

Investigating cosmic discordances

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The Λ CDM model

The model that has now practically been selected as the “standard” cosmological model is the Lambda Cold Dark Matter (Λ CDM) model, that provides an amazing description of a wide range of astrophysical and astronomical data.

Over the last few years, the parameters governing Λ CDM have been constrained with unprecedented accuracy.

However, despite its incredible success, Λ CDM still cannot explain key concepts in our understanding of the structure and evolution of the Universe, at the moment based on **unknown quantities**. At the moment, their physical evidence comes solely from cosmology and astrophysics without strong theoretical motivations.

The Λ CDM model

Unknown quantities:

- an early stage of accelerated expansion (**Inflation**) which produces the initial, tiny, density perturbations, needed for structure formation.
- a clustering matter component to facilitate structure formation (**Dark Matter**),
- an energy component to explain the current stage of accelerated expansion (**Dark Energy**).

The Λ CDM model

Unknown quantities:

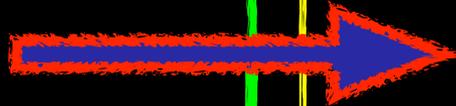
- an early stage of accelerated expansion (**Inflation**) which produces the initial, tiny, density perturbations, needed for structure formation.
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In addition, the Λ CDM model is based on the choice of three, very specific, solutions for these **unknown quantities**, mostly motivated by **computational simplicity**, i.e. the theoretical predictions under Λ CDM for several observables are, in general, easier to compute and include fewer free parameters than most other solutions.

The Λ CDM model

Unknown quantities:

- an early stage of accelerated expansion (**Inflation**) which produces the initial, tiny, density perturbations, needed for structure formation.
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Specific solutions for Λ CDM:

- **Inflation** is given by a single, minimally coupled, slow-rolling scalar field;
- **Dark Matter** is a pressureless fluid made of cold, i.e., with low momentum, and collisionless particles;
- **Dark Energy** is a cosmological constant term.

Warning!

Therefore, the **6 parameter Λ CDM model** can be rightly considered, at best, as an **approximation to a more realistic scenario** that still needs to be fully understood. With the increase in experimental sensitivity, observational evidence for **deviations from Λ CDM is, therefore, expected**.

And, actually, **anomalies and tensions between observations at early cosmological time and measurements at late cosmological time** are present with different statistical significance.

While some proportion of these discrepancies may have a systematic origin, their magnitude and persistence across probes strongly hint at cracks in the standard cosmological scenario and the necessity of new physics. In other words, if not due to systematics, the current anomalies could represent a crisis for the standard cosmological model and their experimental confirmation can bring a revolution in our current ideas of the structure and evolution of the Universe.

These tensions can indicate a failure in Λ CDM model.

CMB constraints

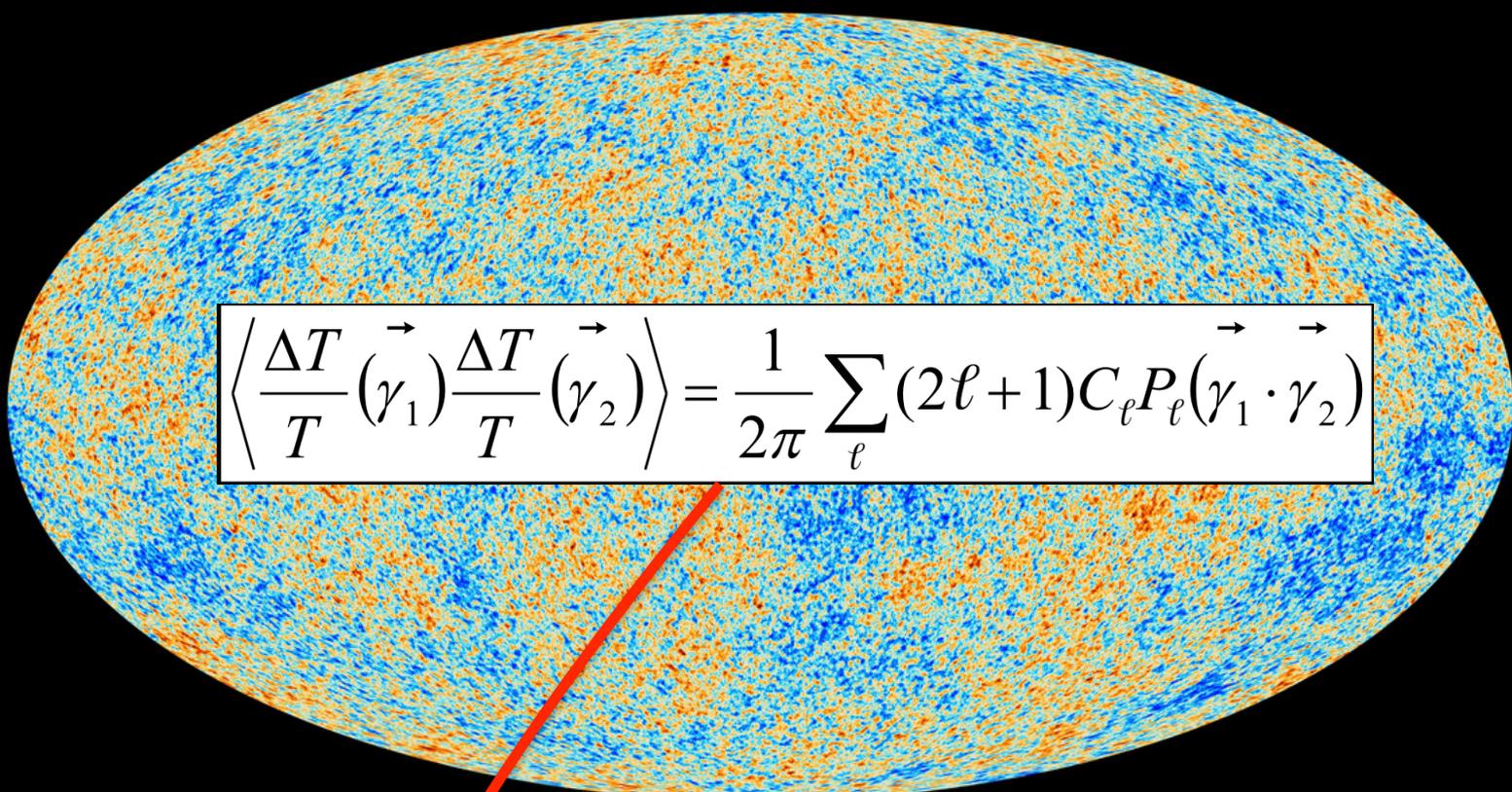
Most of the **anomalies and tensions** are involving the CMB data.



- Frequency range of 30GHz to 857GHz;
- Orbit around L2;
- Composed by 2 instruments:
 - LFI → 1.5 meters telescope; array of 22 differential receivers that measure the signal from the sky comparing with a black body at 4.5K.
 - HFI → array of 52 bolometers cooled to 0.1K.

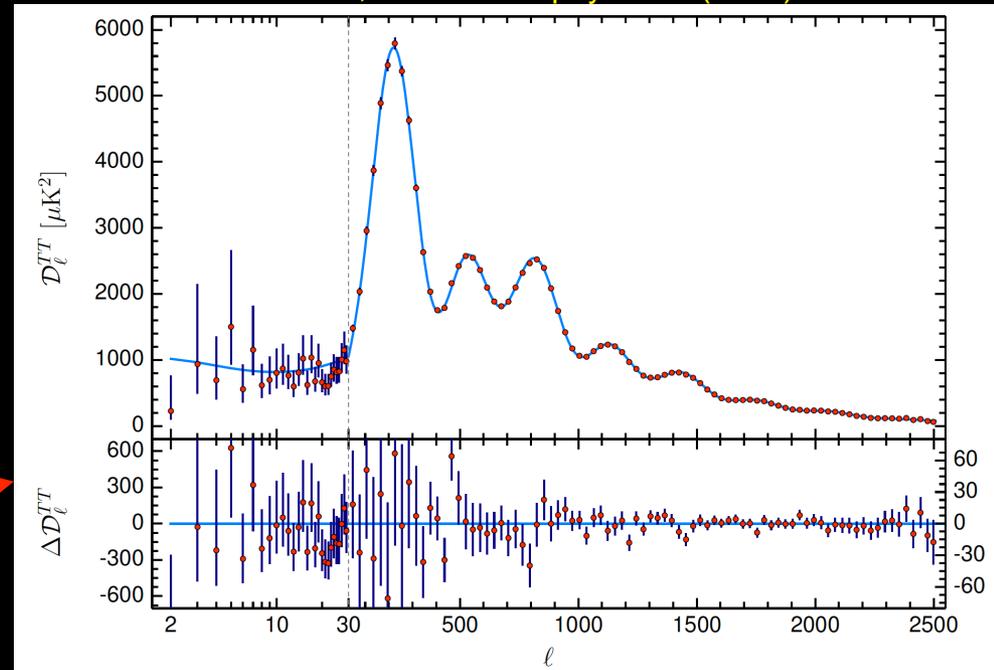
Planck 2018, Astron.Astrophys. 641 (2020) A6

2018 Planck results are a wonderful confirmation of the flat standard Λ CDM cosmological model, but are **model dependent!**


$$\left\langle \frac{\Delta T}{T}(\vec{\gamma}_1) \frac{\Delta T}{T}(\vec{\gamma}_2) \right\rangle = \frac{1}{2\pi} \sum_{\ell} (2\ell + 1) C_{\ell} P_{\ell}(\vec{\gamma}_1 \cdot \vec{\gamma}_2)$$

From the map of the CMB anisotropies we can extract the temperature angular power spectrum.

Planck 2018, *Astron. Astrophys.* 641 (2020) A6



Cosmological parameters:
($\Omega_b h^2$, $\Omega_m h^2$, H_0 , n_s , τ , A_s)

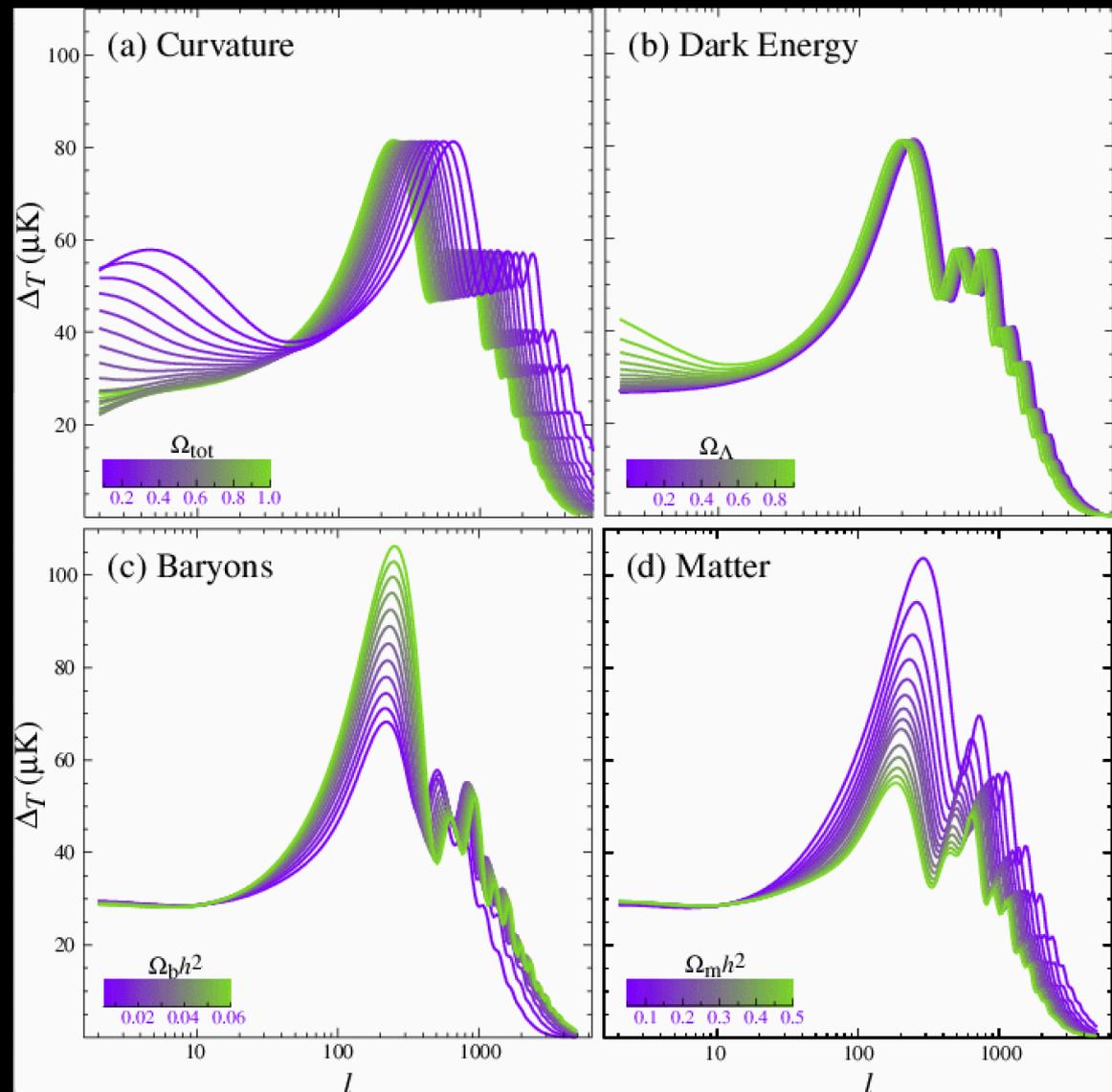


Theoretical model

Wayne Hu's tutorial

We choose a set of cosmological parameters that describes our **theoretical model** and compute the angular power spectra.

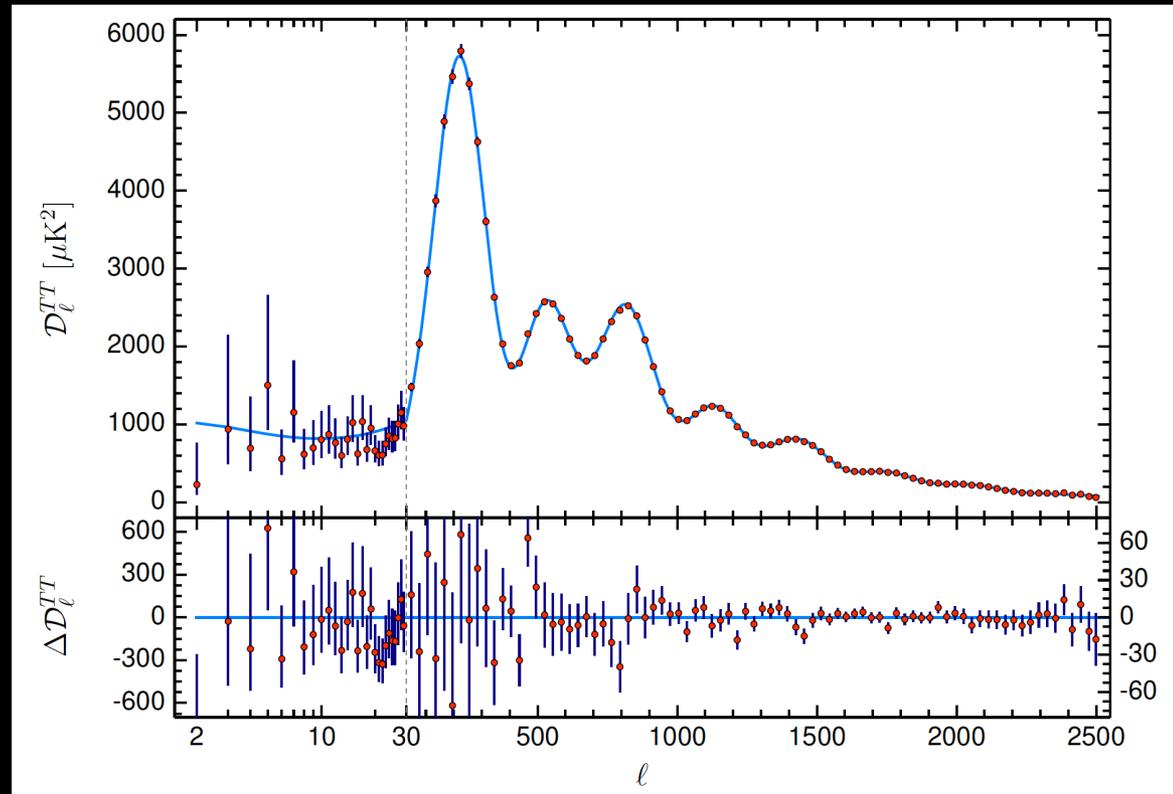
Because of the correlations present between the parameters, variation of different quantities can produce similar effects on the CMB.



Cosmological parameters:
($\Omega_b h^2$, $\Omega_m h^2$, H_0 , n_s , τ , A_s)

Theoretical model

We compare the angular power spectra we computed with the data and, using a bayesian analysis, we get a combination of cosmological parameter values in agreement with these.

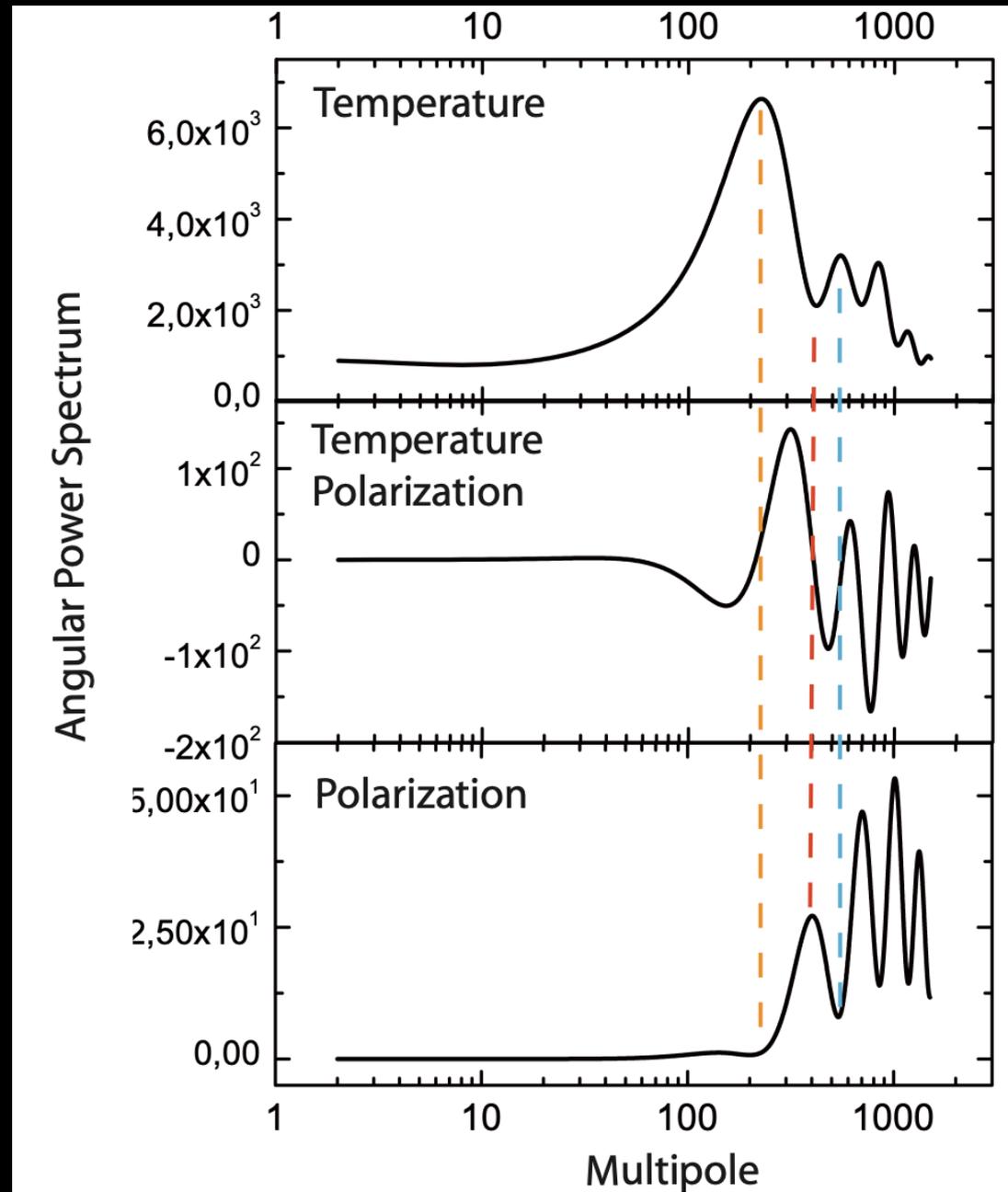


Planck 2018, Astron.Astrophys. 641 (2020) A6

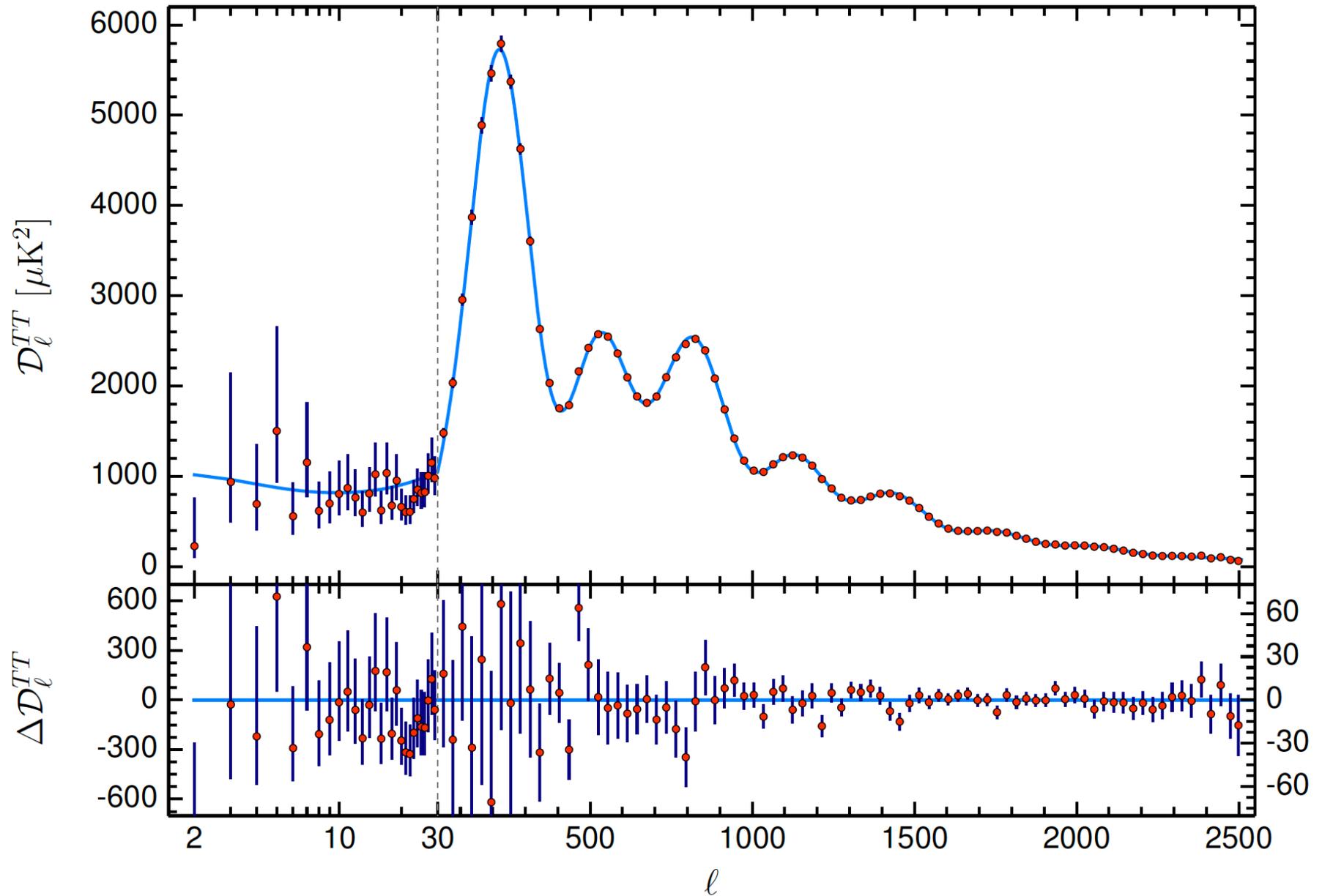
Parameter constraints

We can extract 4 independent angular spectra from the CMB:

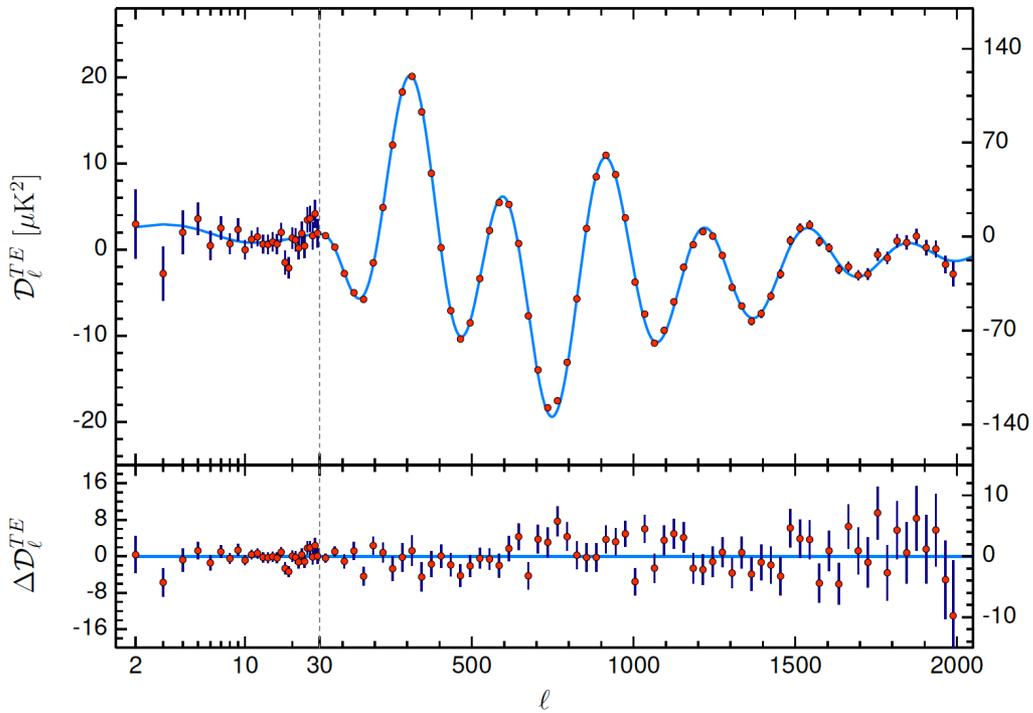
- Temperature
- Cross Temperature Polarization E
- Polarization type E (density fluctuations)
- Polarization type B (gravitational waves)



Planck satellite experiment



Planck satellite experiment

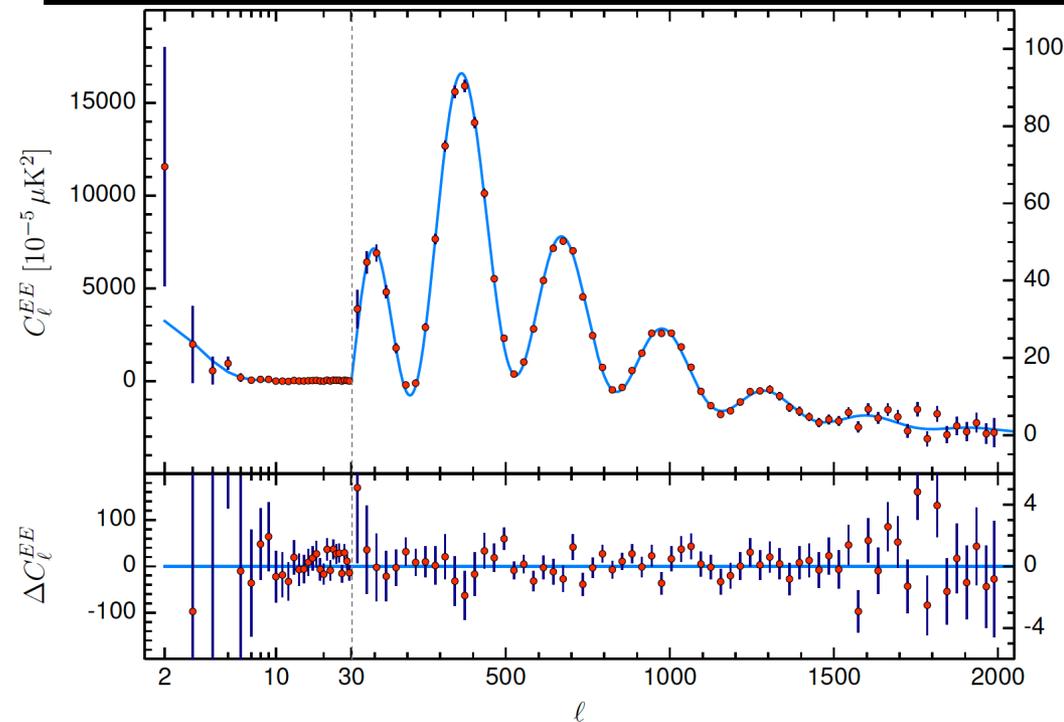


The theoretical spectra in light blues are computed from the best-fit base- Λ CDM theoretical spectrum fit to the Planck TT,TE,EE+lowE+lensing likelihood.

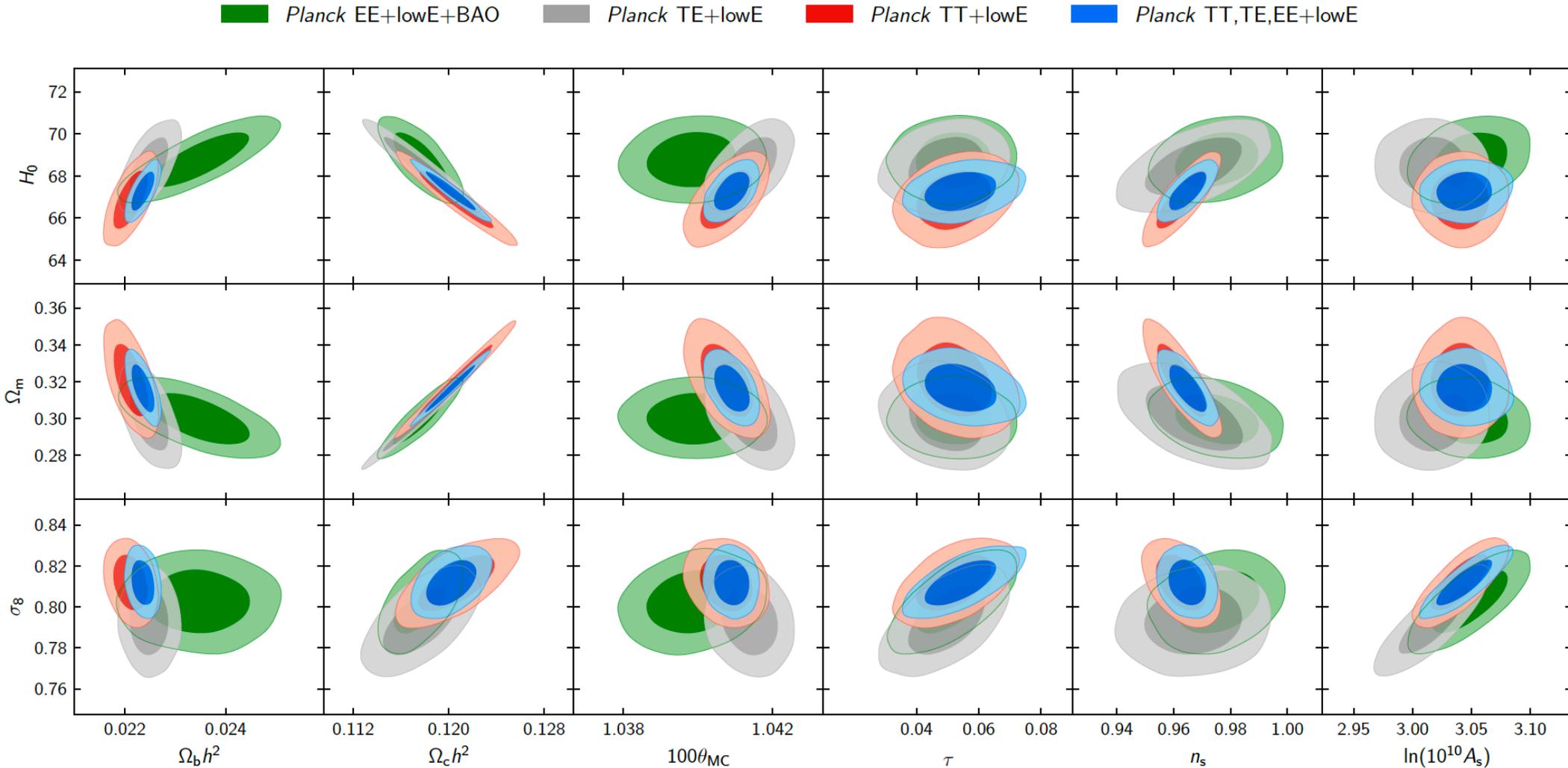
Residuals with respect to this theoretical model are shown in the lower panel in each plot.

Planck 2018, *Astron.Astrophys.* 641 (2020) A6

Polarization spectra



CMB constraints



Planck 2018, *Astron.Astrophys.* 641 (2020) A6

Constraints on parameters of the LCDM model from the separate Planck EE, TE, and TT high- l spectra combined with low- l polarization (lowE), and, in the case of EE also with BAO, compared to the joint result using Planck TT,TE,EE+lowE.

CMB constraints

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022	0.02249 ± 0.00025	0.0240 ± 0.0012	0.02236 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1206 ± 0.0021	0.1177 ± 0.0020	0.1158 ± 0.0046	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04077 ± 0.00047	1.04139 ± 0.00049	1.03999 ± 0.00089	1.04090 ± 0.00031	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0522 ± 0.0080	0.0496 ± 0.0085	0.0527 ± 0.0090	$0.0544^{+0.0070}_{-0.0081}$	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.040 ± 0.016	$3.018^{+0.020}_{-0.018}$	3.052 ± 0.022	3.045 ± 0.016	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9626 ± 0.0057	0.967 ± 0.011	0.980 ± 0.015	0.9649 ± 0.0044	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s ⁻¹ Mpc ⁻¹] . .	66.88 ± 0.92	68.44 ± 0.91	69.9 ± 2.7	67.27 ± 0.60	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.679 ± 0.013	0.699 ± 0.012	$0.711^{+0.033}_{-0.026}$	0.6834 ± 0.0084	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.321 ± 0.013	0.301 ± 0.012	$0.289^{+0.026}_{-0.033}$	0.3166 ± 0.0084	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1434 ± 0.0020	0.1408 ± 0.0019	$0.1404^{+0.0034}_{-0.0039}$	0.1432 ± 0.0013	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09589 ± 0.00046	0.09635 ± 0.00051	$0.0981^{+0.0016}_{-0.0018}$	0.09633 ± 0.00029	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8118 ± 0.0089	0.793 ± 0.011	0.796 ± 0.018	0.8120 ± 0.0073	0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$.	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011

Planck 2018, Astron.Astrophys. 641 (2020) A6

2018 Planck results are a wonderful confirmation of the flat standard Λ CDM cosmological model, but are **model dependent!**

- The cosmological constraints are obtained assuming a cosmological model.
- The results are affected by the degeneracy between the parameters that induce similar effects on the observables.

The most statistically significant and persisting **anomalies and tensions** of the CMB are:

- H_0 with local measurements
- A_L internal anomaly
- S8 with cosmic shear data
- Ω_k different from zero

See [Di Valentino et al. arXiv:2008.11283 \[astro-ph.CO\]](#), [arXiv:2008.11284 \[astro-ph.CO\]](#), [arXiv:2008.11285 \[astro-ph.CO\]](#), [arXiv:2008.11286 \[astro-ph.CO\]](#) for an overview.

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The H0 tension

We are referring to “Hubble tension” as the disagreement between

- The Planck constraints **assuming Λ CDM**:

$$H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc in } \Lambda\text{CDM}$$

Planck 2018, *Astron.Astrophys.* 641 (2020) A6

- the local measurements obtained by the SH0ES collaboration.

