Early universe cosmology of Yukawa interactions and primordial black holes (?)

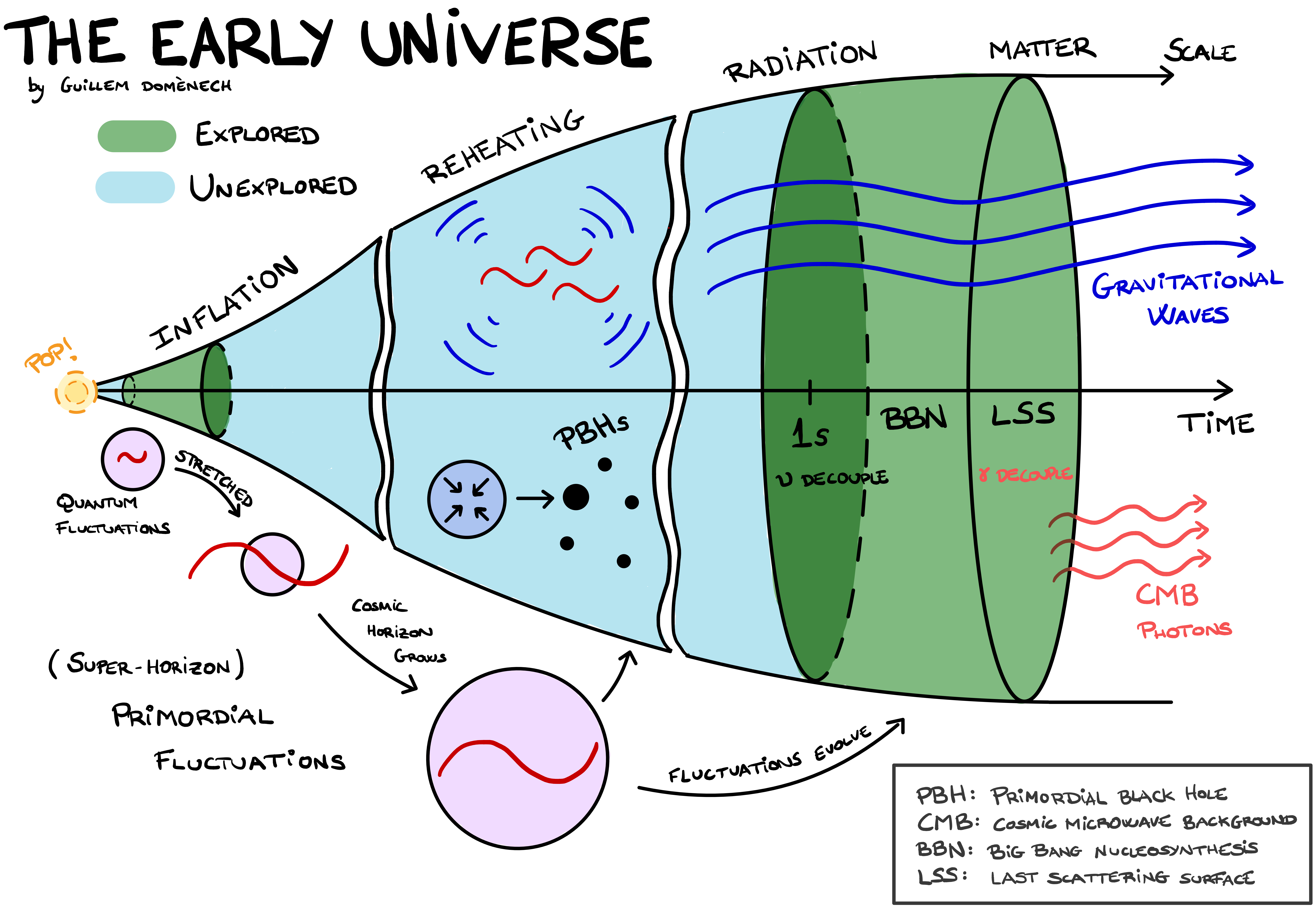
Based on [**2104.05271** & **2304.13053**]
with: D. Inman, A. Kusenko & M. Sasaki

by **Guillem Domènech**
(ITP Hannover)

Seminar at MPIK, Heidelberg,
June 17th, 2024

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Leibniz Universität Hannover



THE EARLY UNIVERSE
by Guillem Domènech
Explored
Unexplored
Pop!
Inflation
Reheating
Radiation
Matter
Scale
Time
Gravitational Waves
PBHs
1s
ν decouple
BBN
LSS
γ decouple
CMB Photons
Quantum Fluctuations
Stretched
Cosmic Horizon Grows
(Super-Horizon) Primordial Fluctuations
Fluctuations evolve
PBH: Primordial Black Hole
CMB: Cosmic Microwave Background
BBN: Big Bang Nucleosynthesis
LSS: Last Scattering Surface

Why?

General (important) questions:

- Can we probe the physics during & after **cosmic inflation**?
- What is **dark matter**? Particles or (Primordial) **Black Holes**?
- (New) **gravitational waves probes**: all LIGO/VIRGO BHs astrophysical? PTAs?
- DM physics seeds of supermassive black holes? Early galaxies?
- How to test new cosmology and beyond SM physics in the very early universe?

My motivation: Primordial black holes are nice!

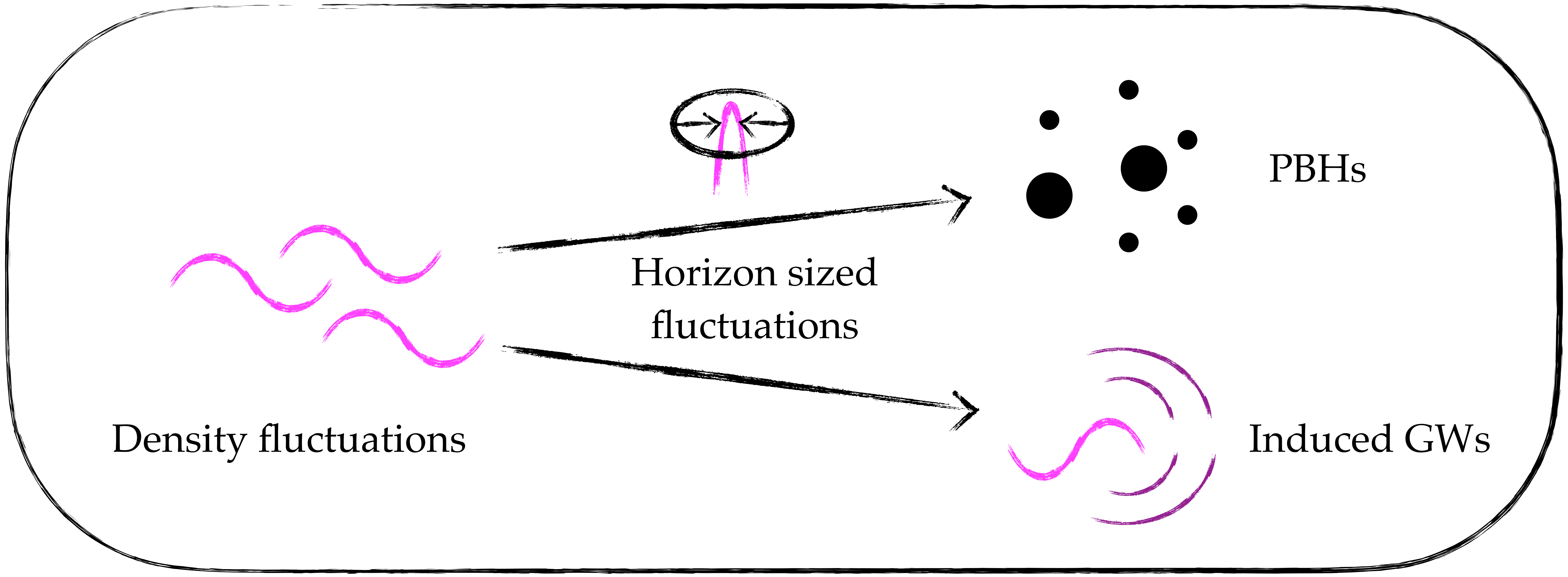
Most common mechanism: **collapse of large primordial fluctuations**

[Carr & Hawking 1974]

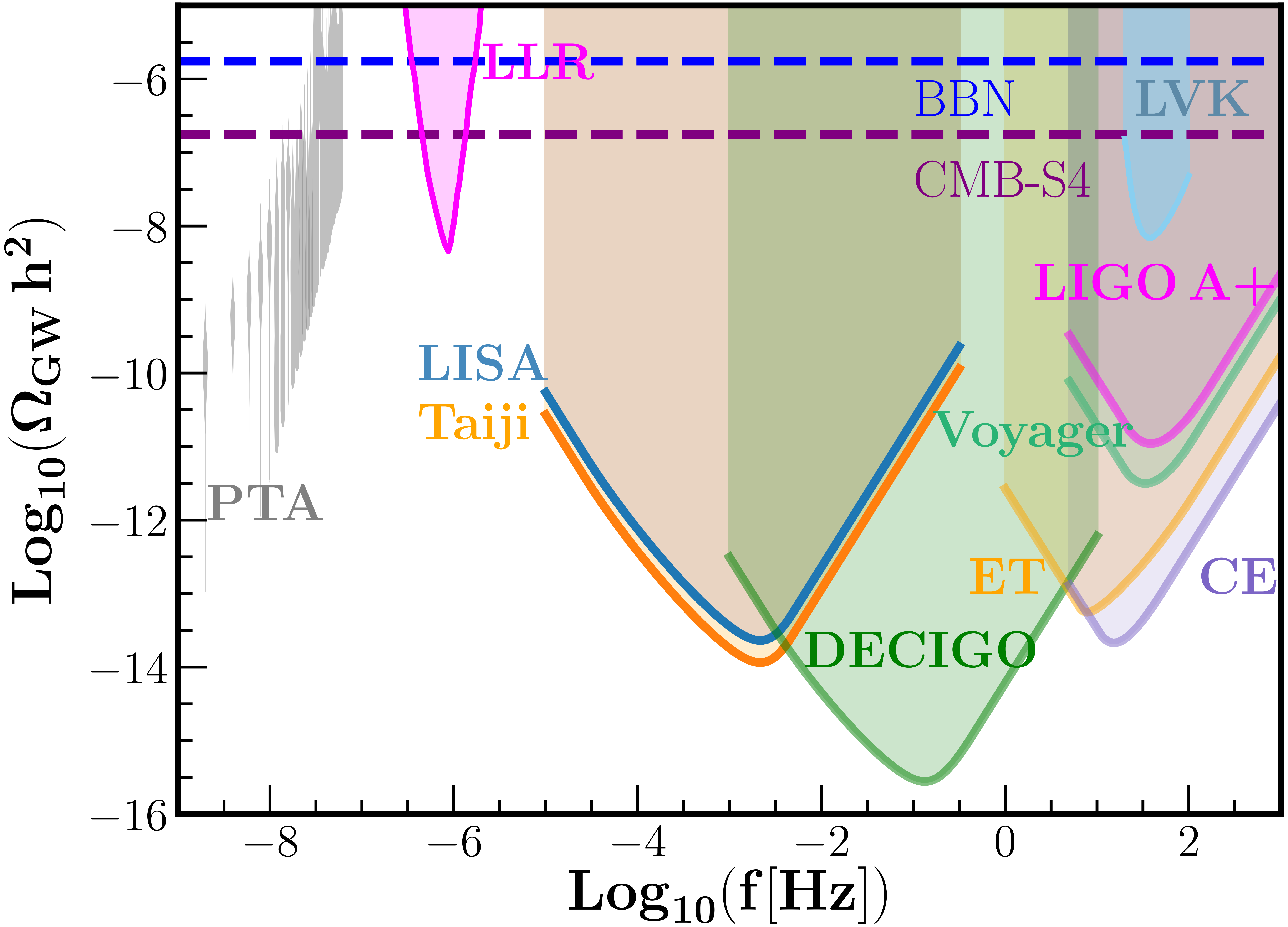
Connection to (or test of) cosmic inflation

Predicts (nice) observable GW signal (induced GWs)

[Review: Domènech 2109.01398]



Exciting evidence from PTA!



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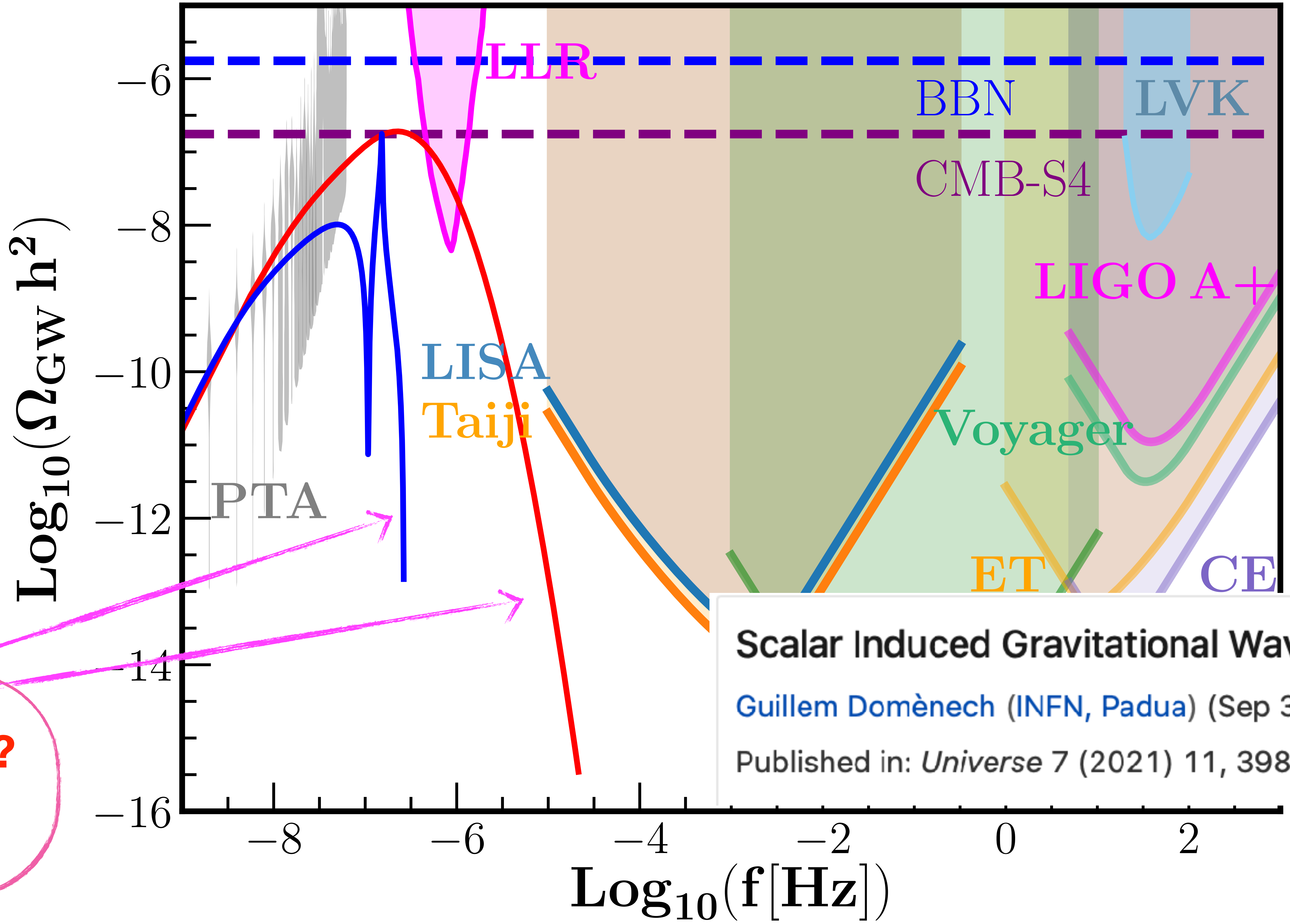


Induced GWs?

PBHs?

Scalar Induced Gravitational Waves Review

Guillem Domènech (INFN, Padua) (Sep 3, 2021)

Published in: *Universe* 7 (2021) 11, 398 • e-Print: 2109.01398 [gr-qc]

My motivation: Primordial black holes are nice!

BUT:

- What if GW signal is not as expected?
- Other mechanisms to form BH in early universe?
  Astrophysical-like channels?

Yukawa forces can be MUCH stronger than gravity!

[Amendola, Rubio & Wetterich: 1711.09915]

[Flores & Kusenko: 2008.12456]

Primordial black holes from fifth forces

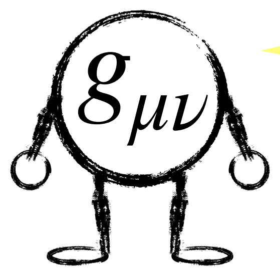
Luca Amendola,[1,*] Javier Rubio,[1,†] and Christof Wetterich[1,‡]

[1] *Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

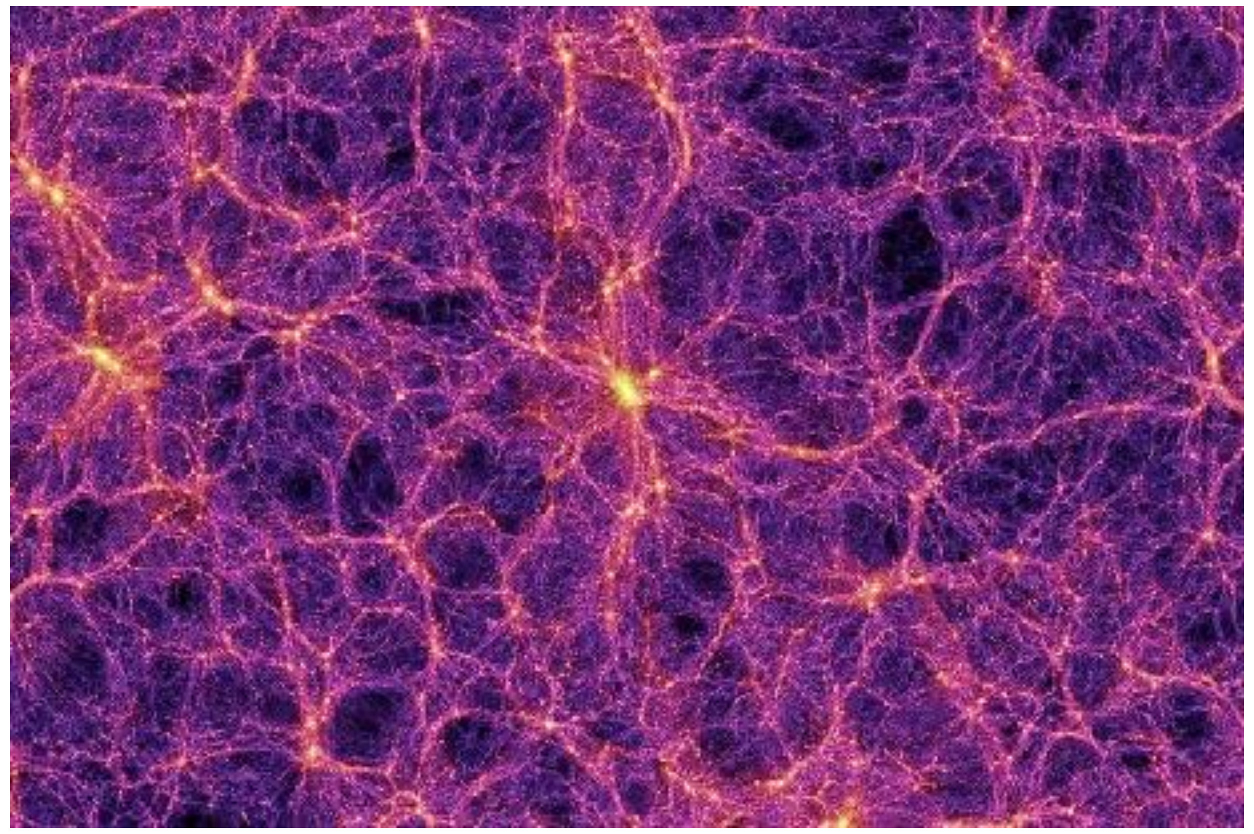
Primordial black holes can be produced by a long range attractive fifth force stronger than gravity, mediated by a light scalar field interacting with nonrelativistic "heavy" particles. As soon as the energy fraction of heavy particles reaches a threshold, the fluctuations rapidly become nonlinear. The overdensities collapse into black holes or similar screened objects, without the need for any particular feature in the spectrum of primordial density fluctuations generated during inflation. We discuss whether such primordial black holes can constitute the total dark matter component in the Universe.

