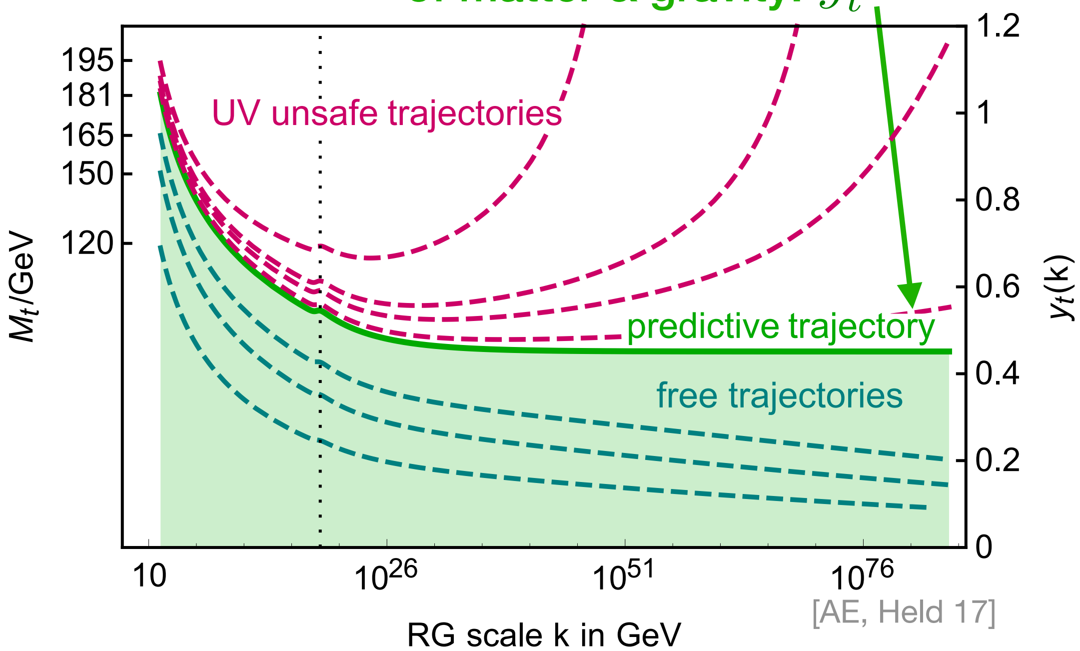
**two regimes in grav. coupling space:**

$f_y(\Lambda) > 0$




$f_y(\Lambda) < 0$

balance of qm fluc's of matter & gravity: $y_t^* > 0$



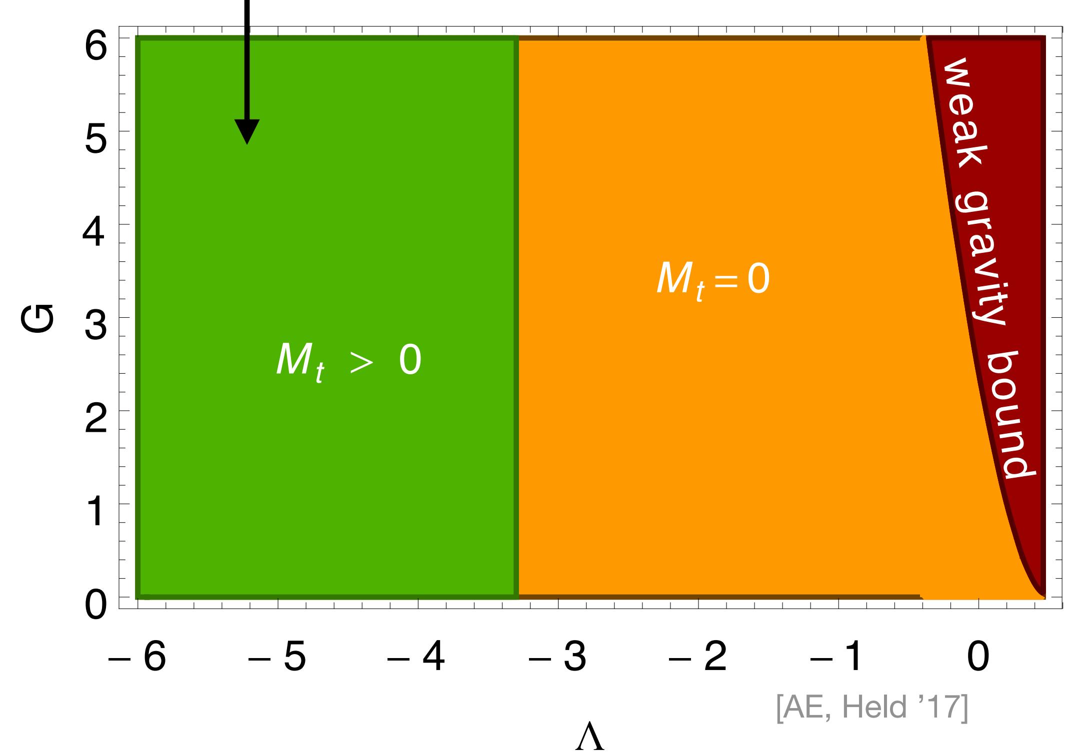

**asymptotic safety in top Yukawa coupling**

$y_t$: **free parameter in SM** $M_t \approx 172.4\,\mathrm{GeV}$ [CMS '16]

$$\beta_{y_t} = \frac{1}{32\pi^2}\left(9y_t^3 \qquad\qquad\right) + G\, y_t\, f_y(\Lambda)$$

[AE, Held, Pawlowski '16; AE, Held 17]

**$M_t(G,\Lambda)$ uniquely fixed**

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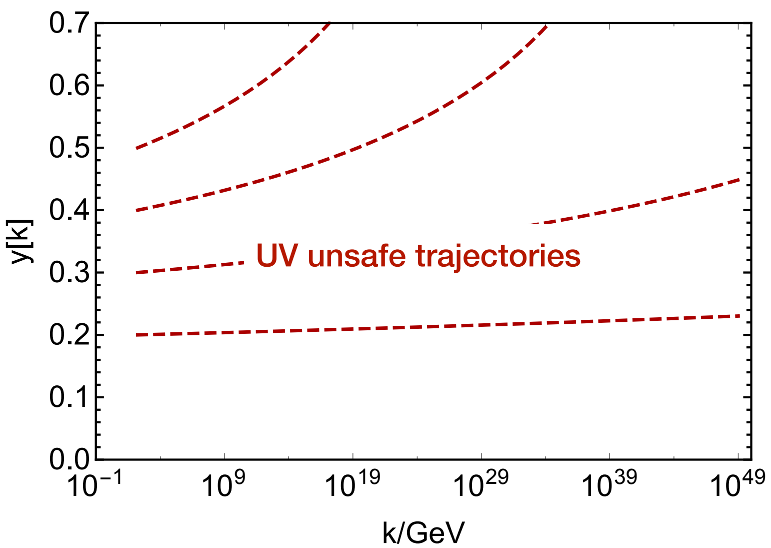