The so called R19:

$$H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$$

Riess et al. *Astrophys.J.* 876 (2019) 1, 85

4.40

or the updated one R20 using the parallax measurements of Gaia EDR3:

$$H_0 = 73.2 \pm 1.3 \text{ km/s/Mpc}$$

Riess et al., *Astrophys.J.Lett.* 908 (2021) 1, L6

4.20

The H0 tension

CMB Polarization
Measurements
with SPTpol

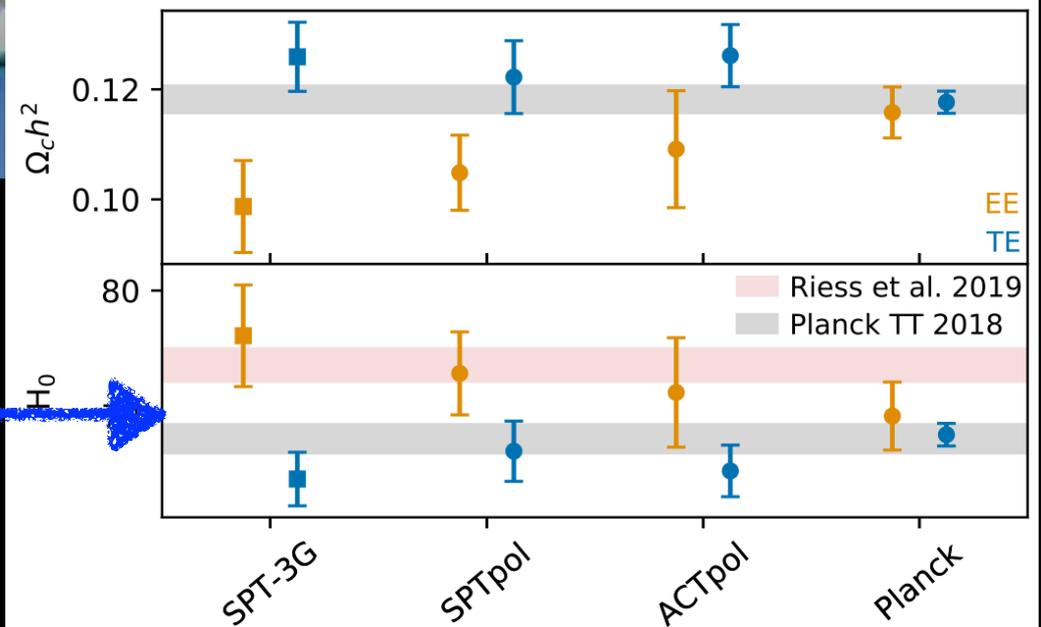
Nicholas Harrington
UC Berkeley



On the same side of Planck, i.e.
preferring smaller values of H_0 we have:

Ground based CMB telescope

SPT-3G:
 $H_0 = 68.8 \pm 1.5$ km/s/Mpc in Λ CDM



The H0 tension

On the same side of Planck, i.e. preferring smaller values of H0 we have:

Ground based CMB telescope

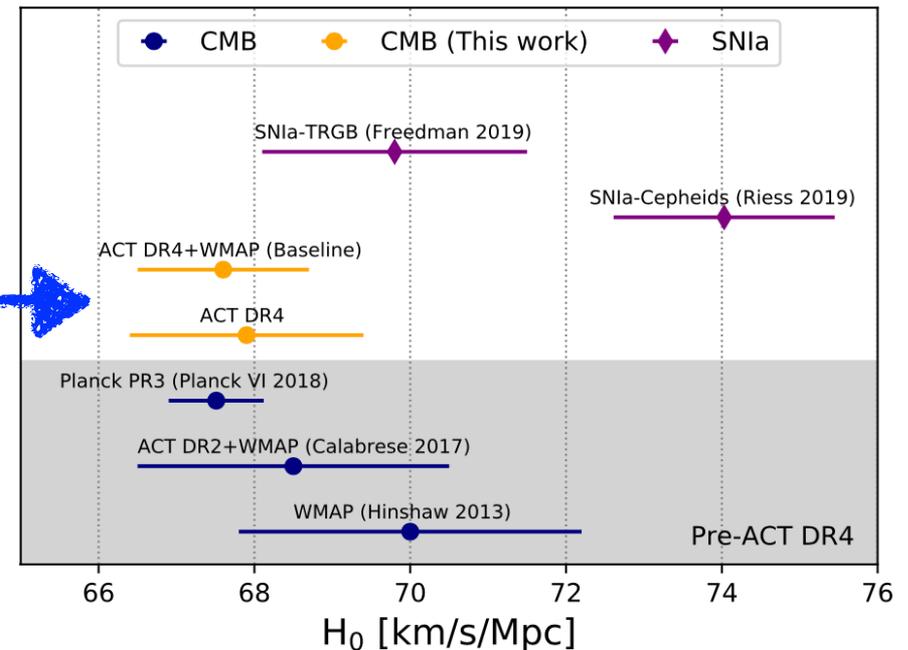


ACT-DR4:

$H_0 = 67.9 \pm 1.5$ km/s/Mpc in Λ CDM

ACT-DR4 + WMAP:

$H_0 = 67.6 \pm 1.1$ km/s/Mpc in Λ CDM



The H0 tension

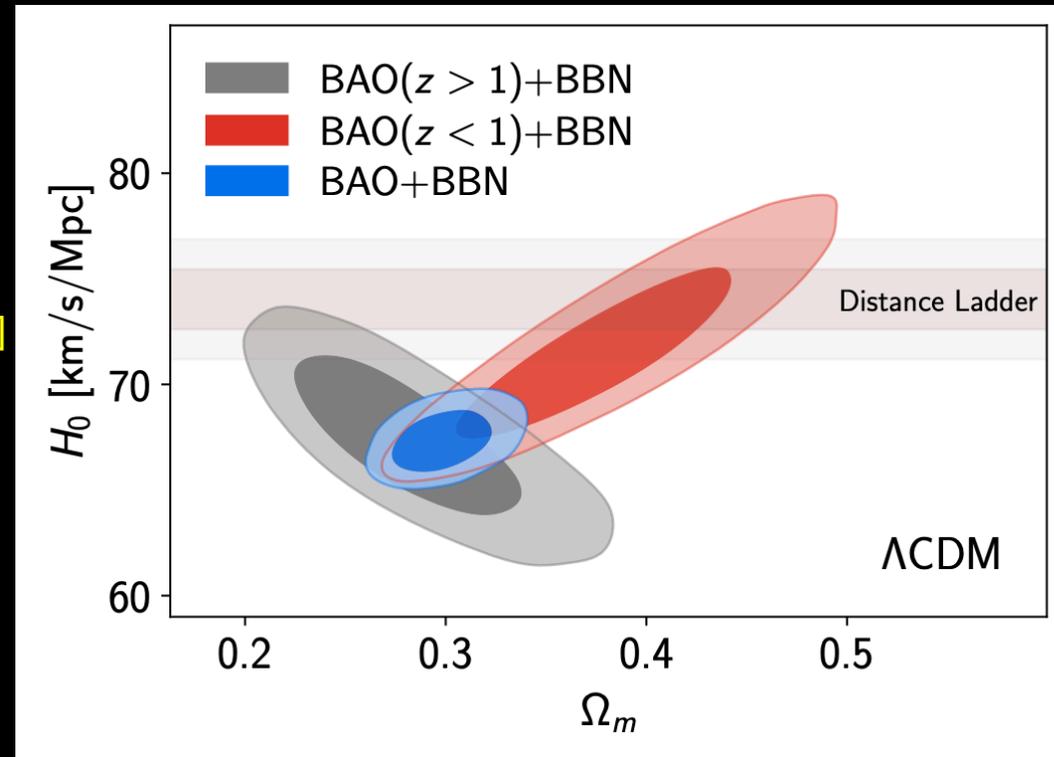
On the same side of Planck, i.e. preferring smaller values of H_0 we have:

BAO+Pantheon+BBN+ θ_{MC} , Planck:
 $H_0 = 67.9 \pm 0.8$ km/s/Mpc

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

BAO+BBN from BOSS and eBOSS:
 $H_0 = 67.35 \pm 0.97$ km/s/Mpc

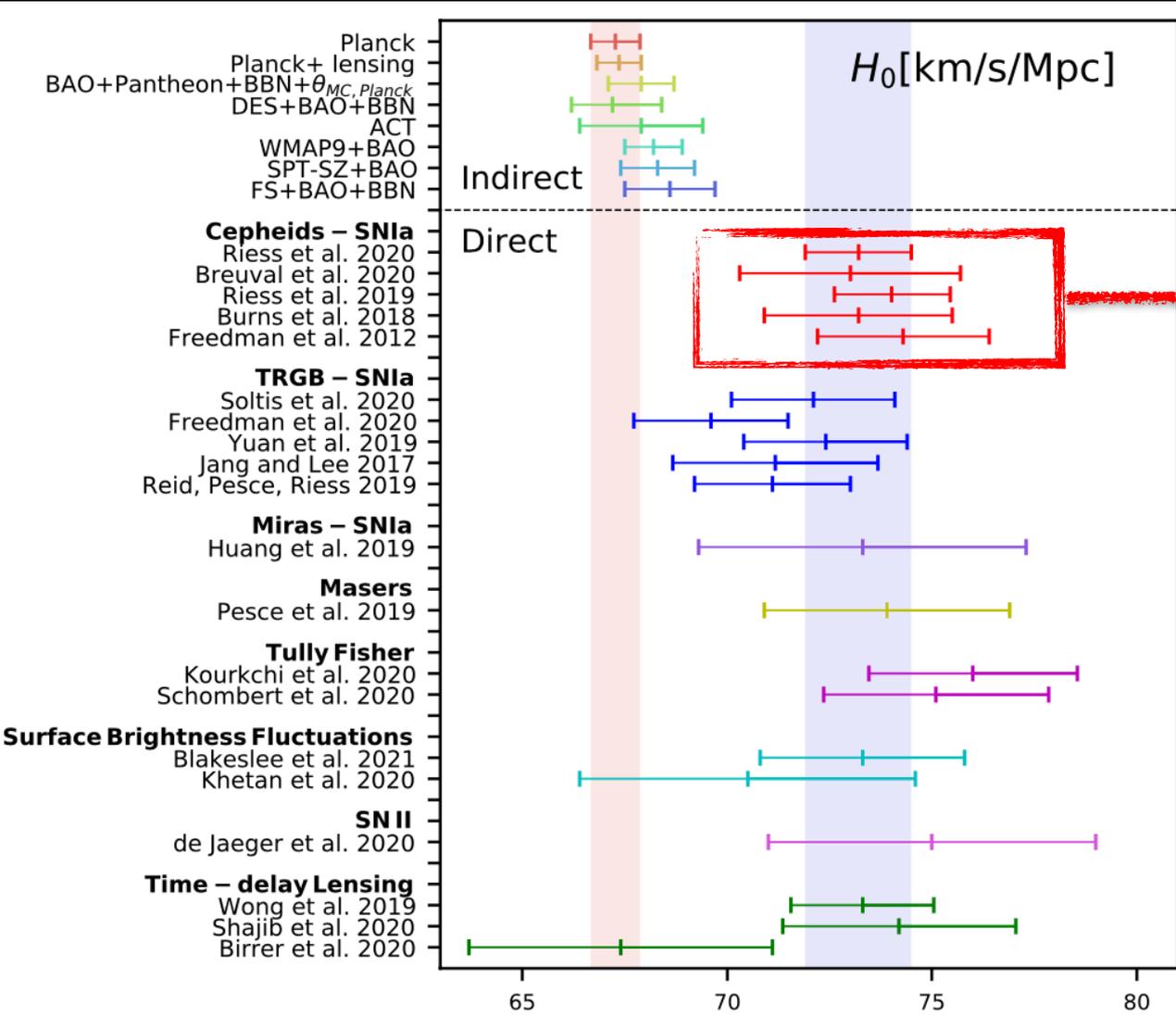
eBOSS, Alam et al., arXiv:2007.08991 [astro-ph.CO]



eBOSS, Alam et al., arXiv:2007.08991 [astro-ph.CO]

Late universe measurements

On the same side of SH0ES, i.e. preferring large values, we have the direct estimates of H_0 .



Cepheids-SN Ia:

$$H_0 = 73.2 \pm 1.3 \text{ km/s/Mpc}$$

Riess et al., arXiv:2012.08534 [astro-ph.CO]

$$H_0 = 73.5 \pm 1.4 \text{ km/s/Mpc}$$

Reid et al., arXiv:1908.05625 [astro-ph.CO]

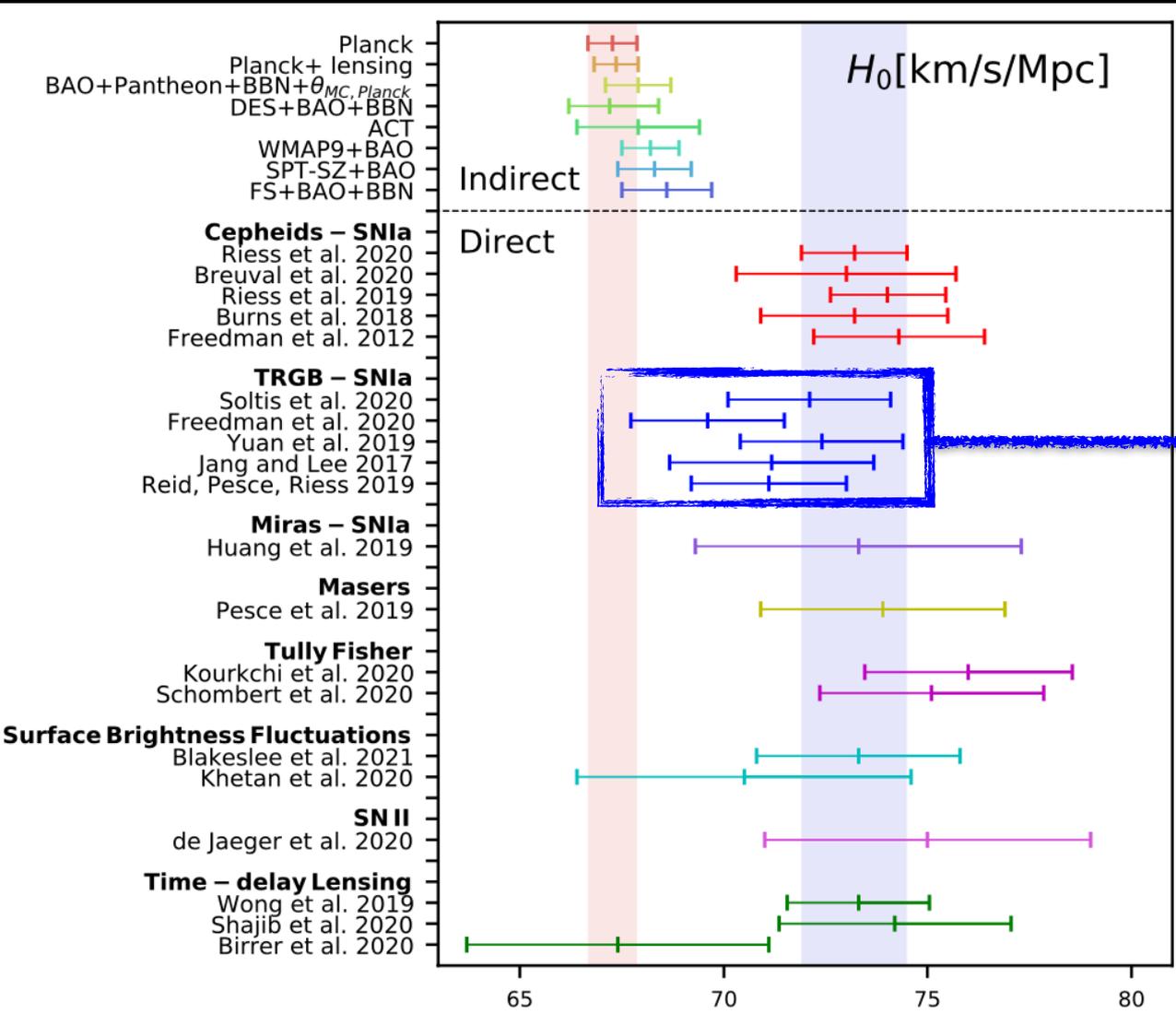
$$H_0 = 73.0 \pm 2.7 \text{ km/s/Mpc}$$

Breival et al., arXiv:2006.08763 [astro-ph.CO]

$$H_0 = 73.2 \pm 2.3 \text{ km/s/Mpc}$$

Burns et al., arXiv:1809.06381 [astro-ph.CO]

Late universe measurements



The Tip of the Red Giant Branch (TRGB) is the peak brightness reached by red giant stars after they stop using hydrogen and begin fusing helium in their core.

$$H_0 = 72.1 \pm 1.2 \text{ km/s/Mpc}$$

Soltis et al., arXiv:2012.09196 [astro-ph.CO]

$$H_0 = 69.6 \pm 1.88 \text{ km/s/Mpc}$$

Freedman et al., arXiv:2002.01550 [astro-ph.CO]

$$H_0 = 72.4 \pm 2.0 \text{ km/s/Mpc}$$

Yuan and Lee., arXiv:1908.00993 [astro-ph.CO]

$$H_0 = 71.17 \pm 2.50 \text{ km/s/Mpc}$$

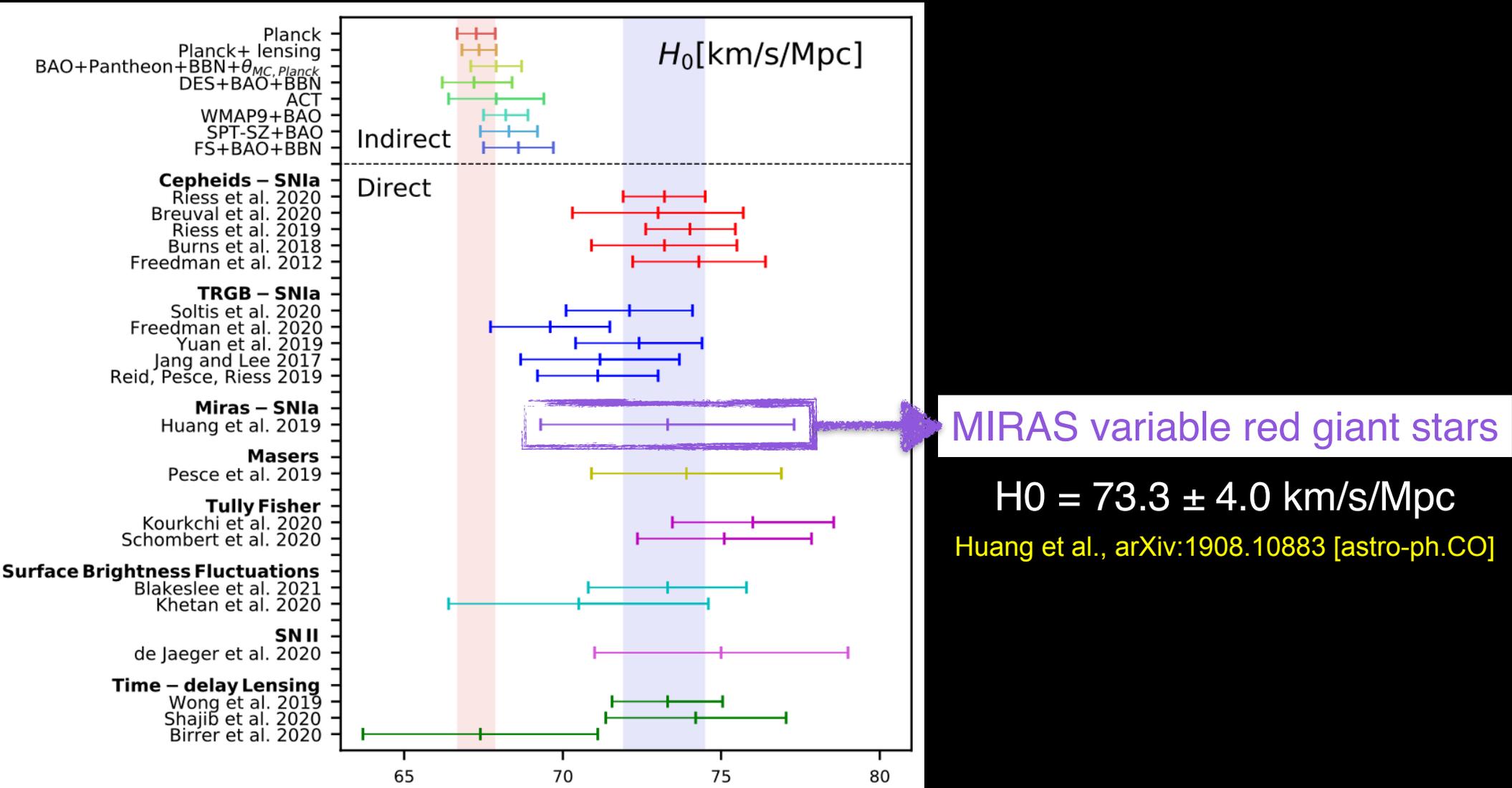
Jang et al., arXiv:1702.01118 [astro-ph.CO]

$$H_0 = 71.1 \pm 1.9 \text{ km/s/Mpc}$$

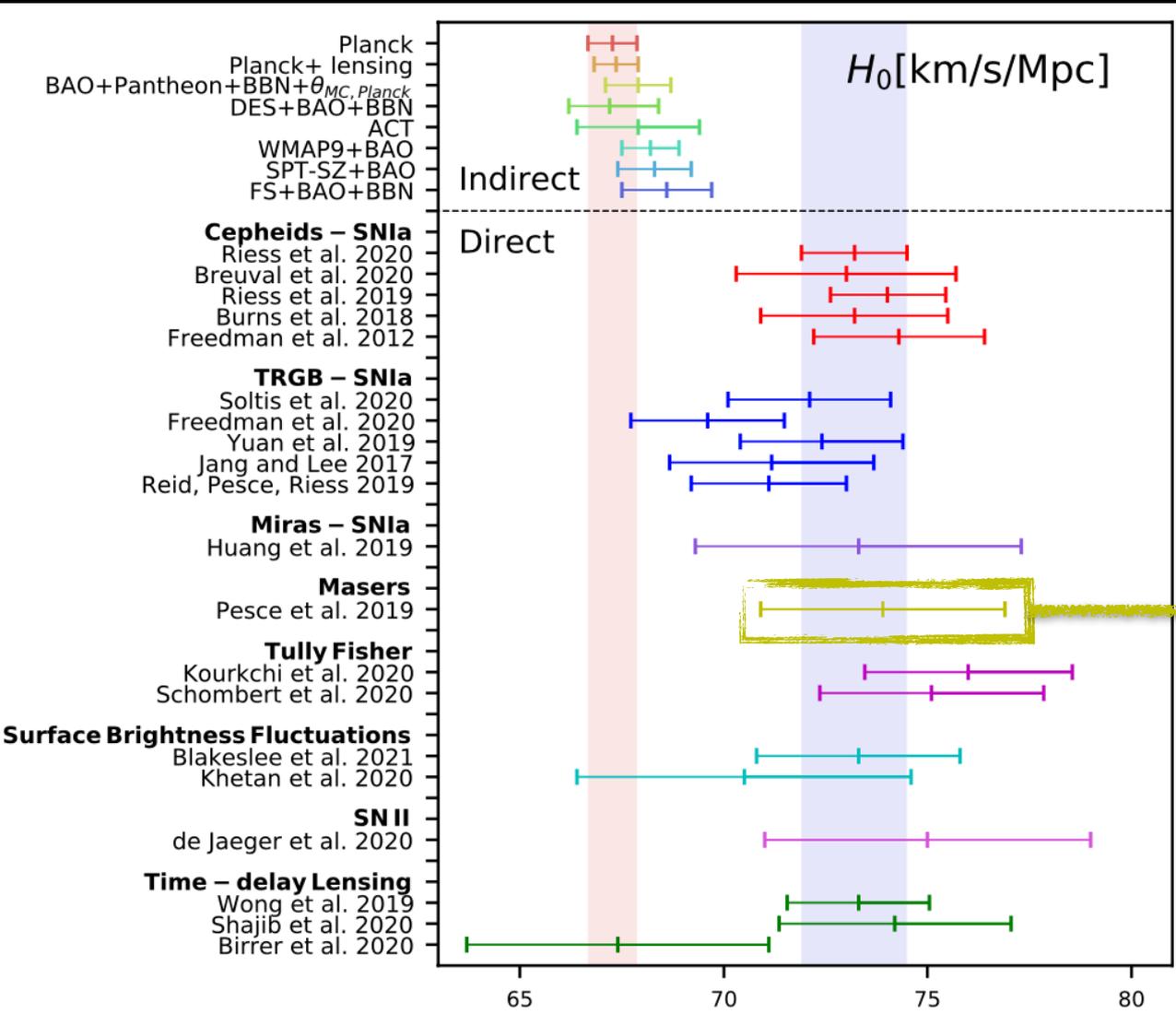
Reid et al., arXiv:1908.05625 [astro-ph.CO]

Di Valentino, MNRAS 2021, arXiv:2011.00246 [astro-ph.CO]

Late universe measurements



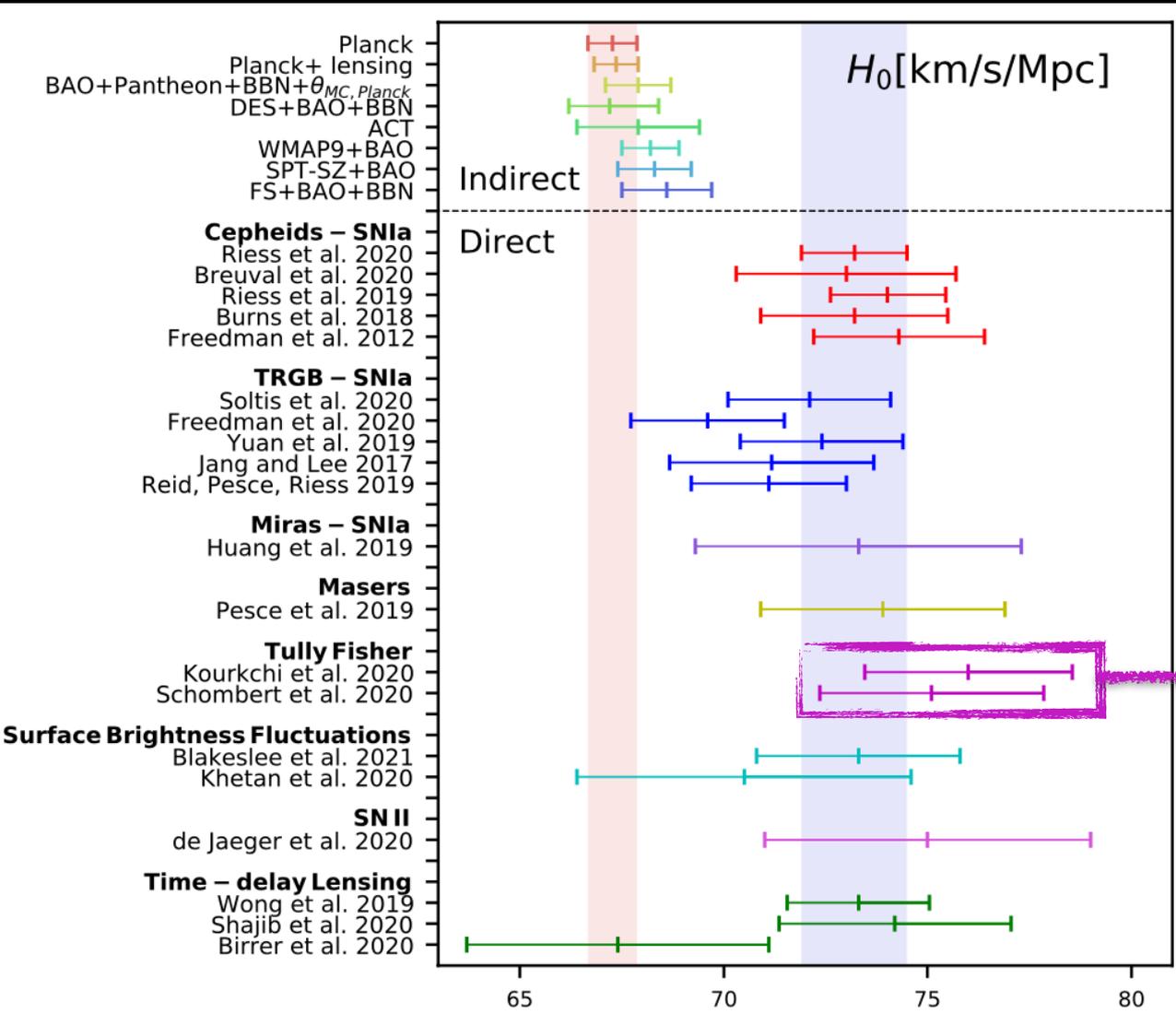
Late universe measurements



Water masers (sources of microwave stimulated emission) in four galaxies at great distances.

$H_0 = 73.9 \pm 3.0$ km/s/Mpc
 Pesce et al. arXiv:2001.09213 [astro-ph.CO]

Late universe measurements

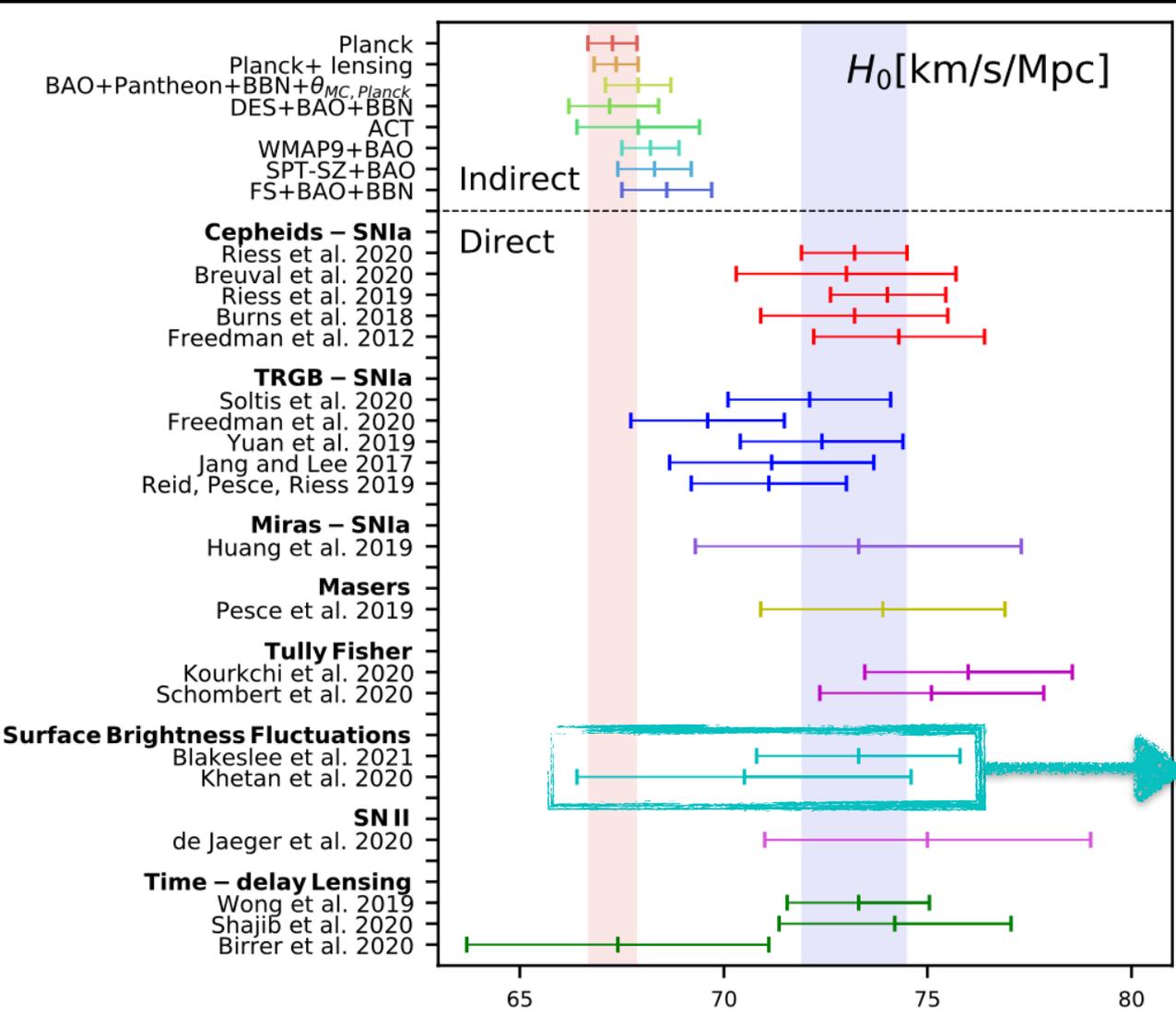


Tully Fisher

$H_0 = 76.00 \pm 2.55$ km/s/Mpc
 Kourkchi et al. arXiv:2004.14499 [astro-ph.CO]

$H_0 = 75.10 \pm 2.75$ km/s/Mpc
 Schombert et al. arXiv:2006.08615 [astro-ph.CO]

Late universe measurements



Surface Brightness Fluctuations

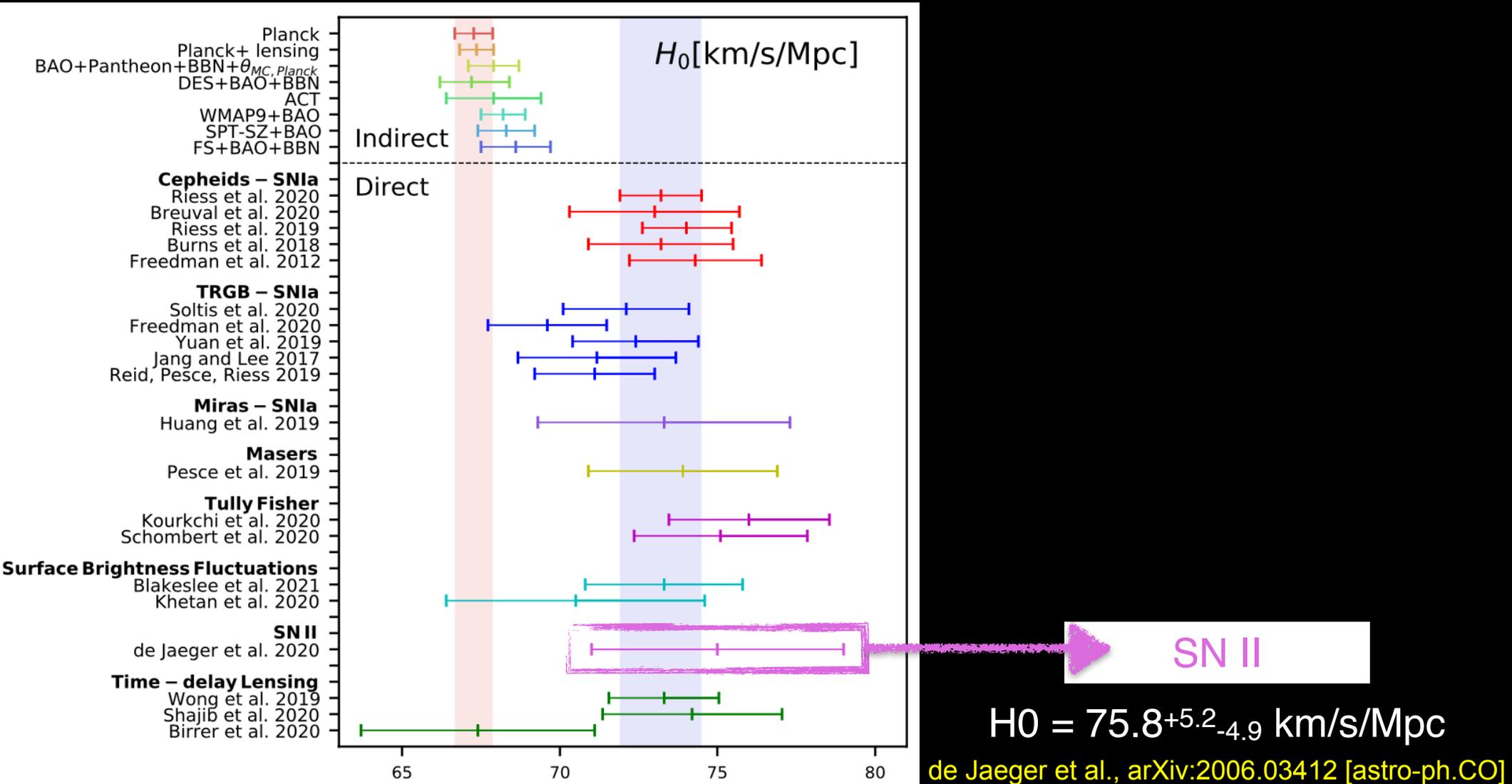
$$H_0 = 73.2 \pm 1.3 \text{ km/s/Mpc}$$

Blakeslee et al., arXiv:2101.02221 [astro-ph.CO]