Main message (& spoiler)

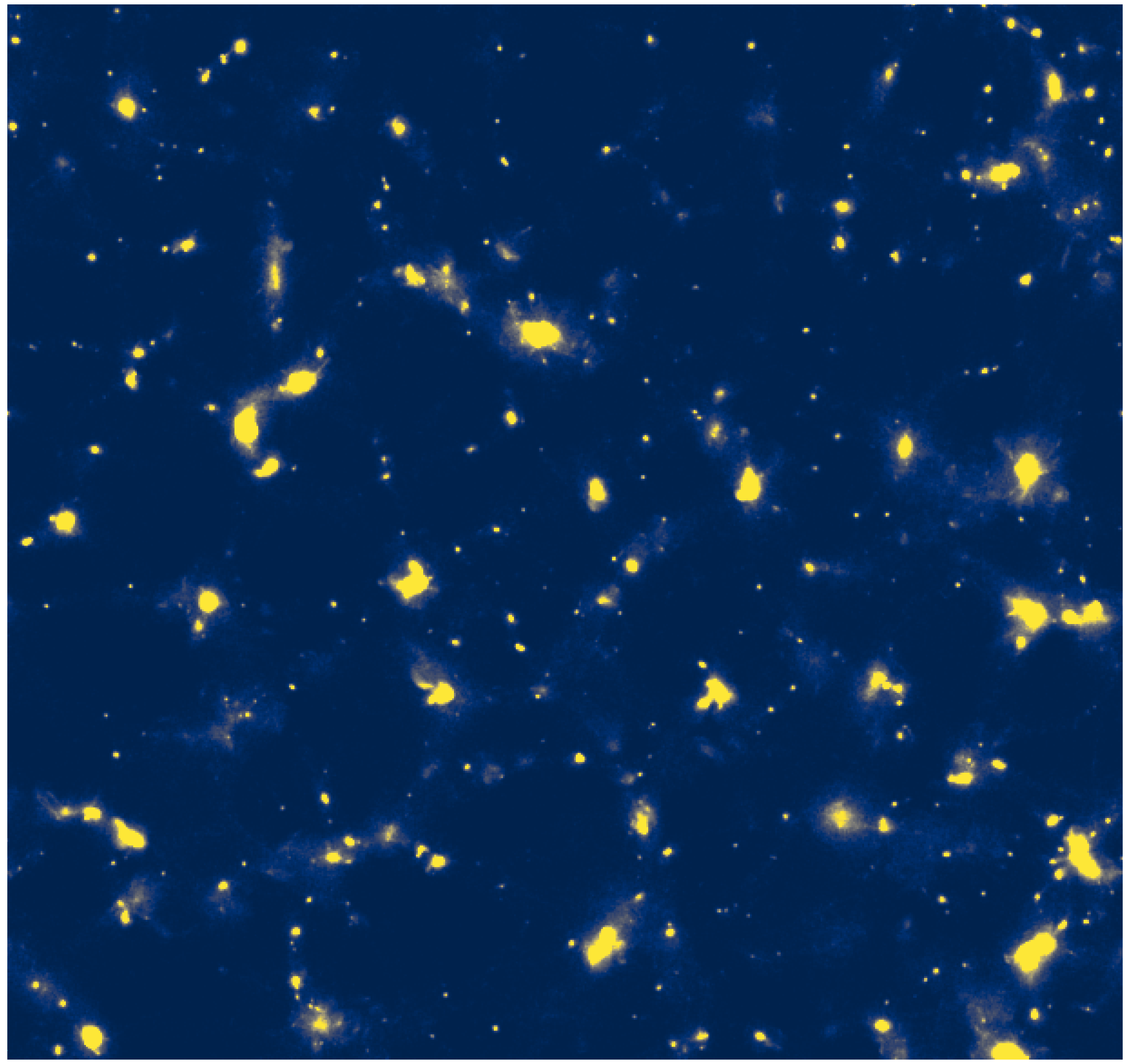
“Yukawa forces can efficiently form structures in the very early universe”



Compare with the “cosmic web”



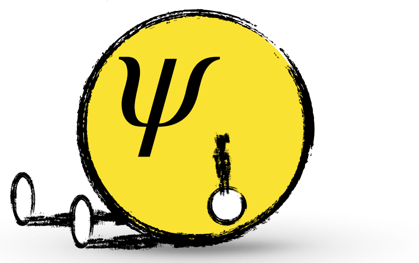
Springel et al. (Virgo Consortium)



All credit to D. Inman

Yellow dots are lumps of non-relativistic fermions hold by Yukawa forces!

Implications for DM and early universe cosmology?



Yukawa vs Gravity: particle interaction

Take 2 fermions with mass m_ψ with Yukawa interaction y

$$F_{\text{gravity}} \sim \frac{Gm_\psi^2}{r^2}$$

$$F_{\text{yukawa}} \sim \frac{y^2}{r^2} e^{-r\ell}$$

$$\frac{F_{\text{yukawa}}}{F_{\text{gravity}}} \sim \frac{y^2 M_{\text{pl}}^2}{m_\psi^2} = \beta^2$$

This can be in general (always?) very large!

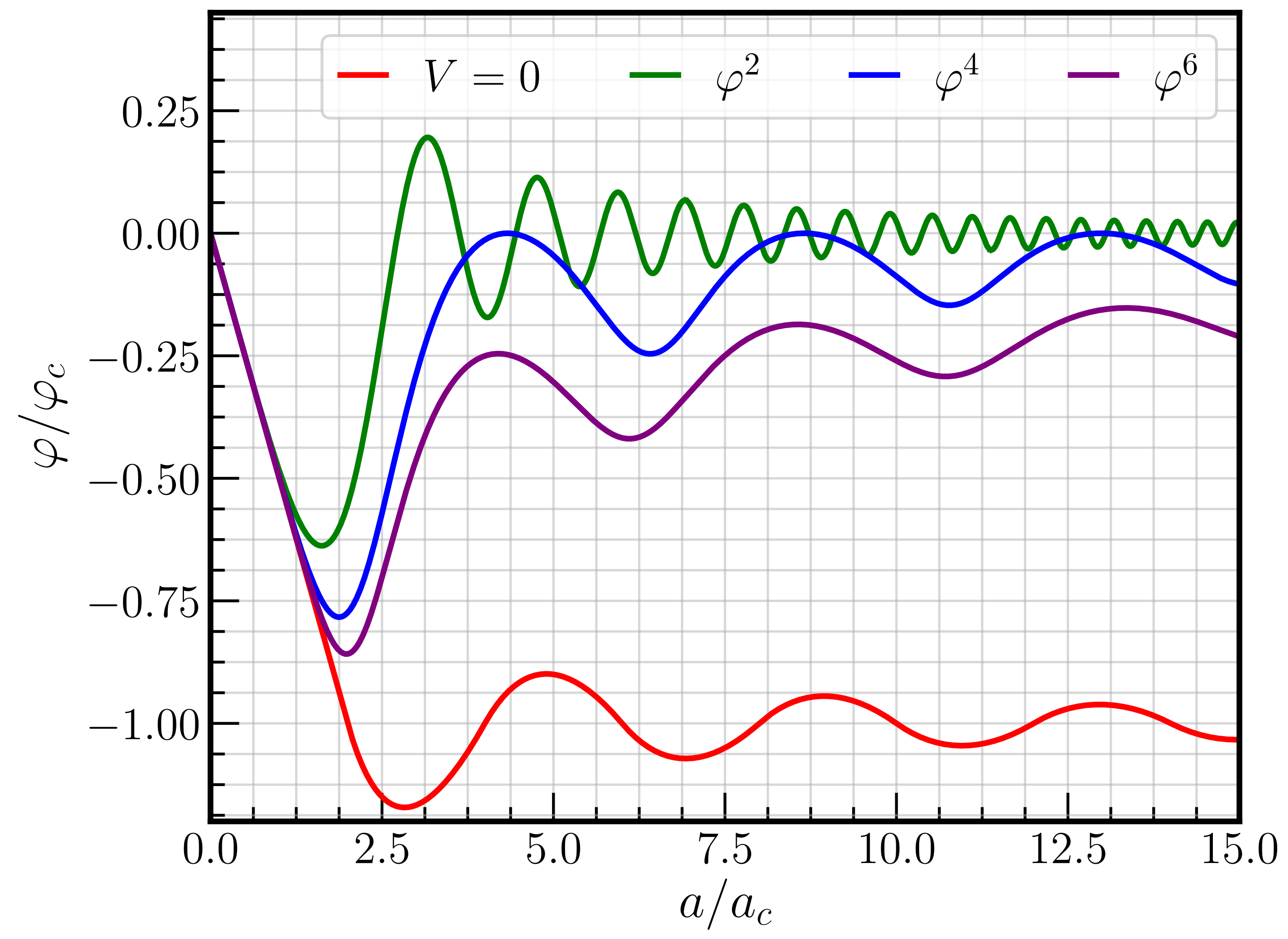
At some point in the past Yukawa force was long range: $\ell \gg H^{-1}$

$$H^{-1} = 10^{-4}\,\text{cm} \left(\frac{T}{10^4\,\text{GeV}} \right)^{-2}$$

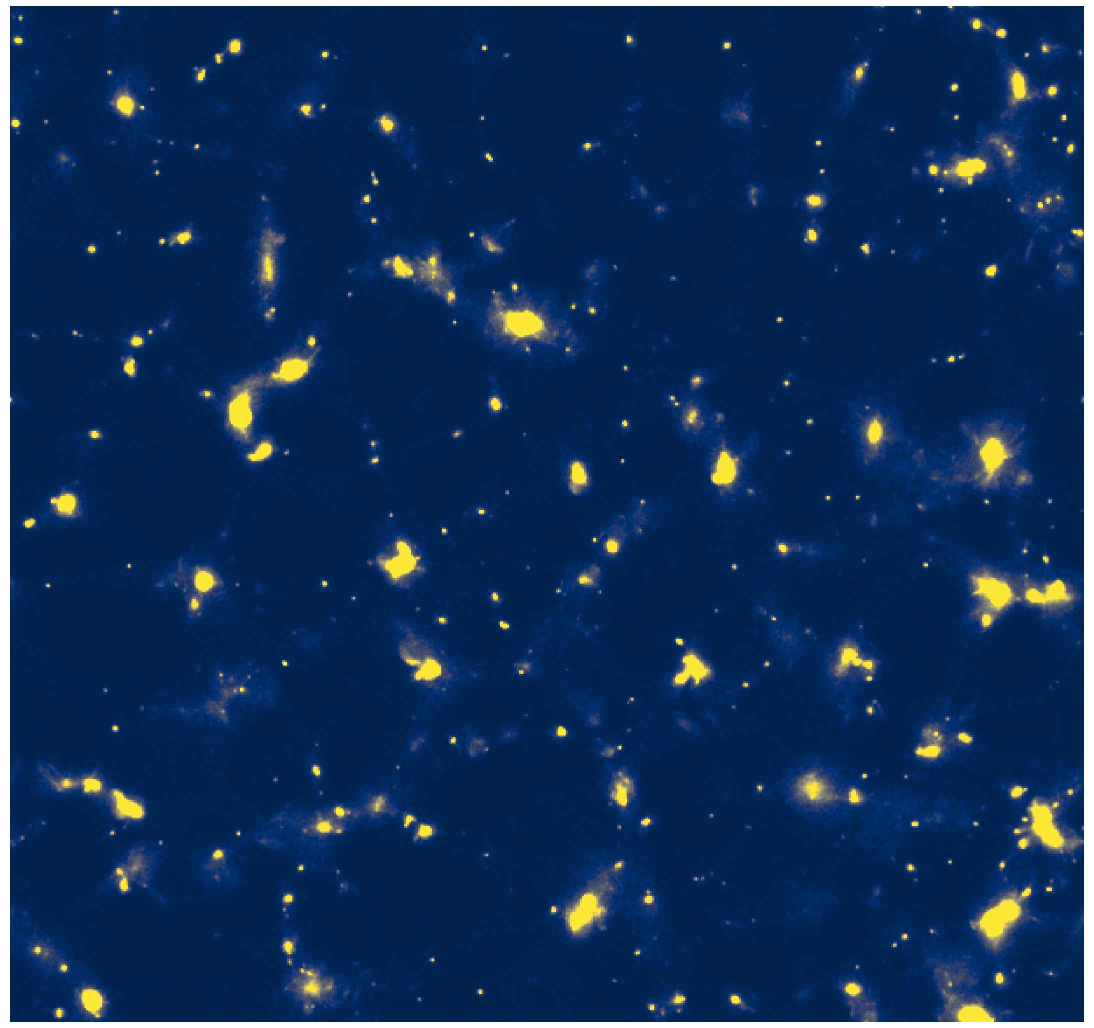
Can this form structures or BH in the radiation dominated universe?

Overview

1. Basics: Fermions, Yukawa & Early universe



2. Fluctuations and N-Body simulations:



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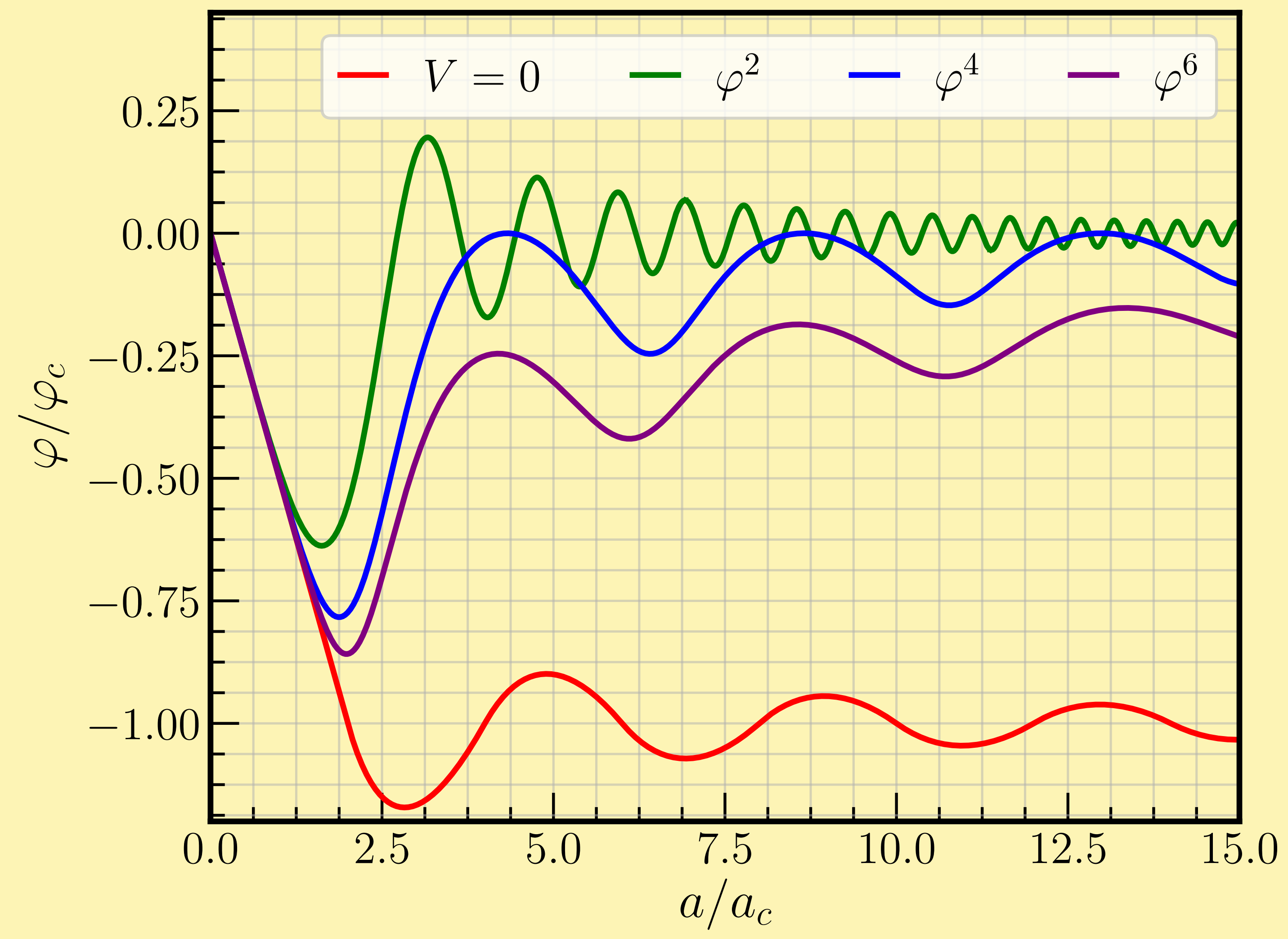

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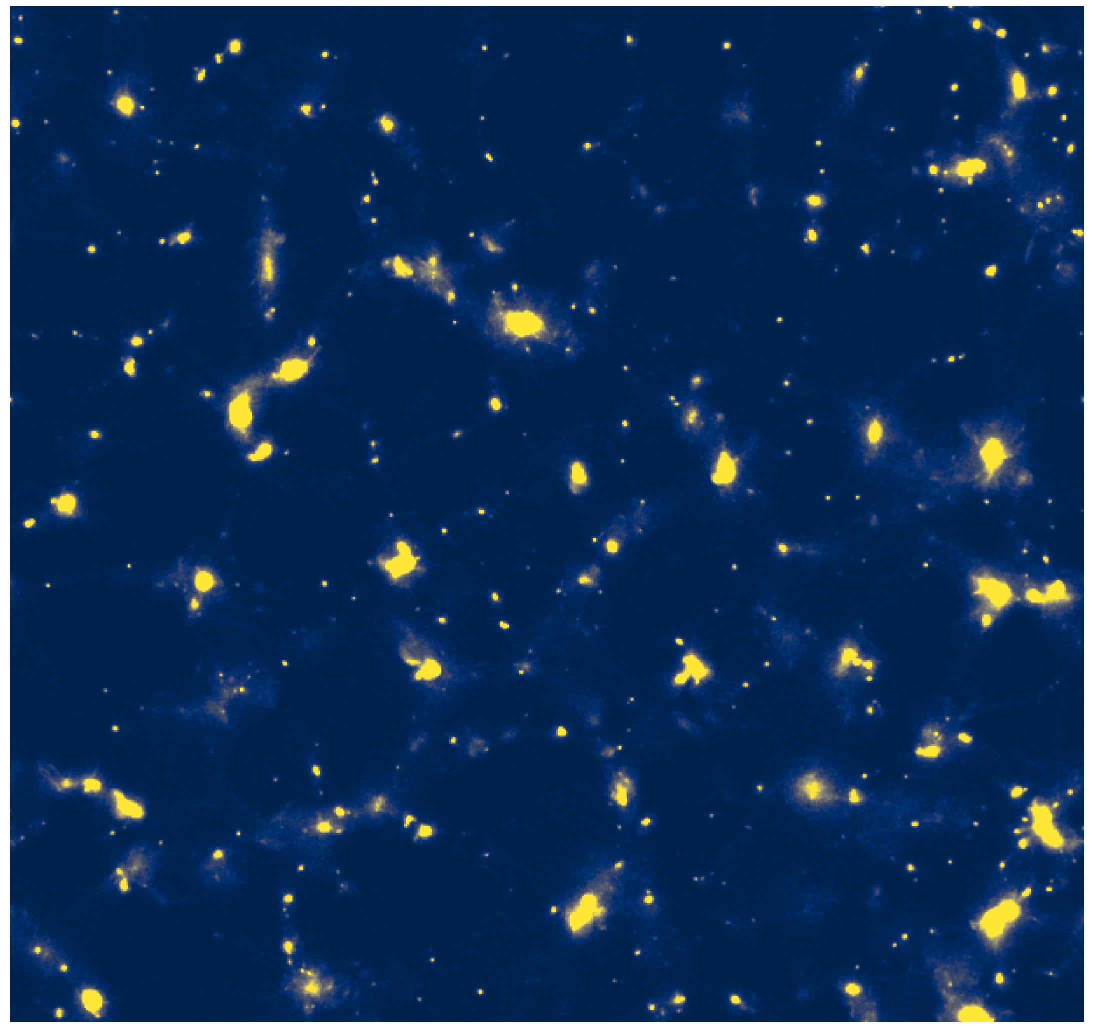