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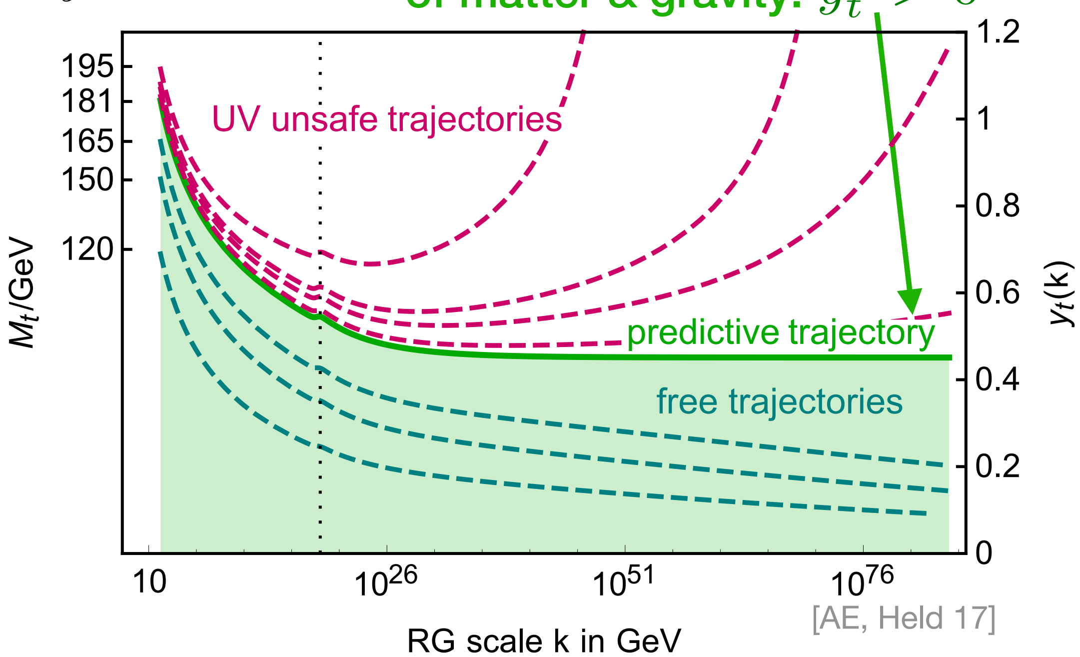


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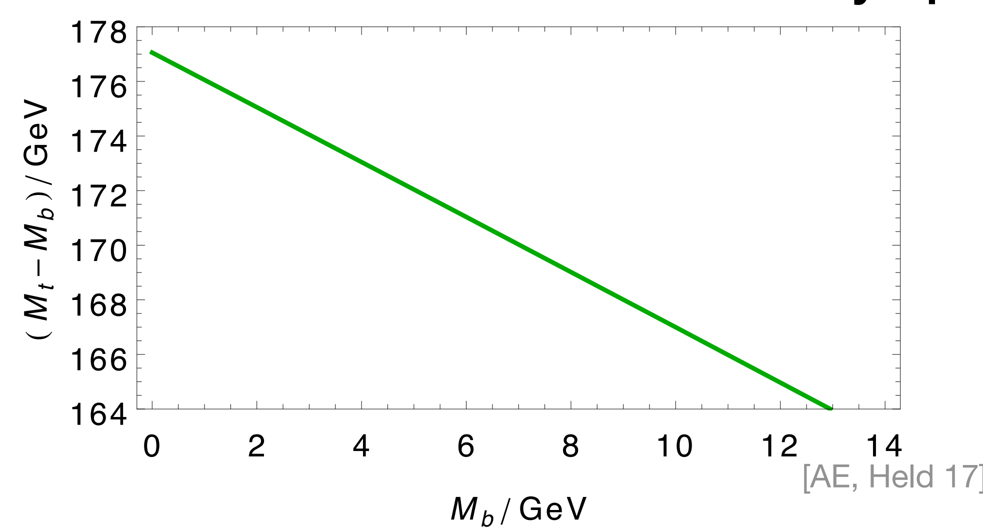
[AE, Held 17]

$$\beta_{y_b} = \frac{1}{32\pi^2}\left(9y_b^3 + 3y_t^2\, y_b\right) + G\, y_b\, f_y(\Lambda)$$

$$y_t^* > 0,\; y_b^* = 0$$

**irrelevant: in IR:** $y_t(G, \Lambda, y_b)$ **relevant**

**$M_t$ fixed in terms of $M_b$: mass difference from asymptotic safety!**




**$M_t$, $M_b$ < $M_{Pl}$: chiral symmetry unbroken by QG**

[AE, Gies 11; Meibohm, Pawlowski '15; AE, Lippoldt '16]

# Top mass from asymptotic safety

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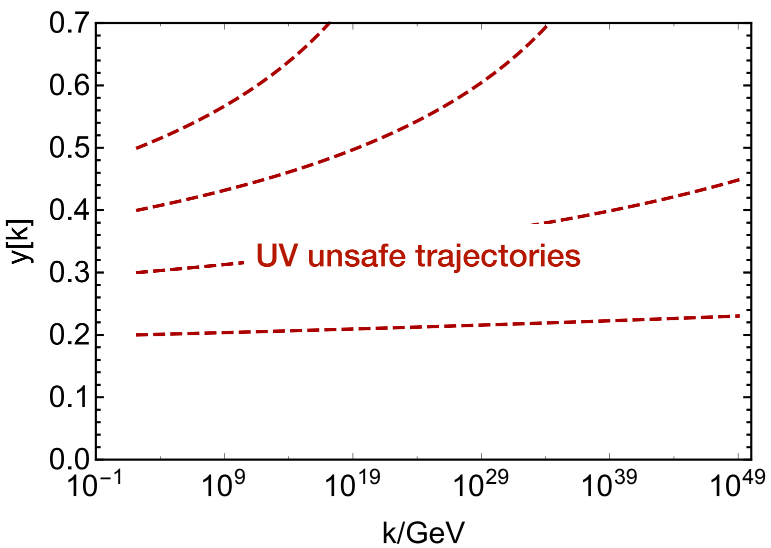