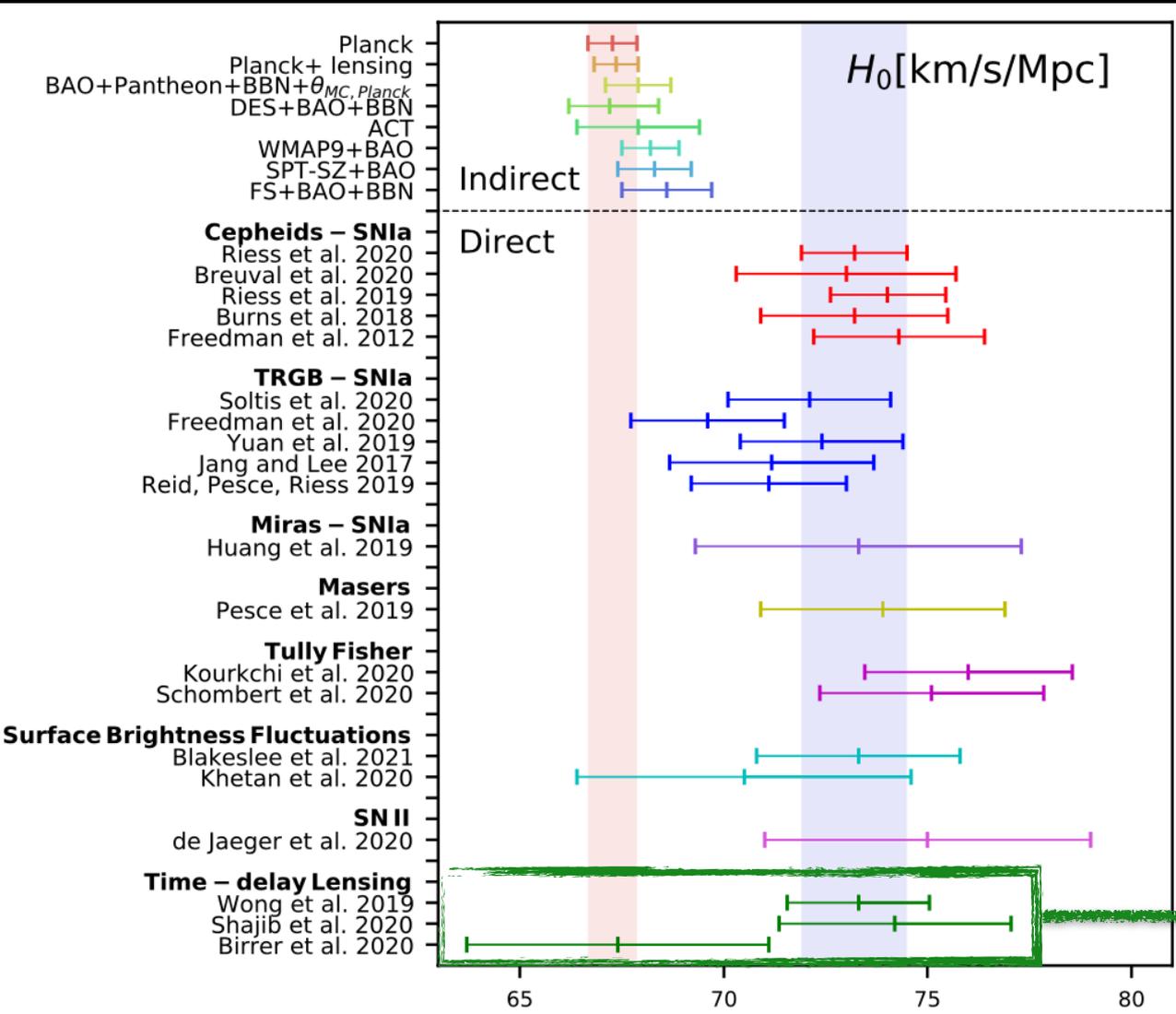
$$H_0 = 70.5 \pm 4.1 \text{ km/s/Mpc}$$

Khetan et al. arXiv:2008.07754 [astro-ph.CO]

Late universe measurements



Late universe measurements



H0LiCOW:

$$H_0 = 73.3^{+1.7}_{-1.8} \text{ km/s/Mpc}$$

Wong et al. [arXiv:1907.04869](https://arxiv.org/abs/1907.04869) [astro-ph.CO]

STRIDES:

$$H_0 = 74.2^{+2.7}_{-3.0} \text{ km/s/Mpc}$$

Shajib et al. [arXiv:1910.06306](https://arxiv.org/abs/1910.06306) [astro-ph.CO]

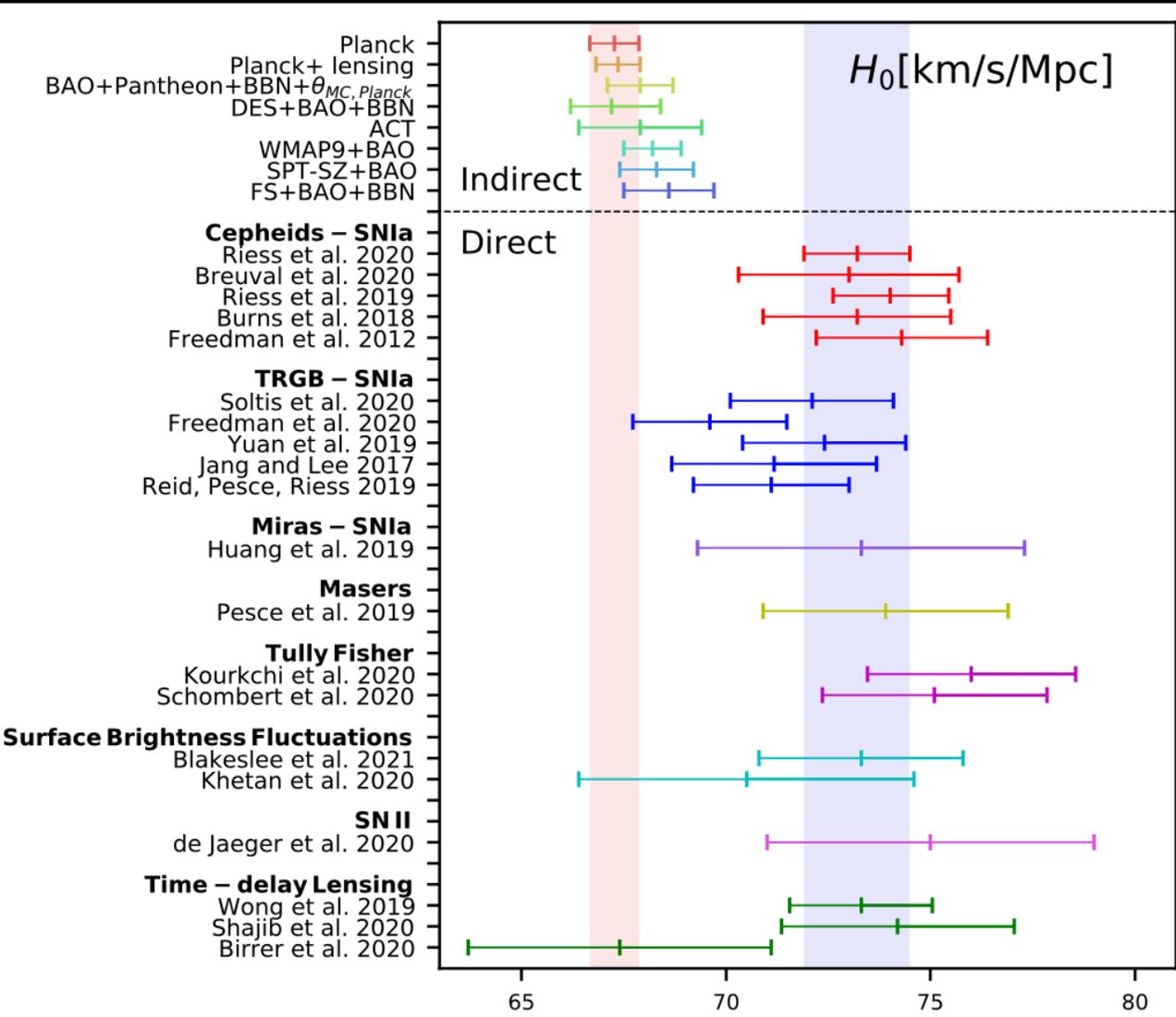
TDCOSMO+SLAC:

$$H_0 = 67.4^{+4.1}_{-3.2} \text{ km/s/Mpc}$$

Birrer et al. [arXiv:2007.02941](https://arxiv.org/abs/2007.02941) [astro-ph.CO]

Strong Lensing: measurement of the time delays of multiple images of quasar systems caused by the strong gravitational lensing from a foreground galaxy.

Late universe measurements

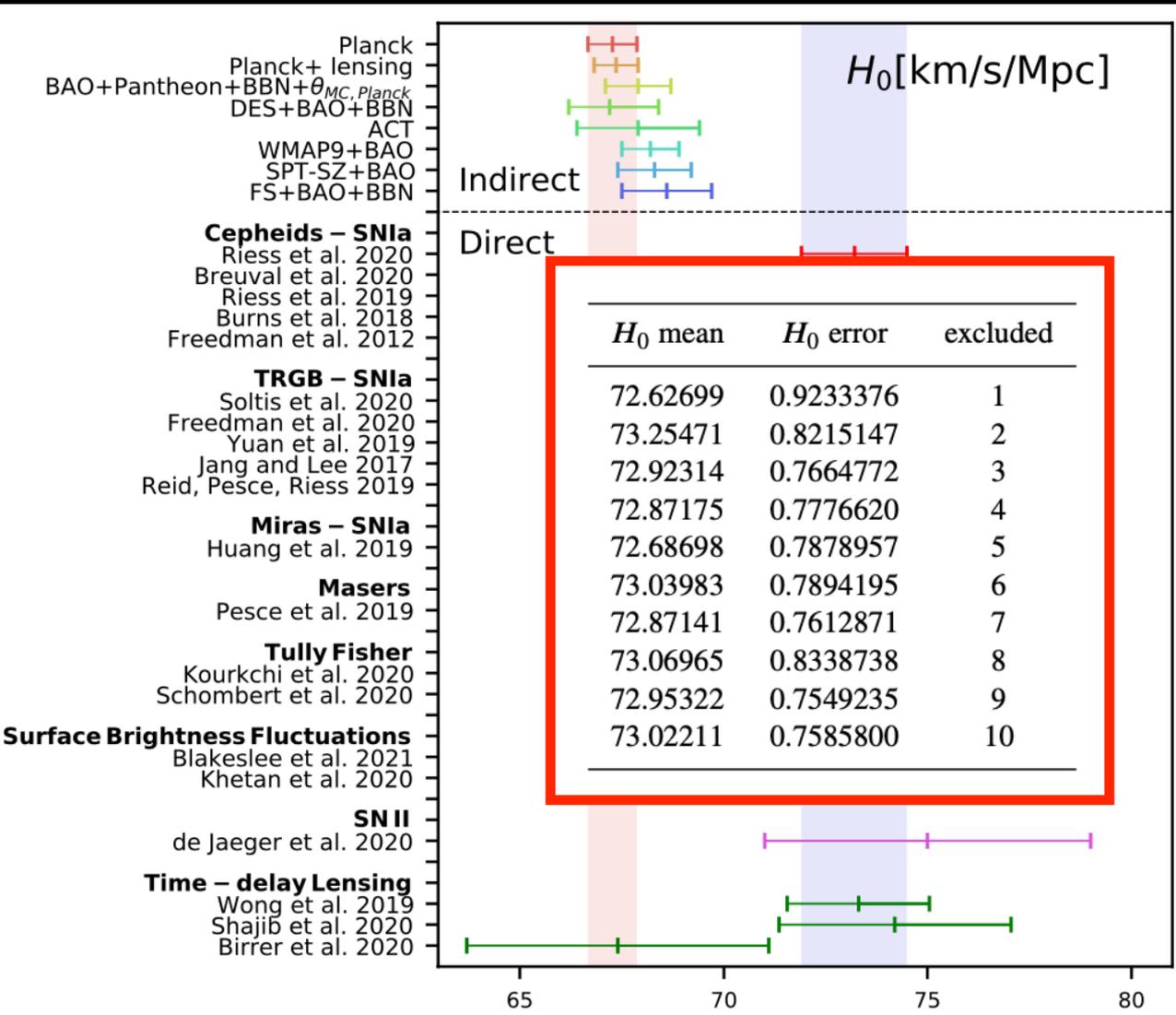


Combining all of them together
(+Standard Sirens and + γ -ray
Attenuation) we obtain our

Optimistic estimate
(5.9σ tension with Planck)

$$H_0 = 72.94 \pm 0.75 \text{ km/s/Mpc}$$

Late universe measurements



Excluding one group of data and taking the result with the largest error bar, i.e. excluding the most precise measurements based on Cepheids-SN Ia, we obtain our

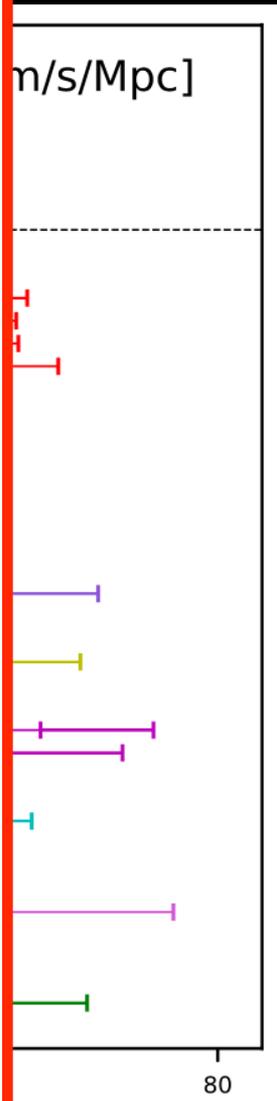
**Conservative estimate
(4.8 σ tension with Planck)**

$$H_0 = 72.63 \pm 0.92 \text{ km/s/Mpc}$$

measurements

Planck
 Planck+ lensing
 BAO+Pantheon+BBN+ $\theta_{MC, Planck}$
 DES+BAO+BBN
 WMAP9+BAO
 SPT-SZ+BAO
 FS+BAO+BBN
Cepheids – SN Ia
 Riess et al. 2016
 Breuval et al. 2017
 Riess et al. 2017
 Burns et al. 2017
 Freedman et al. 2017
TRGB – SN Ia
 Soltis et al. 2017
 Freedman et al. 2017
 Yuan et al. 2017
 Jang and Lee 2017
 Reid, Pesce, Riess 2017
Miras – SN Ia
 Huang et al. 2017
Maser
 Pesce et al. 2017
Tully Fisher
 Kourkchi et al. 2017
 Schombert et al. 2017
Surface Brightness Fluctuation
 Blakeslee et al. 2017
 Khetan et al. 2017
SN Ia
 de Jaeger et al. 2017
Time – delay Lensing
 Wong et al. 2017
 Shajib et al. 2017
 Birrer et al. 2017

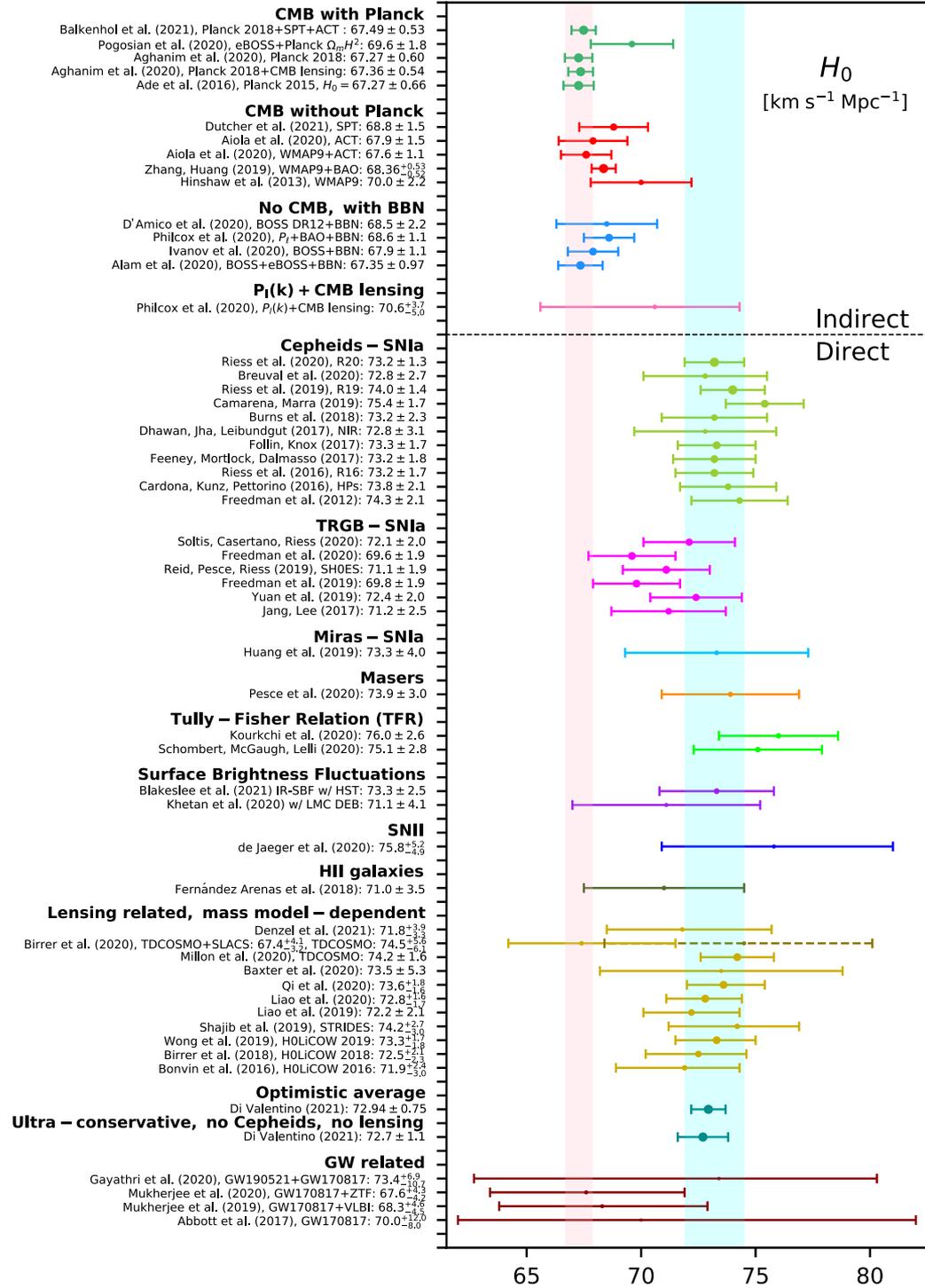
H_0 mean	H_0 error	excluded	excluded
73.05838	1.059989	1	2
72.58911	0.9489663	1	3
72.49379	0.9704452	1	4
72.18593	0.9905548	1	5
72.74182	0.9935880	1	6
72.51725	0.9391693	1	7
72.73386	1.086945	1	8
72.64958	0.9272990	1	9
72.74957	0.9341007	1	10
73.25271	0.8394086	2	3
73.20239	0.8541644	2	4
72.98889	0.8677809	2	5
73.41869	0.8698181	2	6
73.18552	0.8326054	2	7
73.51043	0.9304035	2	8
73.27682	0.8243009	2	9
73.36285	0.8290675	2	10
72.85492	0.7927890	3	4
72.66224	0.8036401	3	5
73.02929	0.8052573	3	6
72.85530	0.7754612	3	7
73.05918	0.8526064	3	8
72.94041	0.7687386	3	9
73.01174	0.7726005	3	10
72.59712	0.8165602	4	5
72.97585	0.8182568	4	6
72.80062	0.7870499	4	7
73.00013	0.7800242	4	9
72.96215	0.7840596	4	10
72.77377	0.8302036	5	6
72.60931	0.7976639	5	7
72.77266	0.8823864	5	8
72.70377	0.7903527	5	9
72.77669	0.7945514	5	10
72.97070	0.7992452	6	7
73.21607	0.8845285	6	8
73.05890	0.7918909	6	9
73.13590	0.7961143	6	10
72.99312	0.8454798	7	8
72.88815	0.7635028	7	9
72.95798	0.7672859	7	10
73.09115	0.8367882	8	9
73.17762	0.8417761	8	10
73.03960	0.7607720	9	10



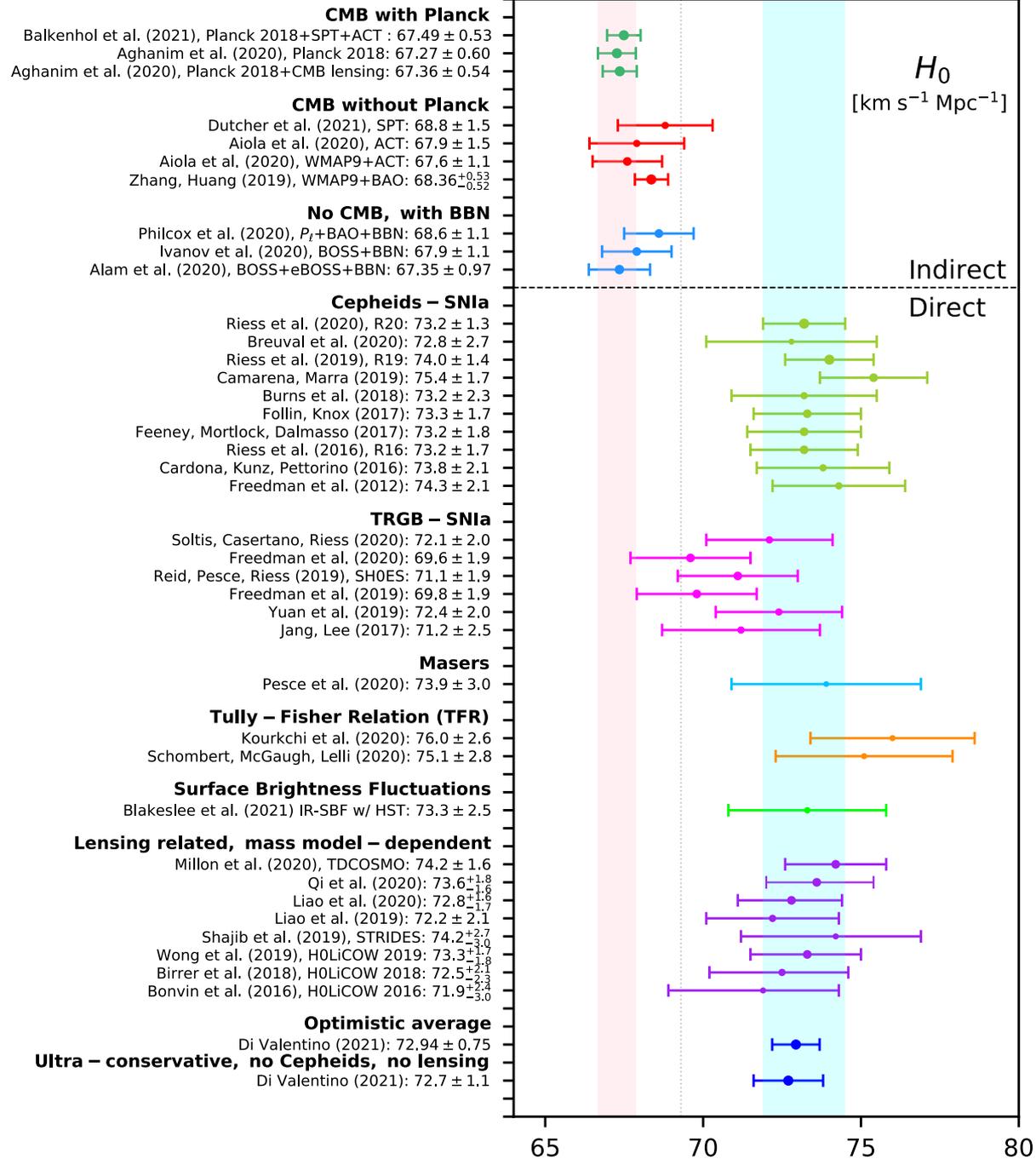
Excluding two groups of data and taking the result with the largest error bar, i.e. excluding the most precise measurements based on Cepheids-SN Ia and Time-delay Lensing, we obtain our

Ultra-conservative estimate (3 σ tension with Planck)

$H_0 = 72.7 \pm 1.1$ km/s/Mpc



High Precision Measures of H_0



Since the Planck constraints are **model dependent**, we can try to expand the cosmological scenario and see which extensions work in solving the tensions between the cosmological probes.

The most **discussed extensions** for solving the H_0 tension are:



the neutrino effective number



the dark energy equation of state

The Neutrino effective number

The expected value is $N_{\text{eff}} = 3.046$, if we assume standard electroweak interactions and three active massless neutrinos. If we measure a $N_{\text{eff}} > 3.046$, we are in presence of extra radiation.

If we compare the Planck 2015 constraint on N_{eff} at 68% cl

$$N_{\text{eff}} = 3.13 \pm 0.32 \quad \text{Planck TT+lowP,}$$

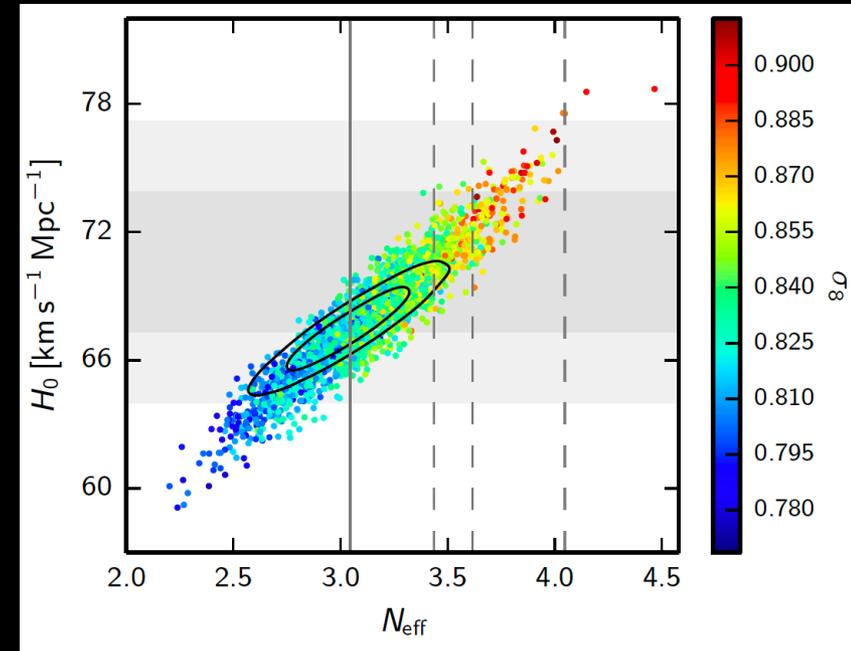
$$N_{\text{eff}} = 3.15 \pm 0.23 \quad \text{Planck TT+lowP+BAO,}$$

with the new Planck 2018 bound,

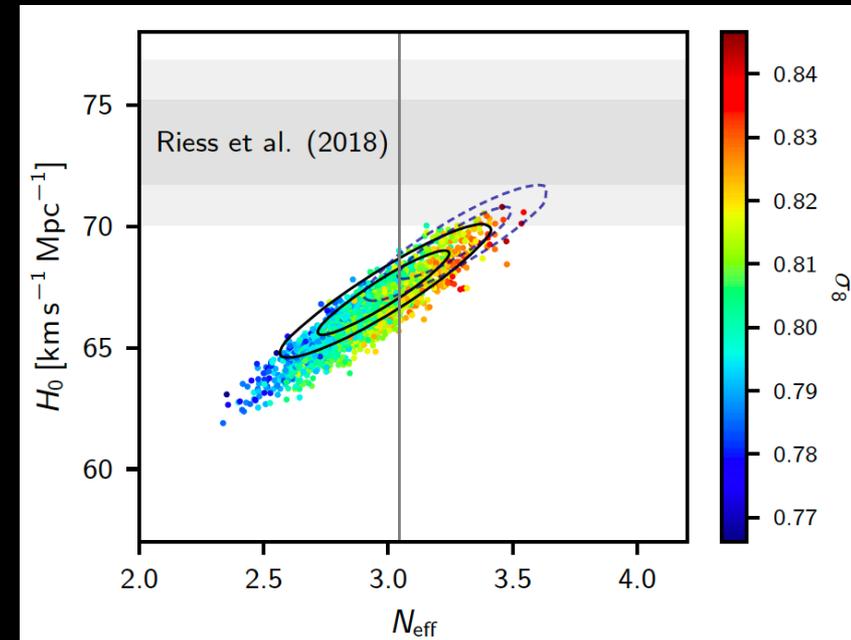
$$N_{\text{eff}} = 2.92^{+0.36}_{-0.37} \quad (95\%, \text{Planck TT,TE,EE+lowE}),$$

we see that the neutrino effective number is now very well constrained.

H_0 passes from 68.0 ± 2.8 km/s/Mpc (2015) to 66.4 ± 1.4 km/s/Mpc (2018), and the tension with R20 increases from 1.7σ to 3.6σ also varying N_{eff} .



Planck collaboration, 2015



Planck collaboration, 2018

The Neutrino effective number

The expected value is $N_{\text{eff}} = 3.046$, if we assume standard electroweak interactions and three active massless neutrinos. If we measure a $N_{\text{eff}} > 3.046$, we are in presence of extra radiation.

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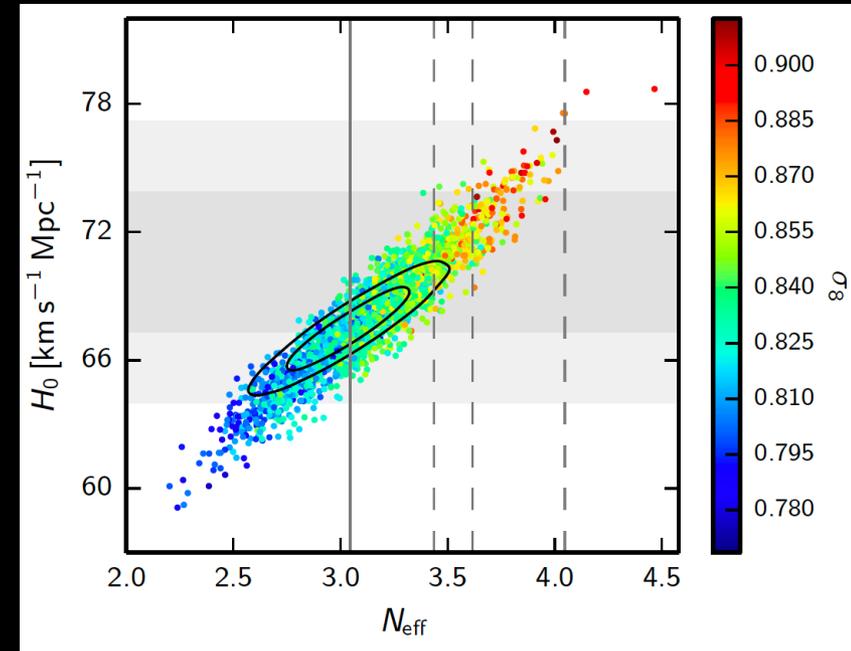
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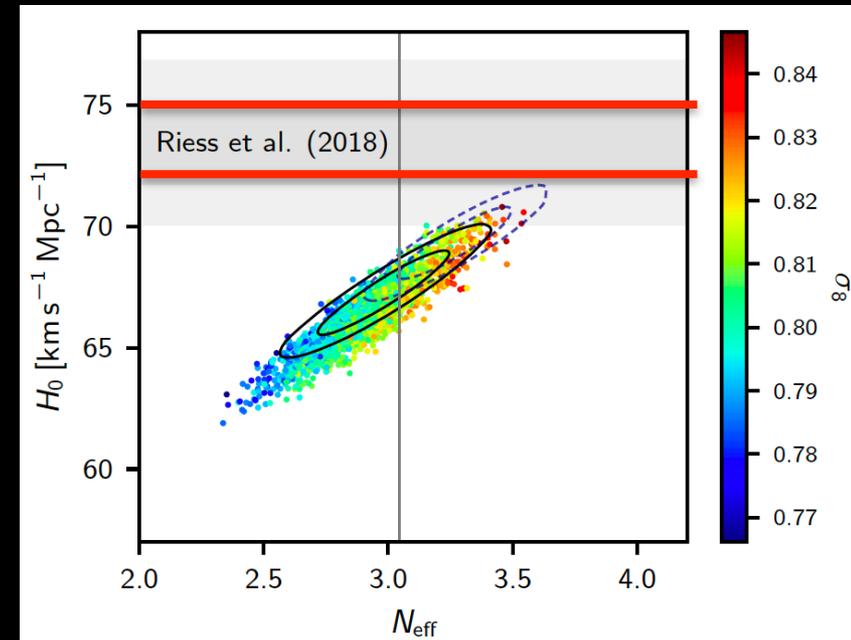
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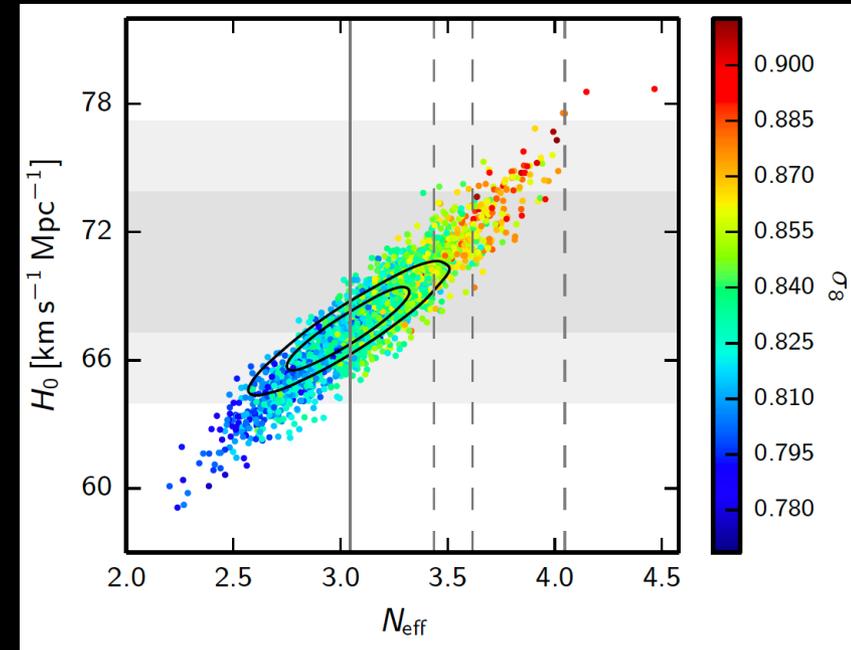
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with the new Planck 2018 bound,

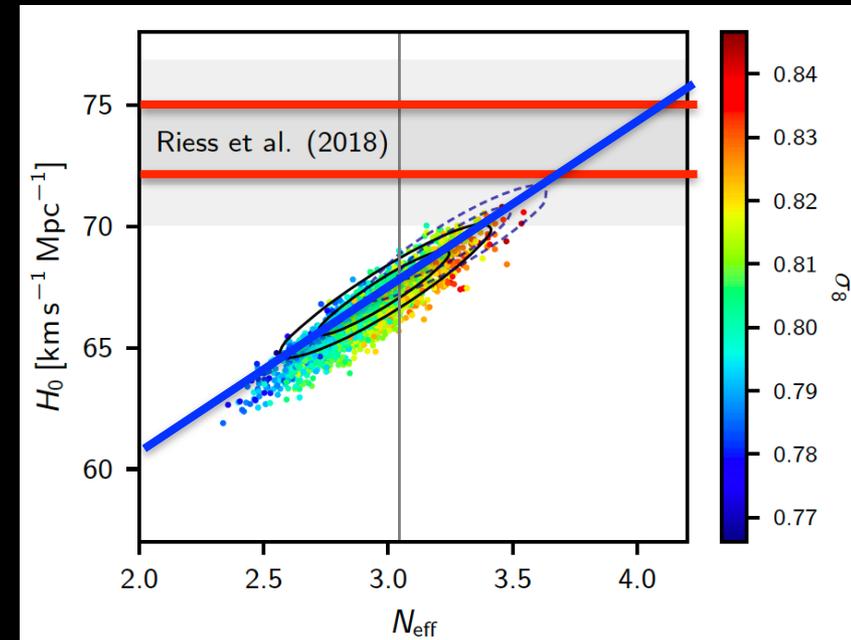
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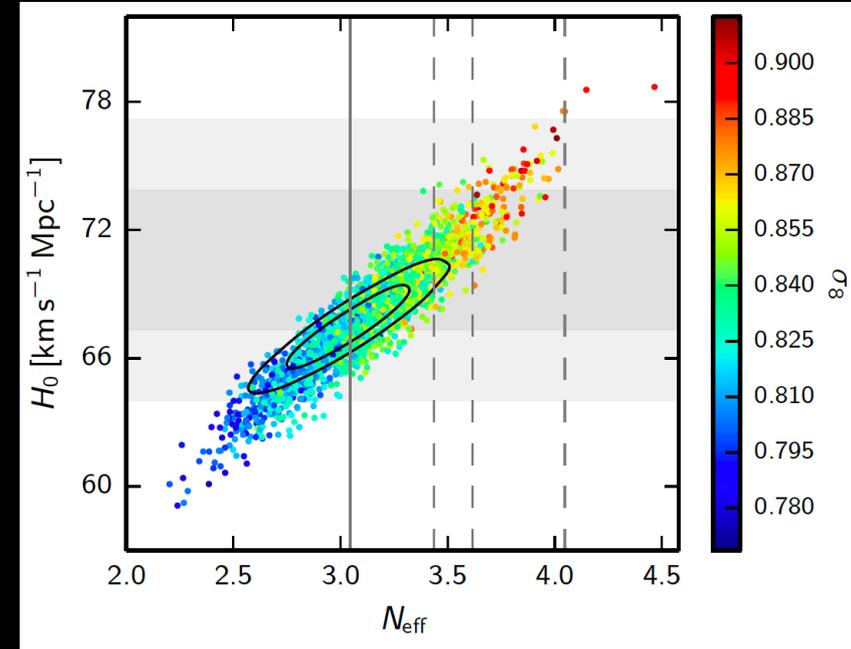
$$N_{\text{eff}} = 3.15 \pm 0.23 \quad \text{Planck TT+lowP+BAO,}$$

with the new Planck 2018 bound,

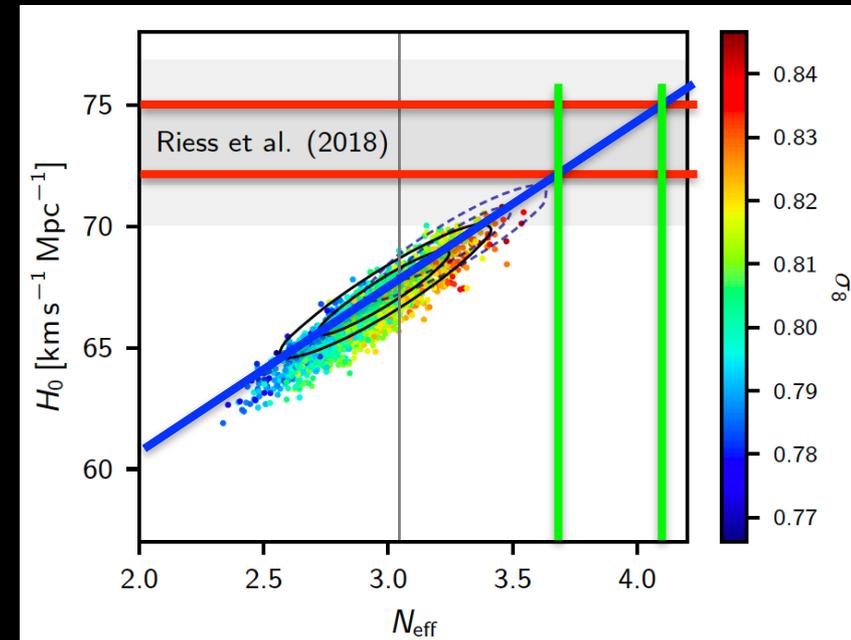
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Planck collaboration, 2015



Planck collaboration, 2018

The Dark energy equation of state

Changing the dark energy equation of state w , we are changing the expansion rate of the Universe:

$$H^2 = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{\text{de}} (1+z)^{3(1+w)} + \Omega_k (1+z)^2 \right]$$

w introduces a geometrical degeneracy with the Hubble constant that will be unconstrained using the CMB data only, resulting in agreement with R20.