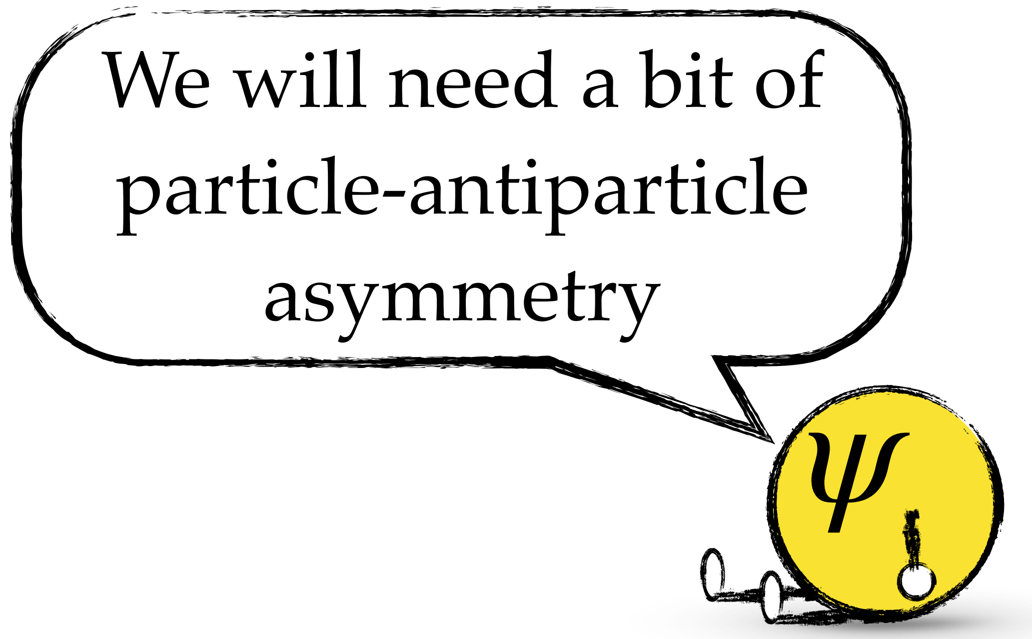
Yukawa interactions: basics

Basic Lagrangian:

$$\mathcal{L}(\psi,\varphi) = \bar{\psi} i \Gamma^\mu D_\mu \psi - \overbrace{\underbrace{|m_\psi + y\varphi|}_{m_{\text{eff}} = m_\psi + y\varphi}\bar{\psi}\psi - V(\varphi)}^{V_{\text{eff}}(\varphi)} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$$

NOTE: Fermion current is conserved: no net particle creation, U(1) symmetry preserved

Conserved fermion number density n_ψ



In cosmology → $n_\psi \propto a^{-3}$

FLRW metric:

$$ds^2 = -dt^2 + a^2 dx^2$$

Yukawa interactions: basics

$$f(\mathbf{p}, m_{\text{eff}}, \mu) = \frac{1}{1 + e^{\frac{E-\mu}{T}}}$$

Thermodynamical considerations:

$$\rho_\psi = \frac{2}{(2\pi)^3 a^3} \int d^3p \, E(\mathbf{p}, m_{\text{eff}}) \left(f(\mathbf{p}, m_{\text{eff}}, \mu) + f(\mathbf{p}, m_{\text{eff}}, -\mu) \right)$$

- Fermions (as perfect fluid) in thermodynamical equilibrium
- "The Universe" is described by grand-canonical ensemble $\Omega = -P_\psi V$

$$d\Omega = -S_\psi dT - N_\psi d\mu - P_\psi dV + Y_\psi d\varphi$$

External force

Energy "conservation":

$$\dot{\rho}_\psi + 3H(\rho_\psi + P_\psi) = -\dot{\varphi} \left(\frac{\partial P_\psi}{\partial \varphi} \right)_{\mu,T}$$

$$\left(\frac{\partial P_\psi}{\partial \varphi} \right)_{\mu,T} = -y\sigma \frac{\rho_\psi - 3P_\psi}{m_{\text{eff}}}$$

Zero for relativistic fermions

Note: no $\left(\frac{\partial \rho_\psi}{\partial \varphi} \right)$

Conserved number density n_ψ ⟶ Conserved entropy density s_ψ

Yukawa interactions: basics

Thermodynamical considerations: $\rho_\psi = \frac{2}{(2\pi)^3 a^3} \int d^3p\, E(\mathbf{p}, m_{\rm eff}) \big(f(\mathbf{p}, m_{\rm eff}, \mu) + f(\mathbf{p}, m_{\rm eff}, -\mu) \big)$

- Fermions (as perfect fluid) in thermodynamical equilibrium

Summary: We can follow the evolution of fermions + scalar field in relativistic/non-relativistic regimes

Energy "conservation":

$$\dot\rho_\psi + 3H(\rho_\psi + P_\psi) = -\dot\varphi \left(\frac{\partial P_\psi}{\partial \varphi} \right)_{\mu,T}$$

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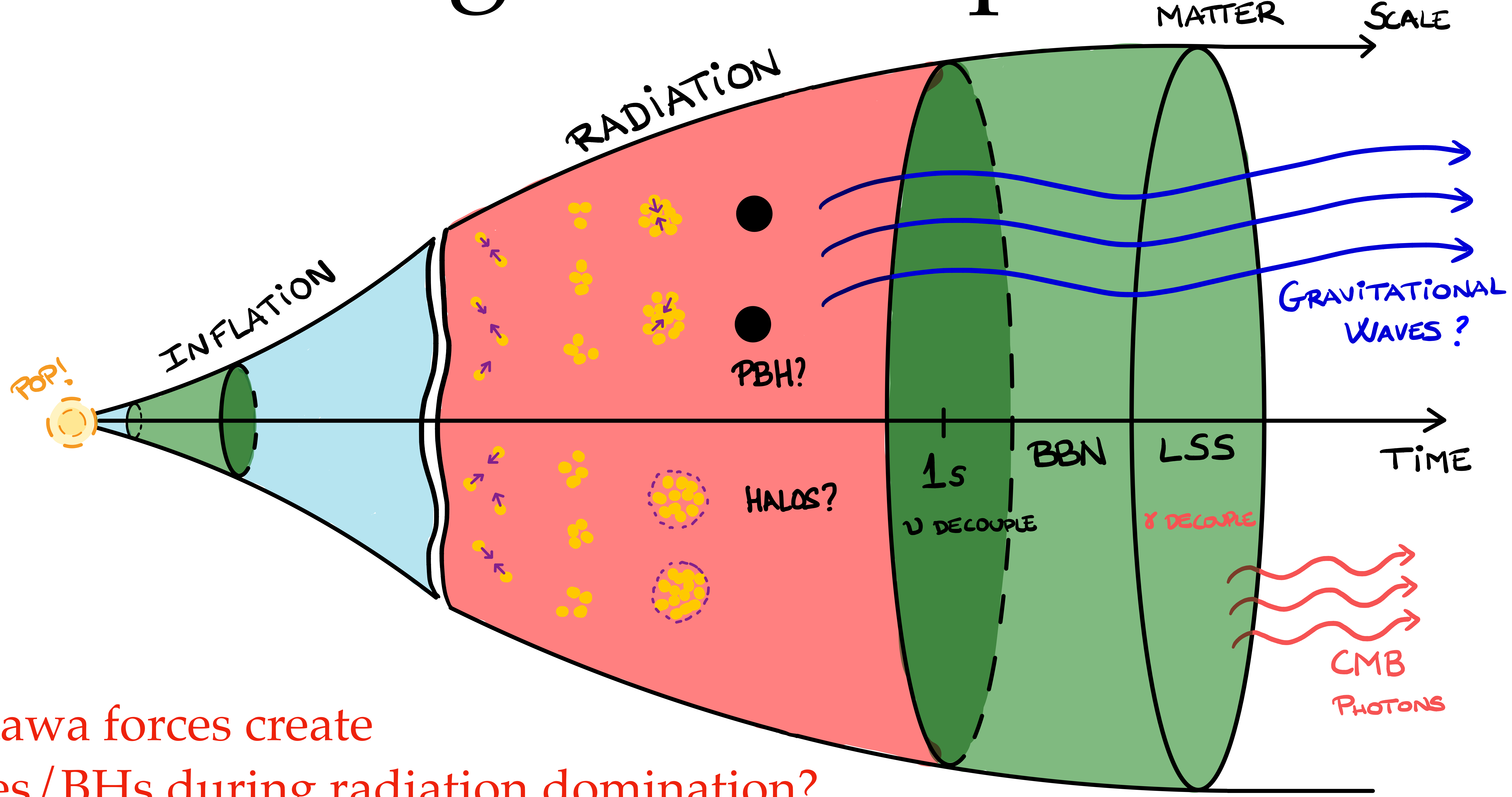
The cosmological set-up



Can Yukawa forces create structures/BHs during radiation domination?

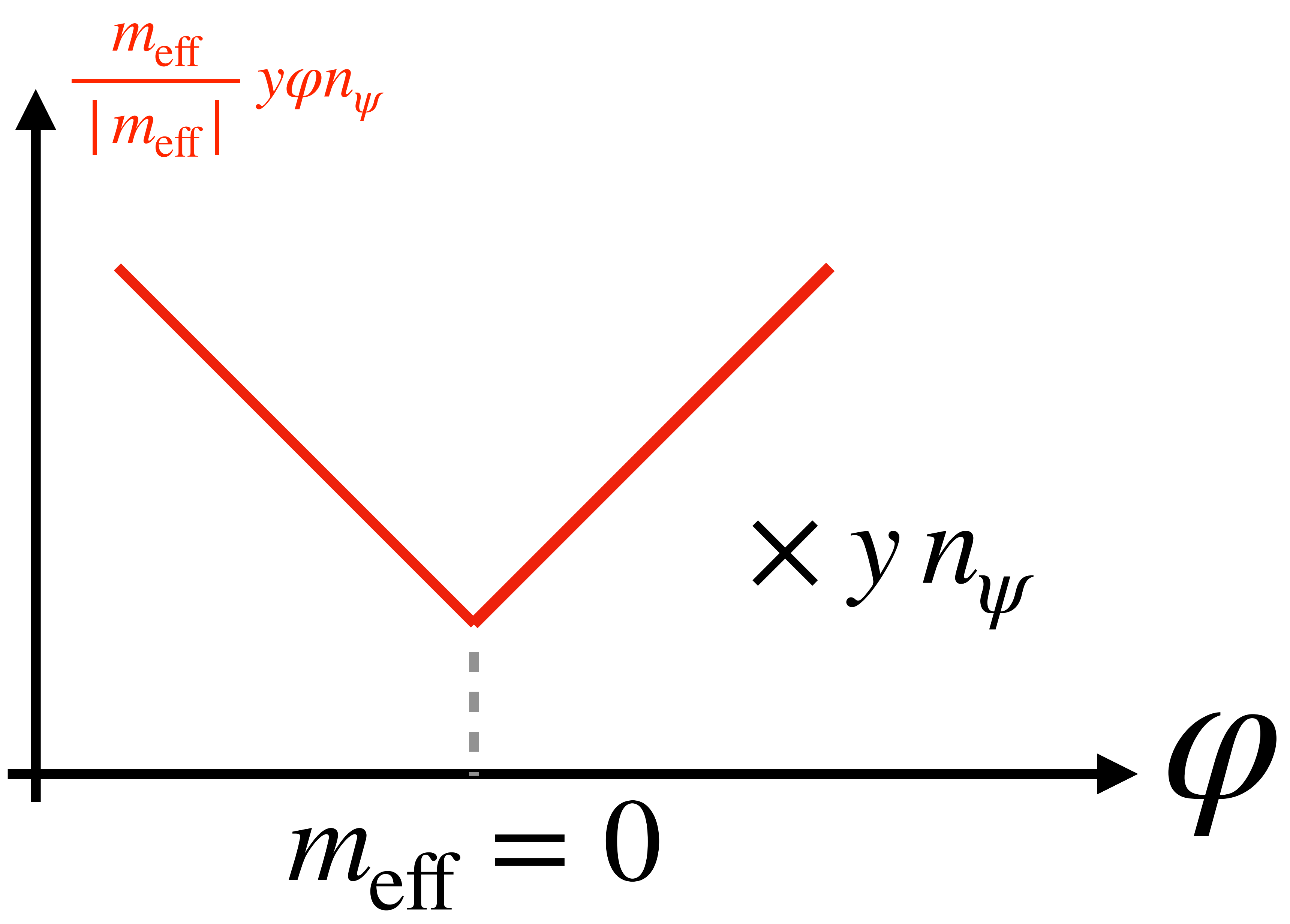
All fermion dynamics determined by the scalar mediator…

Massless Scalar mediator + Non-relativistic fermions

We consider $P_\psi \ll \rho_\psi = m_{\rm eff} n_\psi$ plus Klein-Gordon equation: $m_{\rm eff} = m_\psi + y\varphi$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V_{\rm eff}}{\partial \varphi} = 0 \qquad V_{\rm eff} = \frac{m_{\rm eff}}{|m_{\rm eff}|} y\varphi n_\psi$$

$V_{\rm eff} =$



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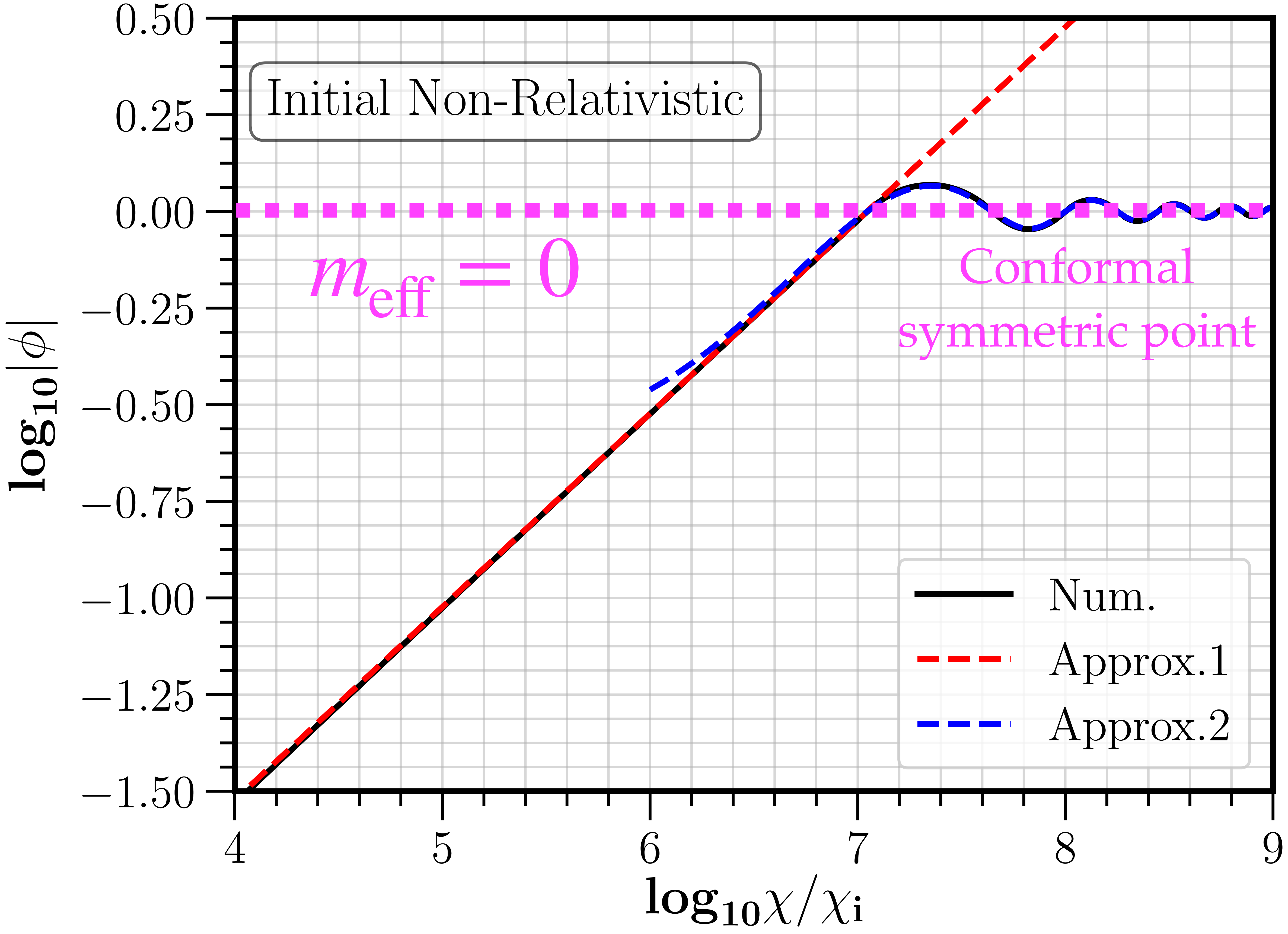
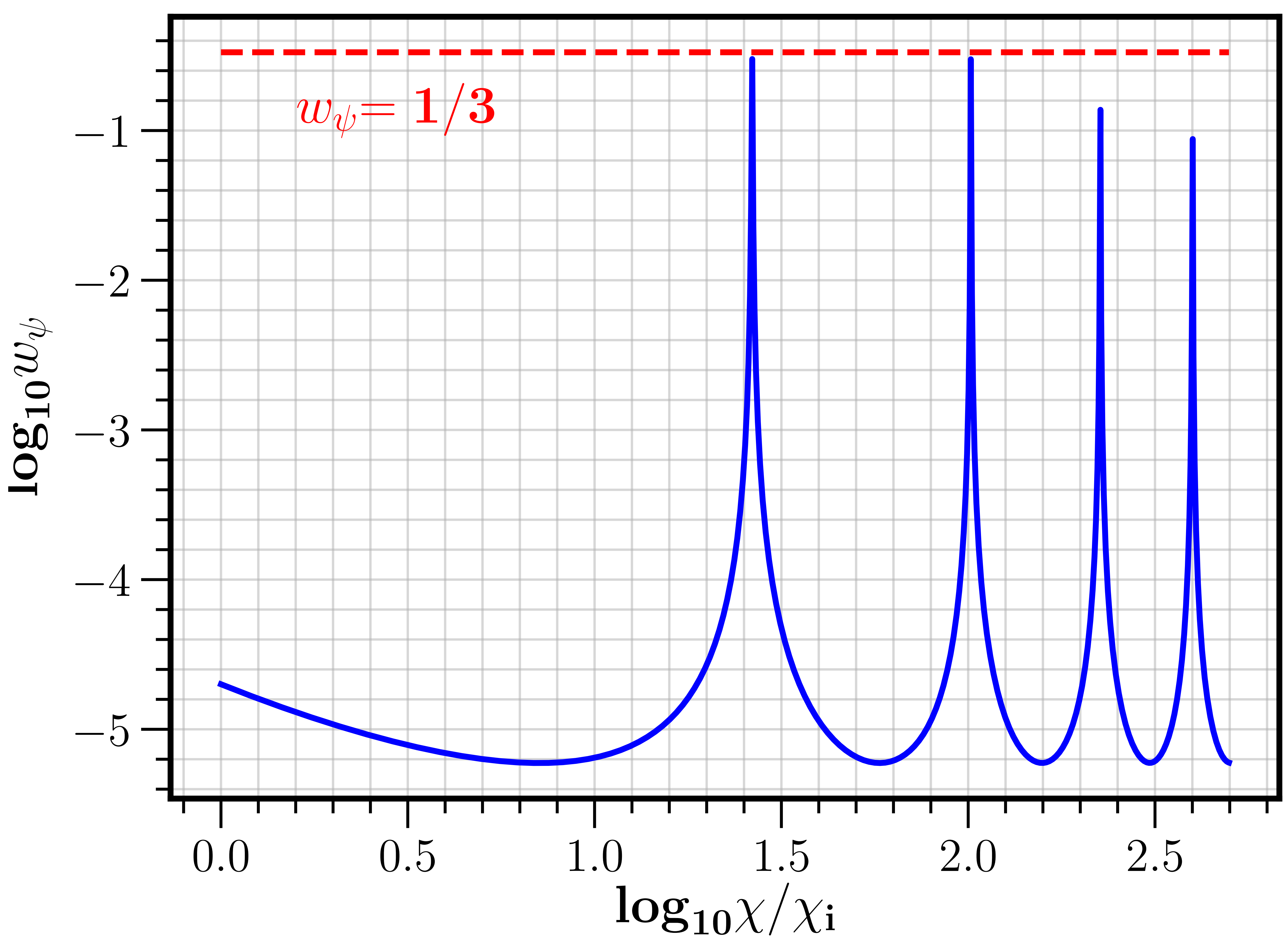
Massless Scalar mediator + Non-relativistic fermions