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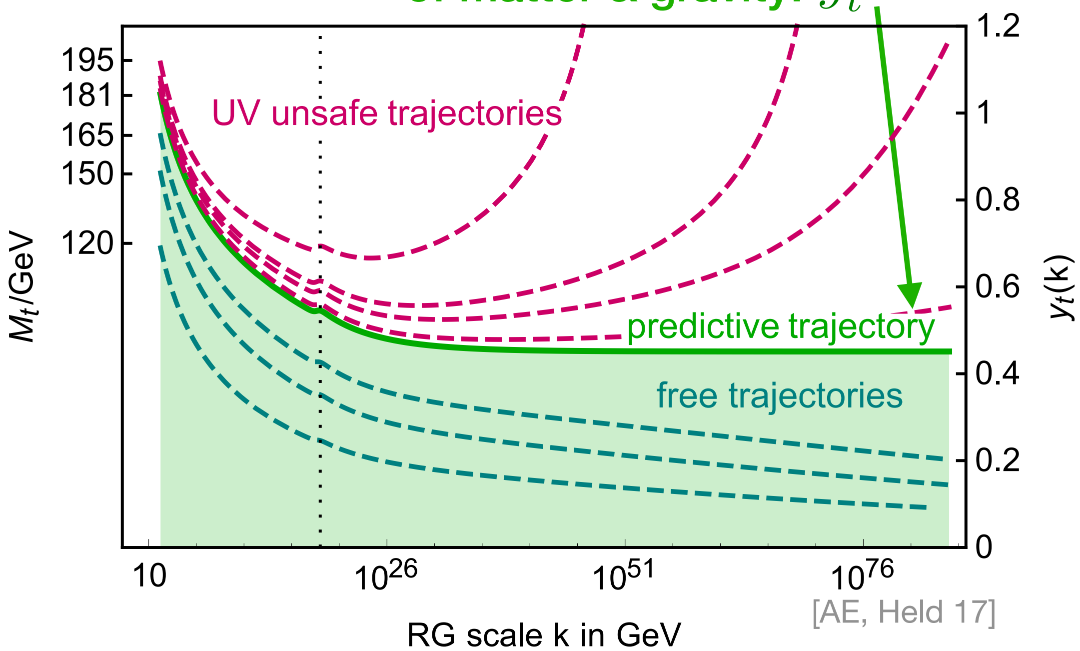


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[AE, Held 17]

**contains running gauge couplings**

$$\beta_{y_b} = \frac{1}{32\pi^2}\left(9y_b^3 + \mathrm{SM}\right) + G\, y_b\, f_y(\Lambda)$$

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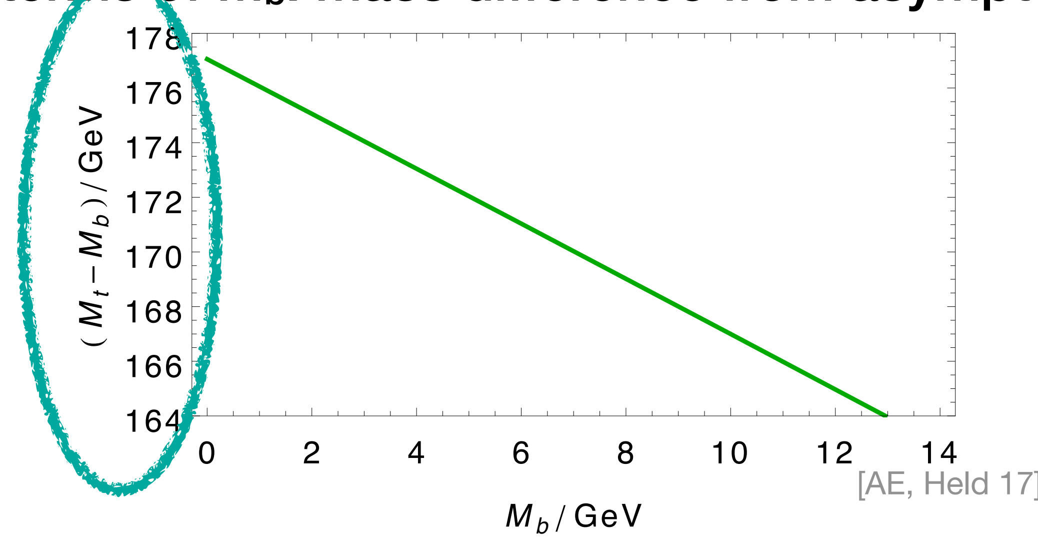
**$M_t$ fixed in terms of $M_b$: mass difference from asymptotic safety!**