We have in 2018 $w = -1.58^{+0.52}_{-0.41}$ with $H_0 > 69.9$ km/s/Mpc at 95% c.l.

Planck data prefer a **phantom dark energy**, with an energy component with $w < -1$, for which the density increases with time in an expanding universe that will **end in a Big Rip**. A phantom dark energy violates the energy condition $\rho \geq |\rho|$, that means that the matter could move faster than light and a comoving observer measure a negative energy density, and the Hamiltonian could have vacuum instabilities due to a negative kinetic energy.

Anyway, there exist models that expect an effective energy density with a phantom equation of state without showing the problems before.

More specific extensions for solving the H0 tension are:

- Early dark Energy (Poulin et al. arXiv:1811.04083, Karwal & Kamionkowski arXiv:1608.01309, Sakstein & Trodden arXiv:1911.11760, Niedermann & Sloth arXiv:1910.10739, Akarsu et al. arXiv:1912.08751, etc...)
- Phenomenologically Emergent Dark Energy (Li & Shafieloo arXiv:1906.08275, Pan et al. arXiv:1907.12551, Li & Shafieloo arXiv:2001.05103, Rezaei et al. arXiv:2004.08168, Liu & Miao arXiv:2002.05563, etc.)
- Modified recombination and reionization histories through heating processes, variation of fundamental constants, or a non-standard CMB temperature-redshift relation (Hart & Chluba arXiv:1705.03925, Yan et al. arXiv:1909.06388, Frusciante et al. arXiv:1912.07586, Braglia et al. arXiv:2004.11161, Ballardini et al. arXiv:2004.14349, Rossi et al. arXiv:1906.10218, etc.)
- Modified Gravity models (Raveri arXiv:1902.01366, Jacques et al. arXiv:1301.3119, Weinberg arXiv:1305.1971, Carneiro et al. arXiv:1812.06064, Paul et al. arXiv:1808.09706, Di Valentino et al. arXiv:1511.00975, Green et al. arXiv:1903.04763, etc...)
- Decaying dark matter (Di Bari et al. arXiv:1303.6267, Choi et al. arXiv:1910.00459, Berezhiani et al. arXiv:1505.03644, Anchordoqui et al. arXiv:1506.08788, Vattis et al. arXiv:1903.06220, etc..)
- Interacting dark sector (Di Valentino et al. arXiv:1704.08342, Kumar and Nunes arXiv:1702.02143, Yang et al. arXiv:1805.08252, Yang et al. arXiv:1809.06883, Yang et al. arXiv:1906.11697, Martinelli et al. arXiv:1902.10694, Di Valentino et al. arXiv:1908.04281, Di Valentino et al. arXiv:1910.09853, etc...)
- Parker Vacuum Metamorphosis (Di Valentino et al., PRD97 (2018) no.4, 043528)
- Vacuum Dynamics (Sola Peracaula et al. arXiv:1705.06723)
- Uber-gravity (Khosravi et al. arXiv:1710.09366)
- Bulk viscosity (Yang et al. arXiv:1906.04162)
- Metastable Dark Energy (Li et al. arXiv:1904.03790)

See our review [Di Valentino et al., arXiv:2103.01183 \[astro-ph.CO\]](#) for a summary of other possible candidates.

Successful models in solving H_0

tension $\leq 1\sigma$ “Excellent models”	tension $\leq 2\sigma$ “Good models”	tension $\leq 3\sigma$ “Promising models”
Dark energy in extended parameter spaces [256]	Early Dark Energy [207]	Early Dark Energy [201]
Dynamical Dark Energy [276]	Phantom Dark Energy [11]	Decaying Warm DM [441]
Metastable Dark Energy [281]	Dynamical Dark Energy [11, 248, 276]	Neutrino-DM Interaction [473]
PEDE [359, 361]	GEDE [364]	Interacting dark radiation [484]
Elaborated Vacuum Metamorphosis [367–369]	Vacuum Metamorphosis [369]	Self-Interacting Neutrinos [667, 668]
IDE [281, 603, 604, 606, 619, 624, 628–630]	IDE [281, 620, 623, 628, 630, 637]	IDE [623]
Self-interacting sterile neutrinos [678]	Critically Emergent Dark Energy [953]	Unified Cosmologies [714]
Generalized Chaplygin gas model [711]	$f(\mathcal{T})$ gravity [781]	Scalar-tensor gravity [822]
Galileon gravity [835, 841]	Über-gravity [58]	Modified recombination [942]
Power Law Inflation [922]	Reconstructed PPS [934]	Super Λ CDM [959]
		Coupled Dark Energy [617]

Table B1. Models solving the H_0 tension with R20 within the 1σ , 2σ and 3σ confidence levels considering the *Planck* dataset only.

Di Valentino et al., arXiv:2103.01183 [astro-ph.CO]

Planck only

What about BAO?

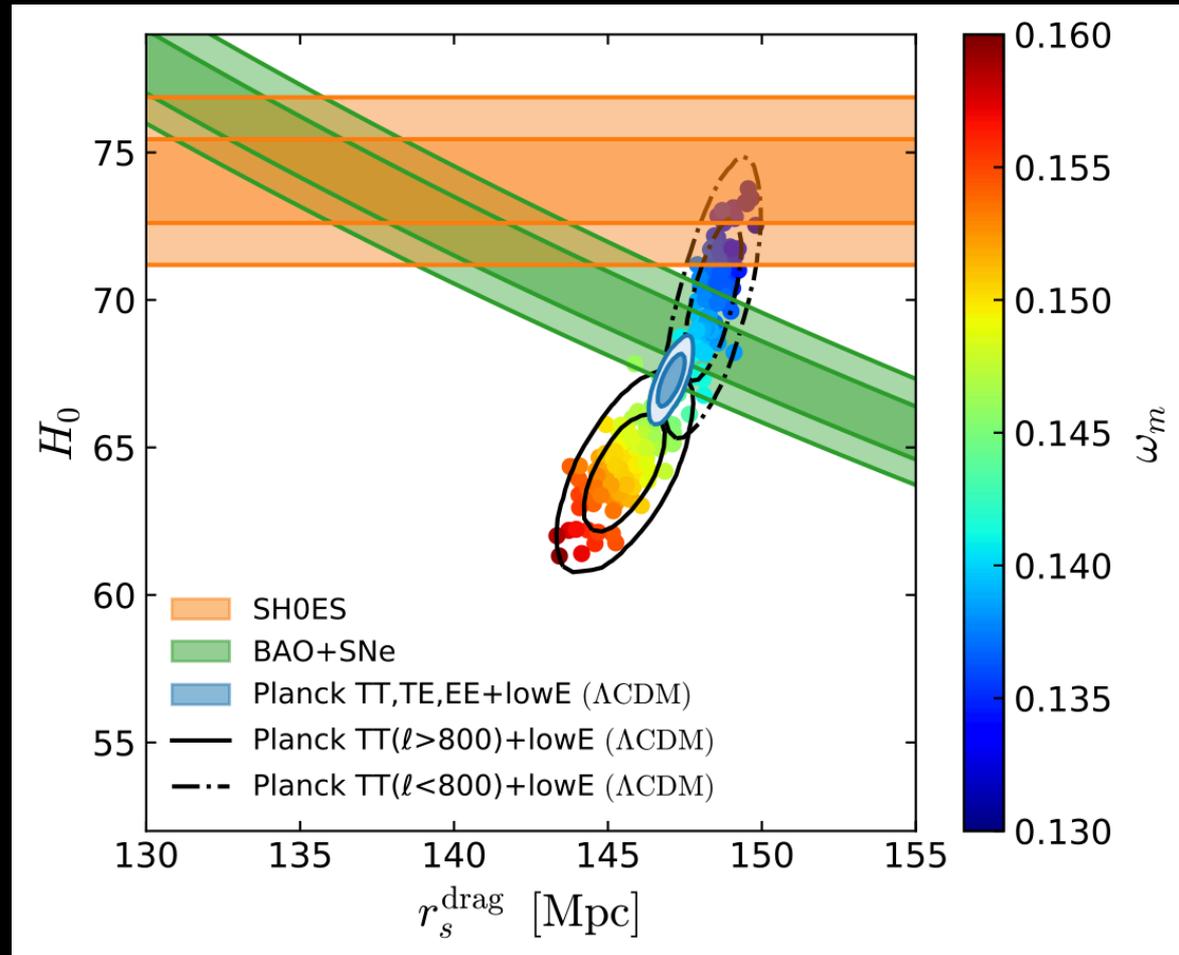
BAO measurements constrain the product of H_0 and the sound horizon r_s .

In order to have a larger H_0 value in agreement with R19,

we need r_s near 137 Mpc.

However, Planck by assuming Λ CDM, prefers r_s near 147 Mpc.

Therefore, a cosmological solution that can increase H_0 and at the same time can lower the sound horizon inferred from CMB data it is promising to put in agreement all the measurements.



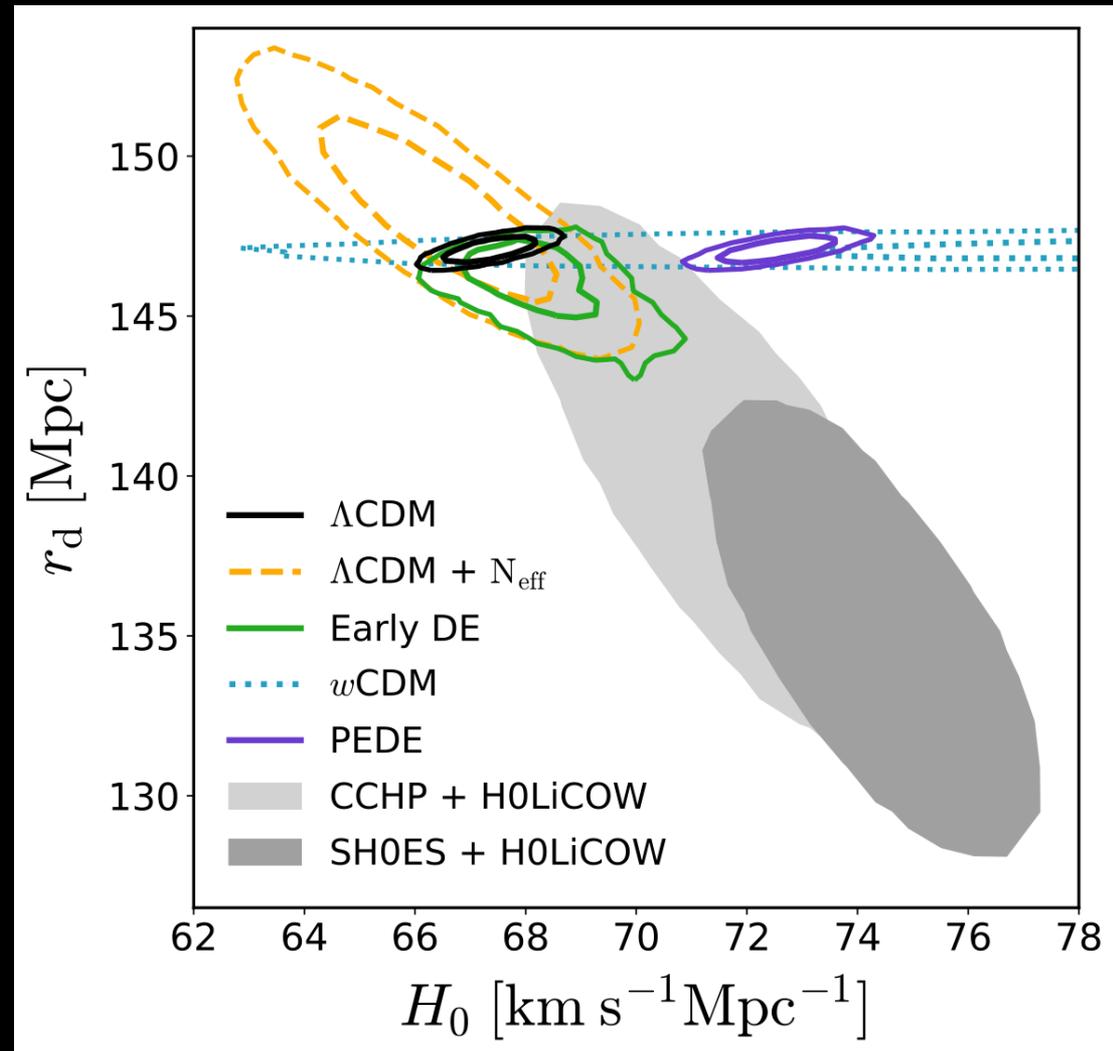
Knox and Millea, *Phys.Rev.D* 101 (2020) 4, 043533

Early vs late time solutions

Here we can see the comparison of the 2σ credibility regions of the CMB constraints and the measurements from late-time observations (SN + BAO + H0LiCOW + SH0ES).

We see that the **late time solutions**, as w CDM, increase H_0 but leave r_s unaltered.

However, the **early time solutions**, as N_{eff} or Early Dark Energy, move in the right direction both the parameters, but can't solve completely the H_0 tension with R19.



Successful models in solving H_0

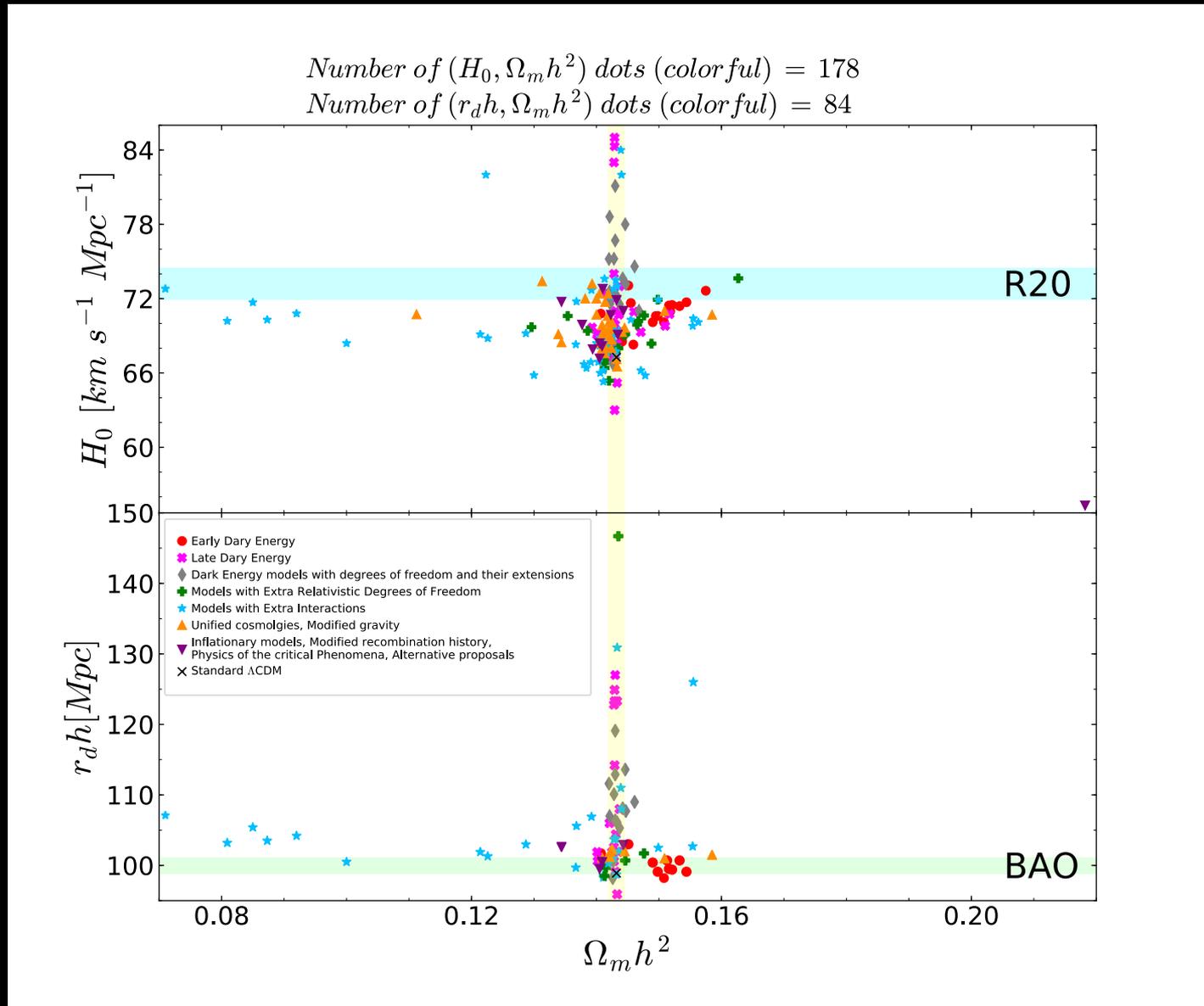
tension $\leq 1\sigma$ “Excellent models”	tension $\leq 2\sigma$ “Good models”	tension $\leq 3\sigma$ “Promising models”
Early Dark Energy [200, 207, 212, 218] Exponential Acoustic Dark Energy [227] Phantom Crossing [282] Late Dark Energy Transition [284] Metastable Dark Energy [281] PEDE [361] Vacuum Metamorphosis [369] Elaborated Vacuum Metamorphosis [368, 369] Sterile Neutrinos [400] Decaying Dark Matter [448] Neutrino-Majoron Interactions [476] IDE [604, 606, 624, 628] DM - Photon Coupling [652] $f(\mathcal{T})$ gravity theory [779] BD- Λ CDM [817] Über-Gravity [58] Galileon Gravity [834] Unimodular Gravity [846] Time Varying Electron Mass [946] Λ CDM [951] Ginzburg-Landau theory [952] Lorentzian Quintessential Inflation [935] Holographic Dark Energy [318]	Early Dark Energy [184, 201, 208, 231] Rock ‘n’ Roll [214] New Early Dark Energy [216] Acoustic Dark Energy [225] Dynamical Dark Energy [276] Running vacuum model [299] Bulk viscous models [307, 308] Holographic Dark Energy [317] Phantom Braneworld DE [345] PEDE [358, 359] Elaborated Vacuum Metamorphosis [368] IDE [626, 637] Interacting Dark Radiation [484] Decaying Dark Matter [438, 441] DM - Photon Coupling [653] Self-interacting sterile neutrinos [678] $f(\mathcal{T})$ gravity theory [784] Über-Gravity [833] VCDM [849] Primordial magnetic fields [948] Early modified gravity [825]	DE in extended parameter spaces [256] Dynamical Dark Energy [248, 276] Holographic Dark Energy [317] Swampland Conjectures [337] MEDE [366] Coupled DM - Dark radiation [501] Decaying Ultralight Scalar [505] BD- Λ CDM [818] Metastable Dark Energy [281] Self-Interacting Neutrinos [667] Dark Neutrino Interactions [683] IDE [601–603, 620, 623, 630, 636] Scalar-tensor gravity [821, 822] Galileon gravity [836, 840] Nonlocal gravity [842] Modified recombination [942] Effective Electron Rest Mass [945] Super Λ CDM [959] Axi-Higgs [947] Self-Interacting Dark Matter [446] Primordial Black Holes [512]

Table B2. Models solving the H_0 tension with R20 within 1σ , 2σ and 3σ using $Planck$ in combination with additional cosmological probes. The H_0 values and the corresponding datasets are discussed in the main text.

Di Valentino et al., arXiv:2103.01183 [astro-ph.CO]

Combination of datasets

Density of the proposed cosmological models:



Di Valentino et al., arXiv:2103.01183 [astro-ph.CO]

At the moment no specific proposal makes a strong case for being highly likely or far better than all others.

IDE can solve the H0 tension

In the standard cosmological framework, the dark matter is assumed to be collisionless. In practice this means that one arbitrarily sets the dark matter interactions to zero when predicting the angular power spectrum of the CMB.

In particular, dark matter and dark energy are described as separate fluids not sharing interactions beyond gravitational ones. However, from a microphysical perspective it is hard to imagine how non-gravitational DM-DE interactions can be avoided, unless forbidden by a fundamental symmetry. This has motivated a large number of studies based on models where DM and DE share interactions other than gravitational.

IDE can solve the H0 tension

At the background level, the conservation equations for the pressureless DM and DE components can be decoupled into two separate equations with an inclusion of an arbitrary function, Q , known as the coupling or interacting function:

$$\begin{aligned}\dot{\rho}_c + 3\mathcal{H}\rho_c &= Q, \\ \dot{\rho}_x + 3\mathcal{H}(1+w)\rho_x &= -Q,\end{aligned}$$

and we assume the phenomenological form for the interaction rate:

$$Q = \xi\mathcal{H}\rho_x$$

proportional to the dark energy density ρ_x and the conformal Hubble rate \mathcal{H} , via a negative dimensionless parameter ξ quantifying the strength of the coupling, to avoid early-time instabilities.

IDE can solve the H0 tension

In this scenario of IDE the tension on H0 between the Planck satellite and R19 is completely solved. The coupling could affect the value of the present matter energy density Ω_m . Therefore, if within an interacting model Ω_m is smaller (because for negative ξ the dark matter density will decay into the dark energy one), a larger value of H0 would be required in order to satisfy the peaks structure of CMB observations, which accurately determine the value of $\Omega_m h^2$.

Parameter	<i>Planck</i>	<i>Planck</i> + <i>R19</i>
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015
$\Omega_c h^2$	< 0.105	< 0.0615
n_s	0.9655 ± 0.0043	0.9656 ± 0.0044
$100\theta_s$	$1.0458^{+0.0033}_{-0.0021}$	1.0470 ± 0.0015
τ	0.0541 ± 0.0076	0.0534 ± 0.0080
ξ	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$
H_0 [km s ⁻¹ Mpc ⁻¹]	$72.8^{+3.0}_{-1.5}$	$74.0^{+1.2}_{-1.0}$

TABLE I. Mean values with their 68% C.L. errors on selected cosmological parameters within the $\xi\Lambda$ CDM model, considering either the *Planck* 2018 legacy dataset alone, or the same dataset in combination with the *R19* Gaussian prior on H_0 based on the latest local distance measurement from *HST*. The quantity quoted in the case of $\Omega_c h^2$ is the 95% C.L. upper limit.

IDE is in agreement with the near universe

Within interacting cosmologies the growth of dark matter perturbations will be larger than in uncoupled models.

This feature will be general for models with negative coupling and in which the energy exchange among the dark sectors is proportional to ρ_x , due to a **suppression of the friction term and an enhancement of the source term** in the differential growth equation.

arXiv.org > astro-ph > arXiv:0905.0492

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 5 May 2009 (v1), last revised 24 Jun 2009 (this version, v2)]

The Growth of Structure in Interacting Dark Energy Models

Gabriela Caldera-Cabral, Roy Maartens, Bjoern Malte Schaefer

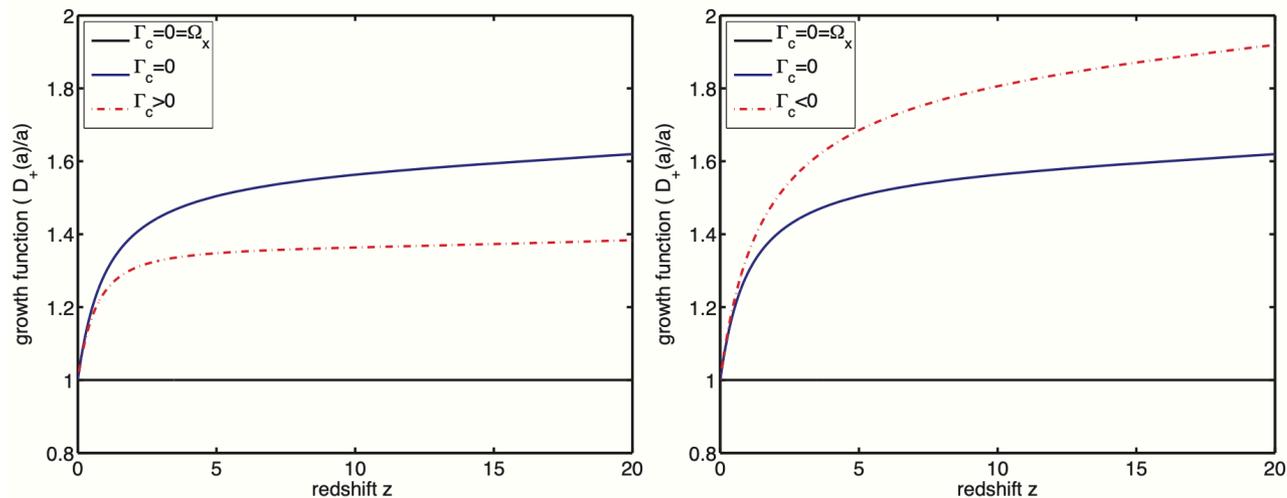
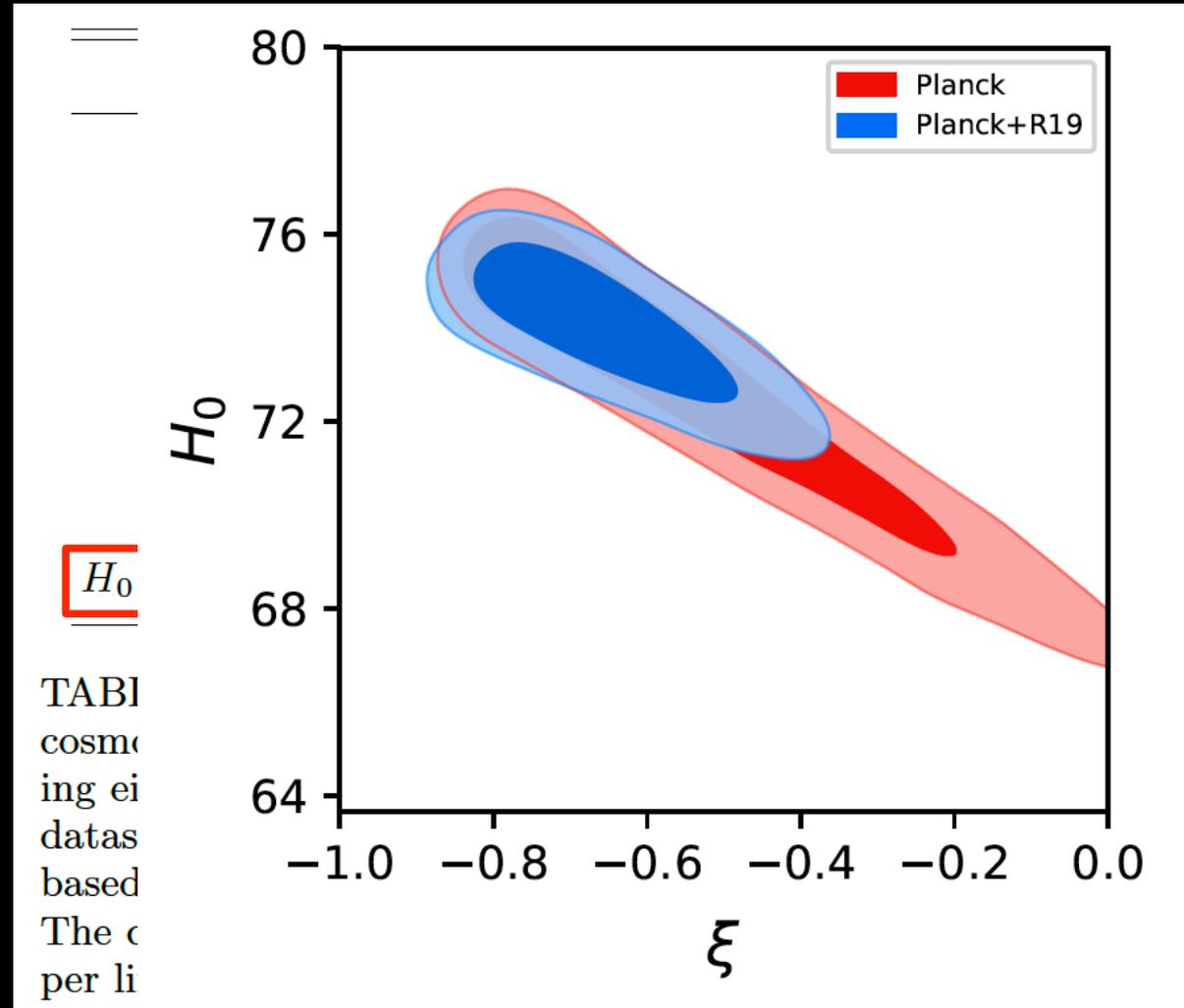


FIG. 2: Linear growth function $D_+ = \delta_c/\delta_{c0}$, normalized to today's value, relative to its value in a pure-matter model ($D_+ = a$). The interacting models (dashed-dotted lines), with $\Gamma_c = \pm 0.3H_0$, are shown in comparison to non-interacting models (solid lines).

IDE can solve the H0 tension

Therefore we can safely combine the two datasets together, and we obtain a **non-zero dark matter-dark energy coupling ξ at more than FIVE standard deviations.**



Bayes factor

Anyway it is clearly interesting to quantify the better **accordance of a model with the data respect to another by using the marginal likelihood also known as the Bayesian evidence.**

Given a vector of parameters θ of a model M and a set of data x , the parameters posterior distribution is given by

$$p(\theta|x, M) = \frac{p(x|\theta, M) \pi(\theta|M)}{p(x|M)}$$

Prior

Likelihood

The marginal likelihood (or evidence) given by

$$E \equiv p(x|M) = \int d\theta p(x|\theta, M) \pi(\theta|M)$$

Given two competing models M_0 and M_1 it is useful to consider the ratio of the likelihood probability **(the Bayes factor):**

$$\ln \mathcal{B} = p(x|M_0)/p(x|M_1)$$

According to the revised Jeffrey's scale by **Kass and Raftery 1995**, the evidence for M_0 (against M_1) is considered as **"positive"** if $|\ln \mathcal{B}| > 1.0$, **"strong"** if $|\ln \mathcal{B}| > 3.0$, and **"very strong"** if $|\ln \mathcal{B}| > 5.0$.

IDE can solve the H0 tension

Computing the Bayes factor for the IDE model with respect to LCDM for the **Planck** dataset we find $\ln B = 1.2$, i.e. a **positive evidence** for the IDE model. If we consider **Planck + R19** we find the extremely high value $\ln B = 10.0$, indicating a **very strong evidence for the IDE model**.

Parameter	<i>Planck</i>	<i>Planck</i> + <i>R19</i>
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015
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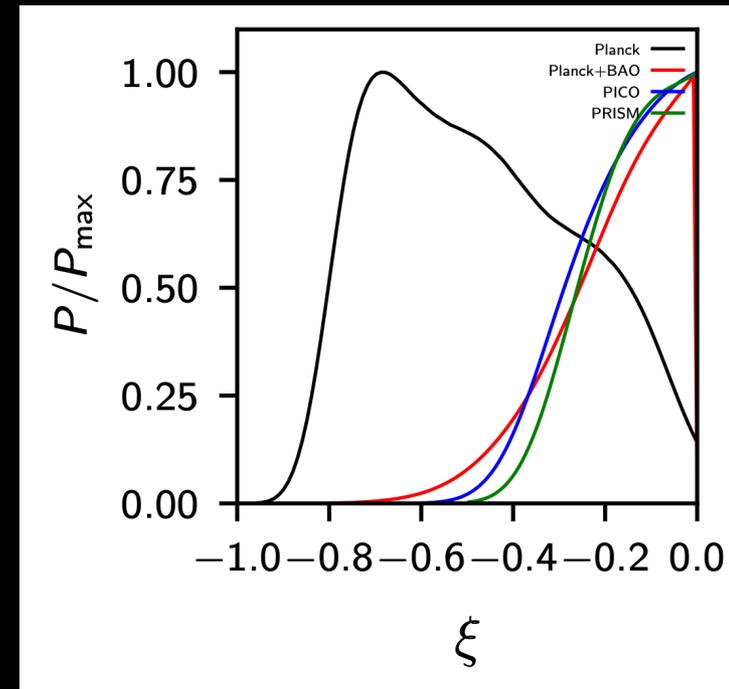
fake IDE detection

Parameters	Fiducial model	Planck	Planck+BAO	PICO	PRISM
$\Omega_b h^2$	0.02236	0.02238 ± 0.00015	0.02230 ± 0.00014	0.022364 ± 0.000029	0.022361 ± 0.000019
$\Omega_c h^2$	0.1202	$0.056^{+0.025}_{-0.047}$	$0.101^{+0.019}_{-0.006}$	$0.100^{+0.019}_{-0.008}$	$0.103^{+0.016}_{-0.007}$
$100\theta_{MC}$	1.04090	$1.0451^{+0.0021}_{-0.0032}$	$1.0419^{+0.0005}_{-0.0011}$	$1.04206^{+0.0005}_{-0.0011}$	$1.04191^{+0.00042}_{-0.00094}$
τ	0.0544	$0.0528^{+0.010}_{-0.009}$	0.0517 ± 0.0098	$0.0543^{+0.0016}_{-0.0019}$	$0.0542^{+0.0017}_{-0.0019}$
n_s	0.9649	0.9652 ± 0.0041	0.9624 ± 0.0036	0.9571 ± 0.0014	0.9657 ± 0.0012
$\ln(10^{10} A_s)$	3.045	$3.041^{+0.020}_{-0.018}$	3.042 ± 0.019	$3.0436^{+0.0030}_{-0.0034}$	3.0435 ± 0.0032
ξ	0	$-0.48^{+0.16}_{-0.30}$	> -0.223	> -0.220	> -0.195

Di Valentino & Mena, Mon.Not.Roy.Astron.Soc. 500 (2020) 1, L22-L26, arXiv:2009.12620

For a **simulated Planck-like experiment**, due to the strong correlation present between the standard and the exotic physics parameters, there is a dangerous **detection at more than 3σ for a coupling** between dark matter and dark energy different from zero, even if the fiducial model has $\xi = 0$:

$$-0.85 < \xi < -0.02 \text{ at } 99\% \text{ CL}$$



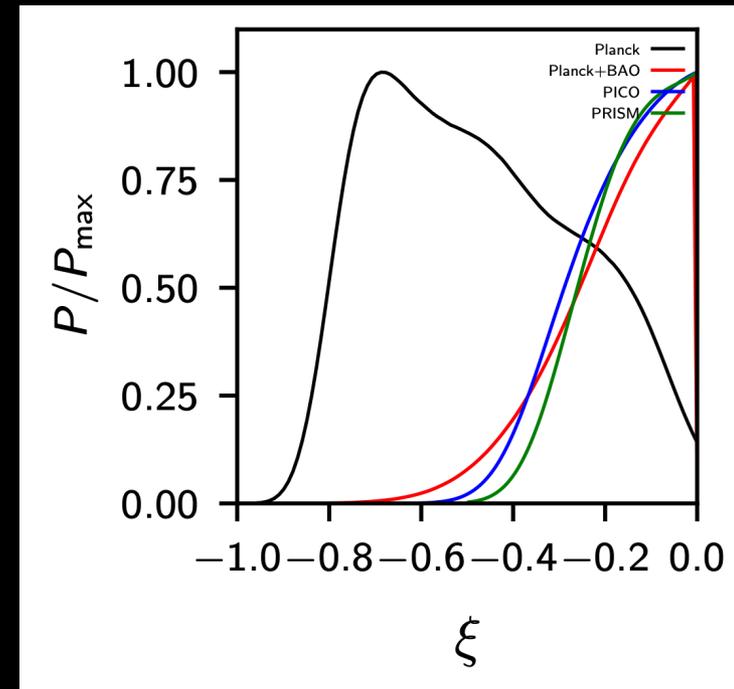
Simulated experiments

fake IDE detection

Parameters	Fiducial model	Planck	Planck+BAO	PICO	PRISM
$\Omega_b h^2$	0.02236	0.02238 ± 0.00015	0.02230 ± 0.00014	0.022364 ± 0.000029	0.022361 ± 0.000019
$\Omega_c h^2$	0.1202	$0.056^{+0.025}_{-0.047}$	$0.101^{+0.019}_{-0.006}$	$0.100^{+0.019}_{-0.008}$	$0.103^{+0.016}_{-0.007}$
$100\theta_{MC}$	1.04090	$1.0451^{+0.0021}_{-0.0032}$	$1.0419^{+0.0005}_{-0.0011}$	$1.04206^{+0.0005}_{-0.0011}$	$1.04191^{+0.00042}_{-0.00094}$
τ	0.0544	$0.0528^{+0.010}_{-0.009}$	0.0517 ± 0.0098	$0.0543^{+0.0016}_{-0.0019}$	$0.0542^{+0.0017}_{-0.0019}$
n_s	0.9649	0.9652 ± 0.0041	0.9624 ± 0.0036	0.9571 ± 0.0014	0.9657 ± 0.0012
$\ln(10^{10} A_s)$	3.045	$3.041^{+0.020}_{-0.018}$	3.042 ± 0.019	$3.0436^{+0.0030}_{-0.0034}$	3.0435 ± 0.0032
ξ	0	$-0.48^{+0.16}_{-0.30}$	> -0.223	> -0.220	> -0.195

Di Valentino & Mena, Mon.Not.Roy.Astron.Soc. 500 (2020) 1, L22-L26, arXiv:2009.12620

The inclusion of **simulated BAO data**, a mock dataset built using the same fiducial cosmological model than that of the CMB, helps in breaking the degeneracy, providing a **lower limit for the coupling ξ** in perfect agreement with zero.