$\rho_\psi = m_{\rm eff} n_\psi \propto a^{-4}$ $\quad \rho_\varphi \propto a^{-4}$

Scaling solution

General scalar mediator + Non-relativistic fermions

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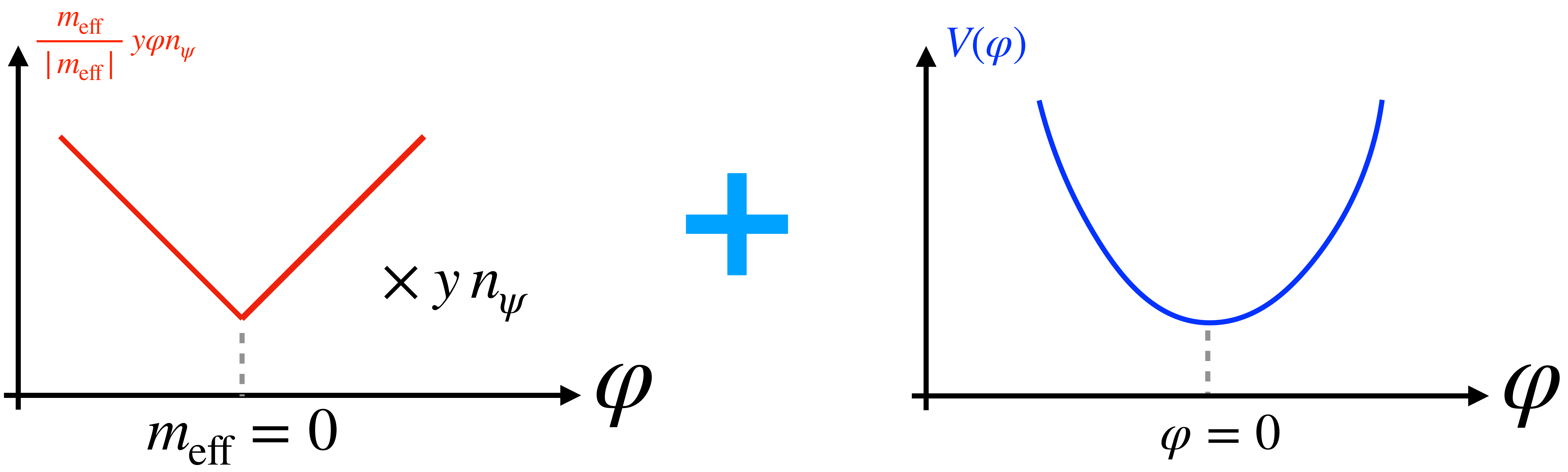
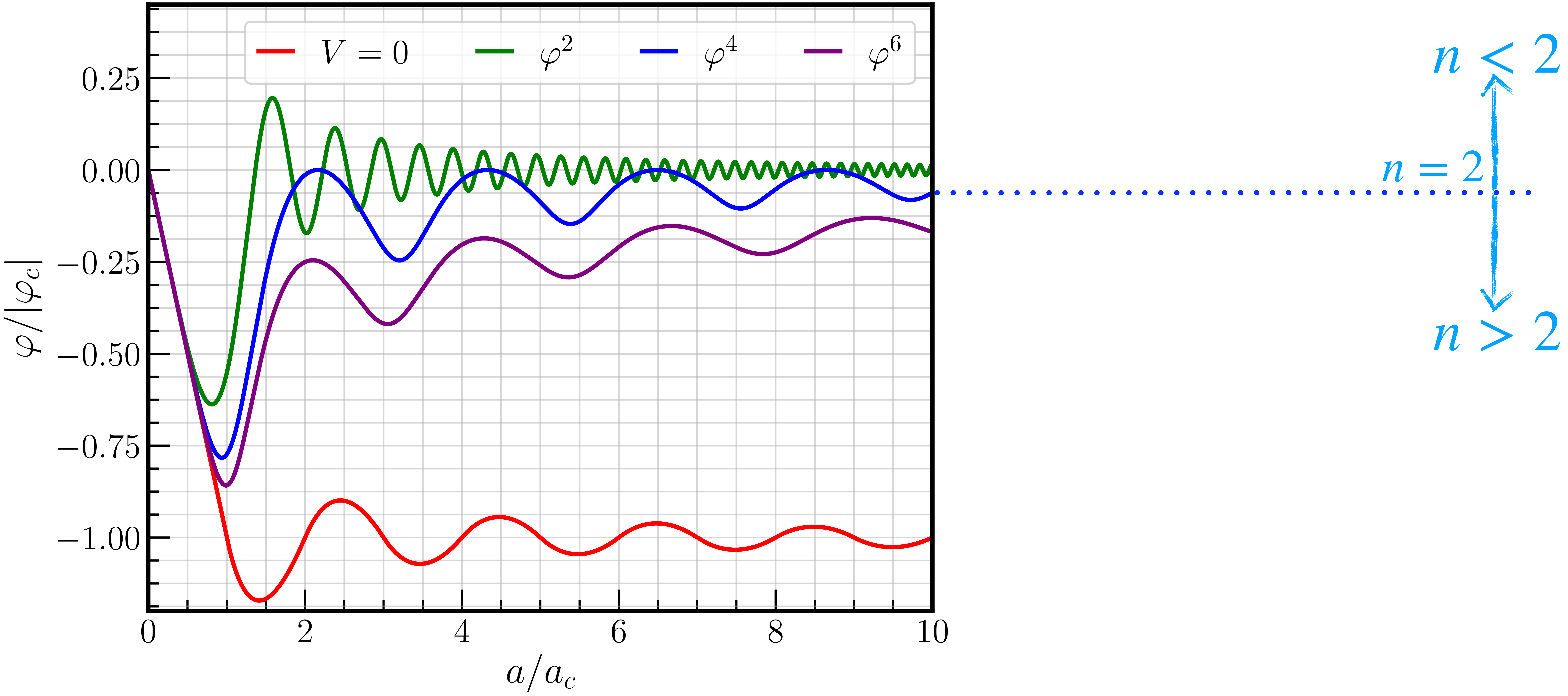


General scalar mediator + Non-relativistic fermions

$$V_{\text{eff}} = \frac{m_{\text{eff}}}{|m_{\text{eff}}|} y\varphi n_\psi + V(\varphi) \qquad V(\varphi) \propto \varphi^{2n}$$



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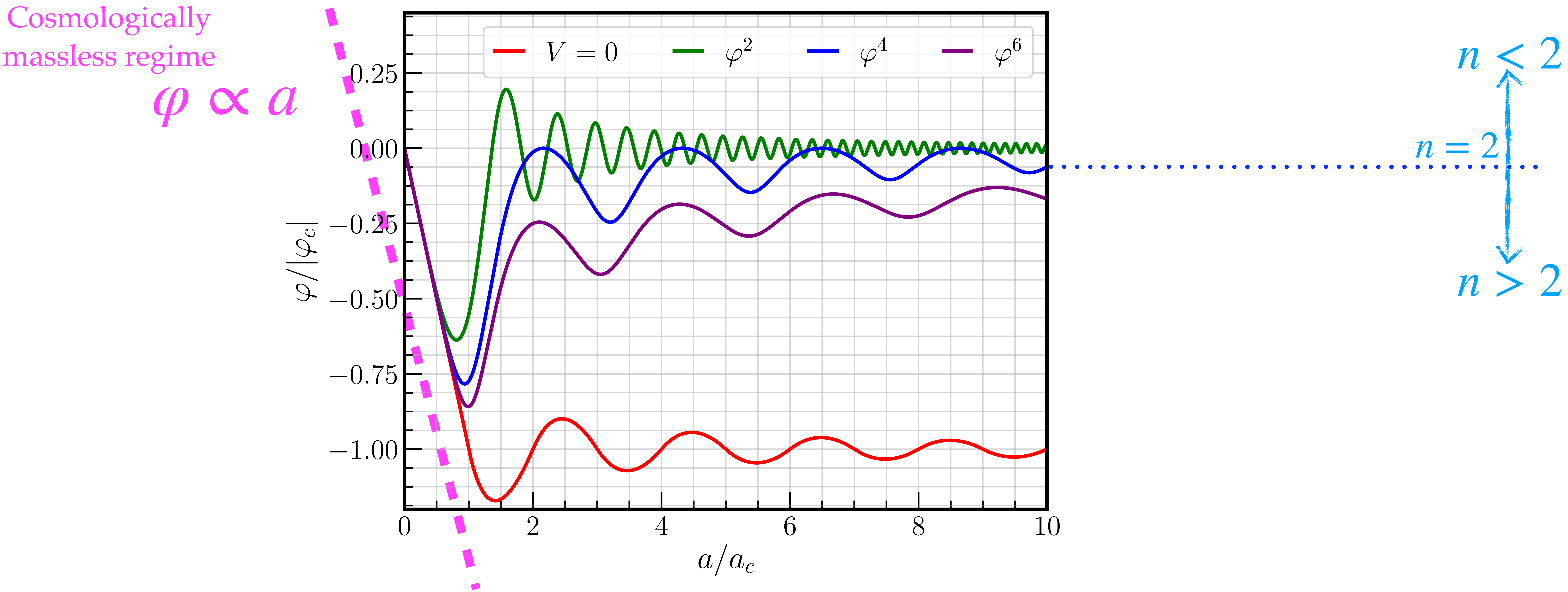

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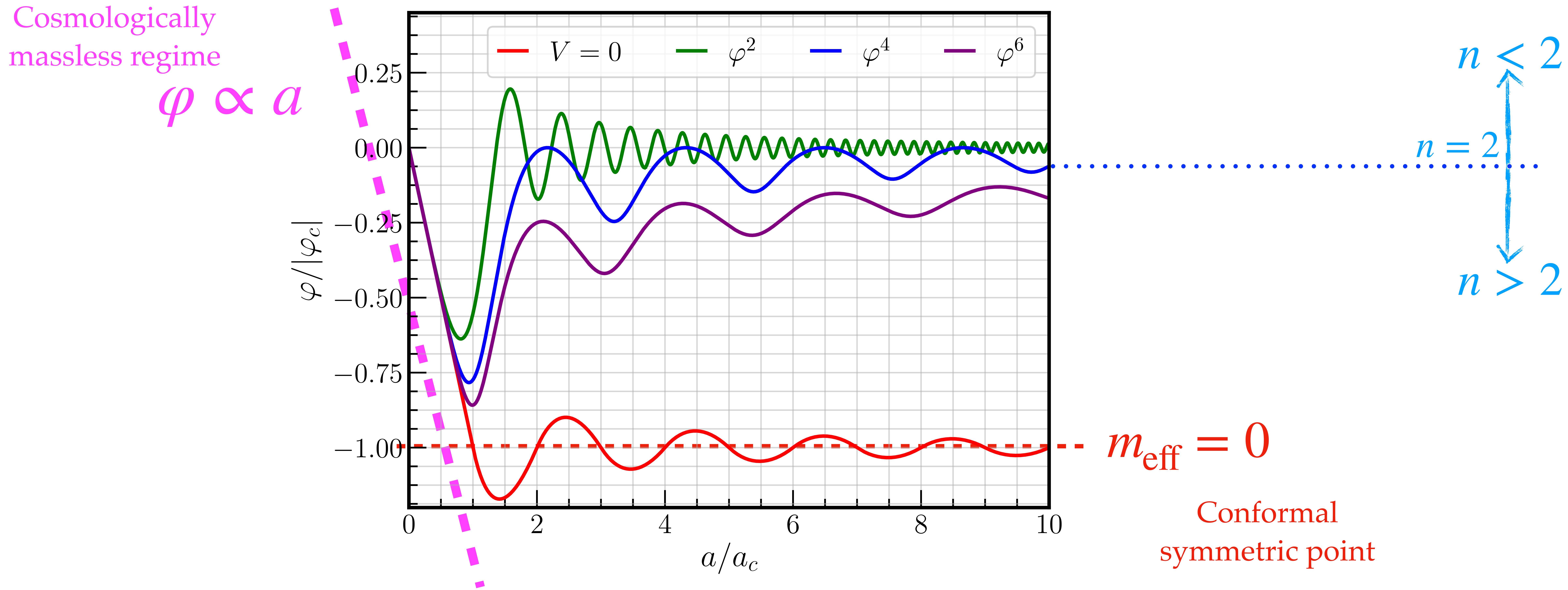

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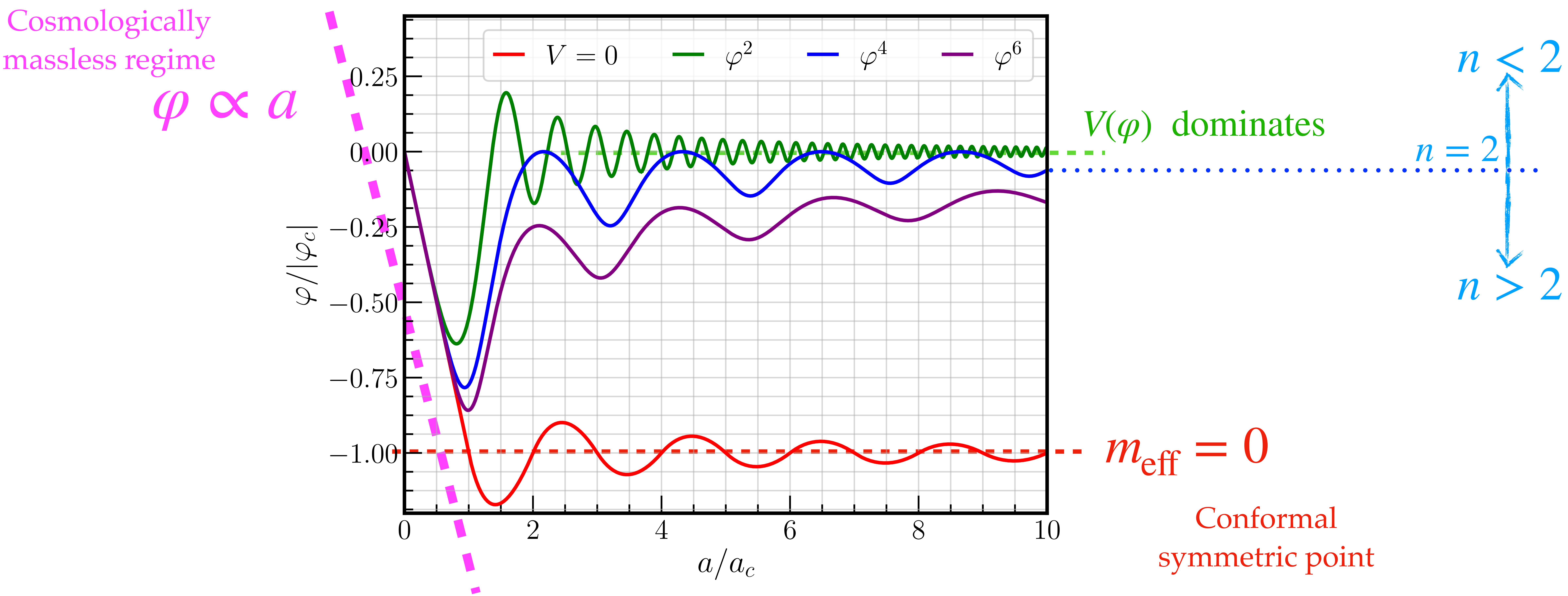

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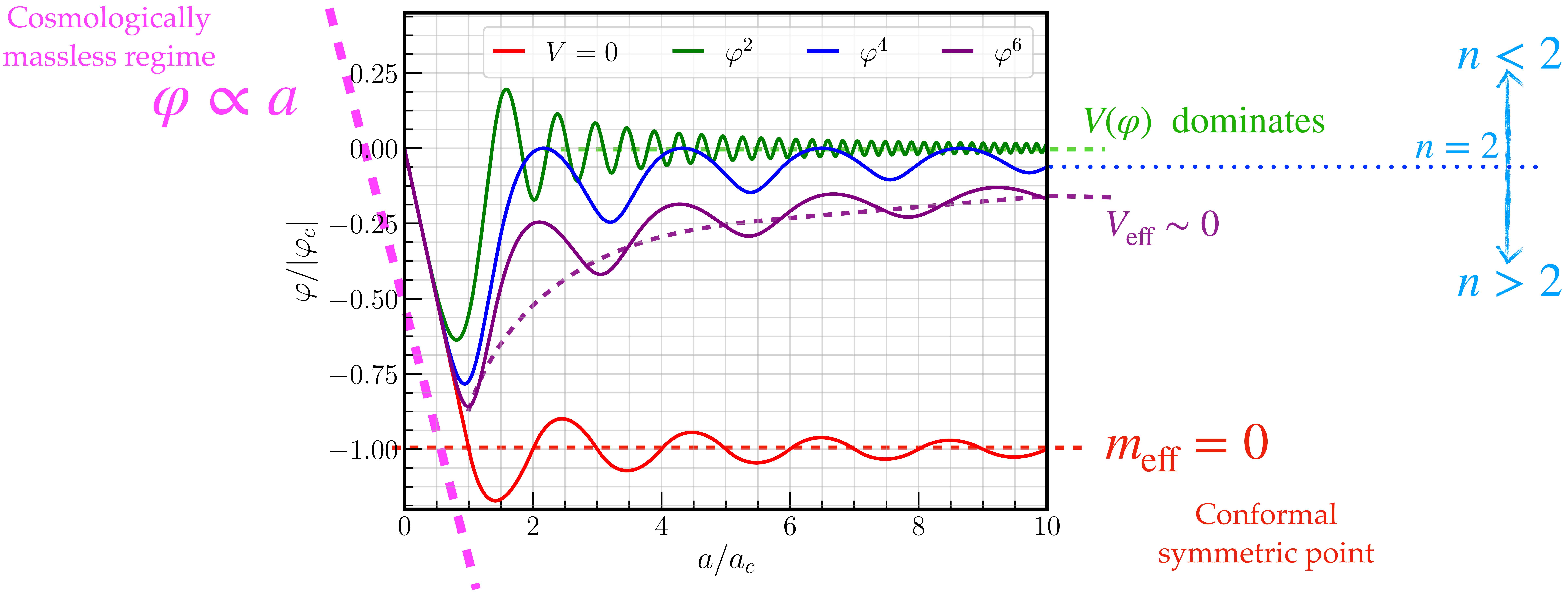

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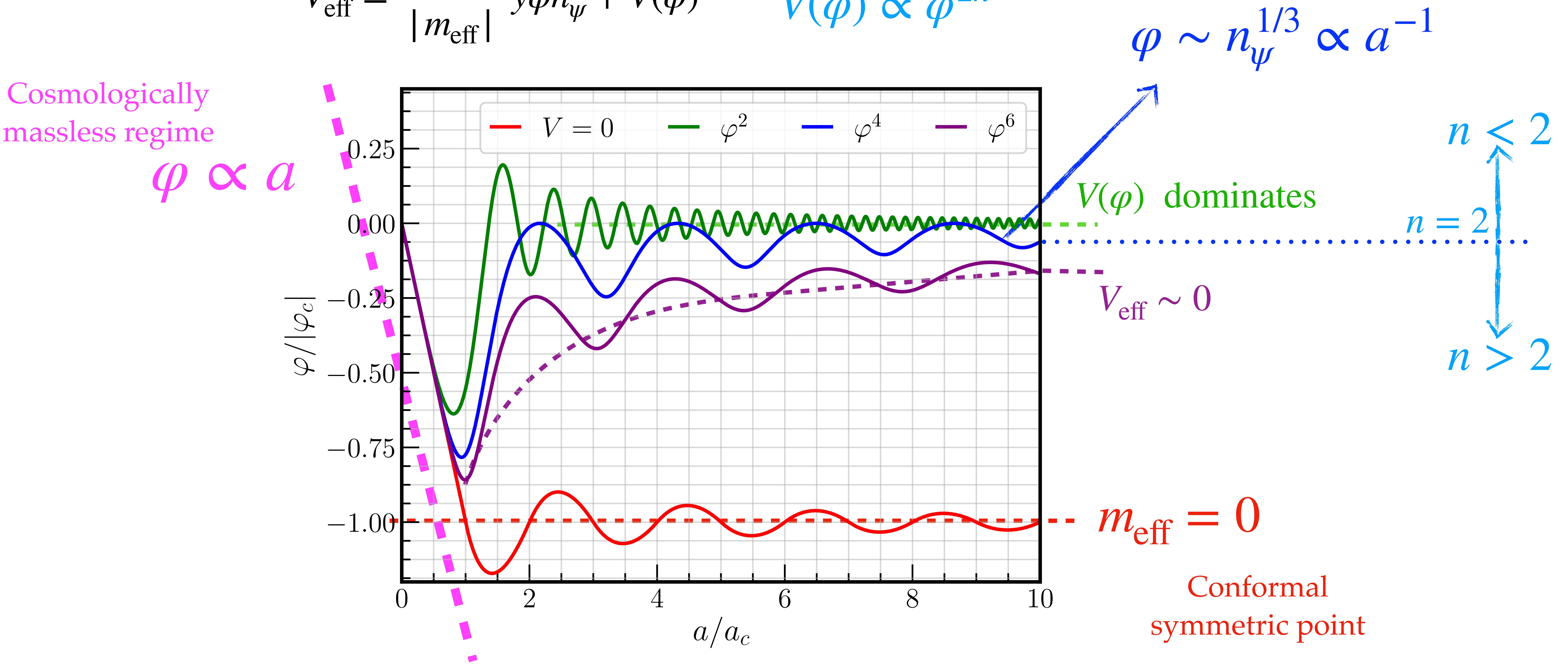