**$M_t$, $M_b$ < $M_{Pl}$: chiral symmetry unbroken by ASQG**

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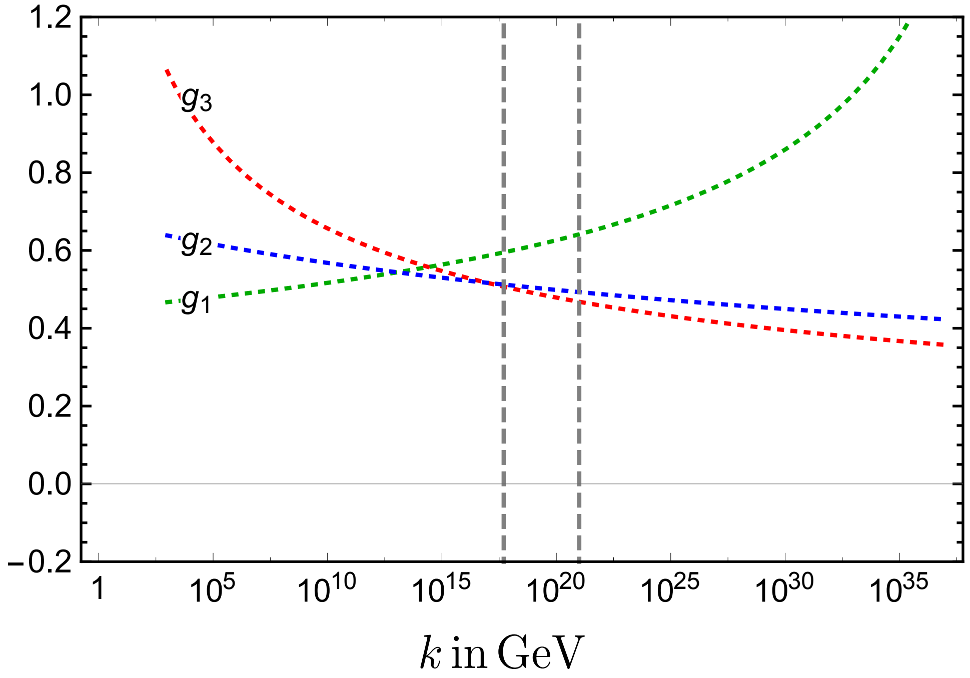
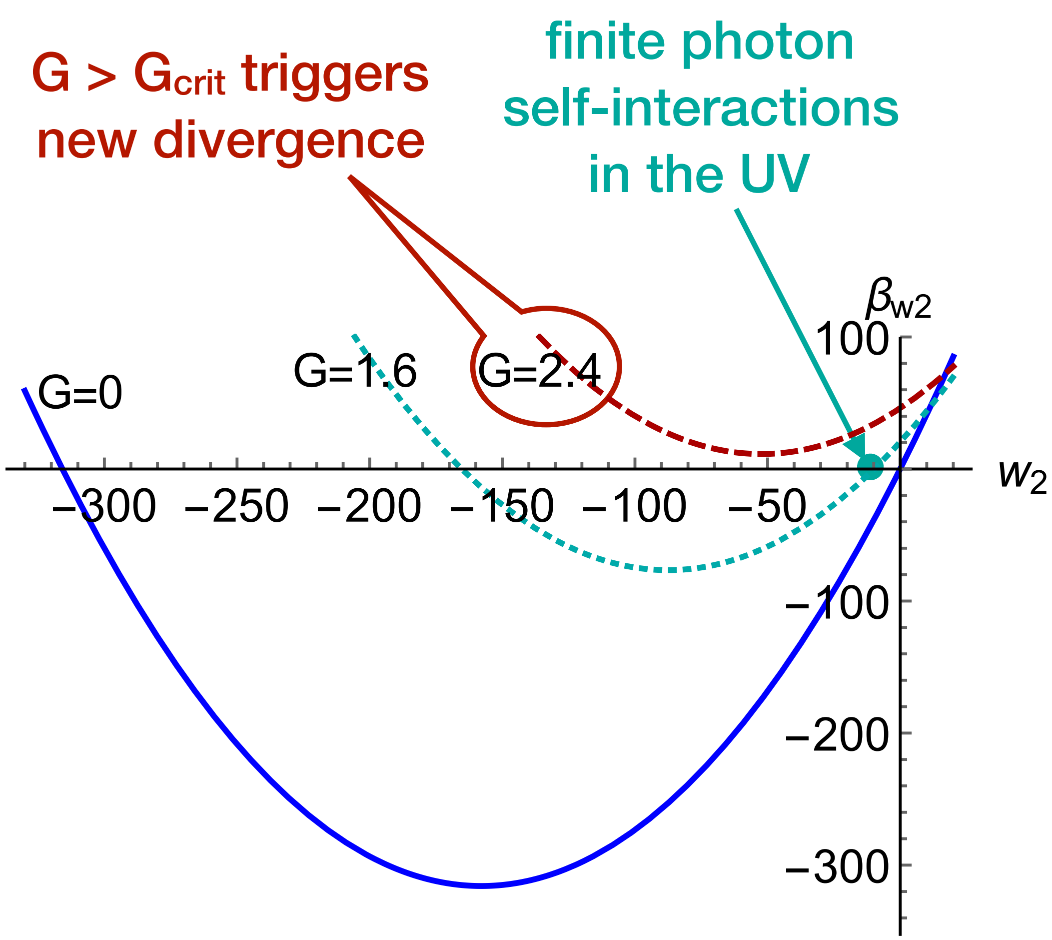
# An asymptotically safe solution to the U(1) triviality problem?

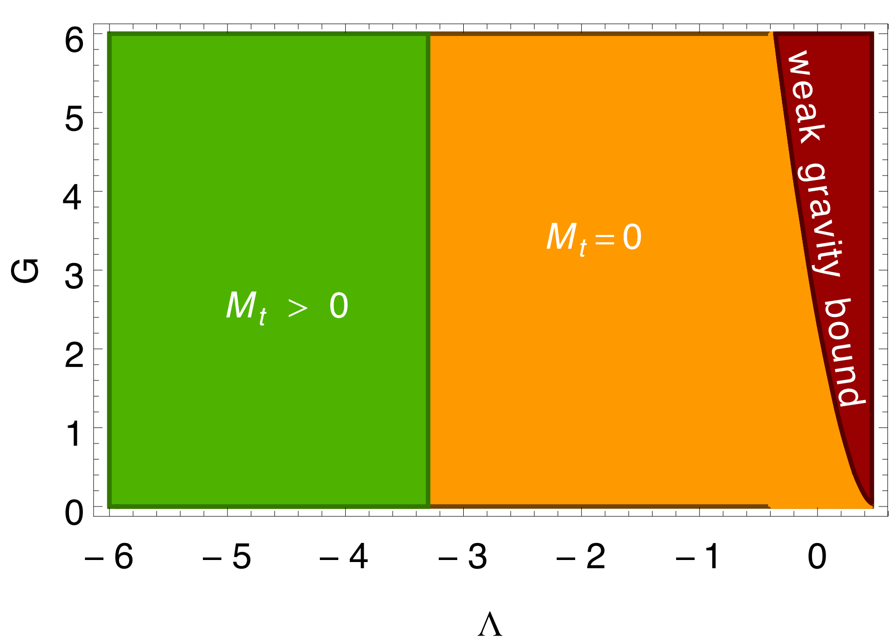



**triviality problem in Abelian hypercharge**

[Gell-Mann, Low ’54; Gockeler et al. 98; Gies, Jaeckel ’04]

# An asymptotically safe solution to the U(1) triviality problem?




**within simple truncations ASQG induces:**

- **asymptotic freedom in all gauge couplings (incl. Abelian hypercharge)**

**truncation: Einstein- Hilbert + $F^2$**

[Daum, Harst, Reuter '10; Folkerts, Litim, Pawlowski '11; Harst, Reuter '11]

$$\beta_{g_1} = \frac{g_1^3}{16\pi^2}\frac{41}{10} - G\, g_1\, f(\Lambda)$$

$+w_2\, F^4$ **QG-induced photon interactions**

[Christiansen, AE '17]

**weak-gravity bound:** [AE, Held, Pawlowski '16; Christiansen, AE '17]

"Strong" quantum gravity triggers new divergences in matter interactions

$G < G_{crit}$ in truncations

see [AE, Held 17]