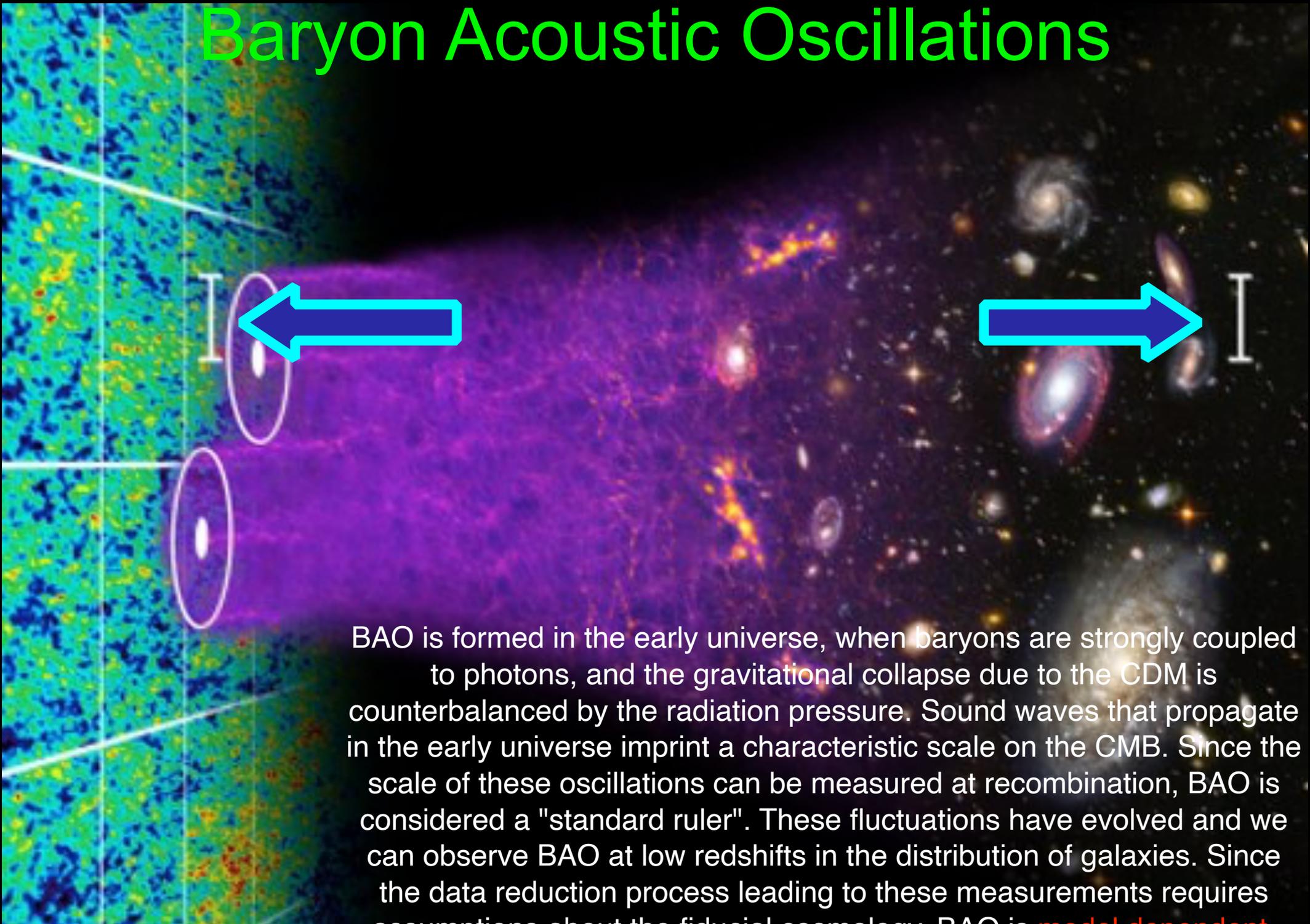
Simulated experiments

IDE can solve the H0 tension

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015	0.02241 ± 0.00014	0.02236 ± 0.00014	0.02235 ± 0.00015
$\Omega_c h^2$	< 0.0634	$0.031^{+0.013}_{-0.023}$	< 0.0675	$0.095^{+0.022}_{-0.008}$	$0.103^{+0.013}_{-0.007}$
$100\theta_{MC}$	$1.0458^{+0.0033}_{-0.0021}$	1.0470 ± 0.0015	$1.0456^{+0.0031}_{-0.0024}$	$1.0424^{+0.0006}_{-0.0013}$	$1.04185^{+0.00049}_{-0.00078}$
τ	0.0541 ± 0.0076	0.0534 ± 0.0080	0.0526 ± 0.0074	0.0540 ± 0.0076	0.0540 ± 0.0076
n_s	0.9655 ± 0.0043	0.9656 ± 0.0044	0.9663 ± 0.0040	0.9647 ± 0.0040	0.9643 ± 0.0042
$\ln(10^{10} A_s)$	3.044 ± 0.016	3.042 ± 0.017	$3.039^{+0.013}_{-0.015}$	3.044 ± 0.016	3.044 ± 0.016
ξ	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$	$-0.51^{+0.12}_{-0.29}$	$-0.22^{+0.21}_{-0.05}$	$-0.15^{+0.12}_{-0.06}$
H_0 [km/s/Mpc]	$72.8^{+3.0}_{+1.5}$	$74.0^{+1.2}_{-1.0}$	$72.8^{+3.0}_{+1.6}$	$69.4^{+0.9}_{-1.5}$	$68.6^{+0.8}_{-1.0}$
σ_8	$2.3^{+0.4}_{-1.4}$	$2.71^{+0.05}_{-1.3}$	$2.2^{+0.4}_{-1.4}$	$1.05^{+0.03}_{-0.24}$	$0.95^{+0.04}_{-0.12}$
S_8	$1.30^{+0.17}_{-0.44}$	$1.44^{+0.17}_{-0.34}$	$1.30^{+0.15}_{-0.42}$	$0.93^{+0.03}_{-0.10}$	$0.892^{+0.028}_{-0.054}$

The addition of low-redshift measurements, as BAO data, still hints to the **presence of a coupling**, albeit at a lower statistical significance. Also for this data sets the **Hubble constant values is larger** than that obtained in the case of a pure LCDM scenario, enough to bring the **H0 tension at 2.4σ** .

Baryon Acoustic Oscillations



BAO is formed in the early universe, when baryons are strongly coupled to photons, and the gravitational collapse due to the CDM is counterbalanced by the radiation pressure. Sound waves that propagate in the early universe imprint a characteristic scale on the CMB. Since the scale of these oscillations can be measured at recombination, BAO is considered a "standard ruler". These fluctuations have evolved and we can observe BAO at low redshifts in the distribution of galaxies. Since the data reduction process leading to these measurements requires assumptions about the fiducial cosmology, BAO is **model dependent**.

IDE can solve the H0 tension

In other words, the tension between Planck+BAO and R19 could be due to a statistical fluctuation in this case.

Moreover, BAO data is extracted under the assumption of Λ CDM, and the modified scenario of interacting dark energy could affect the result. In fact, the full procedure which leads to the BAO constraints carried out by the different collaborations might be not necessarily valid in extended DE models.

For instance, the BOSS collaboration advises caution when using their BAO measurements (both the pre- and post- reconstruction measurements) in more exotic dark energy cosmologies.

BAO constraints themselves might need to be revised in a non-trivial manner when applied to constrain extended dark energy cosmologies.

IDE+w

We can allow for more freedom in the DE sector, varying **at the same time the coupling ξ and the DE equation of state w .**

Model	DE EoS	DM-DE coupling	Energy flow
$\xi\Lambda$ CDM	$w = -0.999$	$\xi < 0$	DM \rightarrow DE
ξp CDM	$w < -1$	$\xi > 0, \quad \xi < -3w$	DE \rightarrow DM
ξq CDM	$w > -1$	$\xi < 0$	DM \rightarrow DE

These are the values allowed for the DE EoS w and the DM-DE coupling ξ ensuring that gravitational instabilities, early-time instabilities, and unphysical values for the DM energy density are avoided. The last column is the direction of energy flow.

Quintessence

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02237 ± 0.00015	0.02241 ± 0.00015	0.02239 ± 0.00014	0.02239 ± 0.00014	0.02236 ± 0.00015
$\Omega_c h^2$	< 0.0433	< 0.0230	< 0.0483	< 0.0543	< 0.0574
$100\theta_{MC}$	$1.0468^{+0.0031}_{-0.0013}$	$1.0482^{+0.0017}_{-0.0008}$	$1.0466^{+0.0031}_{-0.0016}$	$1.0463^{+0.0033}_{-0.0018}$	$1.0461^{+0.0032}_{-0.0019}$
τ	0.0537 ± 0.0077	0.0540 ± 0.0080	0.0530 ± 0.0075	0.0545 ± 0.0078	0.0537 ± 0.0078
n_s	0.9650 ± 0.0042	0.9660 ± 0.0043	0.9658 ± 0.0042	0.9659 ± 0.0041	0.9648 ± 0.0043
$\ln(10^{10} A_s)$	3.043 ± 0.016	3.043 ± 0.016	3.040 ± 0.015	3.044 ± 0.016	3.044 ± 0.016
ξ	$-0.63^{+0.06}_{-0.22}$	$-0.73^{+0.05}_{-0.10}$	$-0.61^{+0.08}_{-0.22}$	$-0.59^{+0.09}_{-0.25}$	$-0.58^{+0.10}_{-0.26}$
w	< -0.839	$-0.949^{+0.013}_{-0.049}$	< -0.839	$-0.842^{+0.086}_{-0.072}$	$-0.842^{+0.090}_{-0.054}$
H_0 [km/s/Mpc]	$69.8^{+4.0}_{+2.5}$	$73.3^{+1.2}_{-1.0}$	$69.9^{+3.7}_{-2.5}$	68.6 ± 1.4	68.3 ± 1.0
σ_8	$2.6^{+0.7}_{-1.7}$	$3.4^{+0.9}_{-1.3}$	$2.5^{+0.6}_{-1.6}$	$2.3^{+0.6}_{-1.4}$	$2.2^{+0.5}_{-1.3}$
S_8	$1.43^{+0.29}_{-0.46}$	$1.63^{+0.31}_{-0.26}$	$1.39^{+0.23}_{-0.44}$	$1.35^{+0.24}_{-0.45}$	$1.33^{+0.20}_{-0.44}$

These are the constraints on the quintessence coupled model ($w > -1$), where the energy flows from the DM to the DE sector. The amount of the **DM mass-energy density today is considerably reduced** as the values of the coupling are increased.

This explains why we have a **non-zero value of the coupling at a rather high significance level (up to 5 standard deviations).**

Moreover, $w > -1$ up to more than 3σ .

Quintessence

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02237 ± 0.00015	0.02241 ± 0.00015	0.02239 ± 0.00014	0.02239 ± 0.00014	0.02236 ± 0.00015
$\Omega_c h^2$	< 0.0433	< 0.0230	< 0.0483	< 0.0543	< 0.0574
$100\theta_{MC}$	$1.0468^{+0.0031}_{-0.0013}$	$1.0482^{+0.0017}_{-0.0008}$	$1.0466^{+0.0031}_{-0.0016}$	$1.0463^{+0.0033}_{-0.0018}$	$1.0461^{+0.0032}_{-0.0019}$
τ	0.0537 ± 0.0077	0.0540 ± 0.0080	0.0530 ± 0.0075	0.0545 ± 0.0078	0.0537 ± 0.0078
n_s	0.9650 ± 0.0042	0.9660 ± 0.0043	0.9658 ± 0.0042	0.9659 ± 0.0041	0.9648 ± 0.0043
$\ln(10^{10} A_s)$	3.043 ± 0.016	3.043 ± 0.016	3.040 ± 0.015	3.044 ± 0.016	3.044 ± 0.016
ξ	$-0.63^{+0.06}_{-0.22}$	$-0.73^{+0.05}_{-0.10}$	$-0.61^{+0.08}_{-0.22}$	$-0.59^{+0.09}_{-0.25}$	$-0.58^{+0.10}_{-0.26}$
w	< -0.839	$-0.949^{+0.013}_{-0.049}$	< -0.839	$-0.842^{+0.086}_{-0.072}$	$-0.842^{+0.090}_{-0.054}$
H_0 [km/s/Mpc]	$69.8^{+4.0}_{+2.5}$	$73.3^{+1.2}_{-1.0}$	$69.9^{+3.7}_{-2.5}$	68.6 ± 1.4	68.3 ± 1.0
σ_8	$2.6^{+0.7}_{-1.7}$	$3.4^{+0.9}_{-1.3}$	$2.5^{+0.6}_{-1.6}$	$2.3^{+0.6}_{-1.4}$	$2.2^{+0.5}_{-1.3}$
S_8	$1.43^{+0.29}_{-0.46}$	$1.63^{+0.31}_{-0.26}$	$1.39^{+0.23}_{-0.44}$	$1.35^{+0.24}_{-0.45}$	$1.33^{+0.20}_{-0.44}$

H_0 shifts towards lower values for the strong anti correlation present with w , that is dominating the impact of ξ , which would instead push H_0 to even larger values as before.

However, in the case of interacting dark energy, quintessence models agree with observations and also **reduce the significance of the Hubble tension**.

Phantom

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02239 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00015	0.02238 ± 0.00014	0.02235 ± 0.00015
$\Omega_c h^2$	$0.132^{+0.005}_{-0.012}$	$0.133^{+0.006}_{-0.012}$	$0.133^{+0.006}_{-0.012}$	$0.134^{+0.007}_{-0.012}$	$0.134^{+0.006}_{-0.012}$
$100\theta_{MC}$	$1.04027^{+0.00064}_{-0.00048}$	$1.04024^{+0.00063}_{-0.00048}$	$1.04029^{+0.00062}_{-0.00051}$	$1.04019^{+0.00060}_{-0.00051}$	$1.04017^{+0.00060}_{-0.00051}$
τ	0.0537 ± 0.0080	0.0542 ± 0.0078	0.0524 ± 0.0072	0.0545 ± 0.0080	0.0542 ± 0.0081
n_s	0.9655 ± 0.0043	0.9650 ± 0.0042	0.9663 ± 0.0041	0.9654 ± 0.0040	0.9643 ± 0.0044
$\ln(10^{10} A_s)$	3.042 ± 0.016	3.044 ± 0.017	3.039 ± 0.014	3.045 ± 0.016	3.045 ± 0.016
ξ	< 0.130	< 0.157	< 0.140	< 0.187	< 0.178
w	$-1.59^{+0.18}_{-0.33}$	-1.264 ± 0.057	$-1.57^{+0.19}_{-0.32}$	$-1.095^{+0.072}_{-0.040}$	$-1.084^{+0.051}_{-0.038}$
H_0 [km/s/Mpc]	> 81.3	74.1 ± 1.4	85^{+10}_{-5}	$68.8^{+1.1}_{-1.5}$	68.33 ± 0.99
σ_8	0.883 ± 0.082	$0.802^{+0.059}_{-0.043}$	0.871 ± 0.083	0.753 ± 0.046	$0.755^{+0.051}_{-0.042}$
S_8	0.742 ± 0.040	$0.778^{+0.032}_{-0.026}$	0.735 ± 0.038	0.790 ± 0.026	0.797 ± 0.027

These are the constraints on the phantom model ($w < -1$), where the energy flows from the DE to the DM sector. Here we have that the value of the **matter density is larger than the Λ CDM model**. Therefore, we obtain an **upper bound on ξ** rather than a preferred region, as the presence of a non-zero coupling increases the value of the matter density.

Phantom

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02239 ± 0.00015	0.02237 ± 0.00015	0.02242 ± 0.00015	0.02238 ± 0.00014	0.02235 ± 0.00015
$\Omega_c h^2$	$0.132^{+0.005}_{-0.012}$	$0.133^{+0.006}_{-0.012}$	$0.133^{+0.006}_{-0.012}$	$0.134^{+0.007}_{-0.012}$	$0.134^{+0.006}_{-0.012}$
$100\theta_{MC}$	$1.04027^{+0.00064}_{-0.00048}$	$1.04024^{+0.00063}_{-0.00048}$	$1.04029^{+0.00062}_{-0.00051}$	$1.04019^{+0.00060}_{-0.00051}$	$1.04017^{+0.00060}_{-0.00051}$
τ	0.0537 ± 0.0080	0.0542 ± 0.0078	0.0524 ± 0.0072	0.0545 ± 0.0080	0.0542 ± 0.0081
n_s	0.9655 ± 0.0043	0.9650 ± 0.0042	0.9663 ± 0.0041	0.9654 ± 0.0040	0.9643 ± 0.0044
$\ln(10^{10} A_s)$	3.042 ± 0.016	3.044 ± 0.017	3.039 ± 0.014	3.045 ± 0.016	3.045 ± 0.016
ξ	< 0.130	< 0.157	< 0.140	< 0.187	< 0.178
w	$-1.59^{+0.18}_{-0.33}$	-1.264 ± 0.057	$-1.57^{+0.19}_{-0.32}$	$-1.095^{+0.072}_{-0.040}$	$-1.084^{+0.051}_{-0.038}$
H_0 [km/s/Mpc]	> 81.3	74.1 ± 1.4	85^{+10}_{-5}	$68.8^{+1.1}_{-1.5}$	68.33 ± 0.99
σ_8	0.883 ± 0.082	$0.802^{+0.059}_{-0.043}$	0.871 ± 0.083	0.753 ± 0.046	$0.755^{+0.051}_{-0.042}$
S_8	0.742 ± 0.040	$0.778^{+0.032}_{-0.026}$	0.735 ± 0.038	0.790 ± 0.026	0.797 ± 0.027

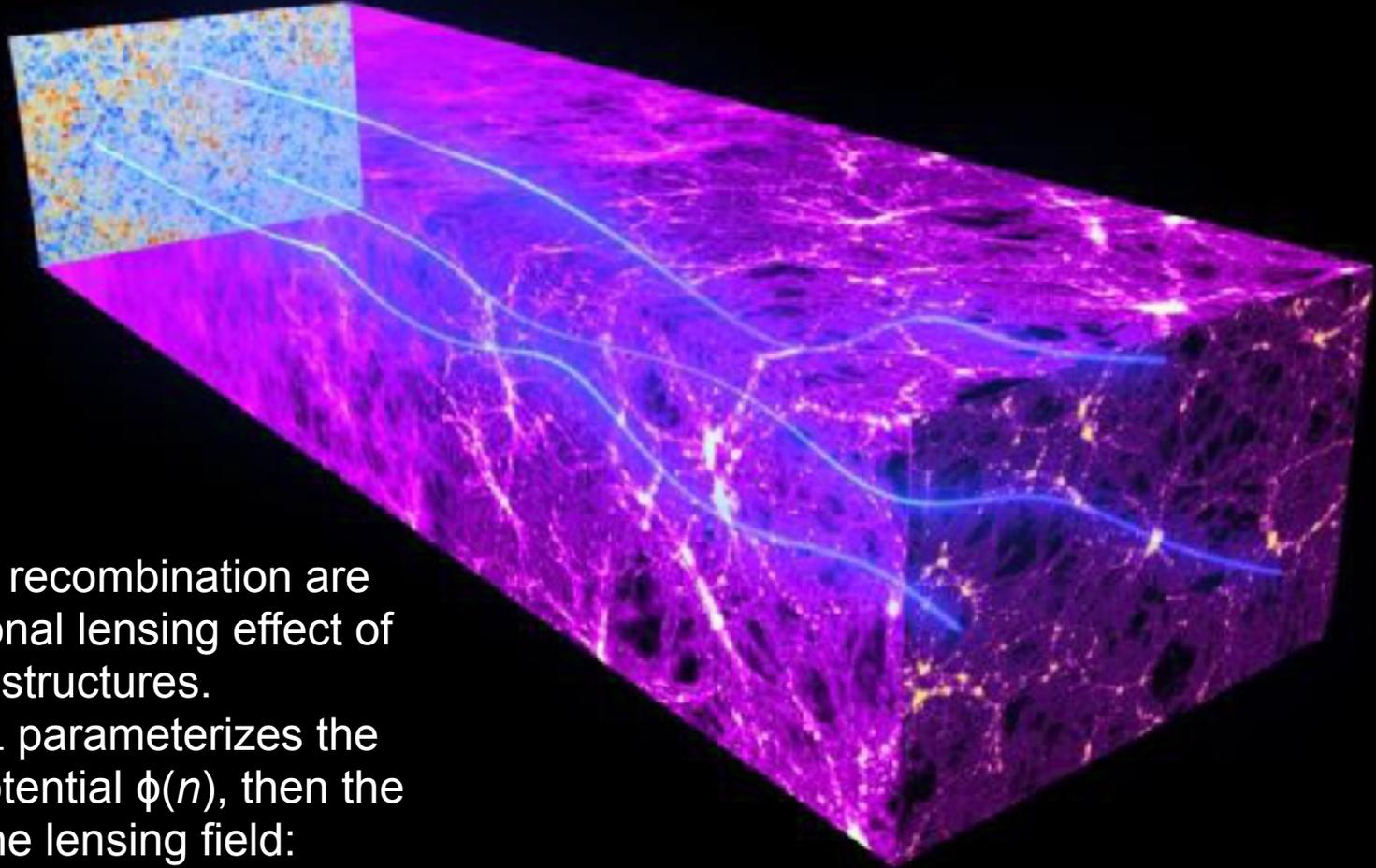
The value of the Hubble constant is also always much larger than in the canonical Λ CDM, because when w is allowed to vary in the phantom region, the parameter H_0 must be increased to not to affect the location of the CMB acoustic peaks. Therefore, the resolution of the H_0 tension is coming from the phantom character of the DE component, rather than from the dark sector interaction itself.

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H_0 with local measurements
- A_L internal anomaly
- S_8 with cosmic shear data
- Ω_k different from zero

See Di Valentino et al. [arXiv:2008.11283 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11283), [arXiv:2008.11284 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11284), [arXiv:2008.11285 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11285), [arXiv:2008.11286 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11286) for an overview.

A_L internal anomaly



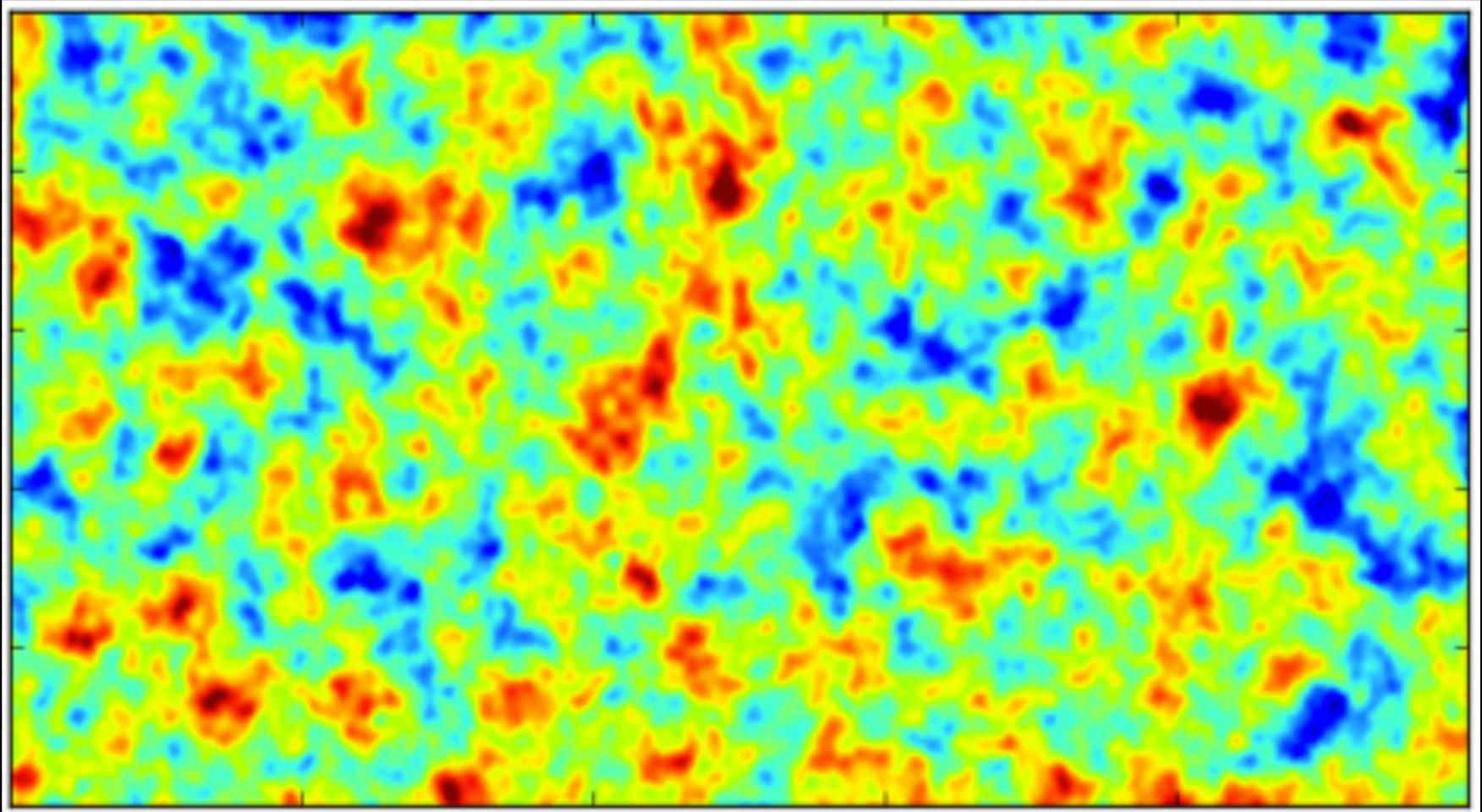
CMB photons emitted at recombination are deflected by the gravitational lensing effect of massive cosmic structures.

The lensing amplitude A_L parameterizes the rescaling of the lensing potential $\phi(n)$, then the power spectrum of the lensing field:

$$C_\ell^{\phi\phi} \rightarrow A_L C_\ell^{\phi\phi}$$

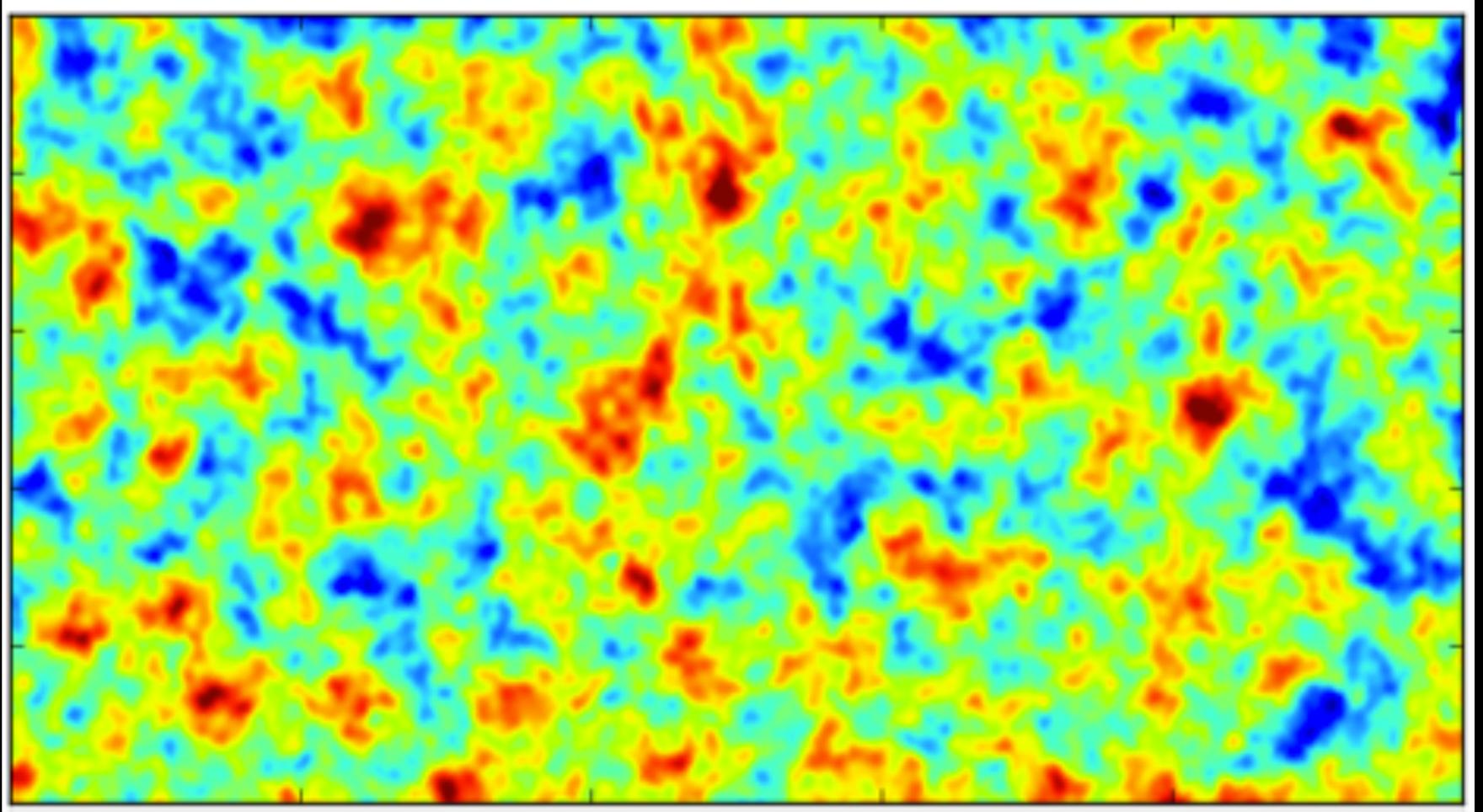
The gravitational lensing deflects the photon path by a quantity defined by the gradient of the lensing potential $\phi(n)$, integrated along the line of sight n , remapping the temperature field.

The CMB lensing



A simulated patch of CMB sky – **before dark matter lensing**

The CMB lensing



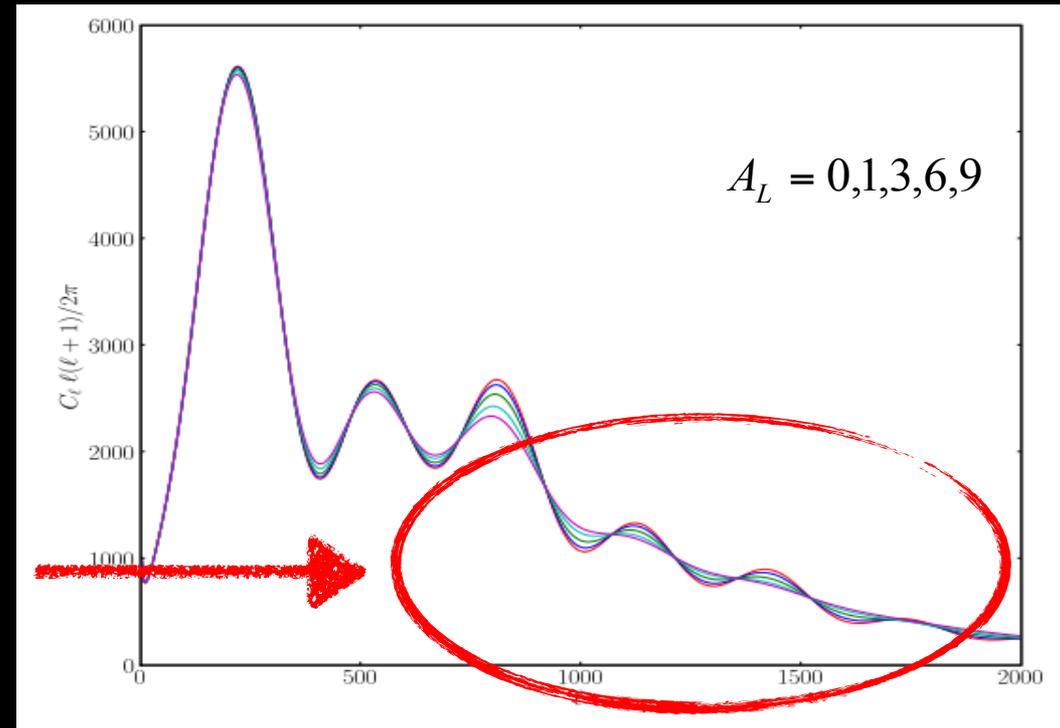
A simulated patch of CMB sky – **after dark matter lensing**

A_L internal anomaly

Its effect on the power spectrum is the smoothing of the acoustic peaks, increasing A_L .

Interesting consistency checks is if the amplitude of the smoothing effect in the CMB power spectra matches the theoretical expectation $A_L = 1$ and whether the amplitude of the smoothing is consistent with that measured by the lensing reconstruction.

If $A_L = 1$ then the theory is correct, otherwise we have a new physics or systematics.