General scalar mediator + Non-relativistic fermions

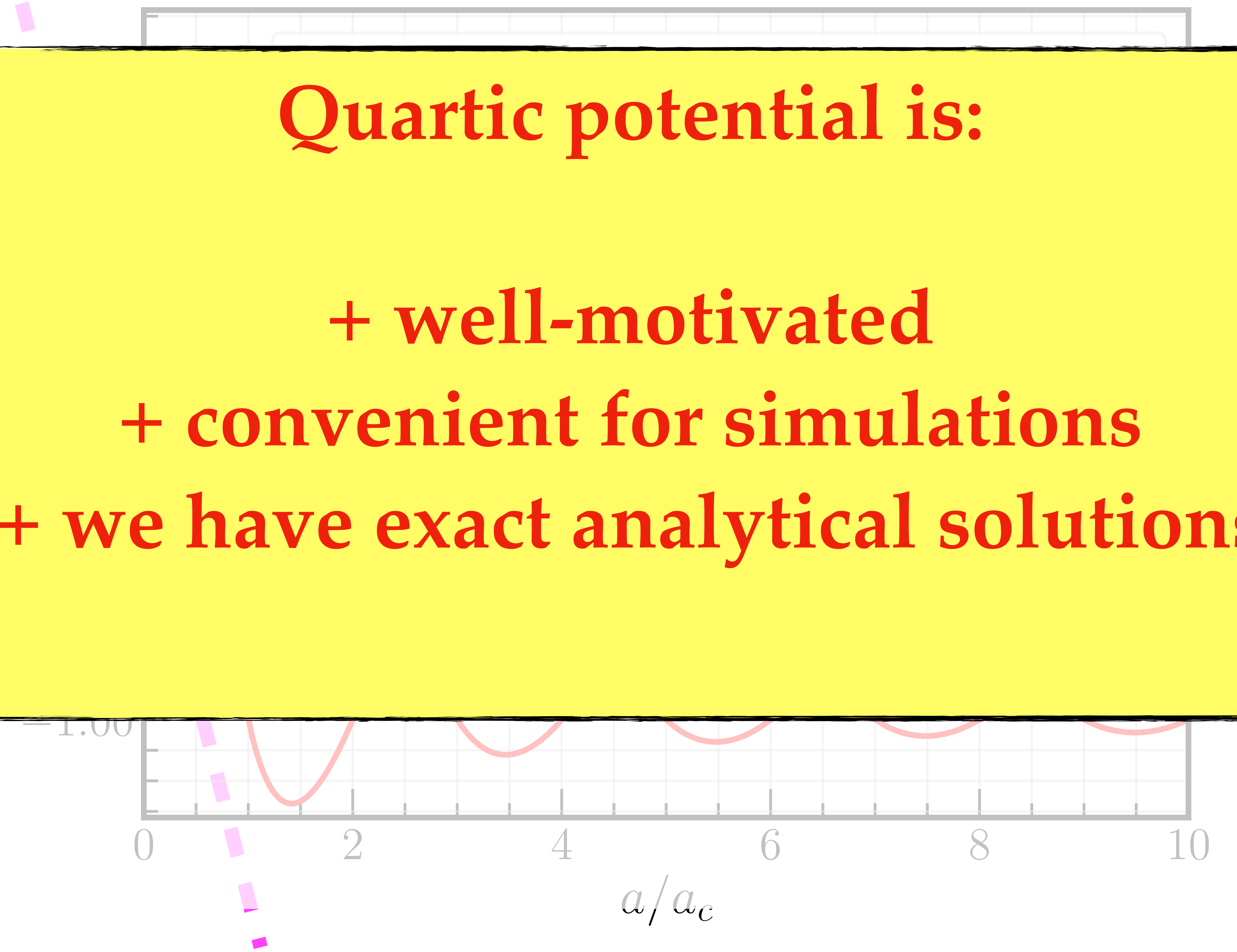
$$V_{\text{eff}} = \frac{m_{\text{eff}}}{|m_{\text{eff}}|} y \varphi n_\psi + V(\varphi)$$

$$V(\varphi) \propto \varphi^{2n}$$

$$\varphi \sim n_\psi^{1/3} \propto a^{-1}$$

Cosmologically massless regime

$\varphi \propto$



) dominates

$n < 2$

$n = 2$

$n > 2$

~ 0

$m_{\text{eff}} = 0$

Quartic potential is:

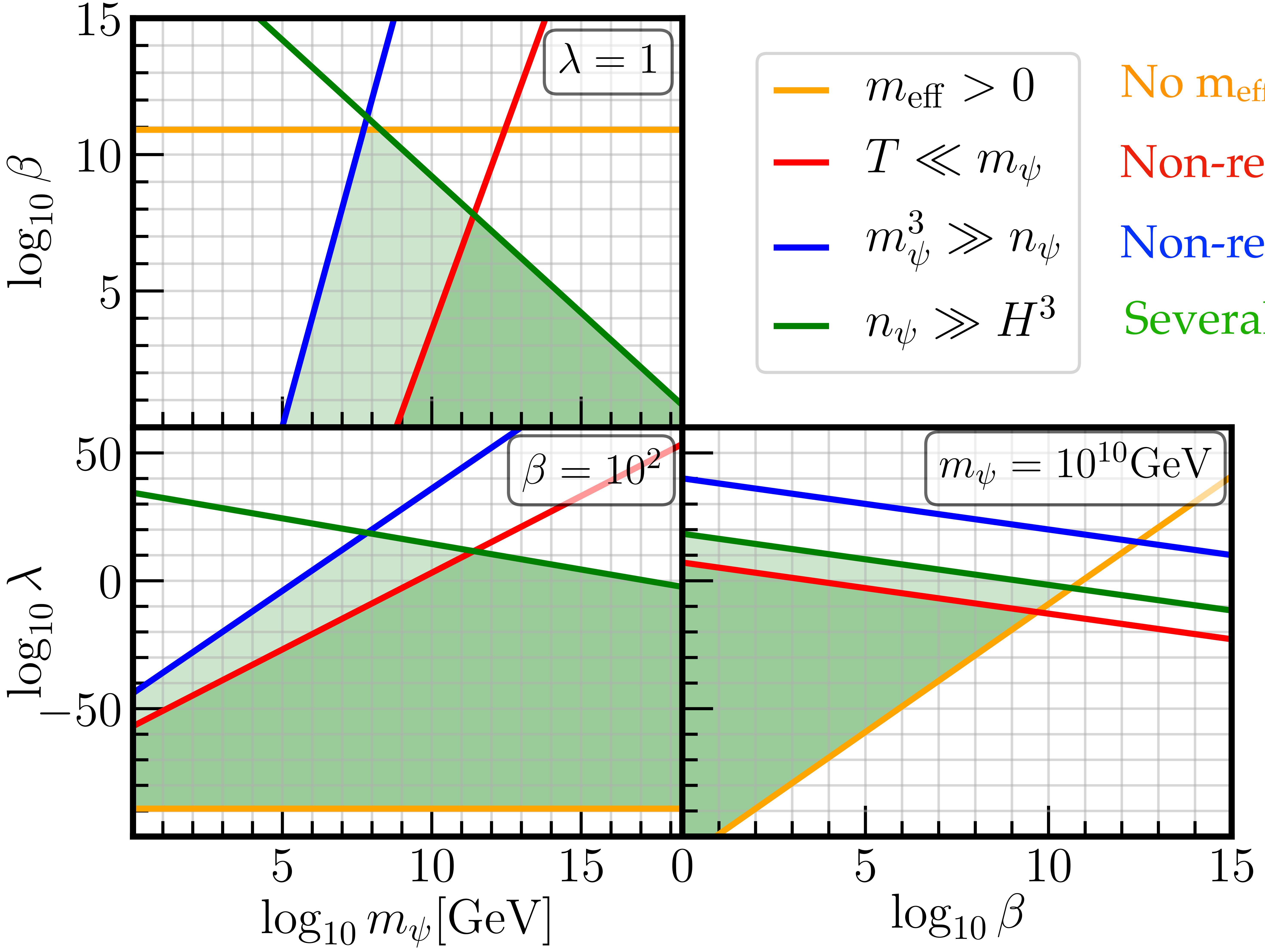
+ well-motivated

+ convenient for simulations

+ we have exact analytical solutions

Non-relativistic fermions: Parameter space

$$V(\varphi) = \frac{1}{4}\lambda\varphi^4$$



No m_{eff}~0 regime

Non-relativistic non-degenerate

Non-relativistic degenerate

Several particles per Hubble volume

Important for the validity of numerical simulations!

$$\beta = \frac{yM_{\mathrm{pl}}}{m_\psi}$$

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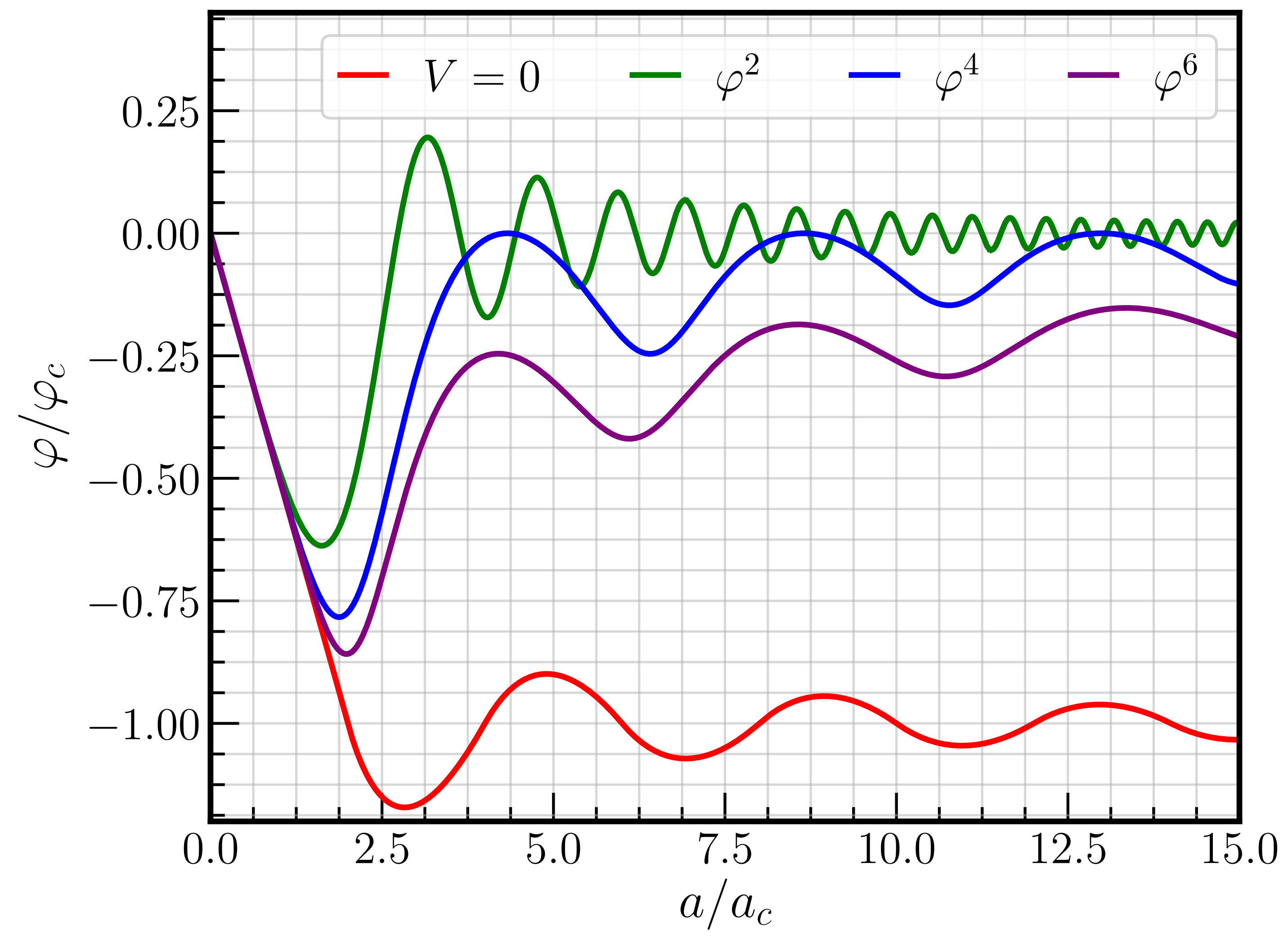

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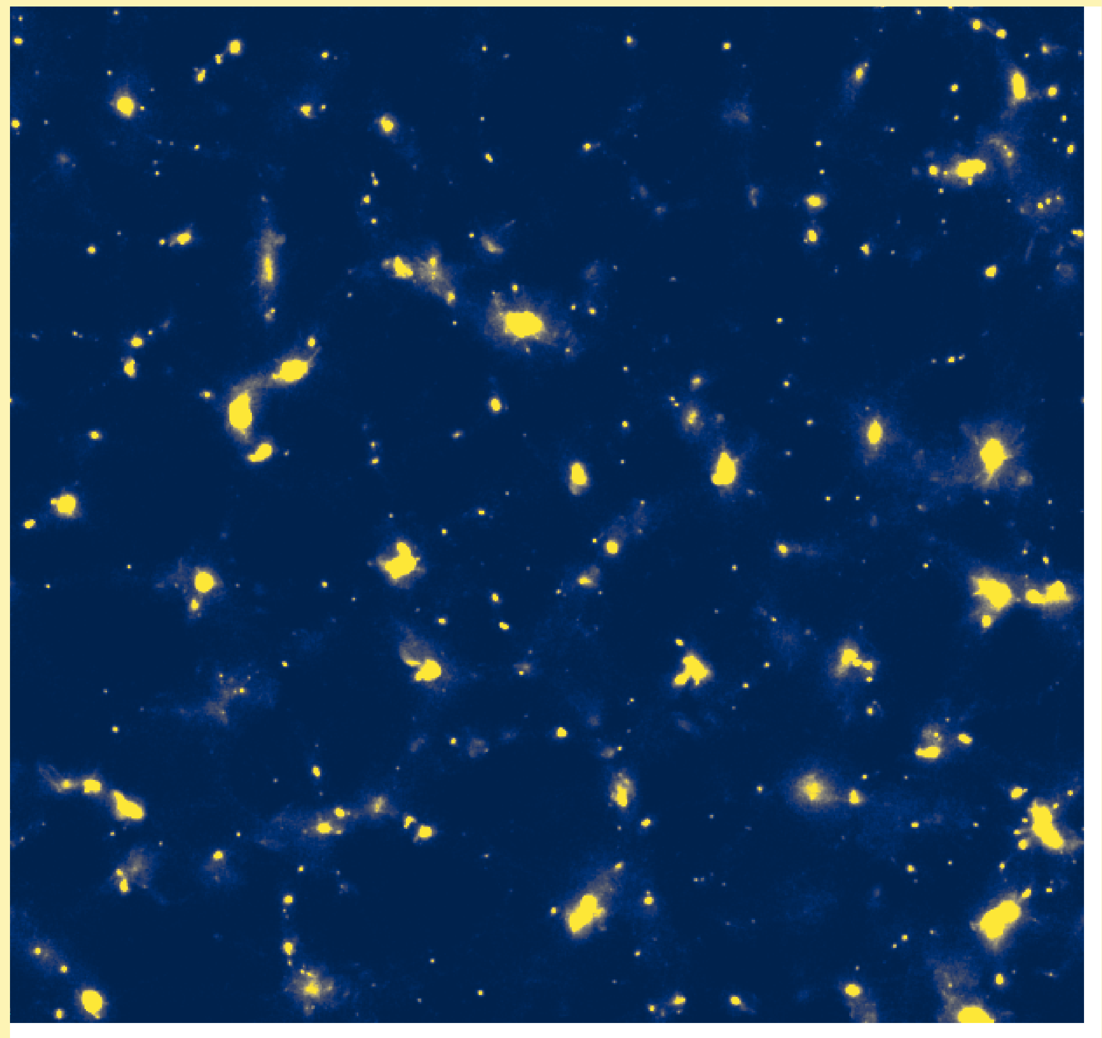

Yukawa vs Gravity: fermion fluctuations

(in comoving variables)

Take continuity + Euler equation (energy + momentum conservation):

$$\delta_\psi' + \vec{\nabla}\left[(1+\delta_\psi)\vec{V}\right] = 0 \qquad \vec{V}' + (2\mathscr{H} + (\ln m_{\rm eff})')\vec{V} + (\vec{V}\cdot\vec{\nabla})\vec{V} + \vec{\nabla}\phi = 0$$

Plus "Poisson" equation: $(\nabla^2 + \ell^{-2})\phi \sim \beta\,\rho_\psi\delta_\psi$

For gravity $\ell = \infty$ and $\beta = 1$

For Yukawa $\ell = a^{-1}V_{\varphi\varphi}^{-1/2}$ and $\beta \gg 1$

Known dynamics of scalar field!

Effect of scalar field on fermions through $m_{\rm eff}$ and ℓ

Yukawa vs Gravity: fermion fluctuations

Combine all at linear level… (use Friedmann Eq. $3H^2 = \rho_{\rm rad}$)

$x = a/a_{\rm eq}$

Gravity Yukawa

$$\delta_\psi'' + \frac{1}{x}\delta_\psi' + \frac{d\ln m_{\rm eff}}{d\ln x}\delta_\psi' = \frac{3}{2x}\delta_\psi\left[1 + \frac{2\beta^2}{1+(k\ell(x))^{-2}}\right]$$

α

Growing mode solution

$m_\psi, \ell = \text{constant}$

$$\delta_\psi \propto I_0(\sqrt{6\alpha x})$$

$\alpha x \ll 1$: $I_0 \sim 1$

$\alpha x \gg 1$: $I_0 \sim e^{\sqrt{6\alpha x}}$ **PBHs?**

General considerations

Results from cosmological perturbations (in quasi-static approx.):

1) Exponential growth for $k \gg aH$ & $k^2 \gg a^2 V_{\varphi\varphi}$

2) Scalar fluctuations only act as an **effective potential** for fermions $\phi_Y \equiv \frac{y}{m_{\rm eff}}\delta\varphi$

Poisson-like equation: $(\nabla^2 - \ell^{-2})\phi_Y = a^2\beta^2 M_{\rm pl}^{-2}\rho_\psi\delta_\psi$

Basic length scale of interaction: $\ell \equiv a^{-1}V_{\varphi\varphi}^{-1/2}$ Growth on scales $k\ell \gg 1$

Known dynamics of scalar field!