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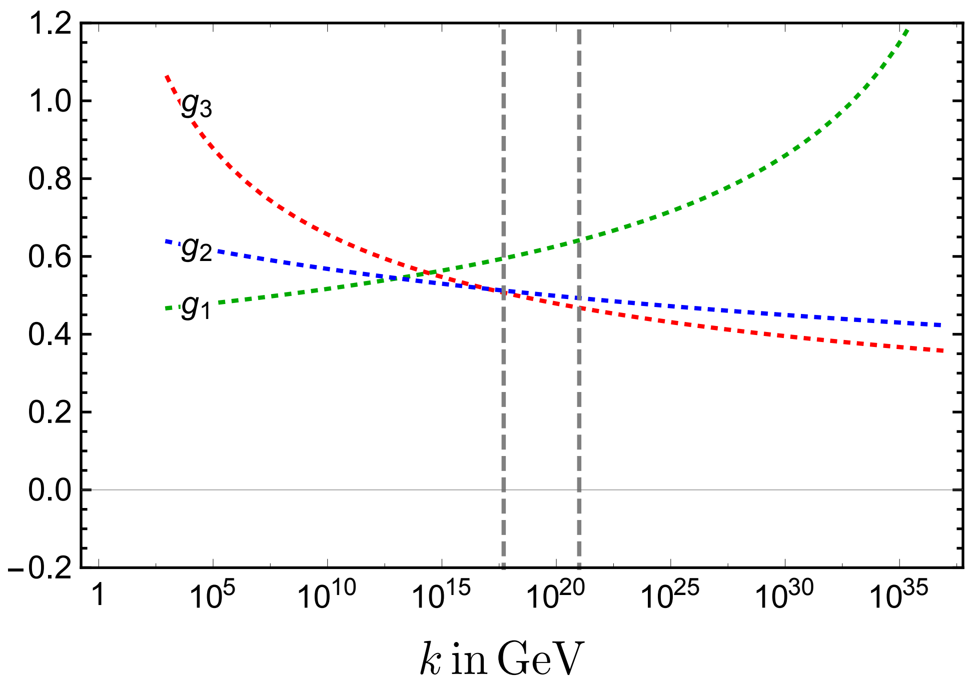