Calabrese et al., Phys. Rev. D, 77, 123531

A_L : a failed consistency check

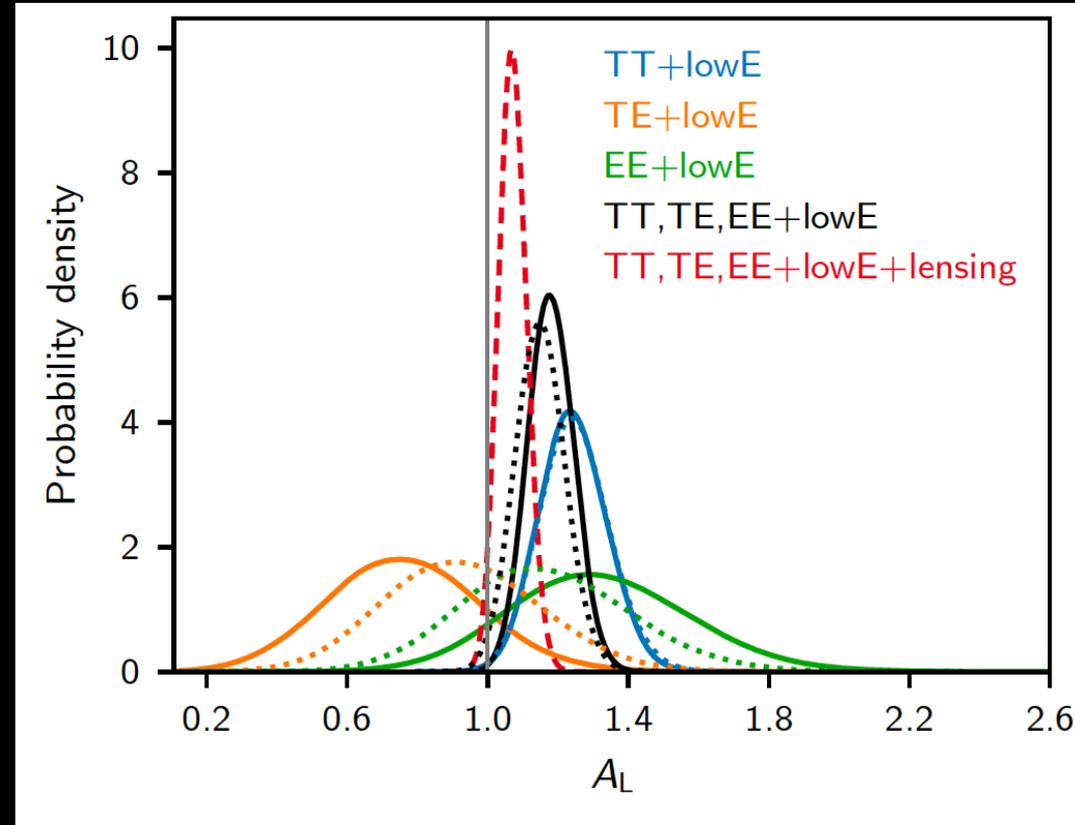
The Planck lensing-reconstruction power spectrum is consistent with the amplitude expected for Λ CDM models that fit the CMB spectra, so the Planck lensing measurement is compatible with $A_L = 1$.

However, the distributions of A_L inferred from the CMB power spectra alone indicate a preference for $A_L > 1$.

The joint combined likelihood shifts the value preferred by the TT data downwards towards $A_L = 1$, but the error also shrinks, increasing the significance of $A_L > 1$ to 2.8σ .

The preference for high A_L is not just a volume effect in the full parameter space, with the best fit improved by $\Delta\chi^2 \sim 9$ when adding A_L for TT+lowE and 10 for TTTEEE+lowE.

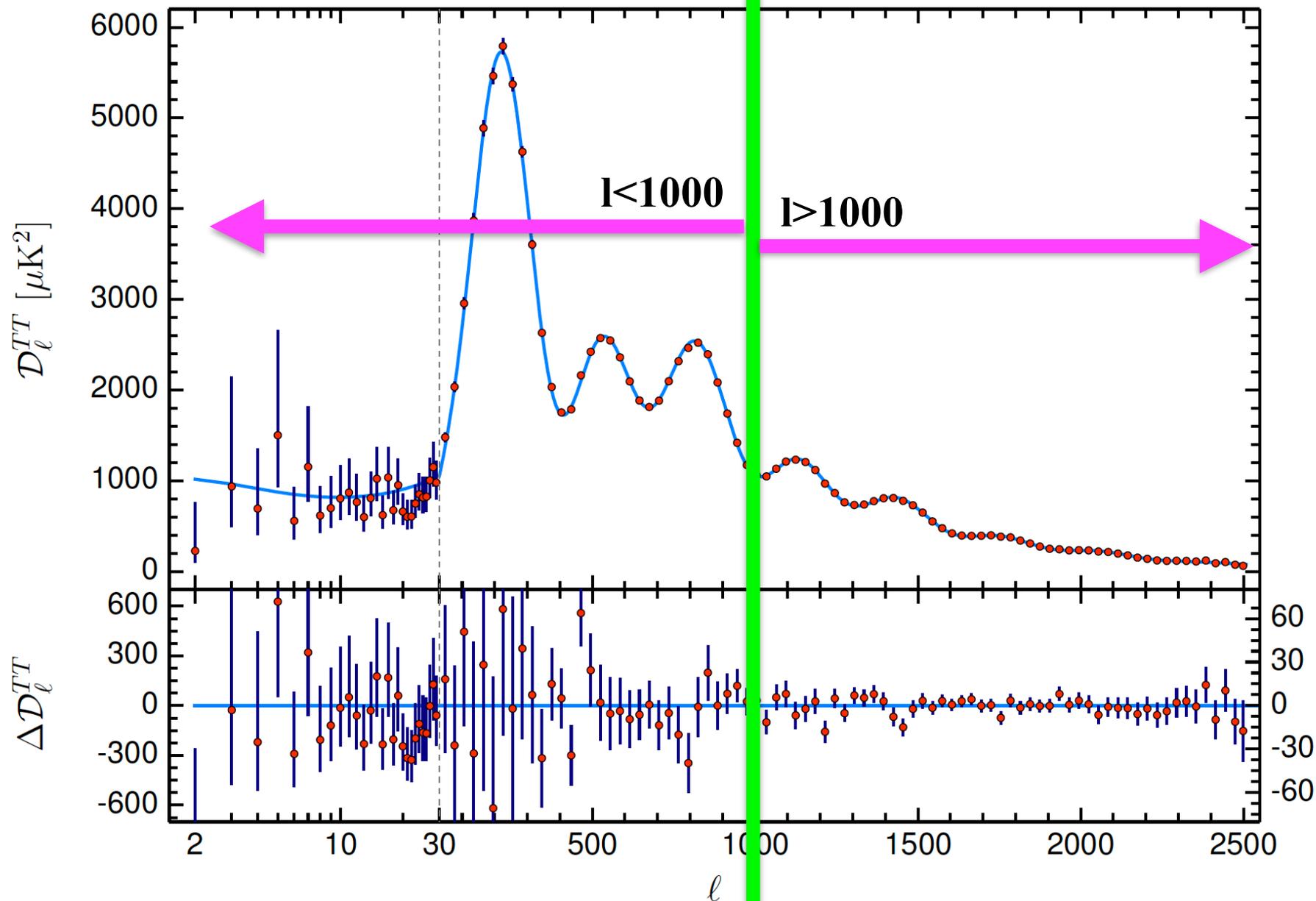
Planck 2018, Astron.Astrophys. 641 (2020) A6



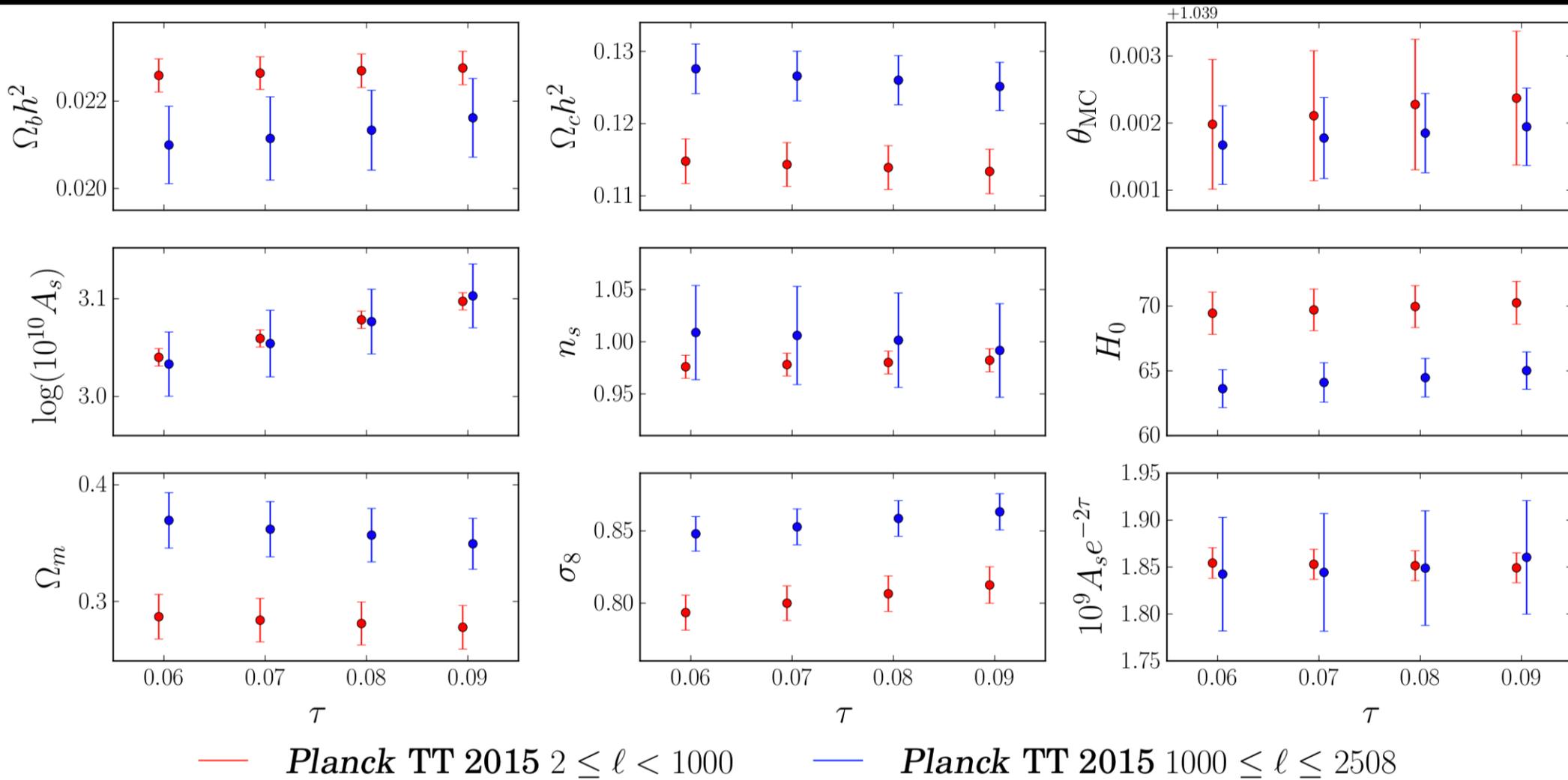
$$A_L = 1.243 \pm 0.096 \quad (68\%, \text{ Planck TT+lowE}),$$

$$A_L = 1.180 \pm 0.065 \quad (68\%, \text{ Planck TT,TE,EE+lowE}),$$

A_L can explain internal tension

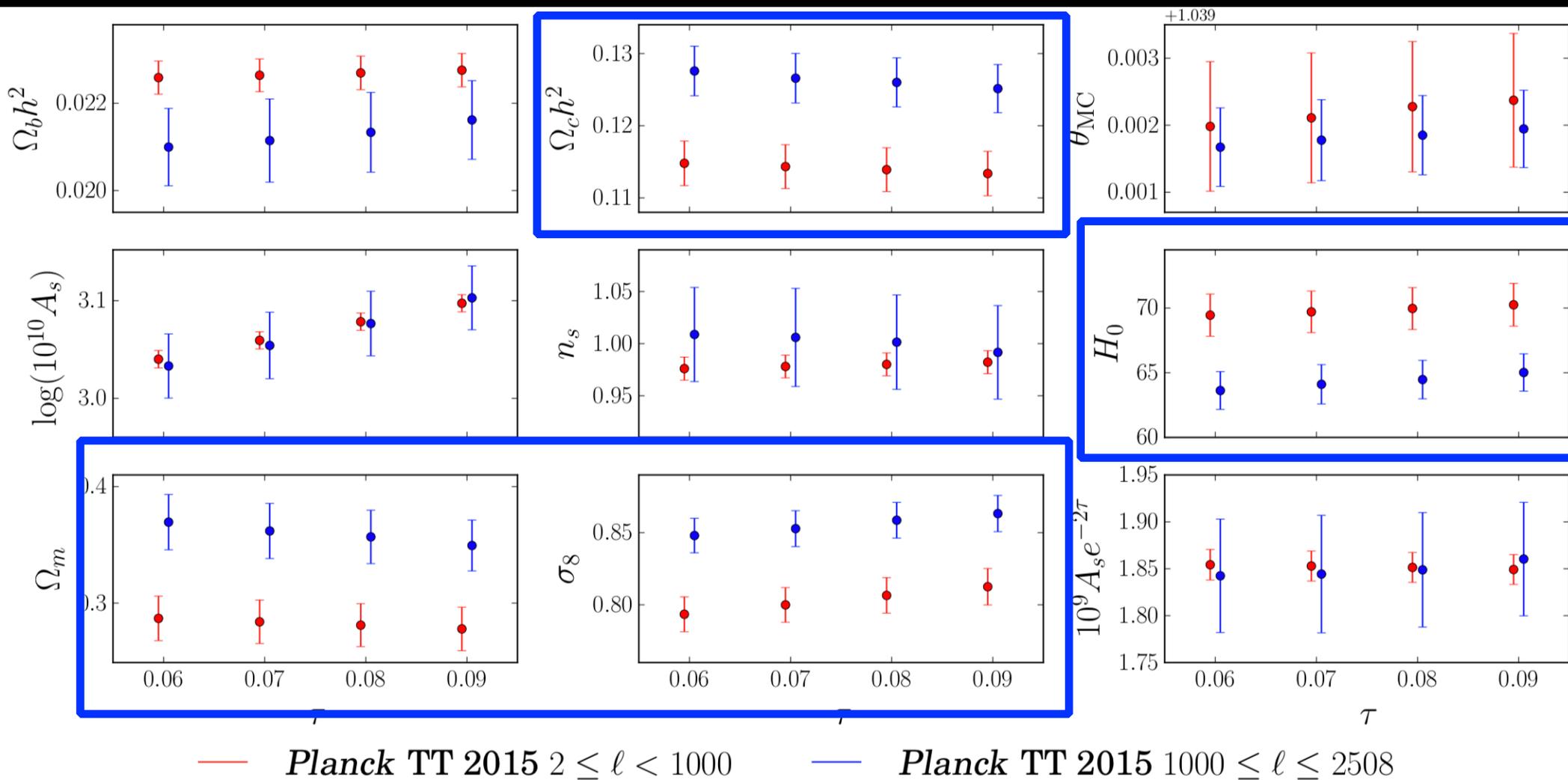


A_L can explain internal tension



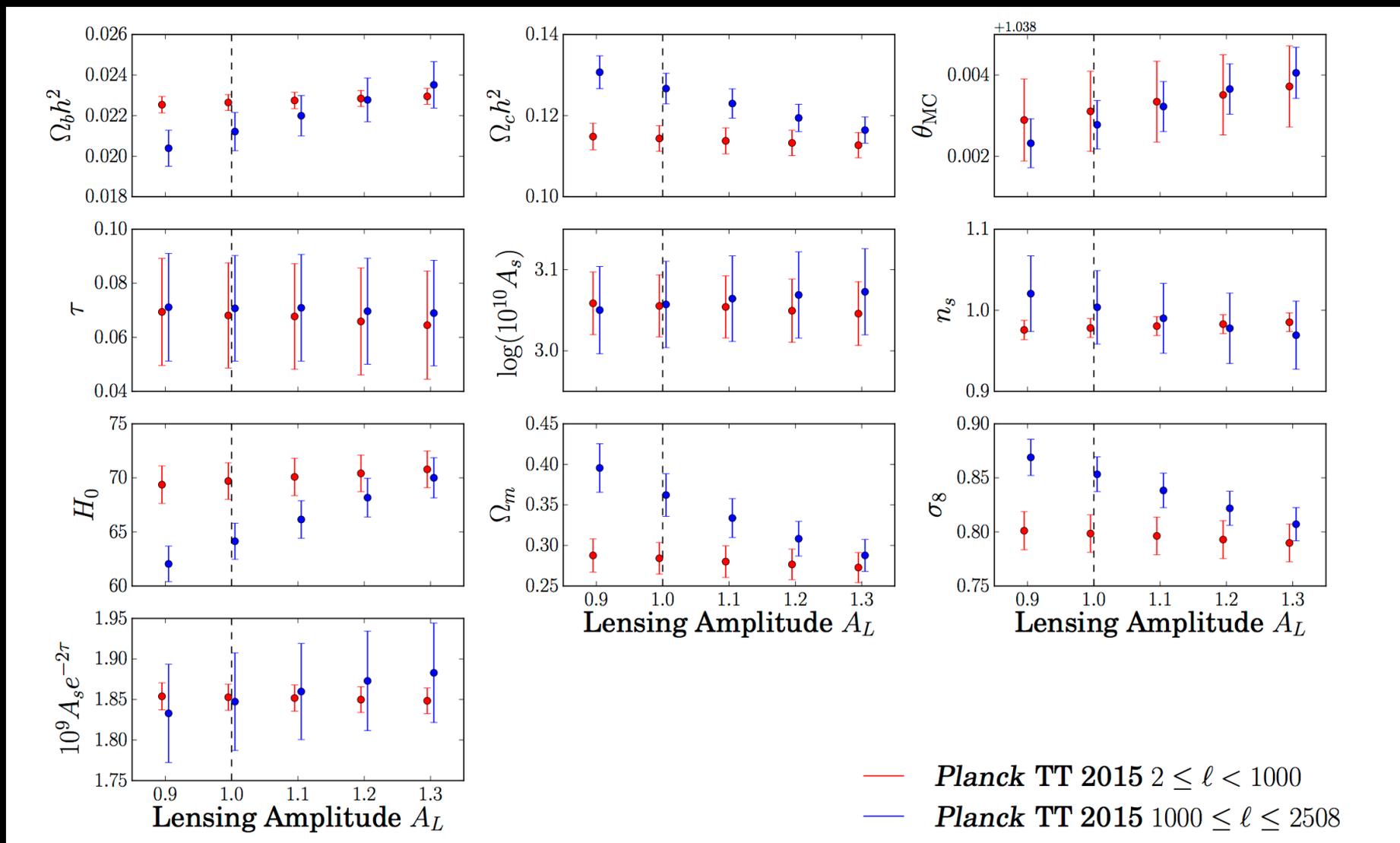
Marginalized 68.3% confidence Λ CDM parameter constraints from fits to the $l < 1000$ and $l \geq 1000$ $Planck$ TT 2015 spectra. Tension at more than 2σ level appears in $\Omega_c h^2$ and derived parameters, including H_0 , Ω_m , and σ_8 .

A_L can explain internal tension



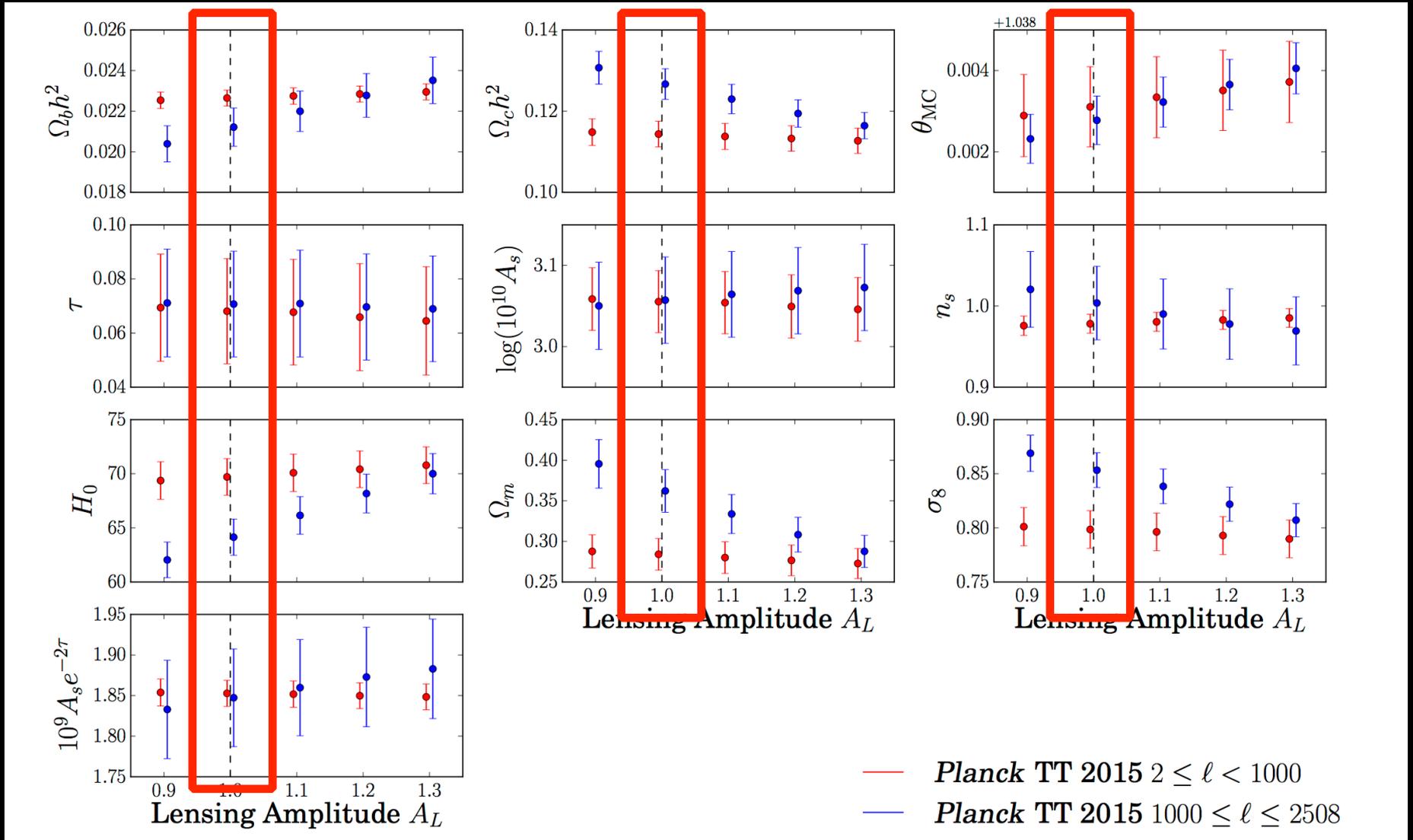
Marginalized 68.3% confidence Λ CDM parameter constraints from fits to the $l < 1000$ and $l \geq 1000$ *Planck* TT 2015 spectra. Tension at more than 2σ level appears in $\Omega_c h^2$ and derived parameters, including H_0 , Ω_m , and σ_8 .

A_L can explain internal tension



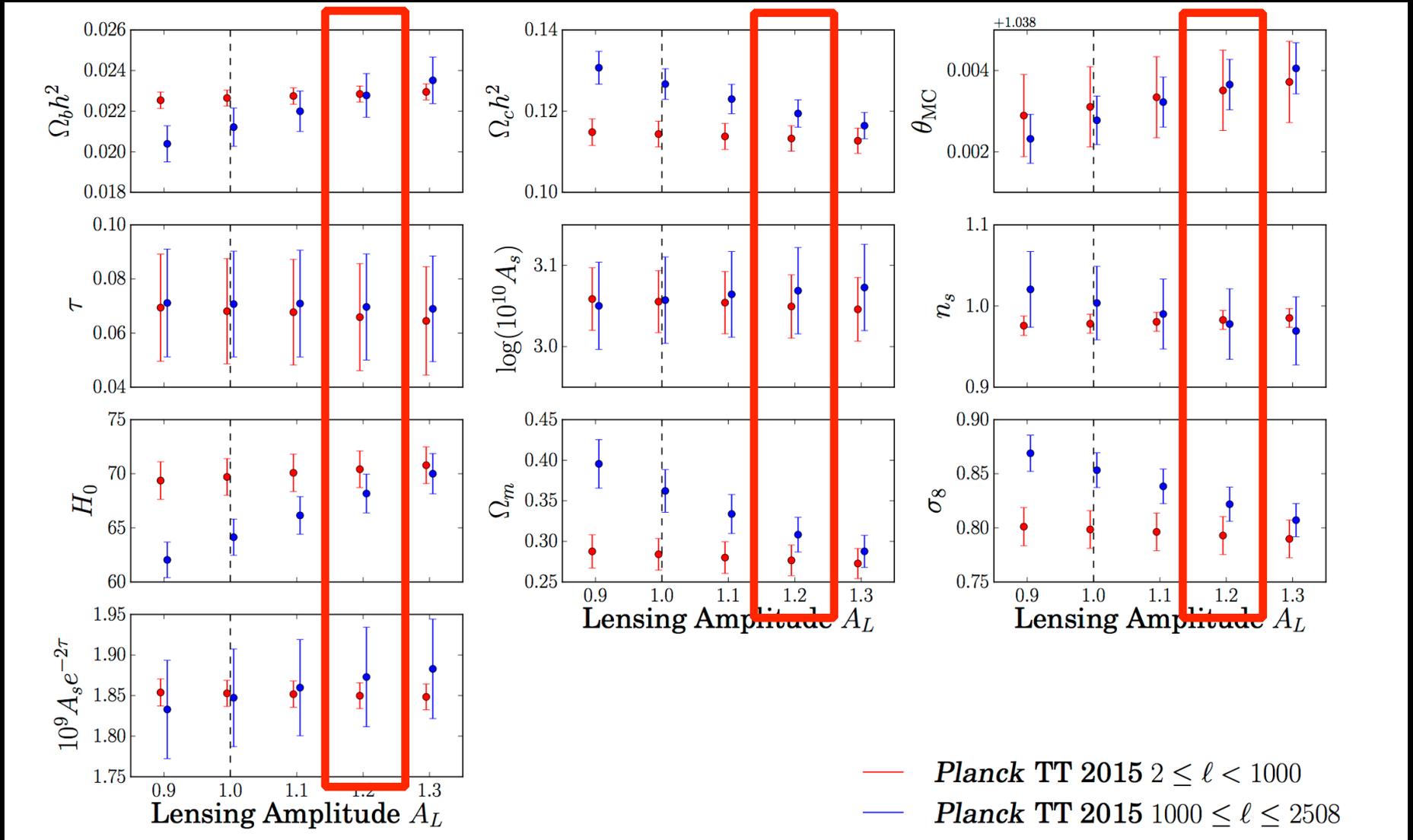
Increasing A_L smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

A_L can explain internal tension



Increasing A_L smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

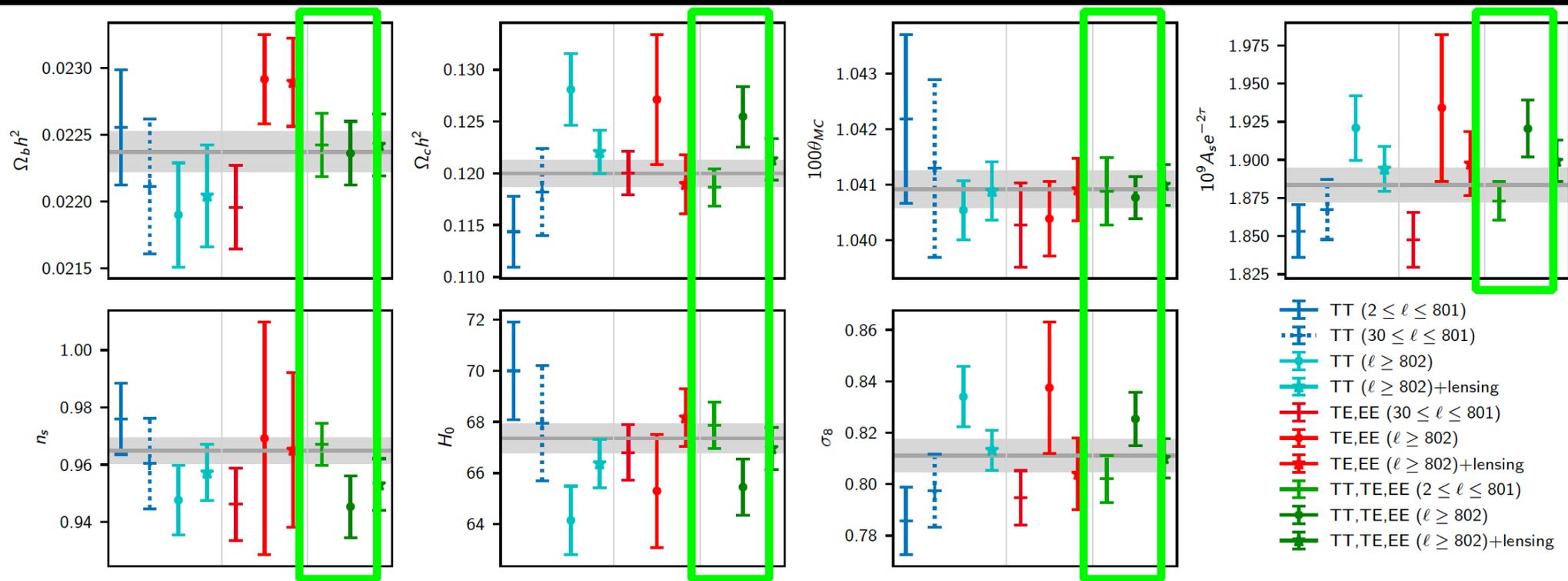
A_L can explain internal tension



Increasing A_L smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

A_L can explain internal tension

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



LCDM 68% marginalized parameter constraints for $l=[2-801]$ (points marked with a cross), $l>802$ (points marked with a circle), and $l>802$ + lensing (points marked with a star). Correcting for the lensing, all the results from high multipoles are in better consistency with the results from lower multipoles.

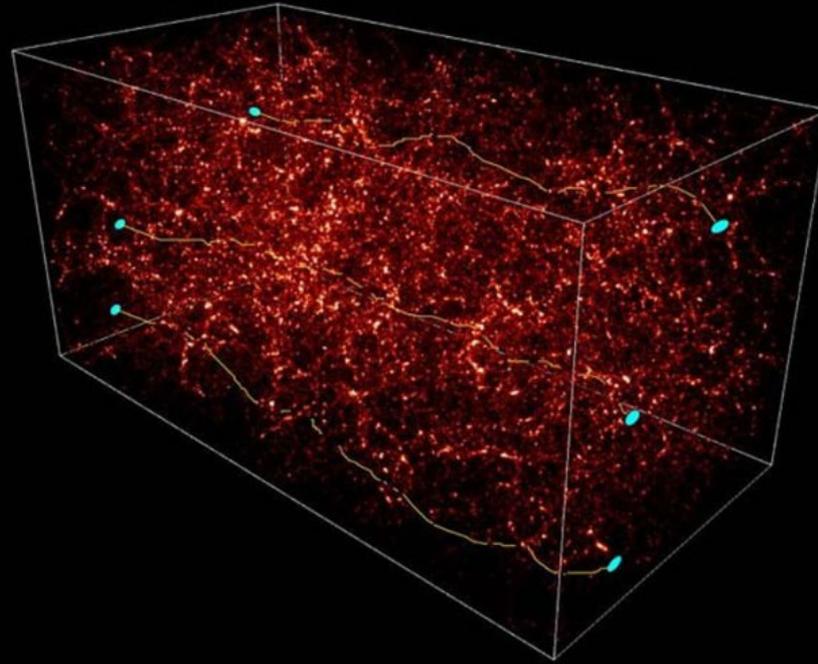
Dotted error bars are the results from $l=[30-801]$, without the large-scale TT likelihood, showing that $l < 30$ pulls the low-multipole parameters further from the joint result.

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H_0 with local measurements
- A_L internal anomaly
- **S8 with cosmic shear data**
- Ω_k different from zero

See Di Valentino et al. [arXiv:2008.11283 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11283), [arXiv:2008.11284 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11284), [arXiv:2008.11285 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11285), [arXiv:2008.11286 \[astro-ph.CO\]](https://arxiv.org/abs/2008.11286) for an overview.

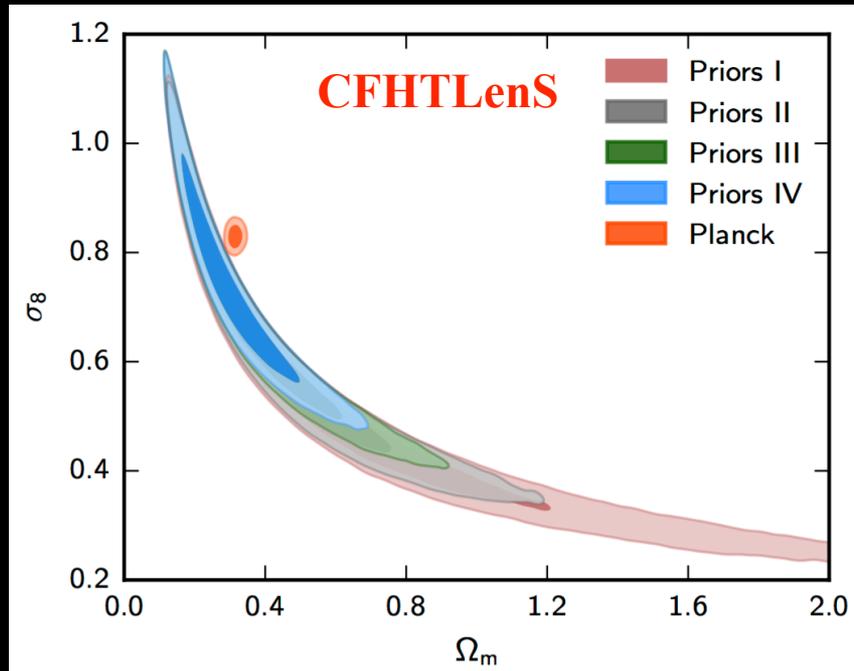
The S8 tension



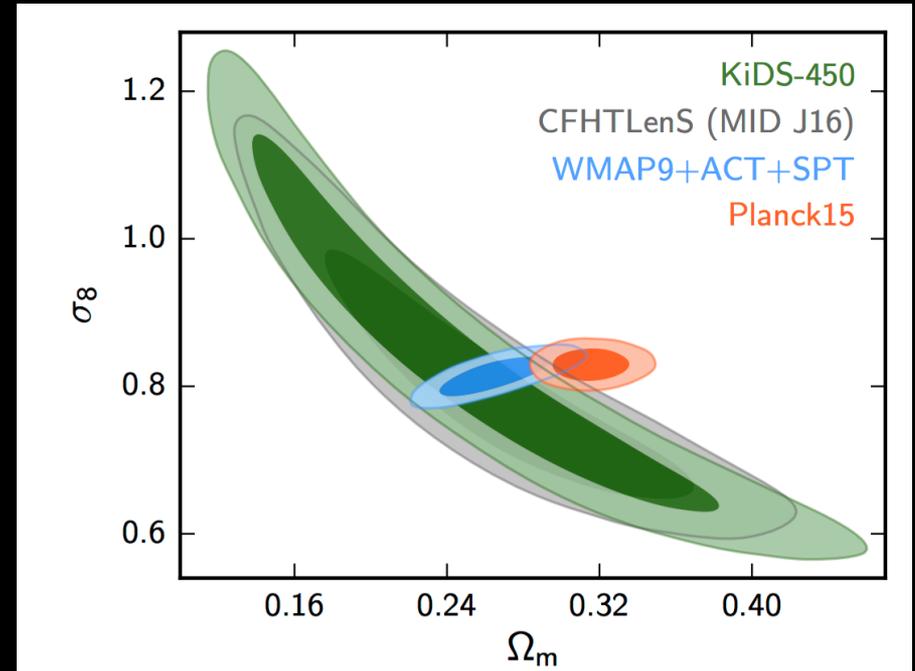
$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

A tension on S8 is present between the Planck data in the Λ CDM scenario and the cosmic shear data.

The S8 tension



Joudaki et al, arXiv:1601.05786



KiDS-450, Hildebrandt et al., arXiv:1606.05338.