General potential:

$V(\varphi) \propto \varphi^{2n}$

$V_{\varphi\varphi} \propto \varphi^{2(n-1)}$

$\ell(n=1) \sim a^{-1}$ (decay) $\quad \varphi \propto a^{-3/2}$

$\ell(n=2) \sim$ constant $\times$ oscillations $\quad \varphi \propto a^{-1}$

$\ell(n>2) \sim a^{\frac{n-2}{2n-1}}$ (grow) $\times$ oscillations $\quad \varphi \propto a^{-3/(2n-1)}$

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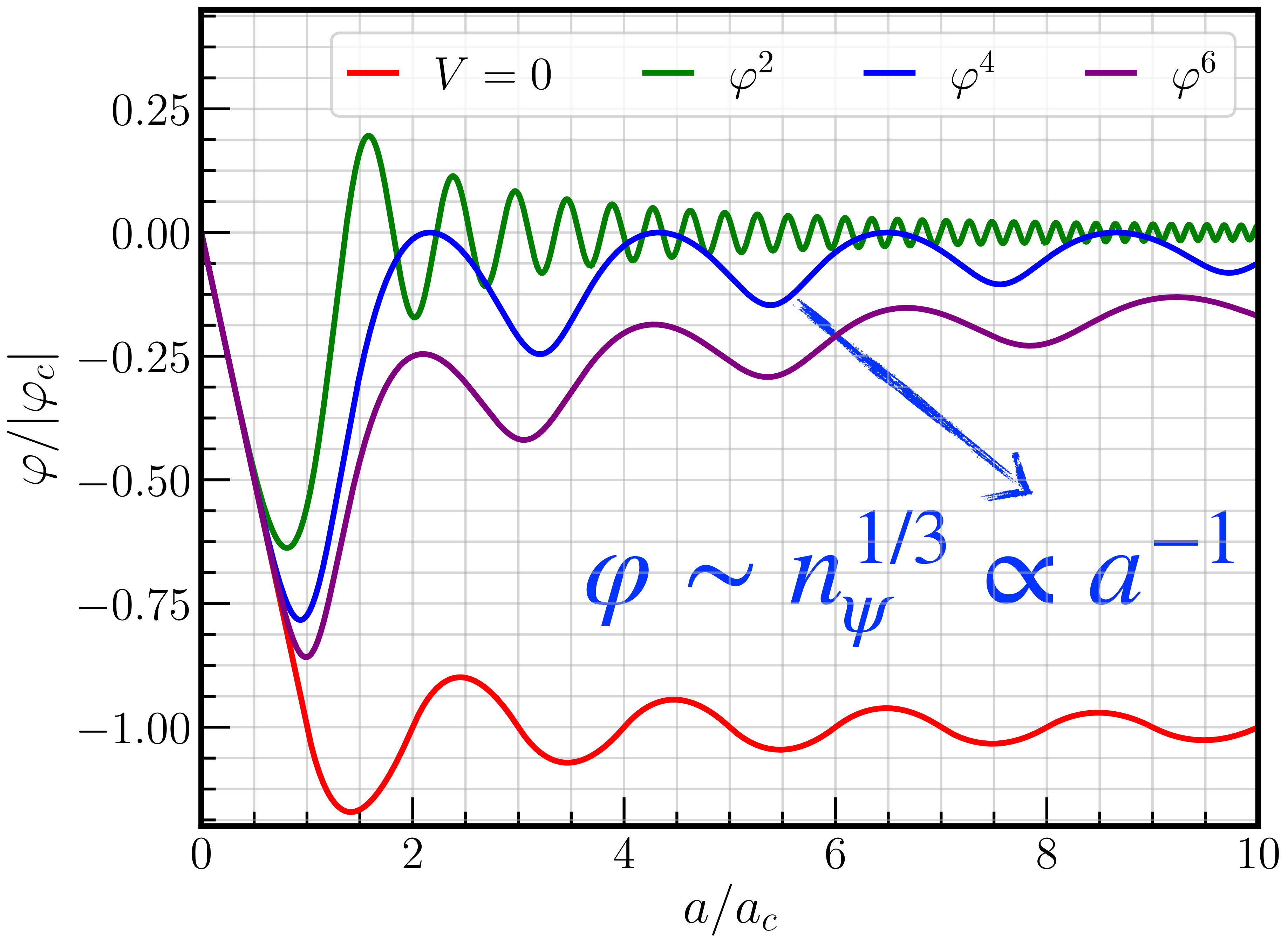
Brief recap

1. Known bg. dynamics of scalar field



2. Known dynamics of interaction scale

$$\ell \equiv a^{-1} V_{\varphi\varphi}^{-1/2} \qquad V(\varphi) \propto \varphi^4$$

$$\ell \sim \text{constant} \times \text{oscillations}$$

3. Affects fermions via Poisson eq.

$$(\nabla^2 - \ell^{-2})\phi_Y = a^2 \beta^2 M_{\rm pl}^{-2} \rho_\psi \delta_\psi$$

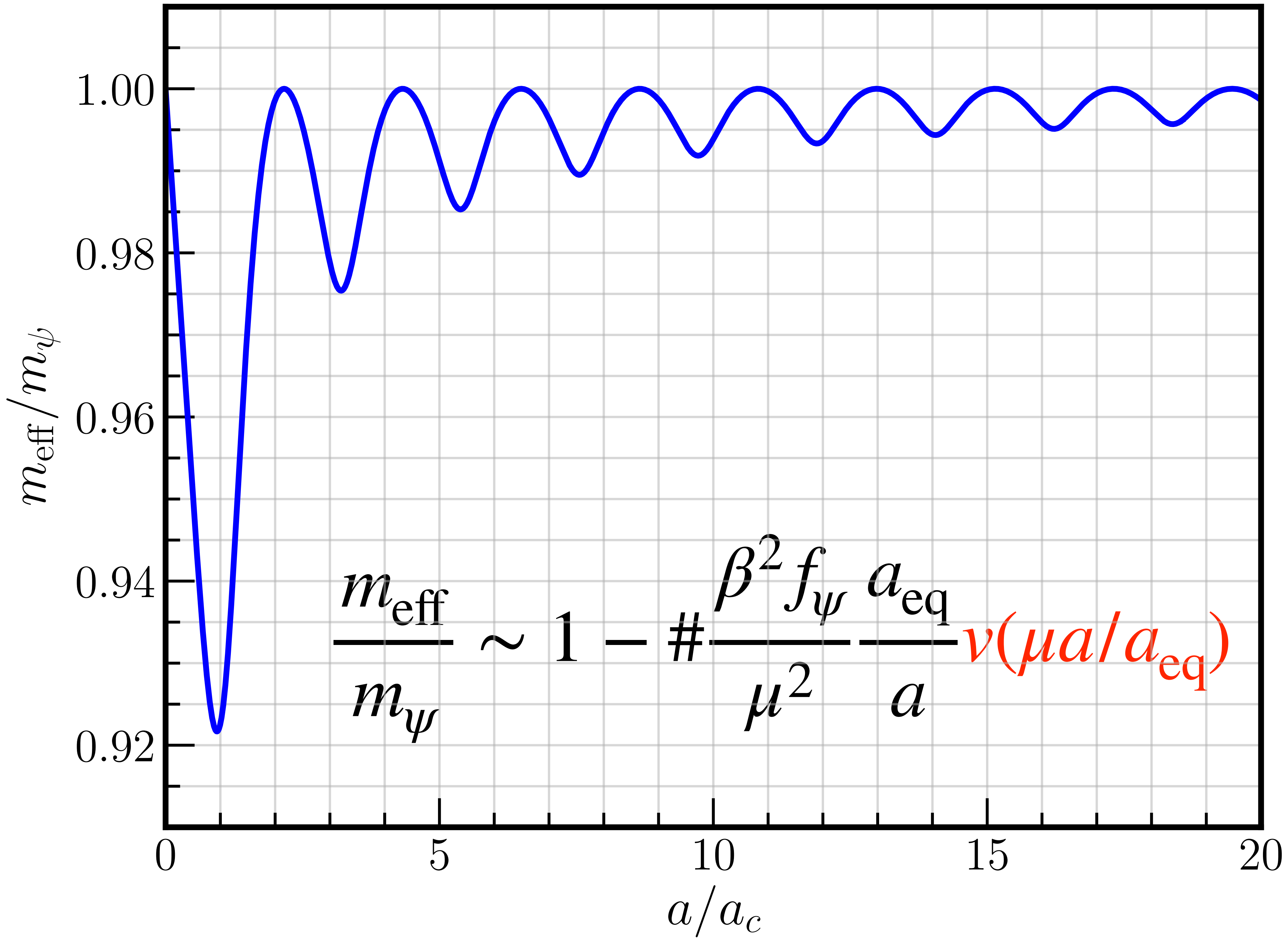
Quartic scalar potential

$$\beta = \frac{yM_{\rm pl}}{m_\psi}$$

$$f_\psi = \frac{\rho_\psi}{\rho_\psi + \rho_m}$$

Physical parameters: $m_\psi, y, \lambda, f_\psi, H_{\rm eq}$ We use matter-radiation equality as reference

Relevant combination: $$\mu = \left(\sqrt{2\lambda}\frac{3f_\psi \beta M_{\rm pl}}{H_{\rm eq}}\right)^{1/3} \approx 3.6\times 10^{18}\lambda^{1/6}(f_\psi\beta)^{1/3}$$



$$\frac{m_{\rm eff}}{m_\psi} \sim 1 - \#\frac{\beta^2 f_\psi}{\mu^2}\frac{a_{\rm eq}}{a}v(\mu a/a_{\rm eq})$$

Oscillating function

$v(\mu a/a_{\rm eq})$

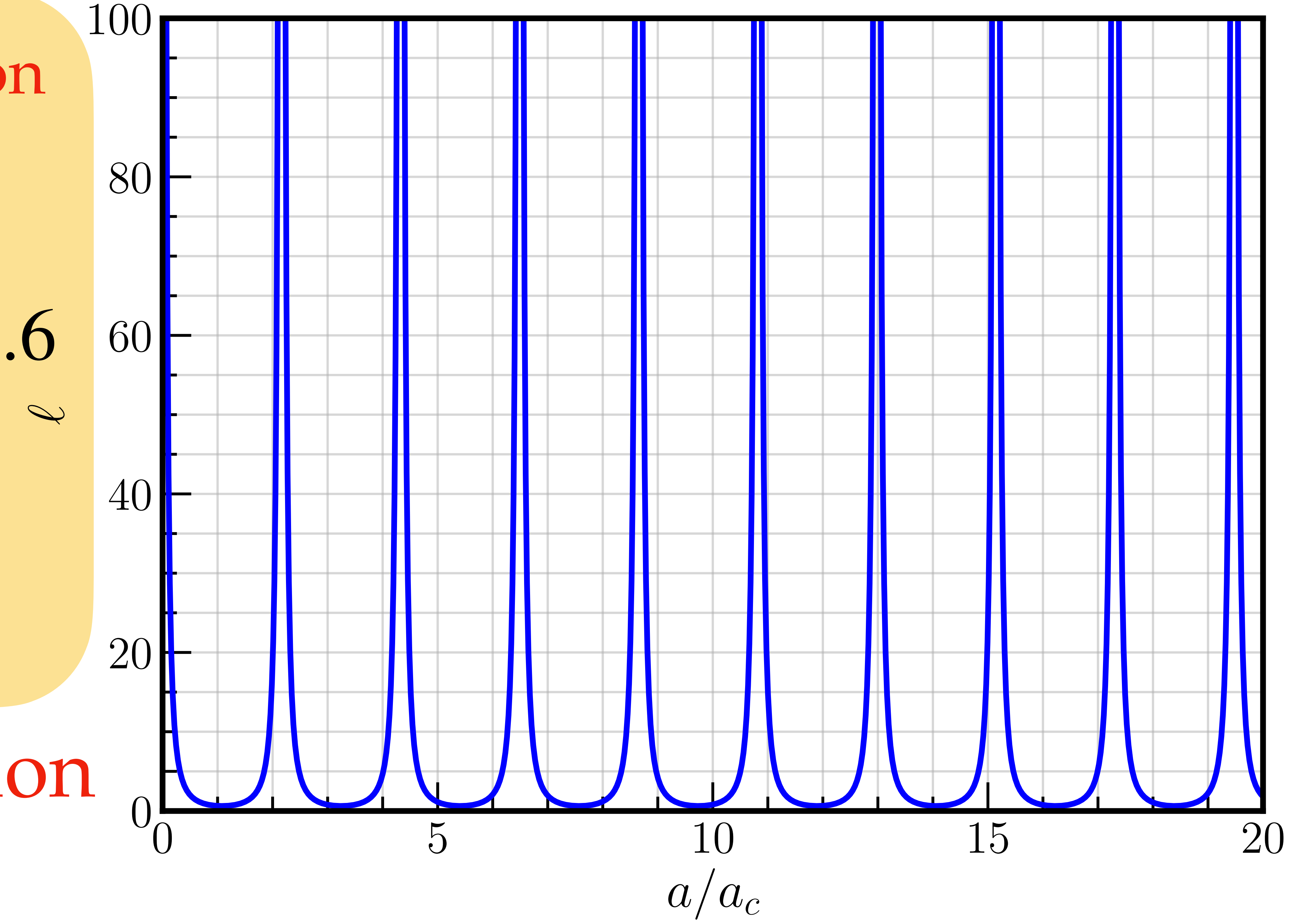
$0 < v(\mu a/a_{\rm eq}) < 1.6$

$m_{\rm eff} \sim m_\psi$

$\mu^{-1} \lesssim k_{\rm eq}\ell < \infty$

Pulse like function

$$\ell^{-1} \sim \mu\, a_{\rm eq}H_{\rm eq} \times v(\mu a/a_{\rm eq})$$



Fermion perturbations

Dependence on β and f_ψ can be totally absorbed into: (also at non-linear level)

$$s = 12\beta^2 f_\psi \frac{a}{a_{\rm eq}}$$

$$\omega = \frac{1}{2}\frac{\mu}{2^{5/6}3^{3/4}f_\psi \beta^2} \approx 4\times 10^{17}\left(\frac{\sqrt{\lambda}}{f_\psi^2\beta^5}\right)^{1/3}$$

Linear equation:

$$s\frac{d^2\delta_\psi}{ds^2} + \left(1+\frac{d\log m_{\rm eff}}{d\log s}\right)\frac{d\delta_\psi}{ds} = \frac{\delta_\psi}{4}\frac{m_{\rm eff}}{m_\psi}\frac{1}{1+(k\ell)^{-2}}$$

$$\frac{m_{\rm eff}}{m_\psi} \sim 1 - \#\frac{1}{s\omega^2}v(\omega s)$$

$$(k\ell)^{-1} \sim (k\bar{\ell})^{-1} \times v(\omega s)$$

Negligible effect for $\omega \gg 1$

Periodic pulse-like function

=> Periodically infinitely long range force

Representative value: $(k_{\rm eq}\bar{\ell})^{-1} \equiv 2^{-1/3}\,3^{1/4}\mu = 6\sqrt{2}f_\psi\beta^2\omega$

Linear + Numerical

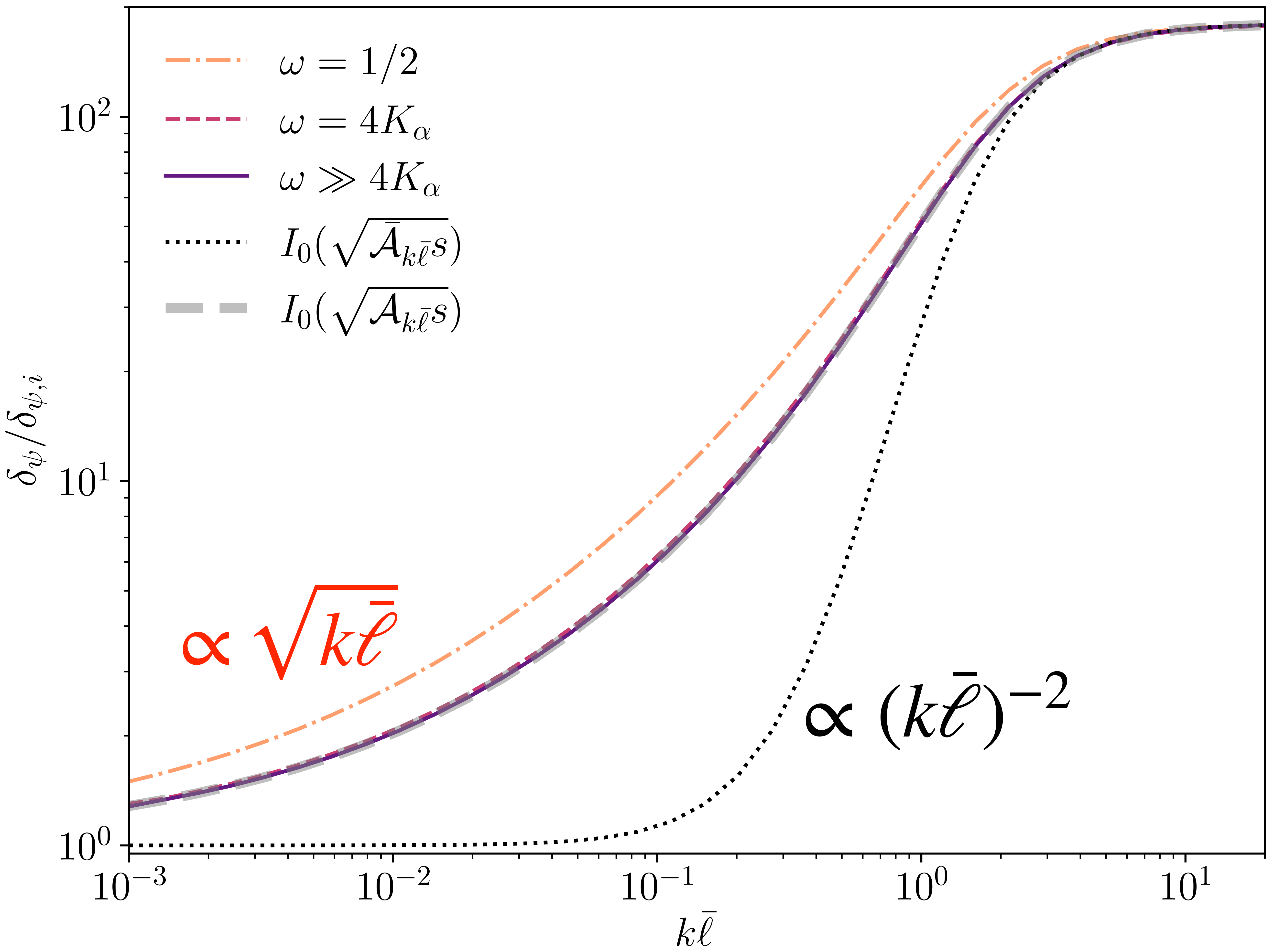
High frequency oscillation-average:

$$\mathscr{A}_{k\bar{\ell}} \equiv \left\langle \frac{1}{1+(k\ell)^{-2}} \right\rangle \approx \left(\frac{1}{1+(k_*/k)^2} \right)^{1/4}$$

Good analytical approximation:

$$\delta_\psi \approx \delta_{\psi,i} I_0(\sqrt{\mathscr{A}_{k\bar{\ell}} s})$$

Well-defined high frequency limit

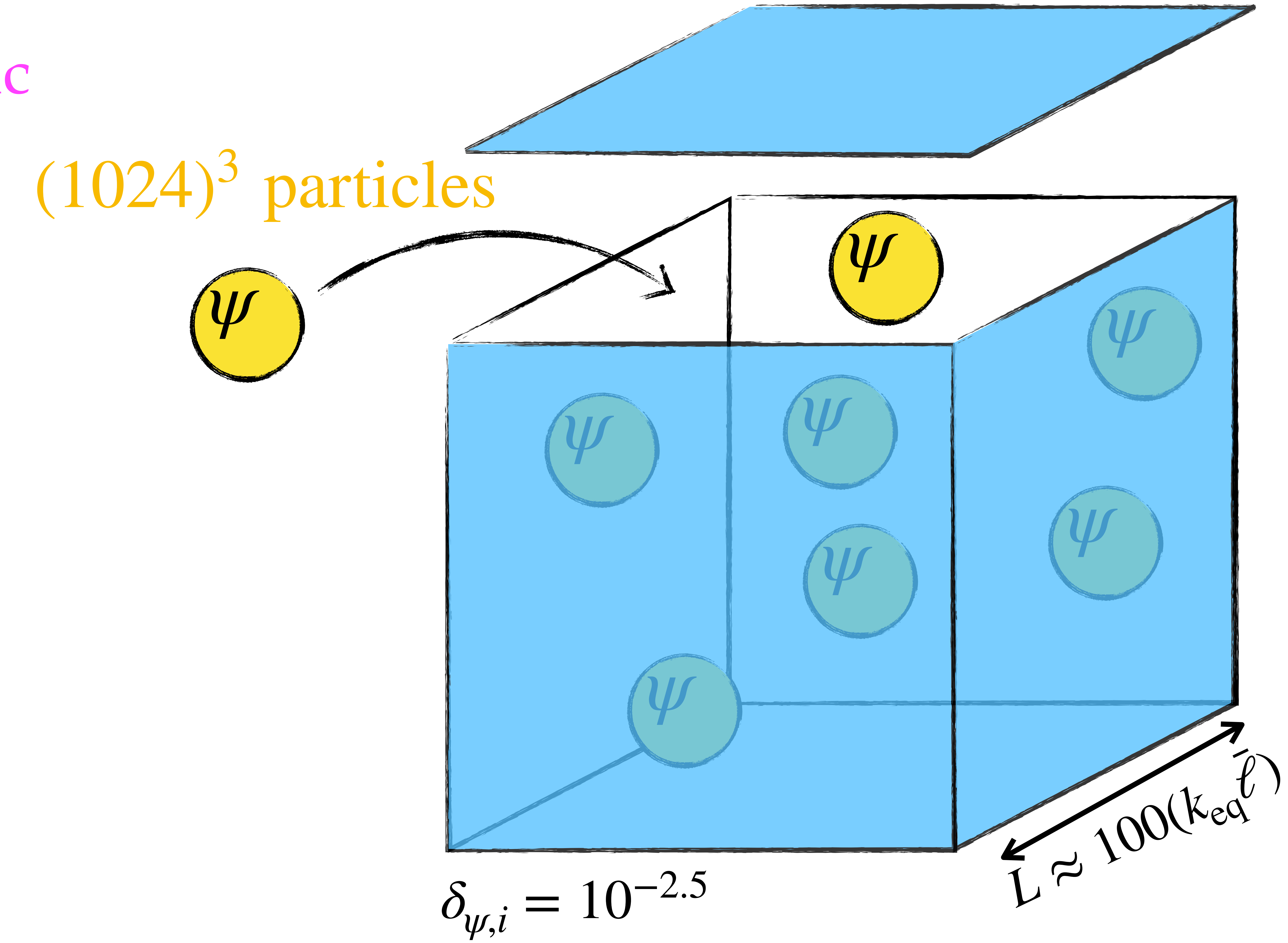


Yukawa interaction longer range than naive estimates

Box & Particles

High frequency limit is scale-free, i.e. independent of L.