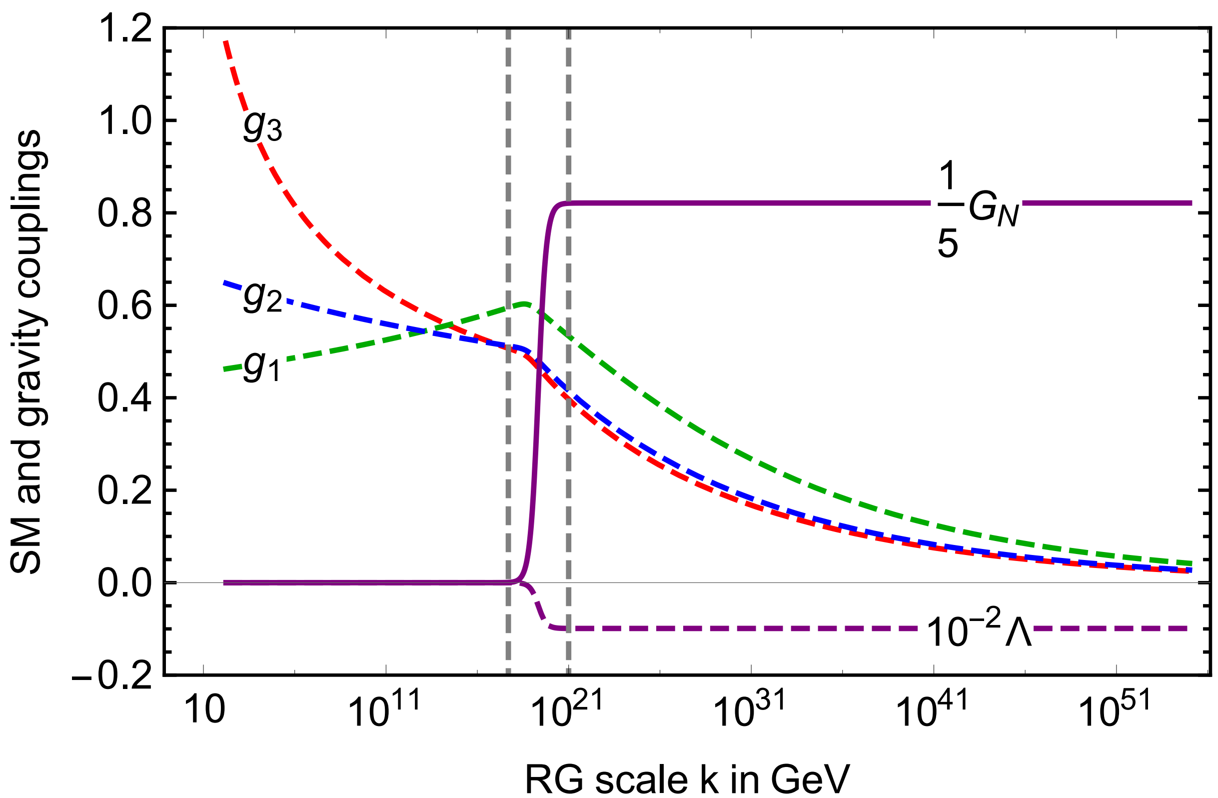


**within simple truncations ASQG induces:**

- **asymptotic freedom in all gauge couplings (incl. Abelian hypercharge)**

$\rightarrow$ **power-law running towards free fixed point**

**truncation: Einstein- Hilbert +** $F^2$

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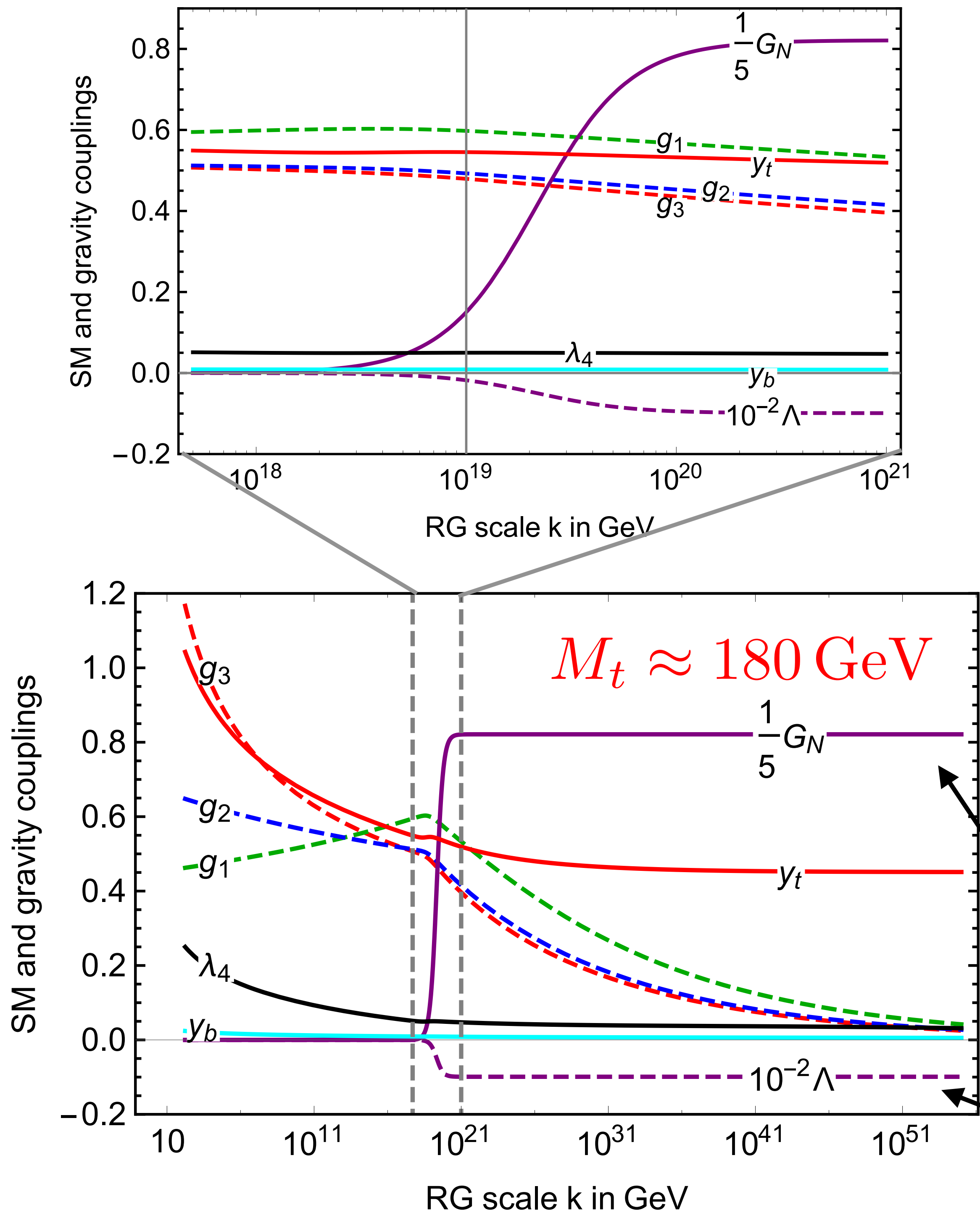



mechanism for Higgs mass:

[Shaposhnikov, Wetterich '09]

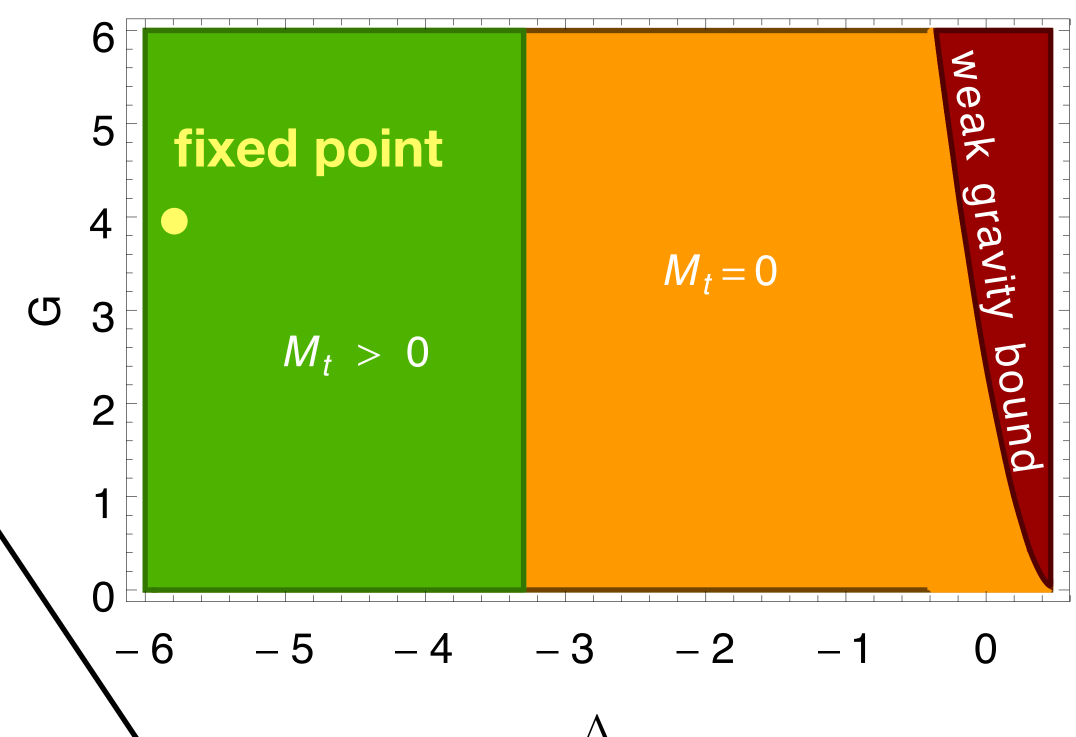
**within simple truncations:**

- **asymptotic freedom in all gauge couplings (incl. Abelian hypercharge)**

[Daum, Harst, Reuter '10; Folkerts, Litim, Pawlowski '11; Harst, Reuter '11, Christiansen, AE '17]

- **asymptotic safety in top Yukawa coupling in part of grav. coupling space**

[AE, Held, Pawlowski '16; AE, Held 05/17, 07/17]




$\beta_G,\ \beta_\Lambda$ **including SM matter fields ***

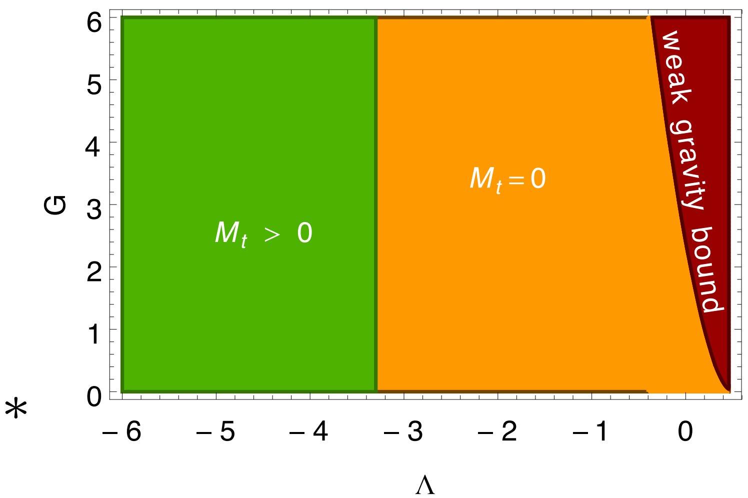
[Dona, AE, Percacci '13]