The **S8 tension** is at about 2.6σ level between Planck assuming Λ CDM and CFHTLenS survey and KiDS-450.

$$S_8 = 0.834 \pm 0.016$$

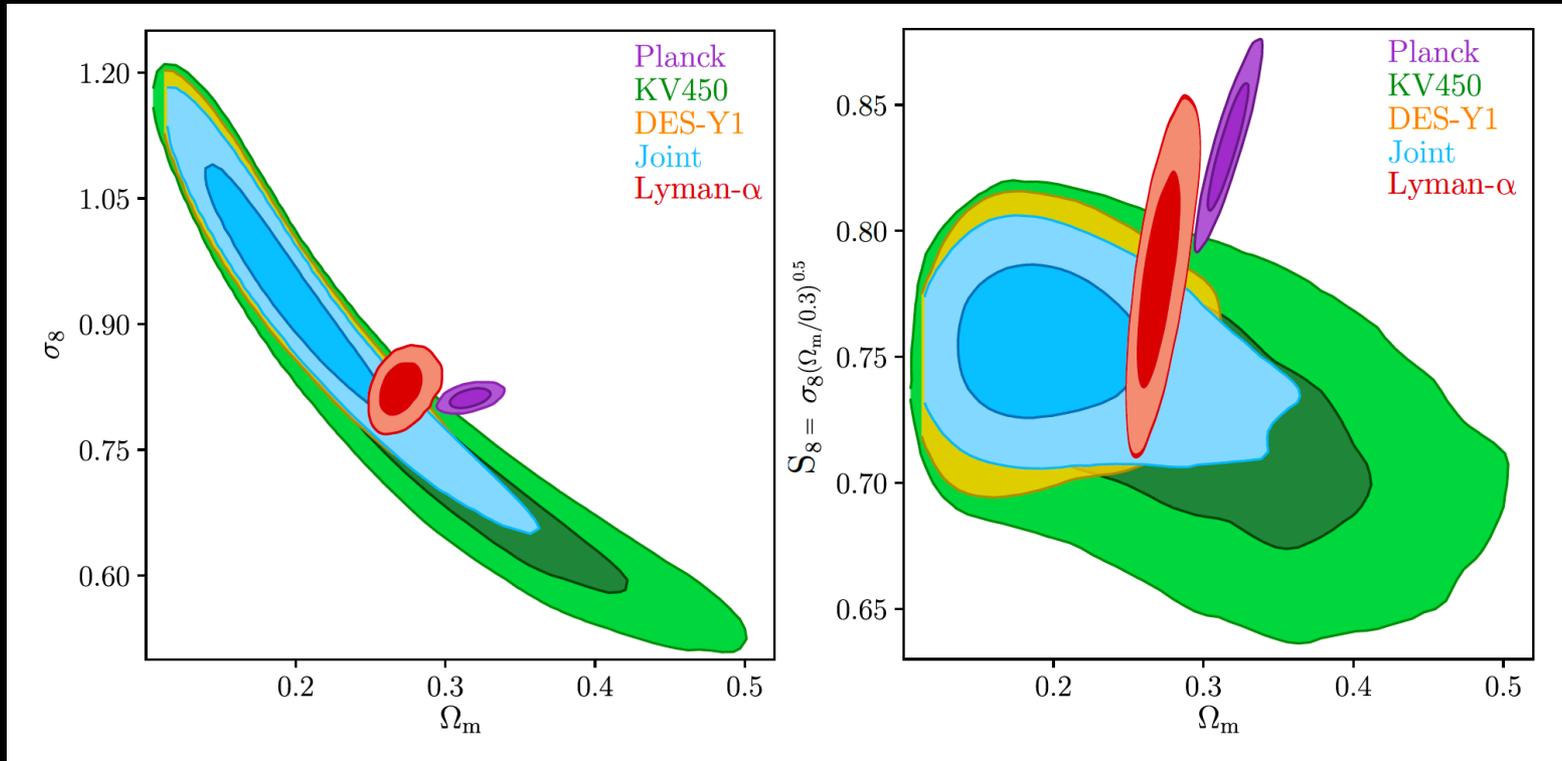
Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

$$S_8 = 0.745 \pm 0.035$$

KiDS-450, Hildebrandt et al., arXiv:1606.05338 [astro-ph.CO]

The S8 tension

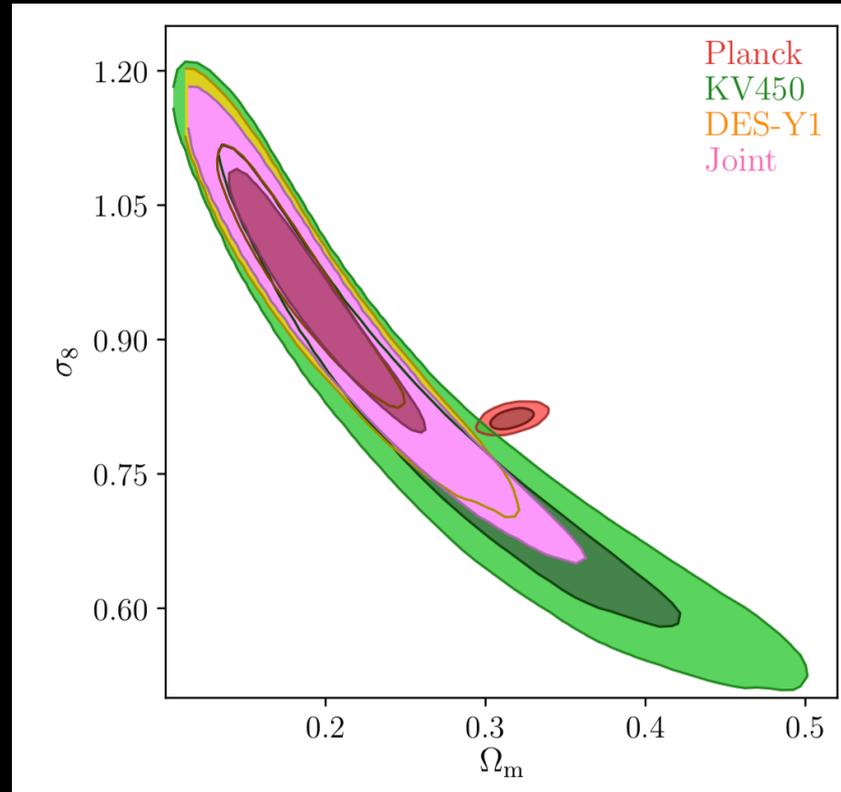
Palanque-Delabrouille et al., arXiv:1911.09073 [astro-ph.CO]



A tension on S_8 at more than 2.5σ is present between Planck assuming Λ CDM and DES-Y1 results including galaxy clustering, and Planck and Ly- α (sharing a similar range of scales).

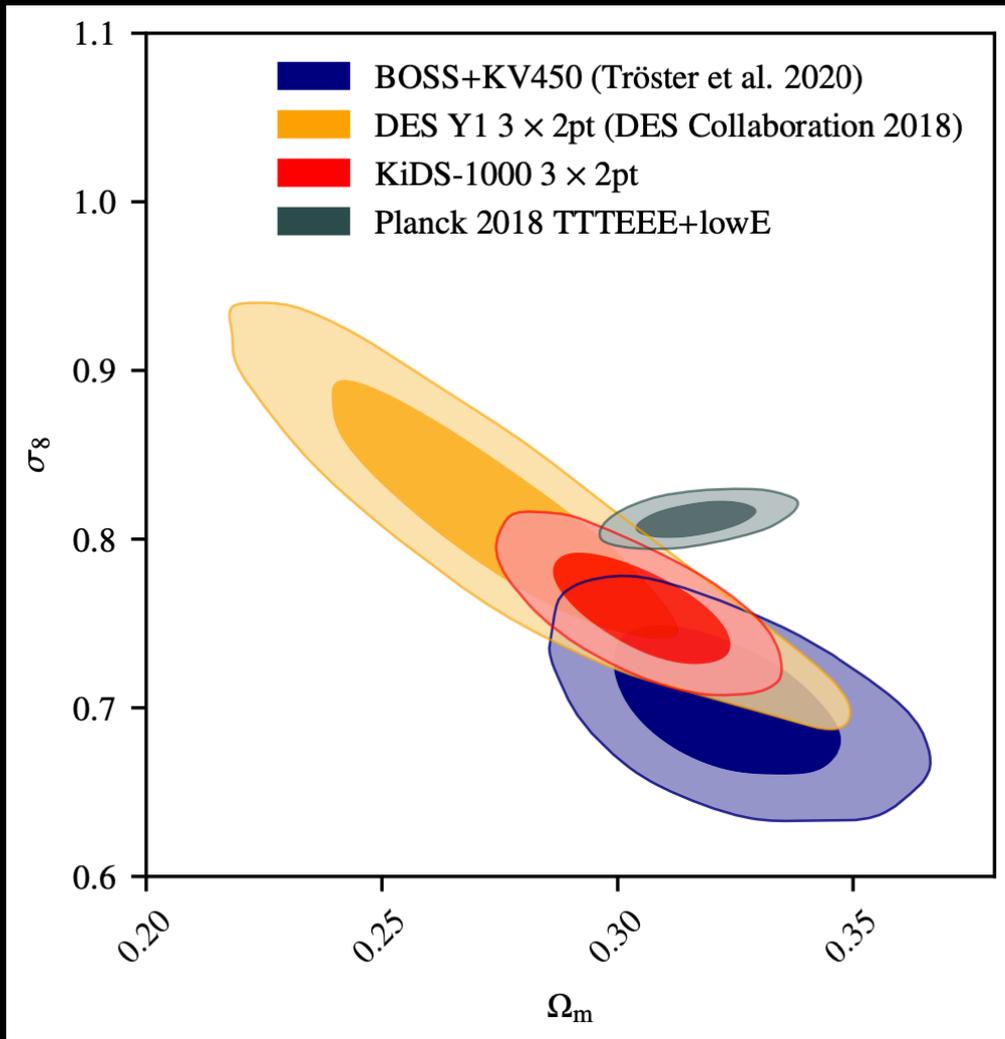
The S8 tension

Asgari et al., arXiv:1910.05336 [astro-ph.CO]



A tension on S8 at 3.2σ is present between Planck assuming Λ CDM and KiDS+VIKING-450 and DES-Y1 combined together.

The S8 tension



The S8 tension is present at 3.4σ between Planck assuming Λ CDM and KiDS+VIKING-450 and BOSS combined together, or 3.1σ with KiDS-1000.

$$S_8 = 0.834 \pm 0.016$$

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

$$S_8 = 0.728 \pm 0.045$$

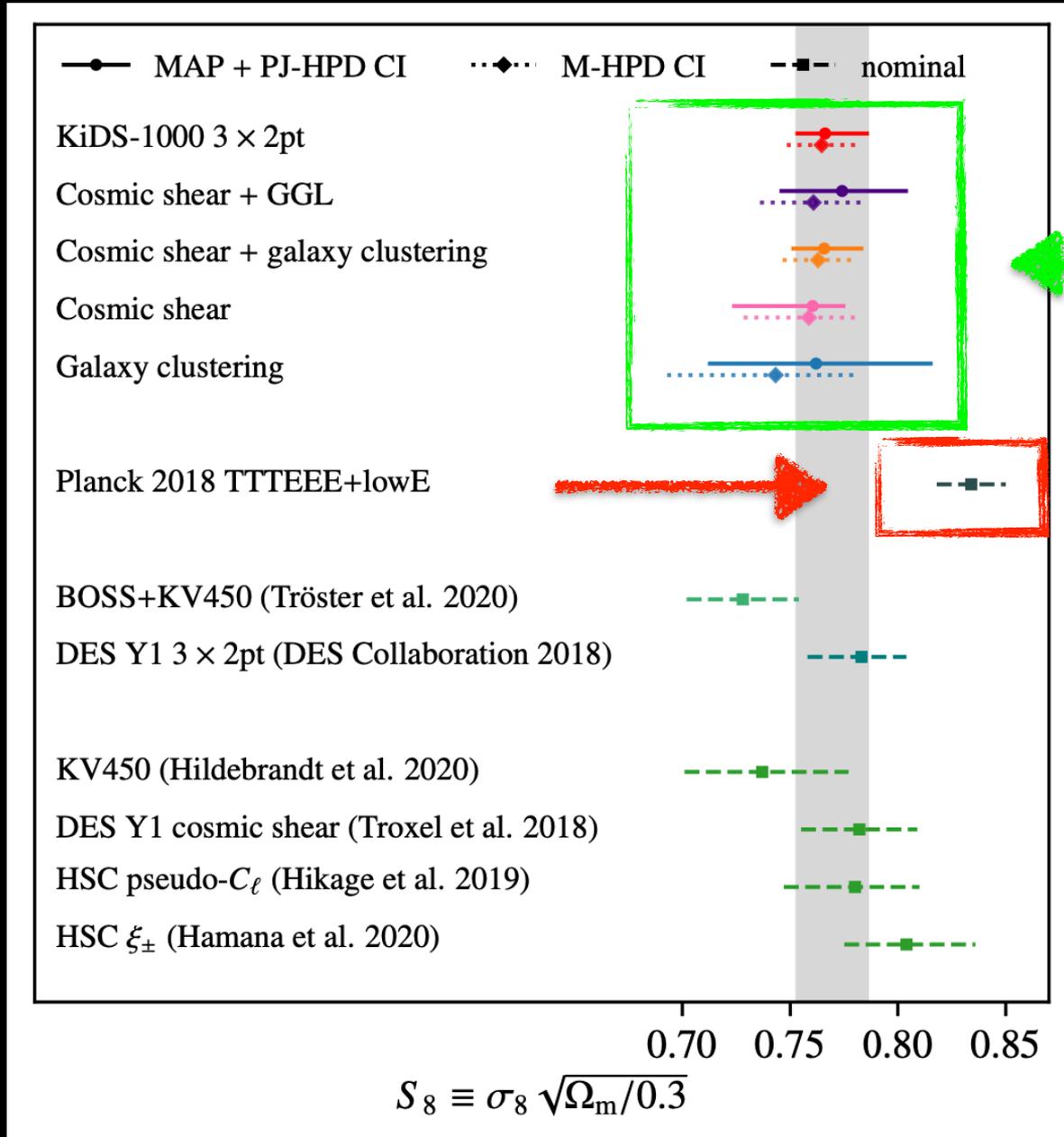
Tröster et al., arXiv:1909.11006 [astro-ph.CO]

$$S_8 = 0.766^{+0.020}_{-0.014}$$

KiDS-1000, Heymans et al., arXiv:2007.15632 [astro-ph.CO]

KiDS-1000, Heymans et al., arXiv:2007.15632 [astro-ph.CO]

The S8 tension



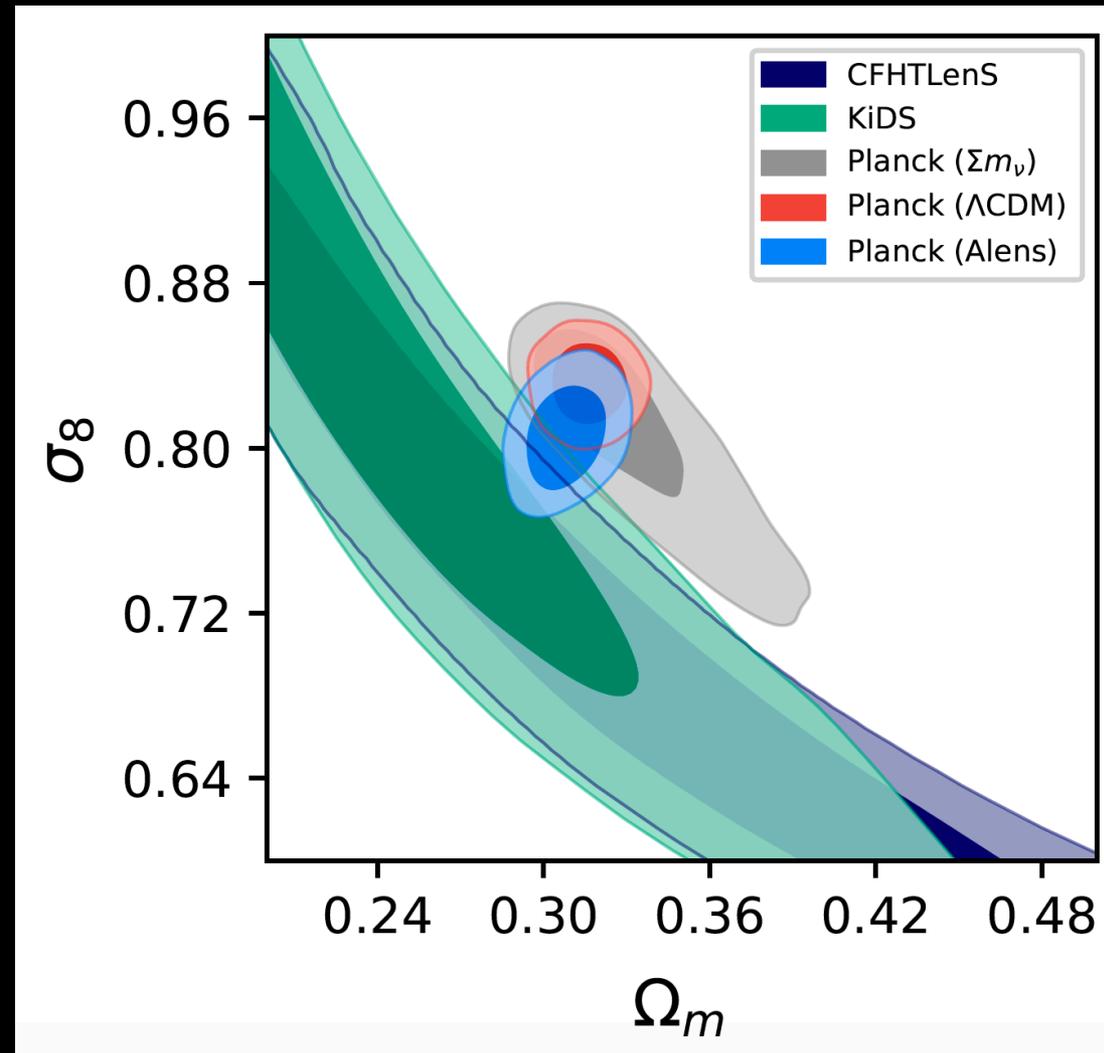
Proposals for solving the S8 tension are:

- Axion monodromy inflation (Meerburg arXiv:1406.3243, etc...)
- Extended parameter spaces involving $\Omega_m > 1$ (Di Valentino et al. arXiv:1507.06646, Di Valentino et al. arXiv:1606.00634, Di Valentino et al. arXiv:1704.00762, Di Valentino et al. arXiv:1908.01391, etc...)
- Active and Sterile Neutrinos (Battye & Moss arXiv:1308.5870 , Bohringer & Chon arXiv:1610.02855, etc...)
- Interacting Dark Energy models (Di Valentino et al. arXiv:1908.04281, Di Valentino et al. arXiv:1910.09853, etc.)
- Decaying dark matter (Chudaykin et al. arXiv:1711.06738, Abellan et al. arXiv:2008.09615, Berezhiani et al. arXiv:1505.03644, Anchordoqui et al. arXiv:1506.08788, Abellan et al. arXiv:2102.12498 [astro-ph.CO], etc..)
- Cannibal dark matter (Heimersheim et al. arXiv:2008.08486, etc...)
- Minimally and non-minimally coupled scalar field models (Davari et al. arXiv:1911.00209, etc...)
- Modified Gravity models (Di Valentino et al. arXiv:1509.07501, Sola Peracaula et al. arXiv:1909.02554, Sola et al. arXiv:2006.04273, etc...)
- Running Vacuum models (Gomez-Valent & Sola arXiv:1711.00692, Lambiase et al. arXiv:1804.07154, Sola et al. arXiv:1506.05793, Sola et al. arXiv:1709.07451, Sola et al. arXiv:1602.02103, etc...)
- Quartessence (Camera et al. arXiv:1704.06277, etc...)

See Di Valentino et al. arXiv:2008.11285 [astro-ph.CO] for a summary of other possible candidates.

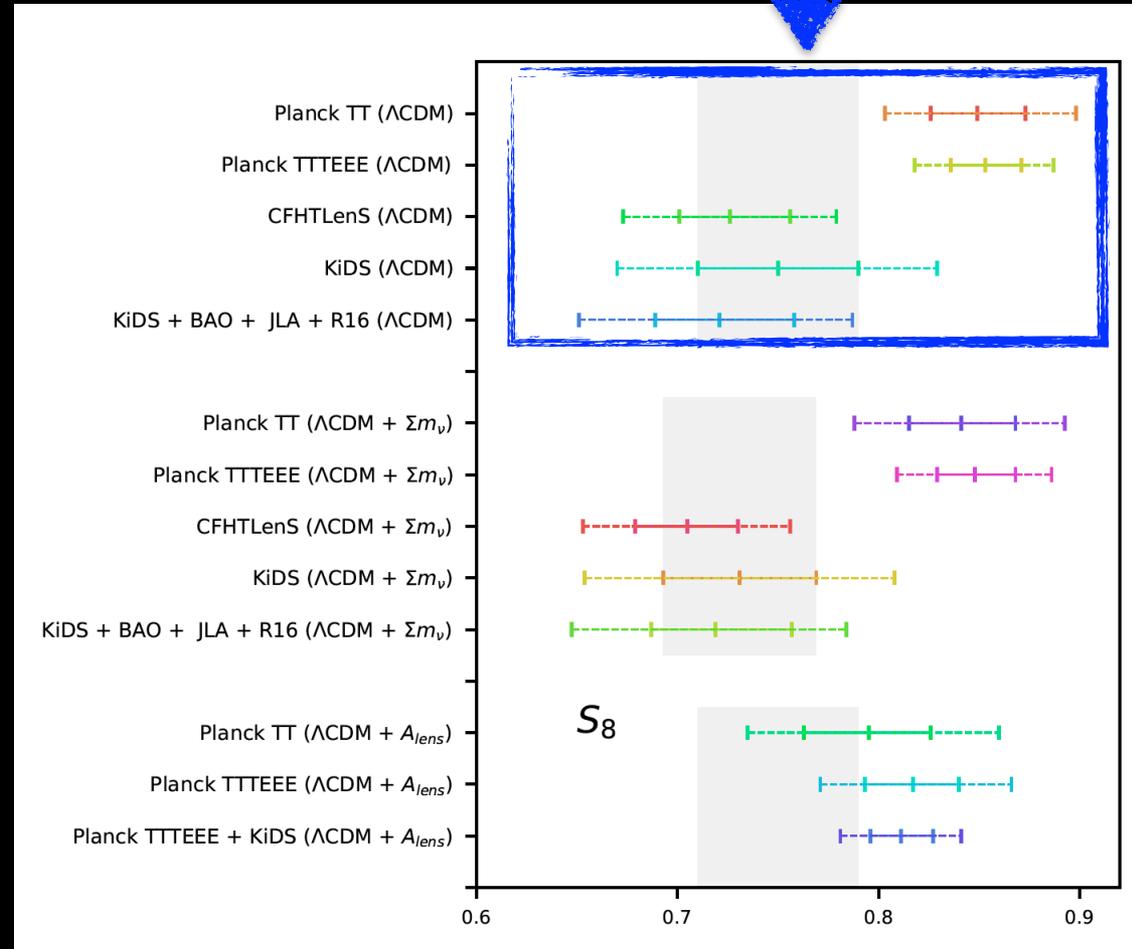
A_L can explain the S8 tension

If we include the additional scaling parameter on the CMB lensing amplitude A_L , we find that this can put in agreement Planck 2015 with the cosmic shear data.



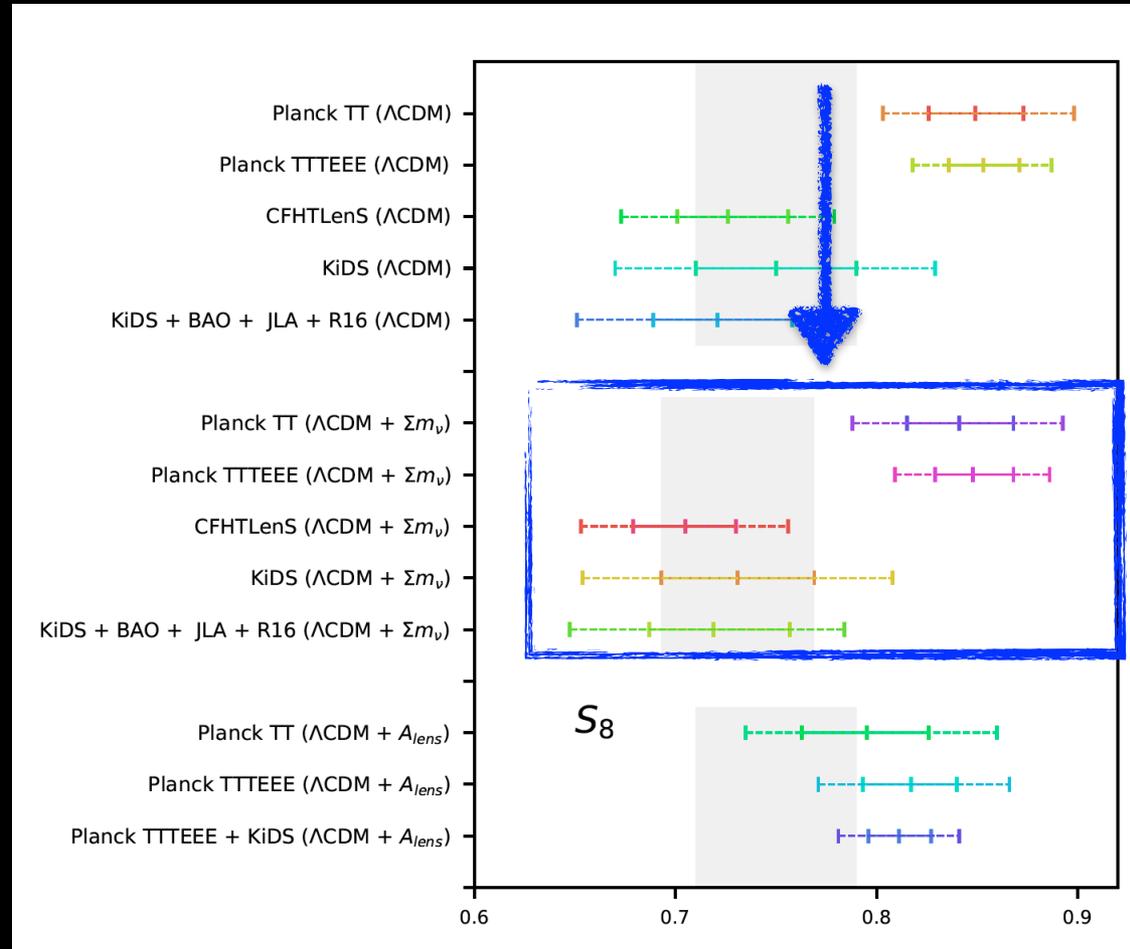
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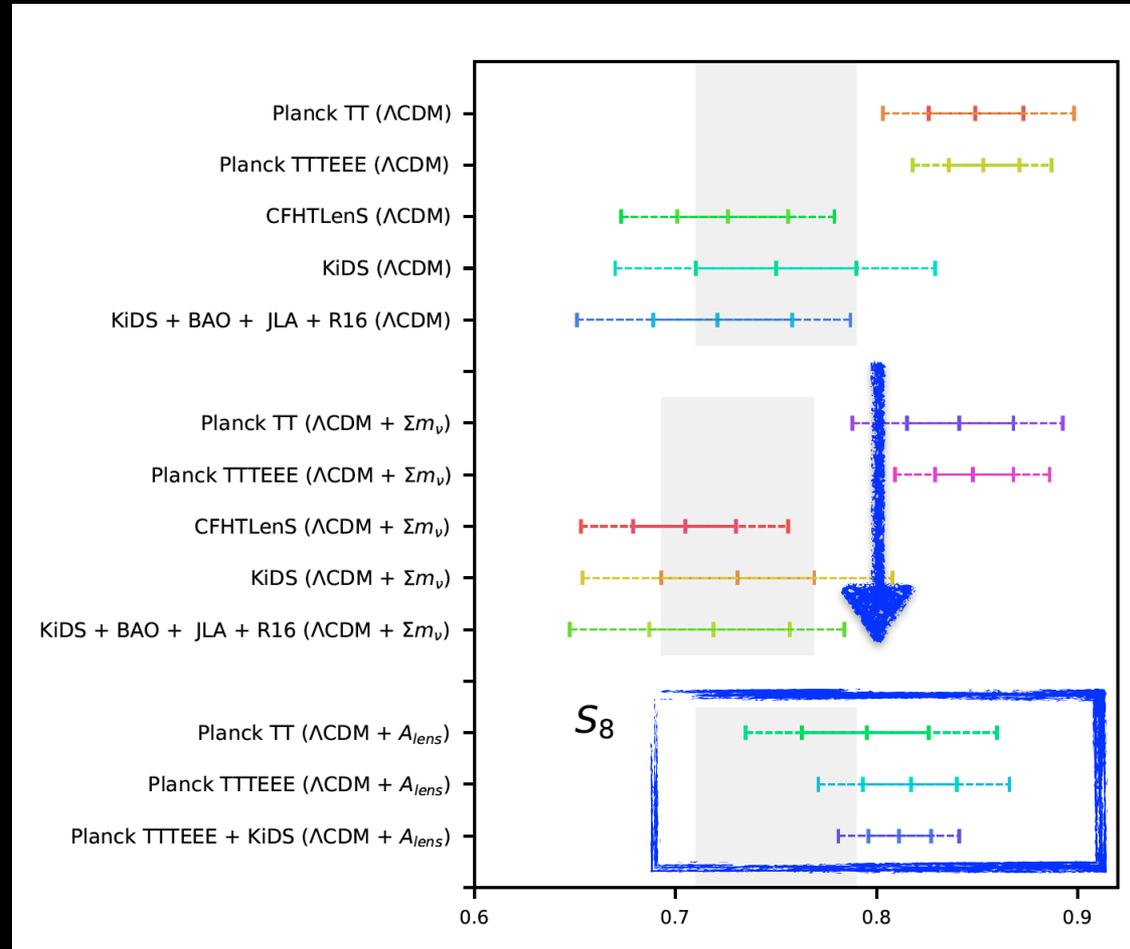
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A_L can explain the S8 tension

If we include the additional scaling parameter on the CMB lensing amplitude A_L , we find that this can put in agreement Planck 2015 with the cosmic shear data.



Beyond six parameters: extending Λ CDM

What happens if we vary all the parameters together?

In practice **we do not try to solve any single tension** with a specific theoretical mechanism, but we allow for a significant number of motivated extensions of Λ CDM, looking for a **possible combination of parameters** that could solve or at least ameliorate, the current discordances.

While this "minimal" 6 parameter approach is justified by the good fit to the data, some of **the assumptions or simplifications made are indeed not anymore fully justified and risk an oversimplification** of the physics that drives the evolution of the Universe.

Beyond six parameters: extending Λ CDM

- The **total neutrino mass** is fixed arbitrary to 0.06eV. However, we know that neutrinos are massive and that current cosmological datasets are sensitive to variations in the absolute neutrino mass scale of order ~ 100 meV.
- The **cosmological constant** offers difficulties in any theoretical interpretation: fixing the dark energy equation of state to -1 is not favoured by any theoretical argument. Moreover, while both matter and radiation evolve rapidly, Λ is assumed not to change with time, so its recent appearance in the standard cosmological model implies an extreme fine-tuning of initial conditions. This fine-tuning is known as the coincidence problem. Therefore it seems reasonable to incorporate in the analysis a possible dynamical dark energy component, constant with redshift w , or redshift dependent $w(z)=w_0+(1-a)w_a$ (CPL).
- Any inflationary model, because it is a dynamical process, predicts a **running of the scalar spectral index**, expected for slow rolling inflation at the level of $(1-n_s)^2 \sim 10^{-3}$.
- The **effective number of relativistic degrees of freedom** N_{eff} could be easily different from the standard expected value of 3.046, for example for the presence of sterile neutrinos or thermal axions.
- We need to take into account the anomalous value for the **lensing amplitude** A_L . While this parameter is purely phenomenological, one should clearly consider it and check if the cosmology obtained is consistent with other datasets.

Beyond six parameters: extending Λ CDM

Cosmological constraints are usually derived under the assumption of a 6 parameters Λ CDM theoretical framework or simple one-parameter extensions.

In [Di Valentino, Melchiorri and Silk, JCAP 2001 \(2020\) no.01, 013](#) we show, using Planck 2018, the cosmological constraints in a significantly extended scenario, varying 11 cosmological parameters simultaneously, updating the results reported in [Di Valentino, Melchiorri and Silk, Phys.Rev. D92 \(2015\) no.12, 121302](#) for the first time:

- the sum of neutrino masses,
- the dark energy equation of state w ,
- the running of the spectral index of primordial perturbations,
- the neutrino effective number,
- the angular power spectrum lensing amplitude, A_{lens} .

Beyond six parameters: extending Λ CDM

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.02228 ± 0.00026	0.02264 ± 0.00026	0.02250 ± 0.00028
$\Omega_c h^2$	0.1172 ± 0.0033	0.1174 ± 0.0035	0.1164 ± 0.0033	0.1175 ± 0.0033	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052	1.04119 ± 0.00050	1.04120 ± 0.00049	1.04111 ± 0.00050
τ	0.0496 ± 0.0086	0.0508 ± 0.0091	$0.0494^{+0.0086}_{-0.0076}$	0.0502 ± 0.0087	$0.0499^{+0.0086}_{-0.0078}$
Σm_ν [eV]	< 0.863	< 0.821	< 0.714	< 0.352	< 0.822
w	-1.27 ± 0.53	$-1.33^{+0.17}_{-0.11}$	-1.33 ± 0.52	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
N_{eff}	2.95 ± 0.24	2.97 ± 0.26	2.85 ± 0.23	3.04 ± 0.23	$2.98^{+0.23}_{-0.25}$
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	1.232 ± 0.090
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.024 ± 0.020	3.030 ± 0.020	$3.028^{+0.020}_{-0.018}$
n_s	0.964 ± 0.012	0.965 ± 0.013	0.958 ± 0.012	0.971 ± 0.012	0.965 ± 0.012
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.0066 ± 0.0082	-0.0041 ± 0.0081	-0.0049 ± 0.0086
H_0 [km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	74^{+10}_{-20}	67.9 ± 1.7	66.9 ± 2.0
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	0.782 ± 0.025	$0.750^{+0.055}_{-0.034}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

In this Table we show the constraints obtained assuming **our extended 11 parameters space**, assuming a constant dark energy equation of state w .

Beyond six parameters: extending Λ CDM

Parameters	Planck	Planck +R19	Planck +lensing	Planck +RAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.02228 ± 0.00026	0.02264 ± 0.00026	0.02250 ± 0.00028
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The significant increase in the number of parameters produces, as expected, a **relaxation in the constraints on the 6 Λ CDM parameters**. It is impressive that despite the increase in the number of the parameters, some of the constraints on key parameters are relaxed **but not significantly altered**. The cold dark matter ansatz remains robust and the baryon density is compatible with BBN predictions.

Beyond six parameters: extending Λ CDM

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.02228 ± 0.00026	0.02264 ± 0.00026	0.02250 ± 0.00028
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α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.0066 ± 0.0082	-0.0041 ± 0.0081	-0.0049 ± 0.0086
H_0 [km/s/Mpc]	73^{+16}_{-20}	74.0 ± 1.4	74^{+16}_{-20}	67.9 ± 1.7	66.9 ± 2.0
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	0.782 ± 0.025	$0.750^{+0.055}_{-0.034}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

We see no evidence for "new physics": we just have (weaker) upper limits on the neutrino mass, the running of the spectral index is compatible with zero, the dark energy equation of state is compatible with $w = -1$, and the neutrino effective number is remarkably close to the standard value $N_{\text{eff}} = 3.046$.

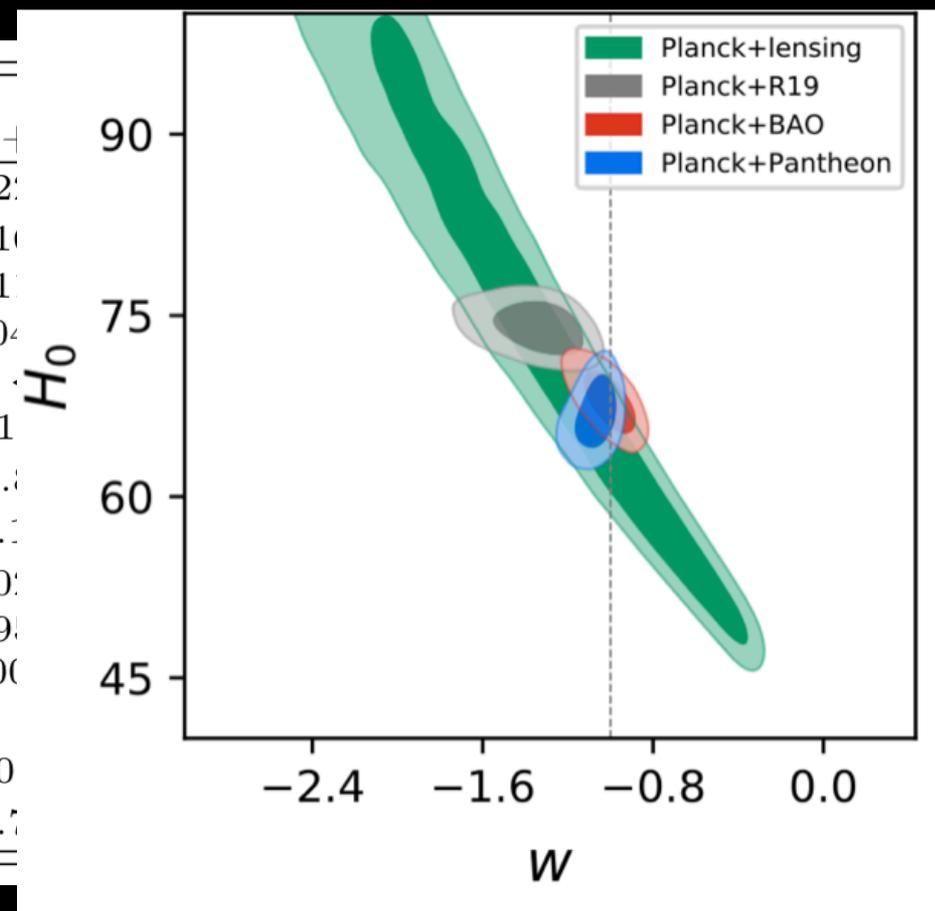
Beyond six parameters: extending Λ CDM

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N_{eff}	2.95 ± 0.24	2.97 ± 0.26	2.85 ± 0.23	3.04 ± 0.23	$2.98^{+0.23}_{-0.25}$
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	1.232 ± 0.090
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.024 ± 0.020	3.030 ± 0.020	$3.028^{+0.020}_{-0.018}$
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S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

We find a **relaxed value for the Hubble constant**, with respect to the one derived under the assumption of Λ CDM. The main reason for this relaxation is the inclusion in the analysis of the **dark energy equation of state w** , that introduces a **geometrical degeneracy** with the matter density and the Hubble constant. In this way, we can solve the existing tensions with the direct measurements.