Hamilton equations for non-relativistic particles:
+radiation domination
+time-dependent mass
+ Yukawa forces



$$\frac{d\vec{x}}{d\eta} \simeq \frac{\vec{p}}{am_{\rm eff}} \qquad \frac{d\vec{p}}{d\eta} \simeq -\, am_{\rm eff}\vec{\nabla}\phi_Y$$

$$(\nabla^2 - \ell^{-2})\phi_Y = a^2\beta^2 M_{\rm pl}^{-2}\rho_\psi\delta_\psi$$

Similar to CDM N-body simulations but not quite

Box & Particles

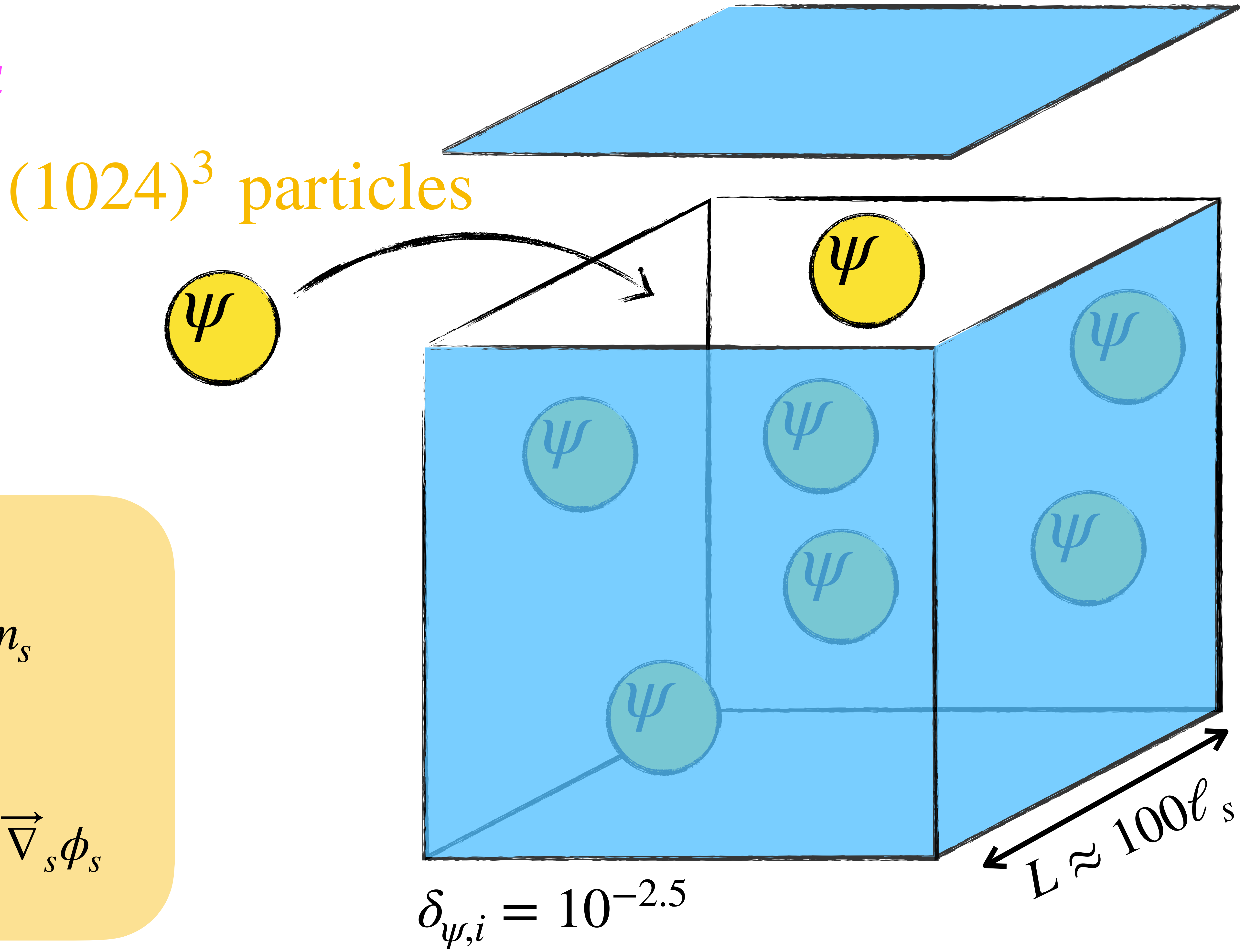
High frequency limit is scale-free, i.e. independent of L.

Hamilton equations for non-relativistic particles:
+radiation domination
+time-dependent mass
+ Yukawa forces

$$\frac{d\vec{x}_s}{dt_s} = \vec{v}_s \qquad \frac{d\vec{p}_s}{dt_s} = \vec{f}_s \qquad \vec{v}_s = \vec{p}_s / m_s$$

$$\left(\nabla_s^2 - \ell_s^{-2}\right)\phi_s = \frac{s}{4}\delta_\psi \qquad \vec{f}_s = -m_s \vec{\nabla}_s \phi_s$$

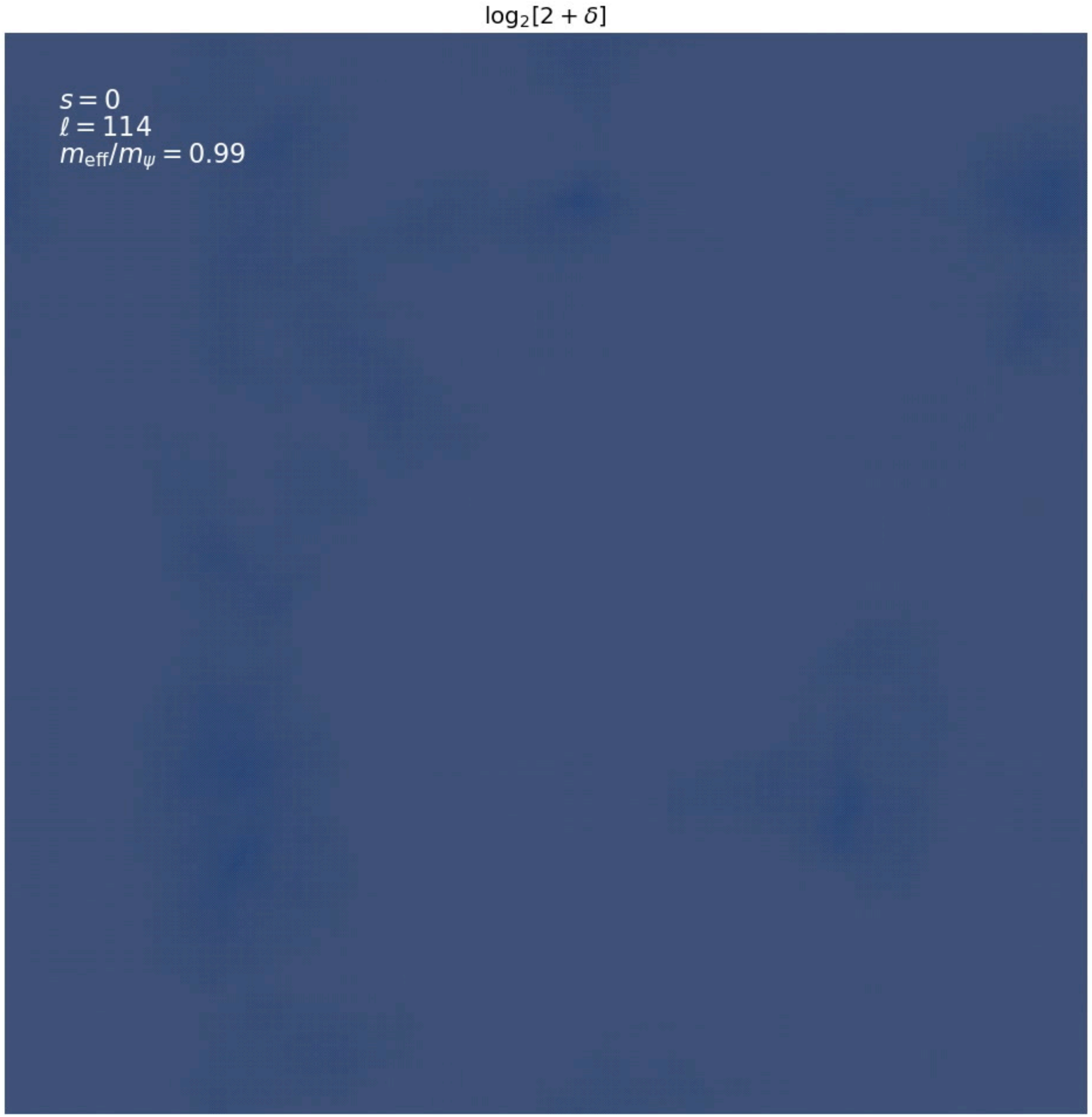
Dimension-less equations



Similar to CDM N-body simulations but not quite

Movie I

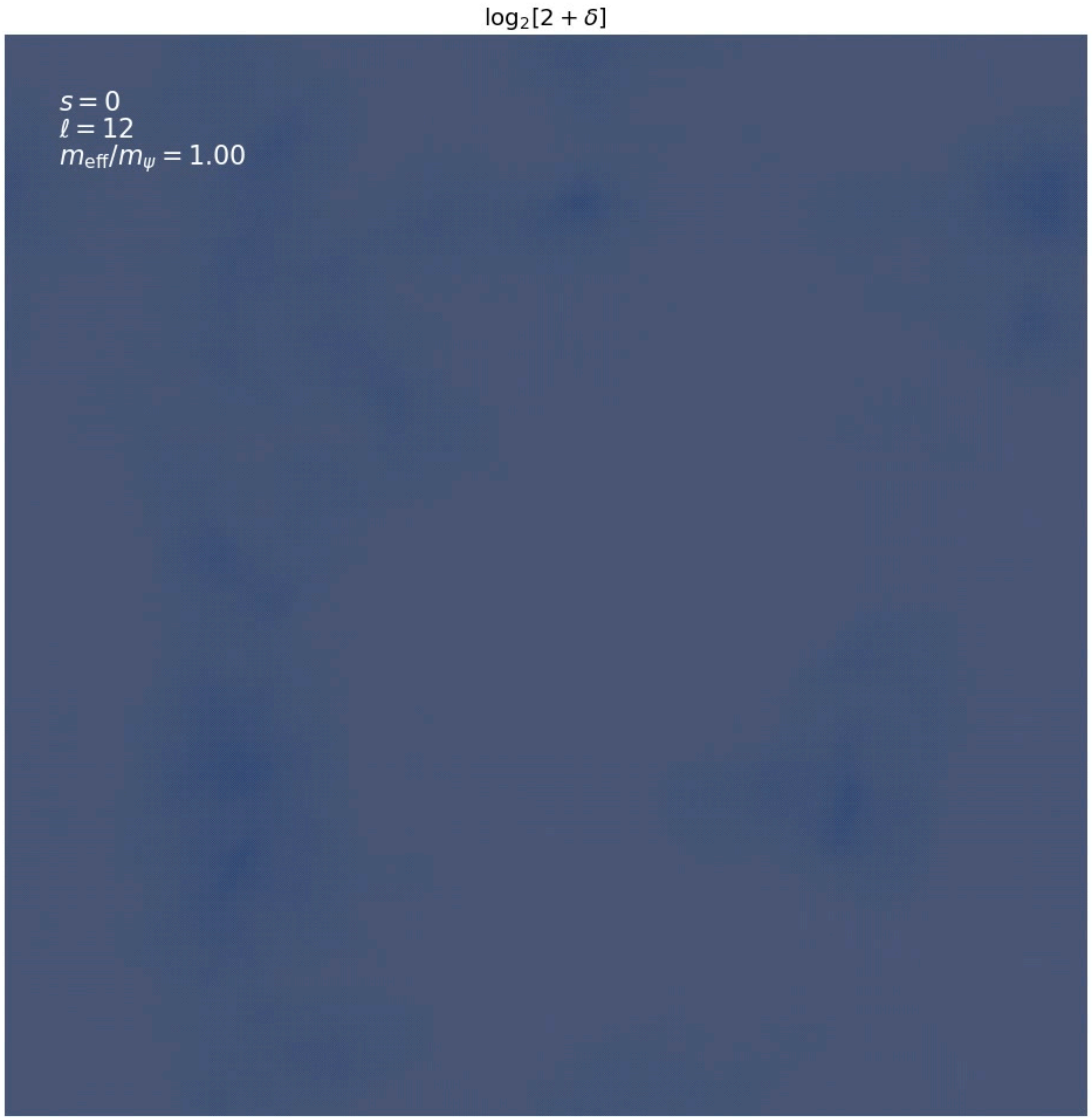
Oscillating ℓ and m_{eff}



All credit to D. Inman

Movie II

Constant ℓ and $m_{\rm eff}$



All credit to D. Inman

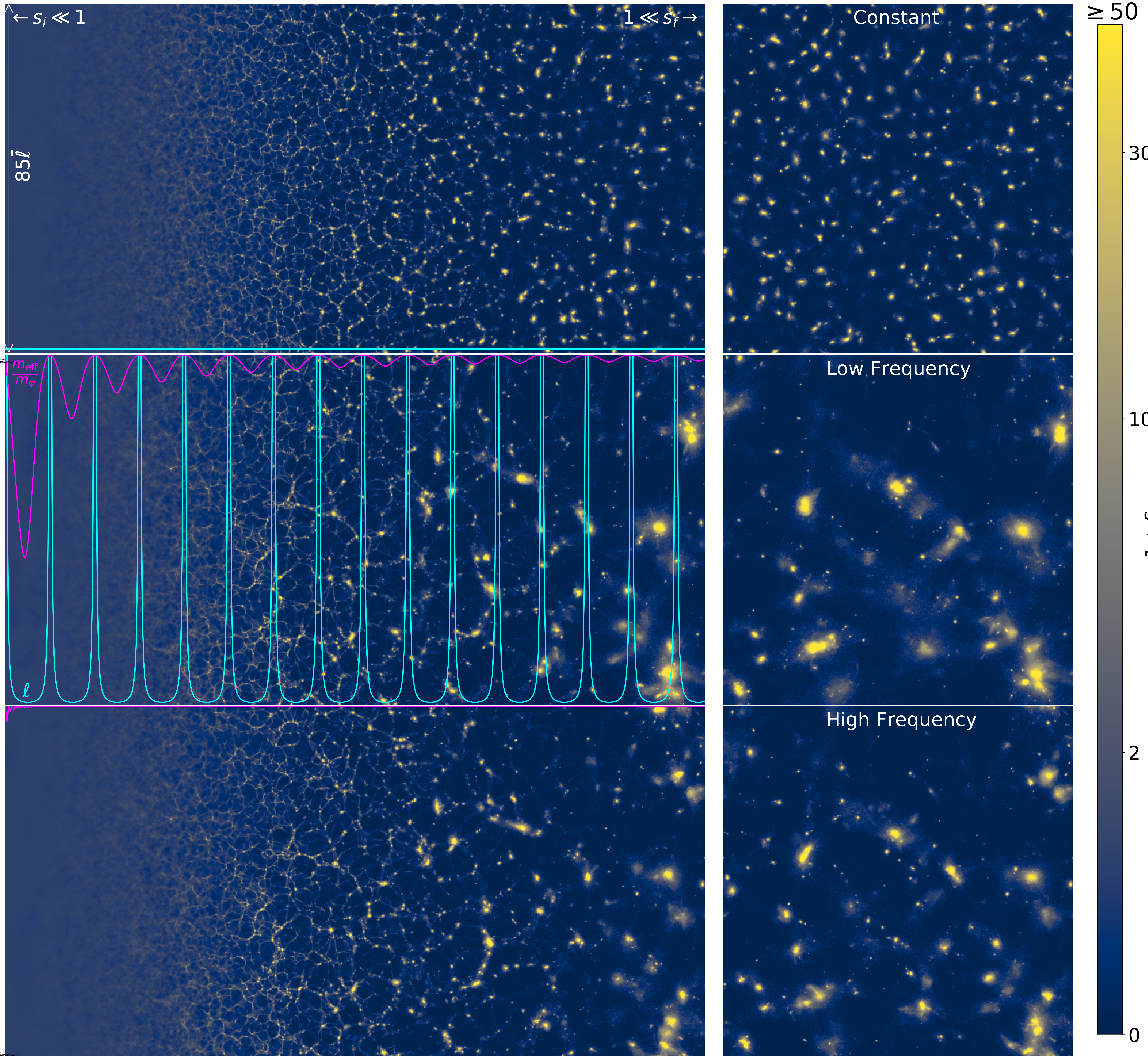
Screenshots

All credit to D. Inman

Constant ℓ and $m_{\rm eff}$

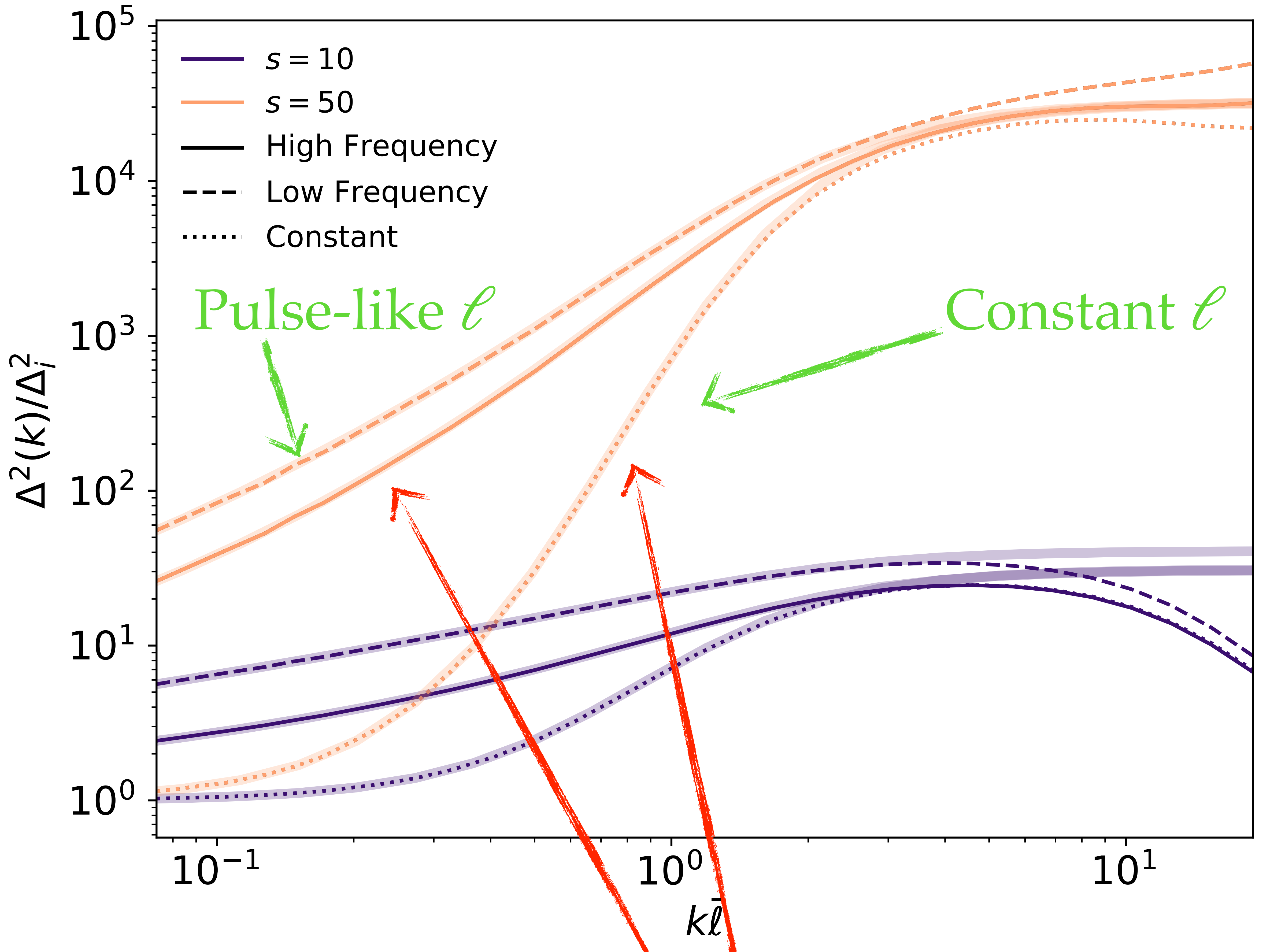
Low-Freq. Oscillating ℓ and $m_{\rm eff}$