*** convergence in fixed-point values: to be tested**

# Learning about the dark sector from asymptotic safety

**dark sector might only couple gravitationally**

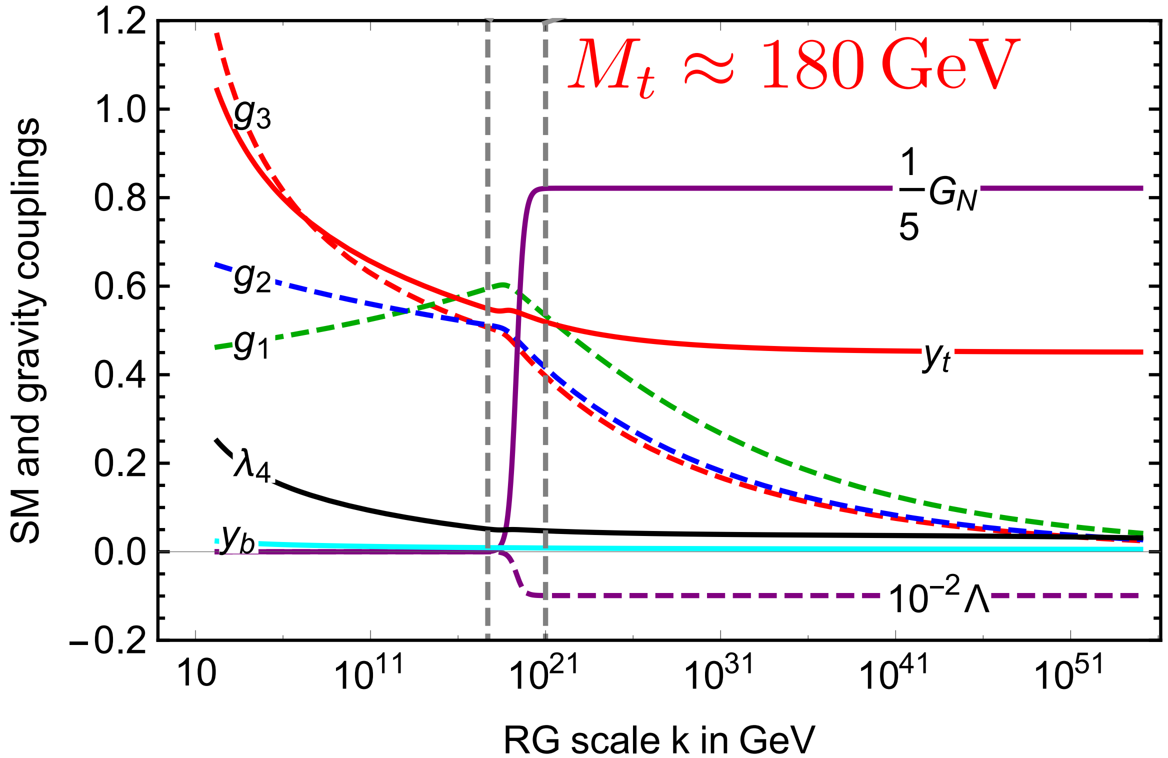
$\longrightarrow$ **direct detection very challenging**

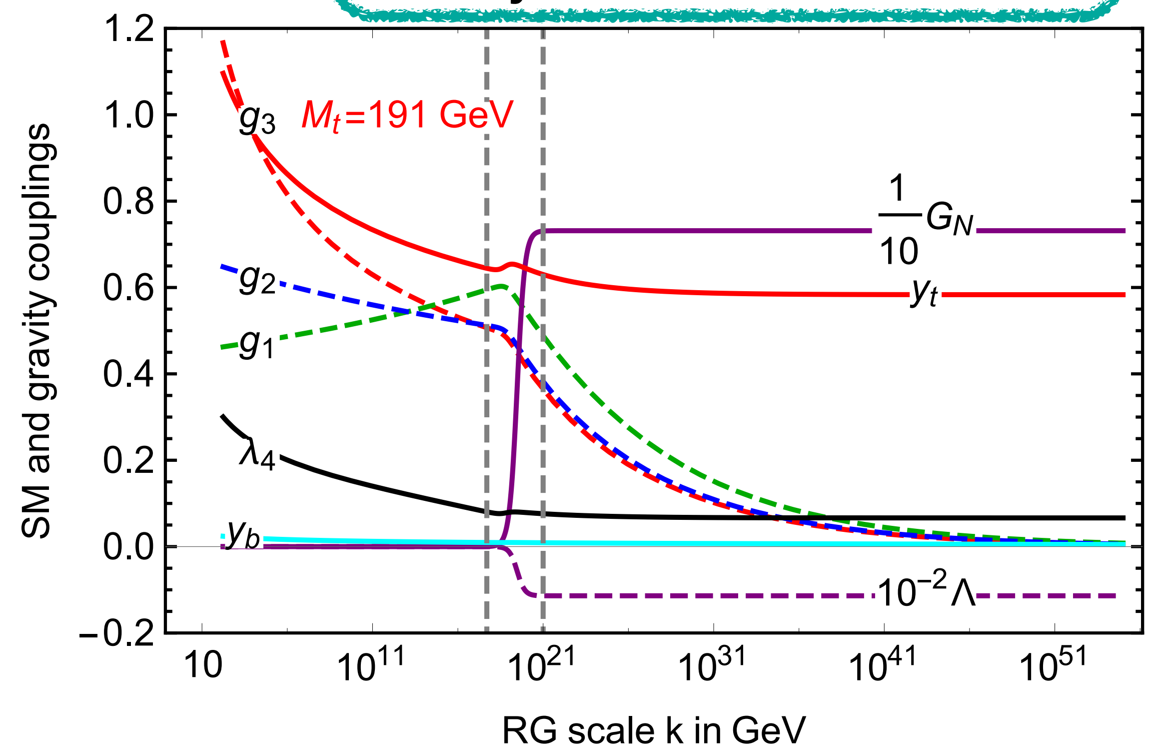



**asymptotic safety: ALL degrees of freedom affect** $G^*,\ \Lambda^*$

$\longrightarrow$ **top-mass value depends on dark sector!** only gravitationally coupled

toy examples: convergence in fixed-point results: to be tested! SM + 3 Weyl fermions + 1 scalar