High-Freq. Oscillating ℓ and $m_{\rm eff}$



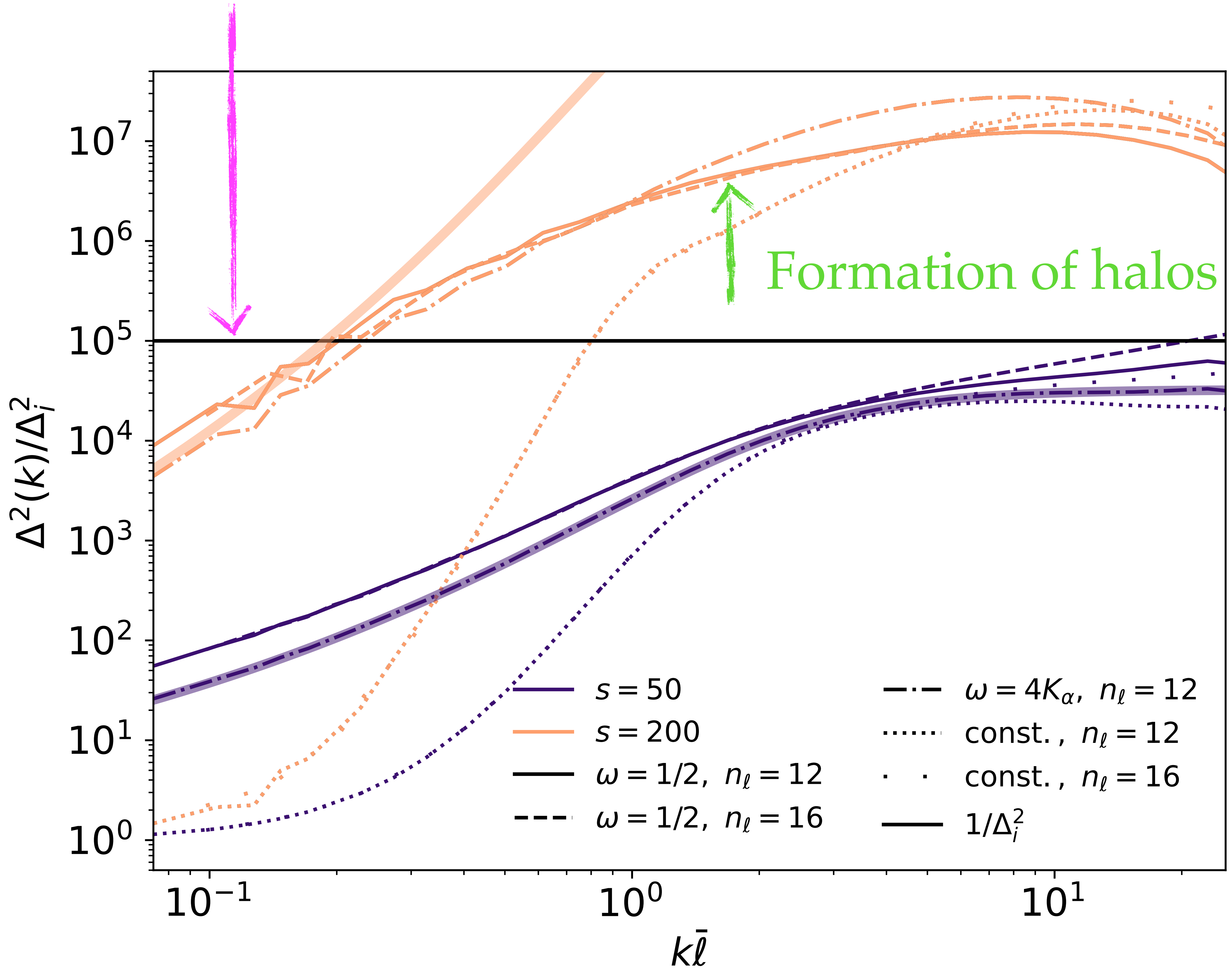
Spectrum of fluctuations

Linear regime



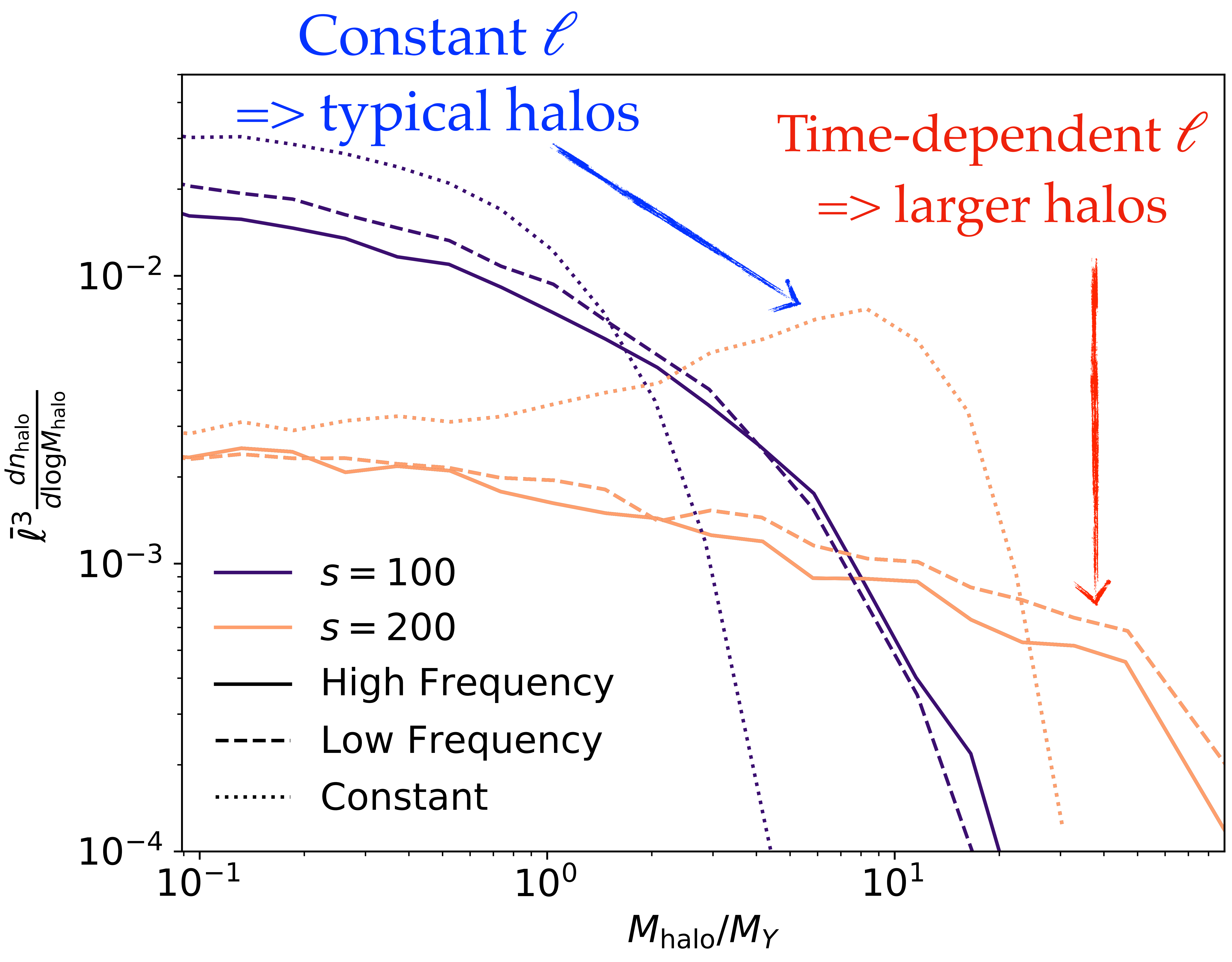
$k\ell \gtrsim 1$

Non-linear in 3 e-folds



Time dependence allows for longer and more massive structures

Halo mass function & Mass estimates



Halo mass function & Mass estimates

Basic length: $\bar{\ell}_Y \approx \frac{0.23\,\mathrm{km}}{(f_\psi \beta \lambda^{1/2})^{1/3}}$

Basic mass: $M_Y \approx \frac{6 \times 10^{-6}\,\mathrm{g}}{\beta\sqrt{\lambda}}$

Maximum halo mass grows with s: $M_{\max}(s) = M_Y \left(k_{\rm nl}(s)\bar{\ell}\right)^{-3}$

E.g. at equality: $M_{\max}(a = a_{\rm eq}) \approx 5 \times 10^{42}\,\mathrm{g}\, \frac{f_\psi^6}{\sqrt{\lambda}} \left(\frac{\beta}{10^5}\right)^{11}$ $R_{\max}(a_0) \sim \left(\frac{M_{\max}}{4\pi\Delta\rho_\psi/3}\right)^{1/3} = 40\,\mathrm{kpc}\frac{f_\psi^{5/3}}{\lambda^{1/6}}\left(\frac{\beta}{10^5}\right)^{11/3}$

Like a galaxy!

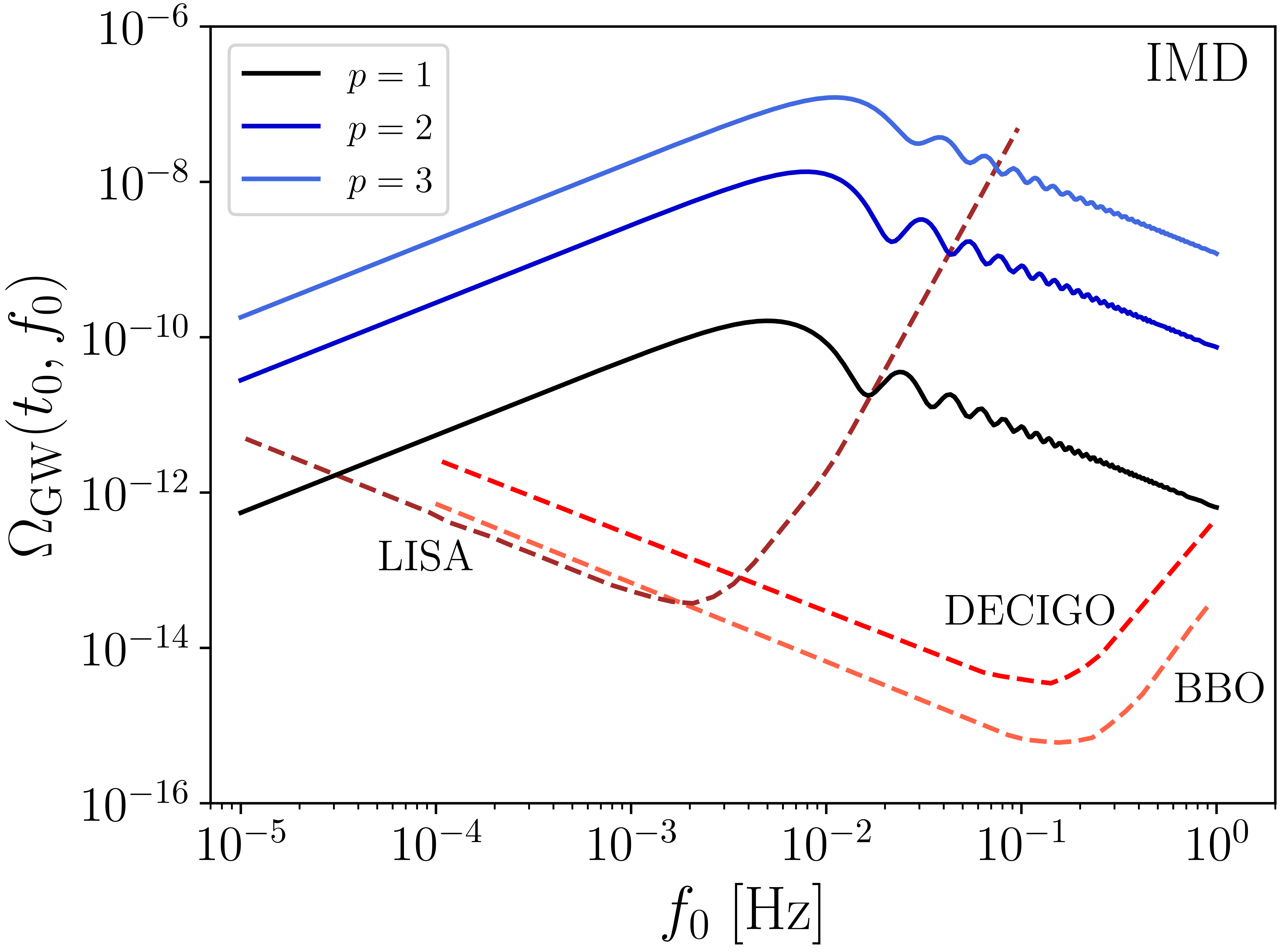
Potential outcomes:

1) Yukawa force overwhelms degeneracy pressure —> Total collapse

2) Fermions slowly decay into Standard Model —> New signatures

GWs signatures?

Flores, Kusenko & Sasaki: 2209.04970



GW from collapsing structures in an early matter domination

Check https://arxiv.org/abs/2404.05430 for recent summary

Halo fates?

Yukawa forces in radiation domination form halos

What next?

- **PBH** formation? If there is efficient radiative cooling yes [Flores & Kusenko 2108.08416]
- **Gravitational waves** from formation? Most likely yes [Flores+ 2209.04970]
- Decay into SM particles? Maybe **magnetogenesis** or **baryogengesis**
  [Durrer & Kusenko 2209.13313] [Flores+ 2208.09789]
- **Dark stars**/halos? [Savastano+ 1906.05300]
- Mixture of heavy fermion dark matter and scalar field (36%) dark matter
  [GD & Sasaki 2104.05271]

Summary

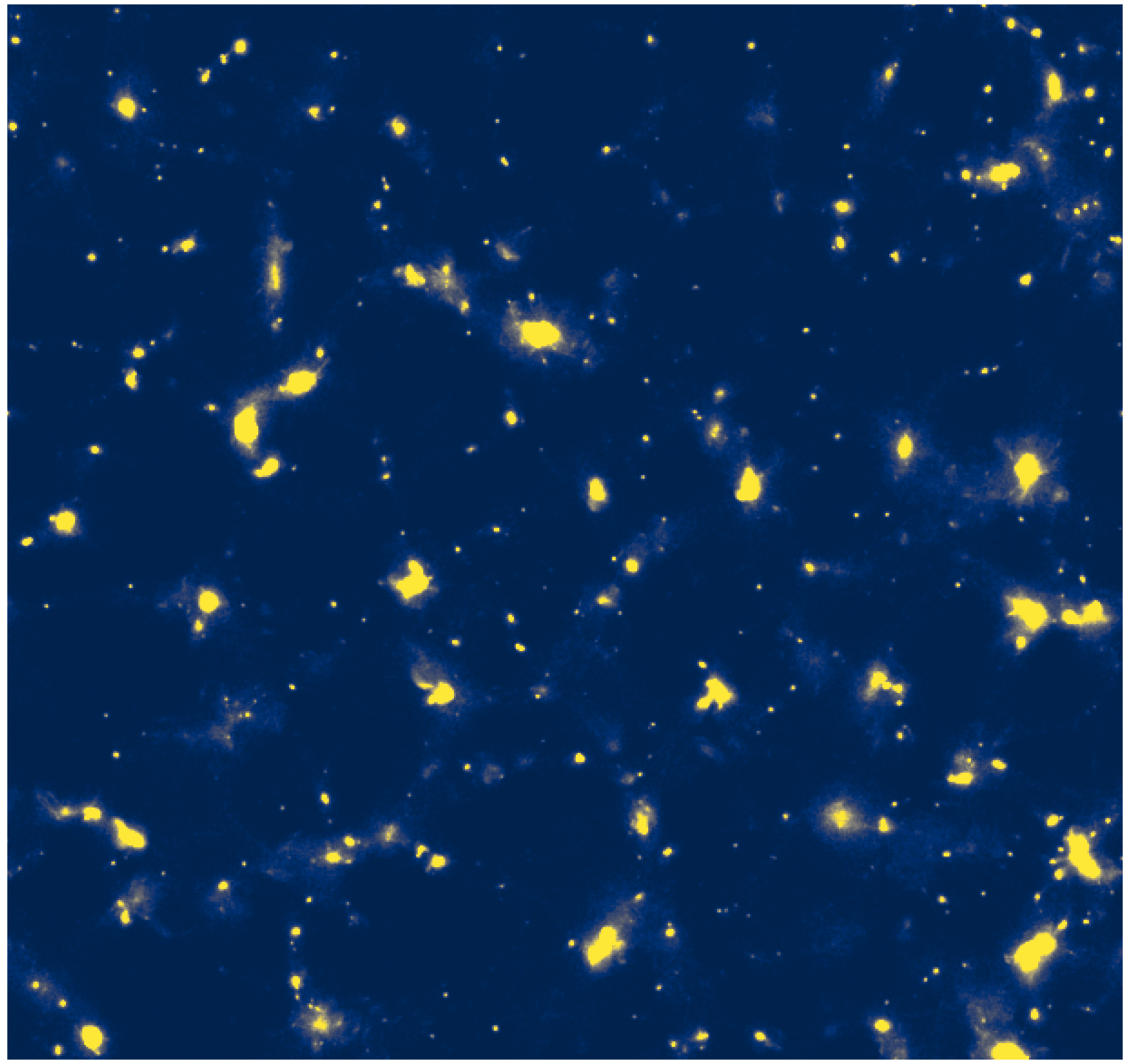
"Yukawa forces can efficiently form structures in the very early universe"

For a quartic scalar field potential

—> (Pulse-like) Longer range interactions

These halos have rich phenomenology

—> PBHs, annihilation, early galaxies…



Early universe cosmology of Yukawa interactions

Based on [2104.05271 & 2304.13053]
with: D. Inman, A. Kusenko & M. Sasaki

by **Guillem Domènech**
(ITP Hannover)

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Leibniz Universität Hannover

Seminar at MPIK, Heidelberg,
June 17th, 2024

THANK YOU!