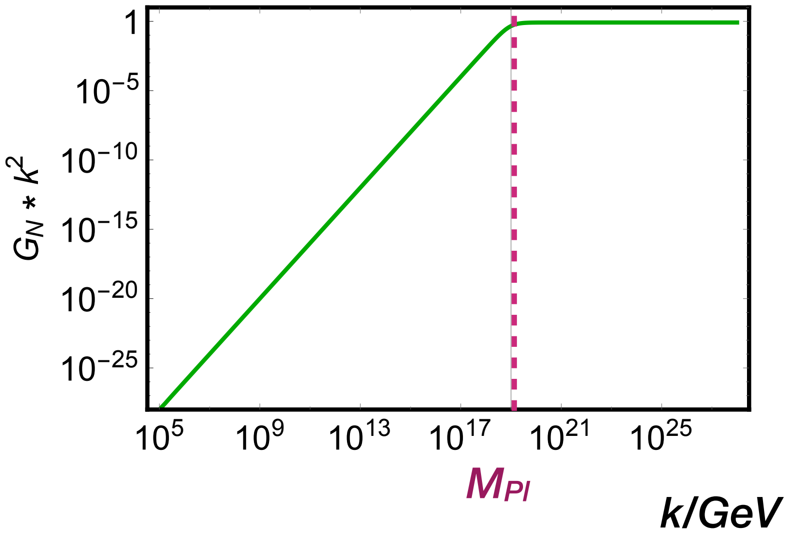


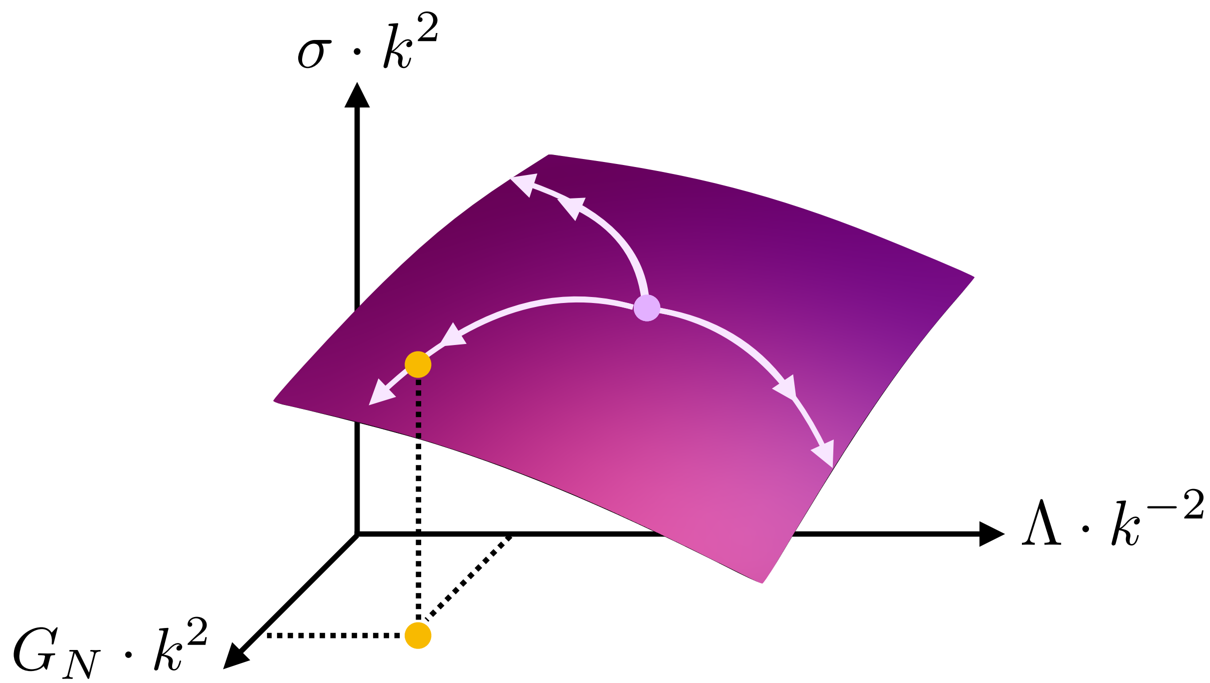




outlook: constrain dark sector by matching top-mass value from AS to measured value

# Conclusions

**Asymptotic safety:**
**Quantum field theory for gravity & matter on all scales**

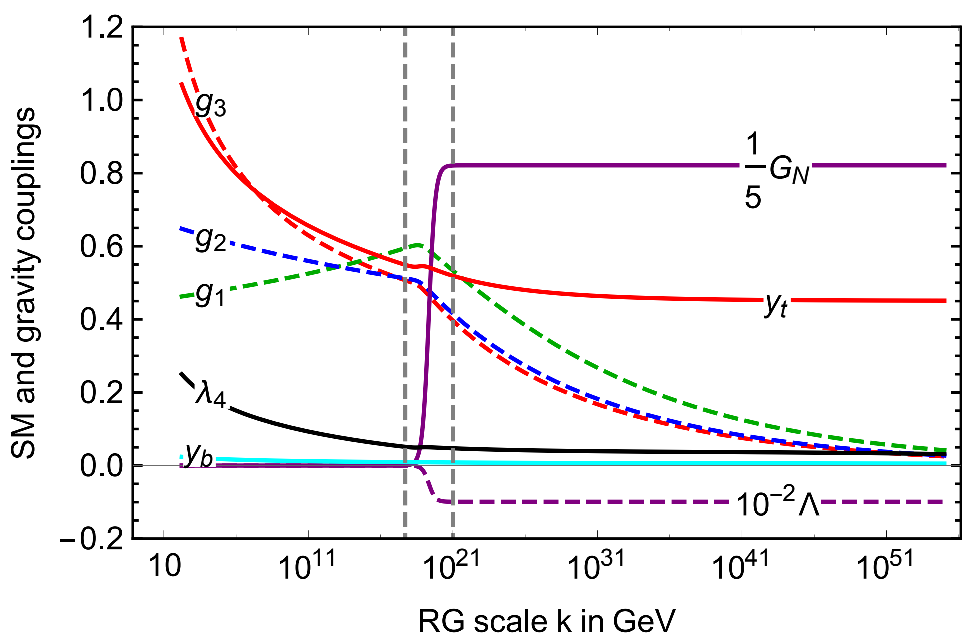



**microscopically:**
**quantum scale- invariance**

**macroscopically:**
**predictions for irrelevant couplings**


**potential consequences:**
**UV completion for Standard Model with fewer free parameters:**
**top-mass value explained,**
**mass-difference to bottom generated**




**outlook:**
- **quantitative convergence**
- **what about the other parameters of the SM?**
- **global stability of Higgs potential & link to Higgs inflation**