Beyond six parameters: extending Λ CDM

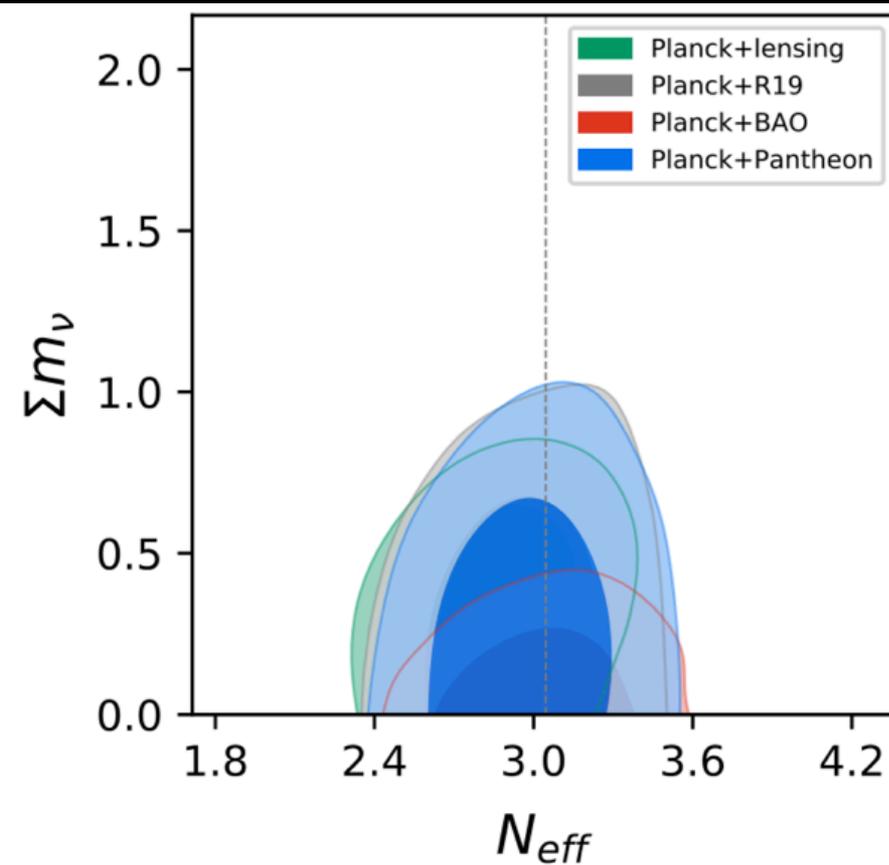
Parameters	Planck	Planck +R19	
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.0224
$\Omega_c h^2$	0.1172 ± 0.0033	0.1174 ± 0.0035	0.110
$100\theta_{MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052	1.041
τ	0.0496 ± 0.0086	0.0508 ± 0.0091	0.04
Σm_ν [eV]	< 0.863	< 0.821	
w	-1.27 ± 0.53	$-1.33^{+0.17}_{-0.11}$	-1
N_{eff}	2.95 ± 0.24	2.97 ± 0.26	2.9
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	1.2
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.0
n_s	0.964 ± 0.012	0.965 ± 0.013	0.9
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.00
H_0 [km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	0
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	0.7



Since now datasets are fully compatible, we combine Planck 2018 with R19 ($H_0=74.03 \pm 1.42$ km/s/Mpc), in order to see which parameter is preferred by the data to solve the tension. **We find a phantom-like dark energy component with an equation of state $w < -1$ at more than three standard deviations, while the neutrino effective number is fully compatible with standard expectations.**

Beyond six parameters: extending Λ CDM

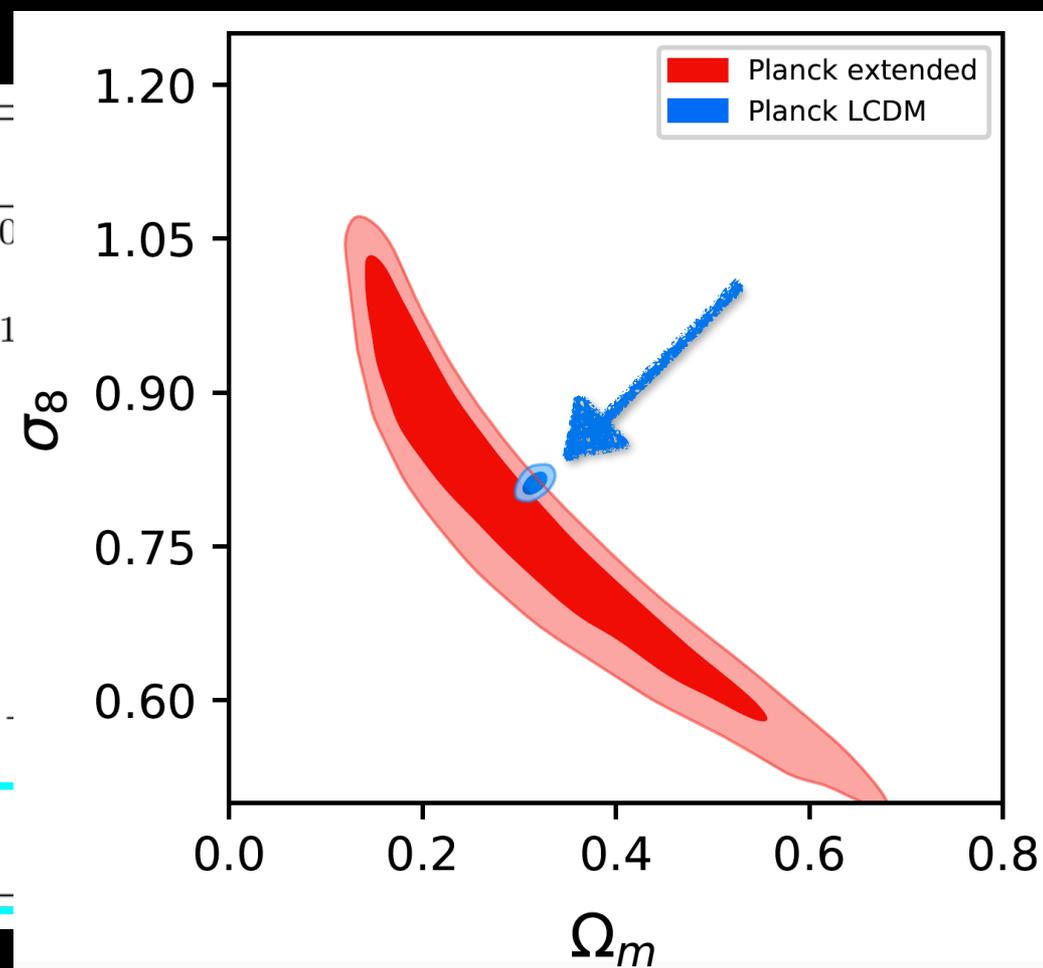
Parameters	Planck	Planck +R19	
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A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	1.
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n_s	0.964 ± 0.012	0.965 ± 0.013	0.9
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.0
H_0 [km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	(
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	0.



Since now datasets are fully compatible, we combine Planck 2018 with R19 ($H_0=74.03 \pm 1.42$ km/s/Mpc), in order to see which parameter is preferred by the data to solve the tension. **We find a phantom-like dark energy component with an equation of state $w < -1$ at more than three standard deviations, while the neutrino effective number is fully compatible with standard expectations.**

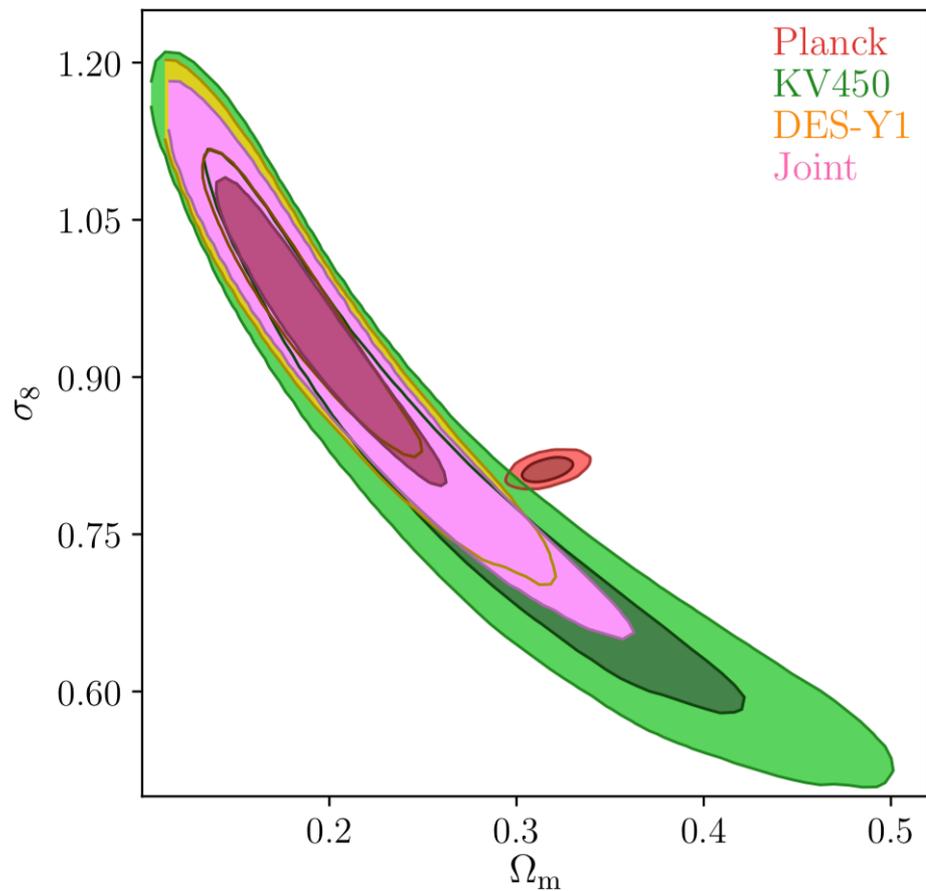
Beyond six parameters: extending Λ CDM

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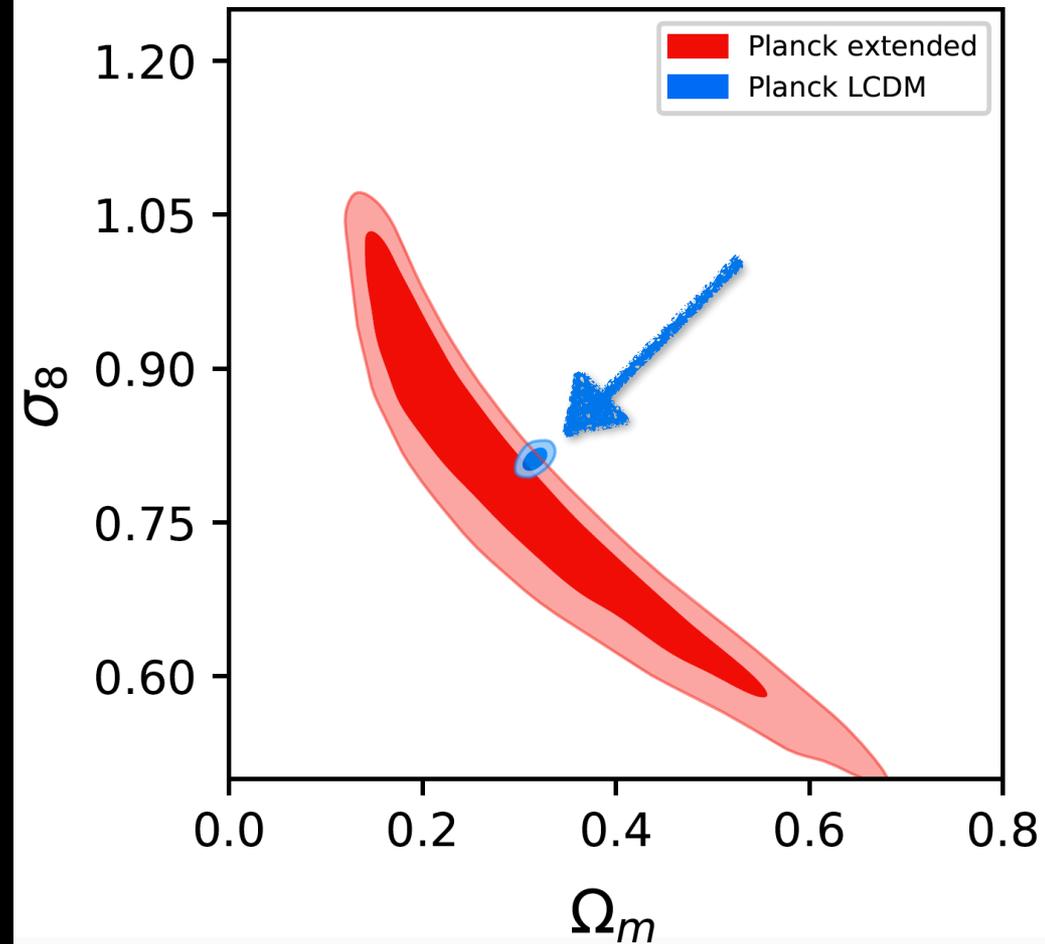


We find relaxed and lower values for the clustering parameter σ_8 and S_8 , with respect to those derived under the assumption of Λ CDM.

Beyond six parameters: extending Λ CDM



Asgari et al., arXiv:1910.05336 [astro-ph.CO]

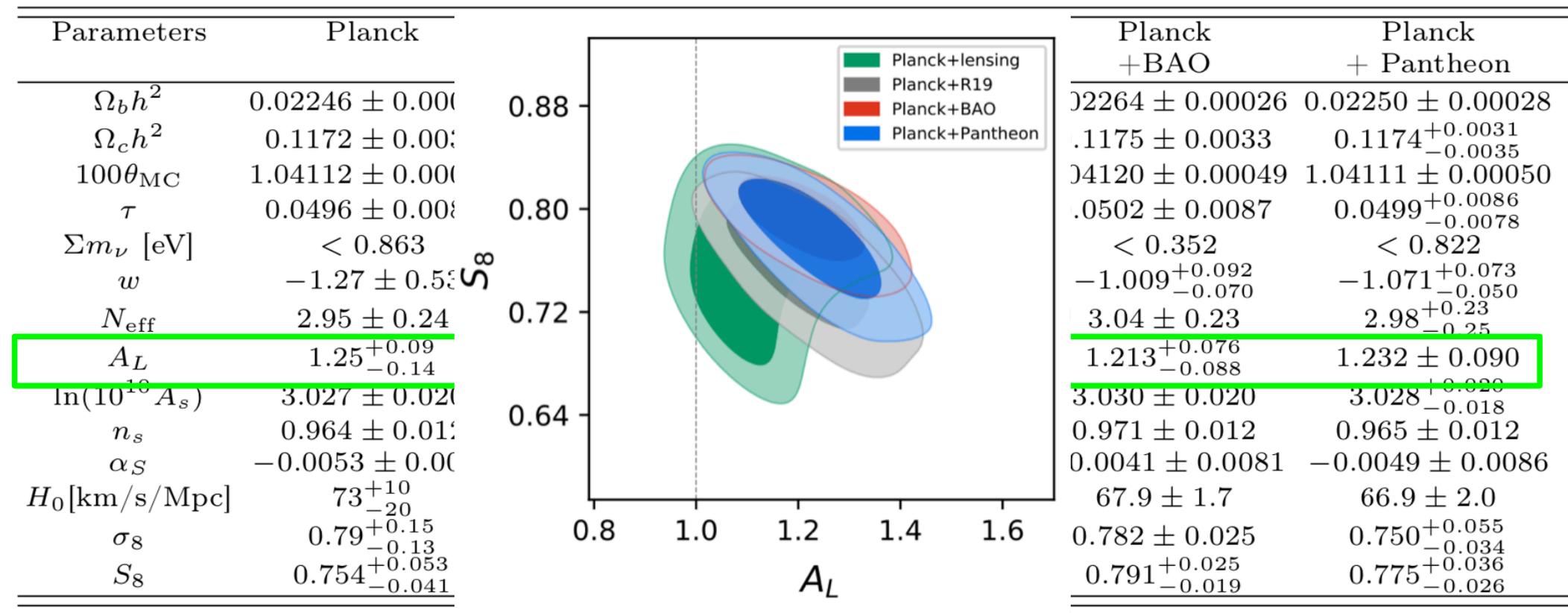


Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

In this way, we can solve the existing S_8 tensions with the CFHTLenS and KiDS-450 cosmic shear surveys.

Beyond six parameters: extending Λ CDM



And in fact, the only notable exception is the angular power spectrum lensing amplitude, A_L that is larger than the expected value at about 3 standard deviations even when combining the Planck data with BAO and supernovae type Ia external datasets.

But...

assuming General Relativity,
is there a **physical explanation**
for A_L ?

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H_0 with local measurements
- A_L internal anomaly
- S_8 with cosmic shear data
- Ω_k different from zero

See Di Valentino et al. [arXiv:2008.11283](https://arxiv.org/abs/2008.11283) [astro-ph.CO], [arXiv:2008.11284](https://arxiv.org/abs/2008.11284) [astro-ph.CO], [arXiv:2008.11285](https://arxiv.org/abs/2008.11285) [astro-ph.CO], [arXiv:2008.11286](https://arxiv.org/abs/2008.11286) [astro-ph.CO] for an overview.

Curvature of the universe

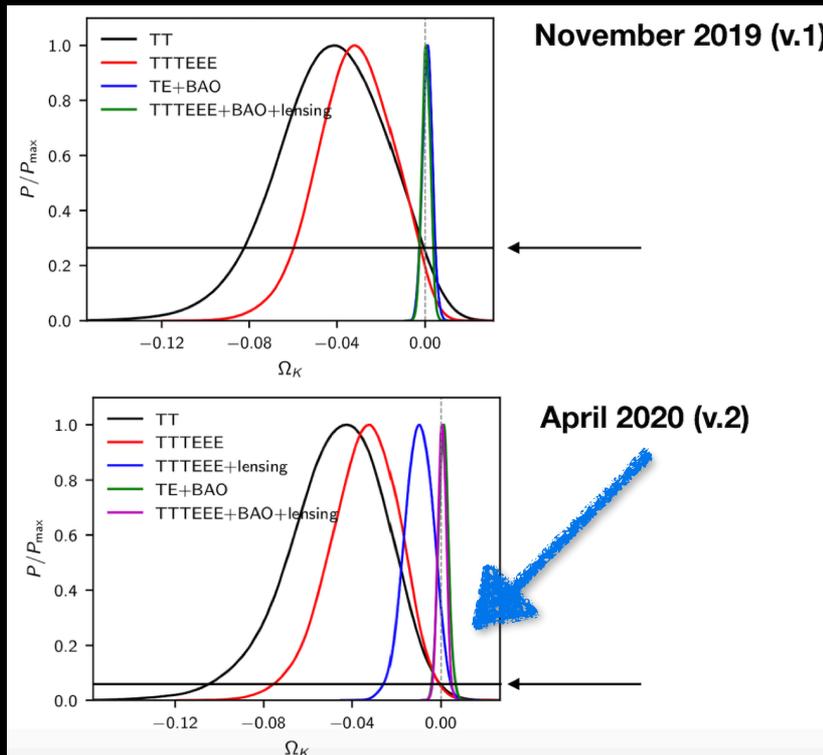
The Λ CDM model assumes that the universe is specially flat. The combination of the Planck temperature and polarization power spectra gives:

$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\%, \text{Planck TT,TE,EE+lowE}),$$

Planck 2018, *Astron.Astrophys.* 641 (2020) A6

a detection of curvature at about 3.4σ ,

with a 99% probability region of $-0.095 \leq \Omega_K \leq -0.007$.



This result has been obtained by using Plik, i.e. the baseline likelihood of Planck, but is NOW confirmed by CamSpec

$$-0.083 \leq \Omega_K \leq -0.001 \text{ at } 99\% \text{ CL.}$$

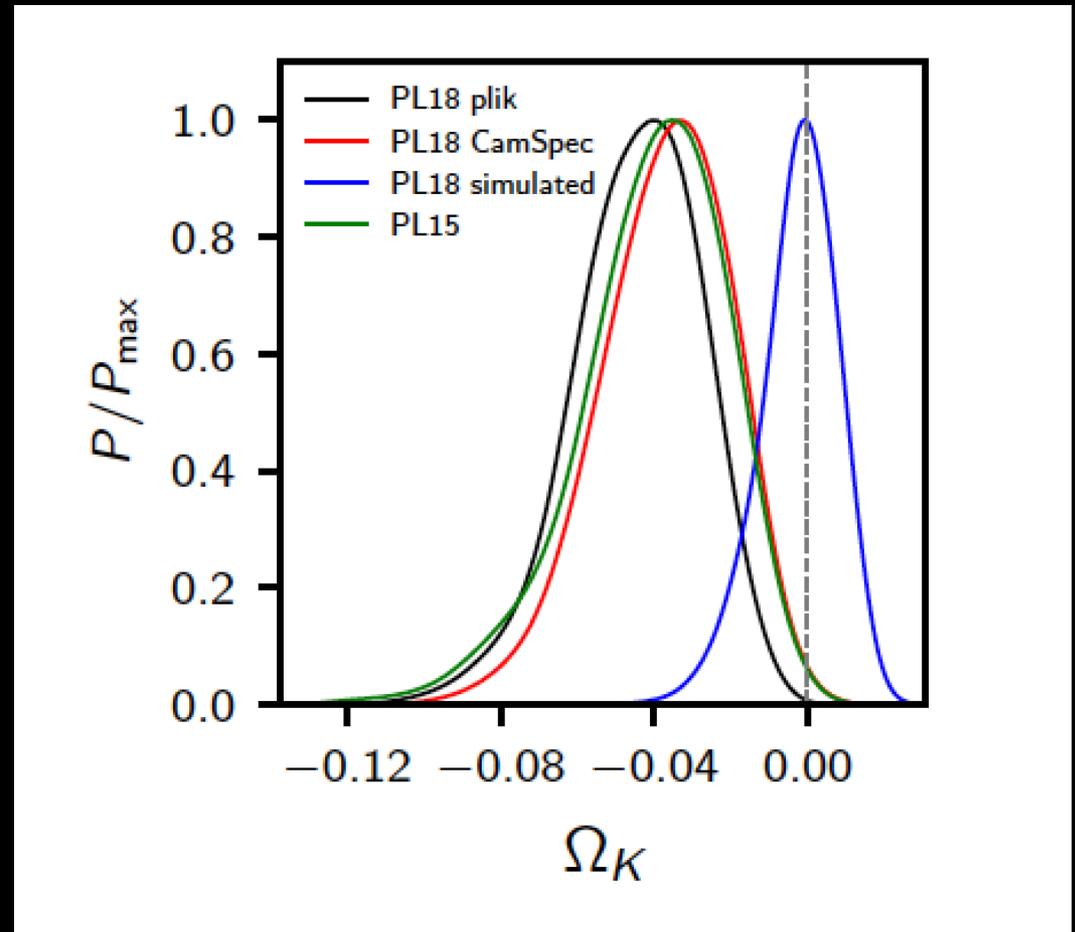
Curvature of the universe

Can Planck provide an **unbiased and reliable estimate** of the curvature of the Universe?

This may not be the case since a "geometrical degeneracy" is present with Ω_m .

When precise CMB measurements at arc-minute angular scales are included, since **gravitational lensing** depends on the matter density, its detection **breaks the geometrical degeneracy**. The Planck experiment with its improved angular resolution offers the unique opportunity of a precise measurement of curvature from a single CMB experiment.

We simulated Planck, finding that such experiment could constrain curvature with a 2% uncertainty, without any significant bias towards closed models.



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

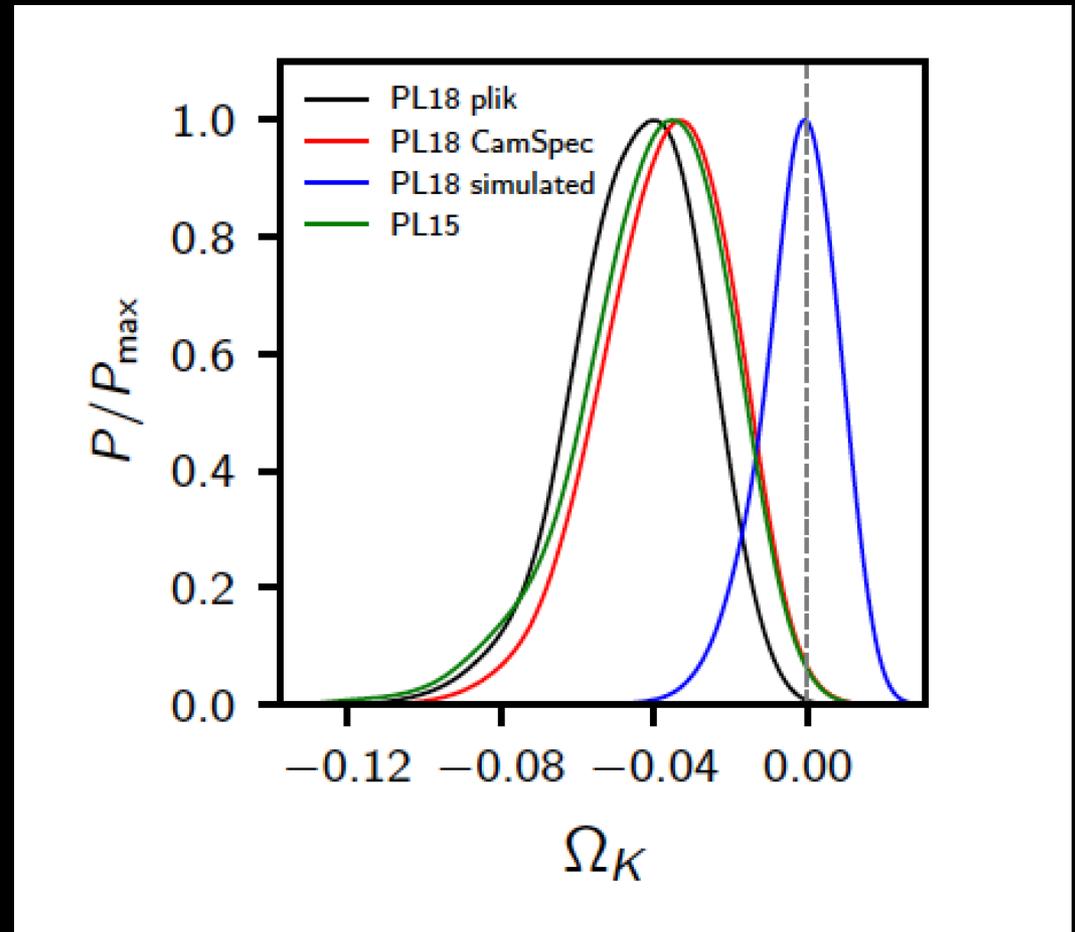
Curvature of the universe

Planck favours a closed Universe ($\Omega_K < 0$) with 99.985% probability.

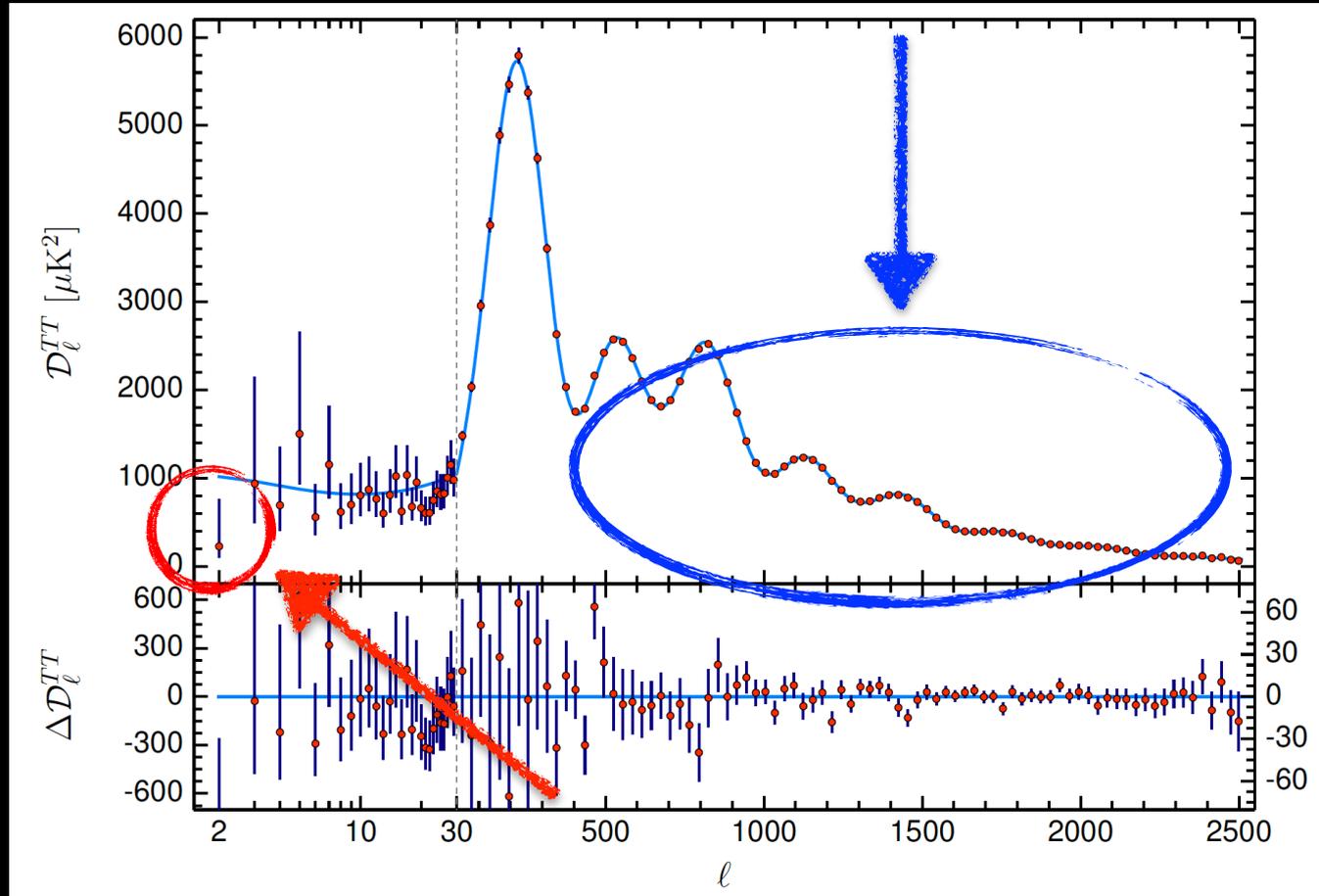
A closed Universe with $\Omega_K = -0.0438$ provides a better fit to PL18 with respect to a flat model.

This is not entirely a volume effect, since the best-fit $\Delta\chi^2$ changes by -11 compared to base Λ CDM when adding the one additional curvature parameter.

The improvement is due also to the fact that closed models could also lead to a large-scale cut-off in the primordial density fluctuations in agreement with the observed low CMB anisotropy quadrupole.



A closed universe fits Planck better than A_L



Planck 2018, Astron.Astrophys. 641 (2020) A6

A model with $\Omega_k < 0$ is slightly preferred with respect to a flat model with $A_L > 1$, because closed models better fit not only the damping tail, but also the low-multipole data, especially the quadrupole.

Astrophysics

[Submitted on 5 Mar 2003 (v1), last revised 30 Jul 2003 (this version, v2)]

Is the Low CMB Quadrupole a Signature of Spatial Curvature?

G. Efstathiou (University of Cambridge)

The temperature anisotropy power spectrum measured with the Wilkinson Microwave Anisotropy Probe (WMAP) at high multipoles is in spectacular agreement with an inflationary Lambda-dominated cold dark matter cosmology. However, the low order multipoles (especially the quadrupole) have lower amplitudes than expected from this cosmology, indicating a need for new physics. Here we speculate that the low quadrupole amplitude is associated with spatial curvature. We show that positively curved models are consistent with the WMAP data and that the quadrupole amplitude can be reproduced if the primordial spectrum truncates on scales comparable to the curvature scale.

Comments: 4 pages, Latex, 2 figs, revised version accepted by MNRAS

Subjects: Astrophysics (astro-ph)

Journal reference: Mon.Not.Roy.Astron.Soc. 343 (2003) L95

DOI: 10.1046/j.1365-8711.2003.06940.x

Cite as: arXiv:astro-ph/0303127

(or arXiv:astro-ph/0303127v2 for this version)

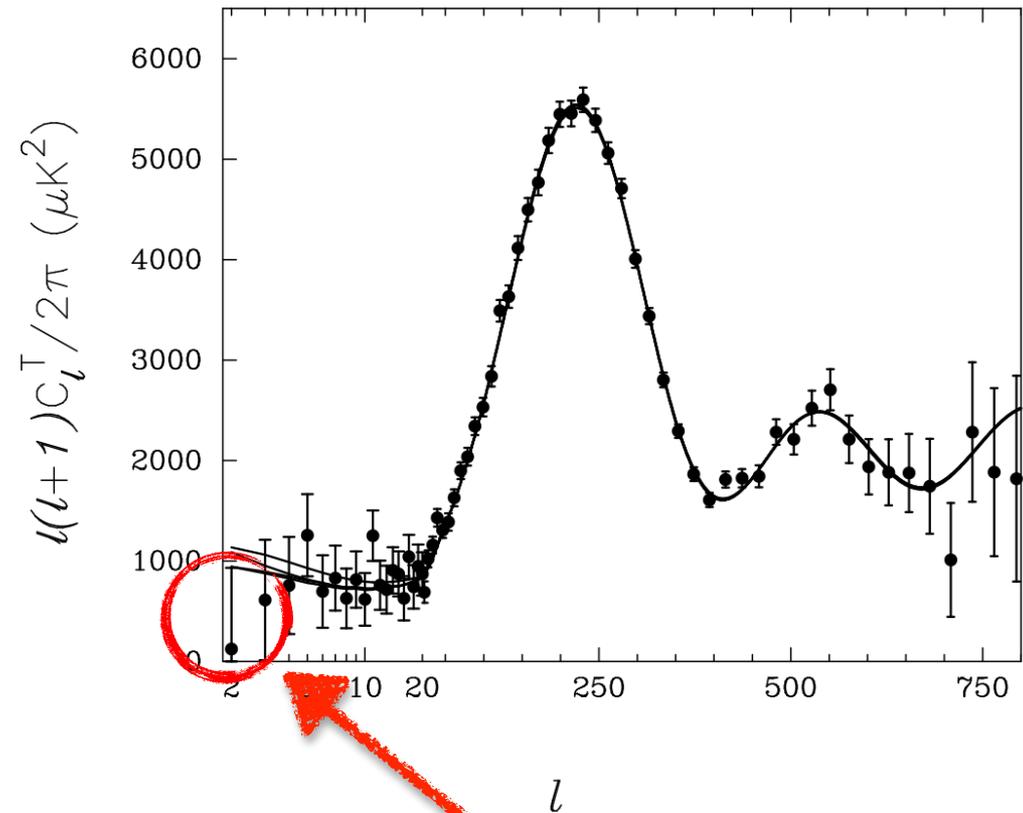
Submission history

From: George Efstathiou [view email]

[v1] Wed, 5 Mar 2003 23:30:33 UTC (21 KB)

[v2] Wed, 30 Jul 2003 10:16:45 UTC (22 KB)

A lower quadrupole than predicted by the Λ CDM was already present in WMAP, and a closed universe to explain this effect was already taken into account.



Curvature of the universe

To better quantify the preference for a closed model, we adopt the deviance information criterion (DIC), which takes into account the Bayesian complexity, that is, the effective number of parameters, of the extended model and is defined as

$$\text{DIC} = 2\overline{\chi_{\text{eff}}^2} - \chi_{\text{eff}}^2$$

where the bar denotes a mean over the posterior distribution. We find that the Planck data yield $\Delta\text{DIC} = -7.4$; that is, a closed Universe with $\Omega_k = -0.0438$ is preferred, with a probability ratio of about 1/41, with respect to a flat model.

Curvature of the universe

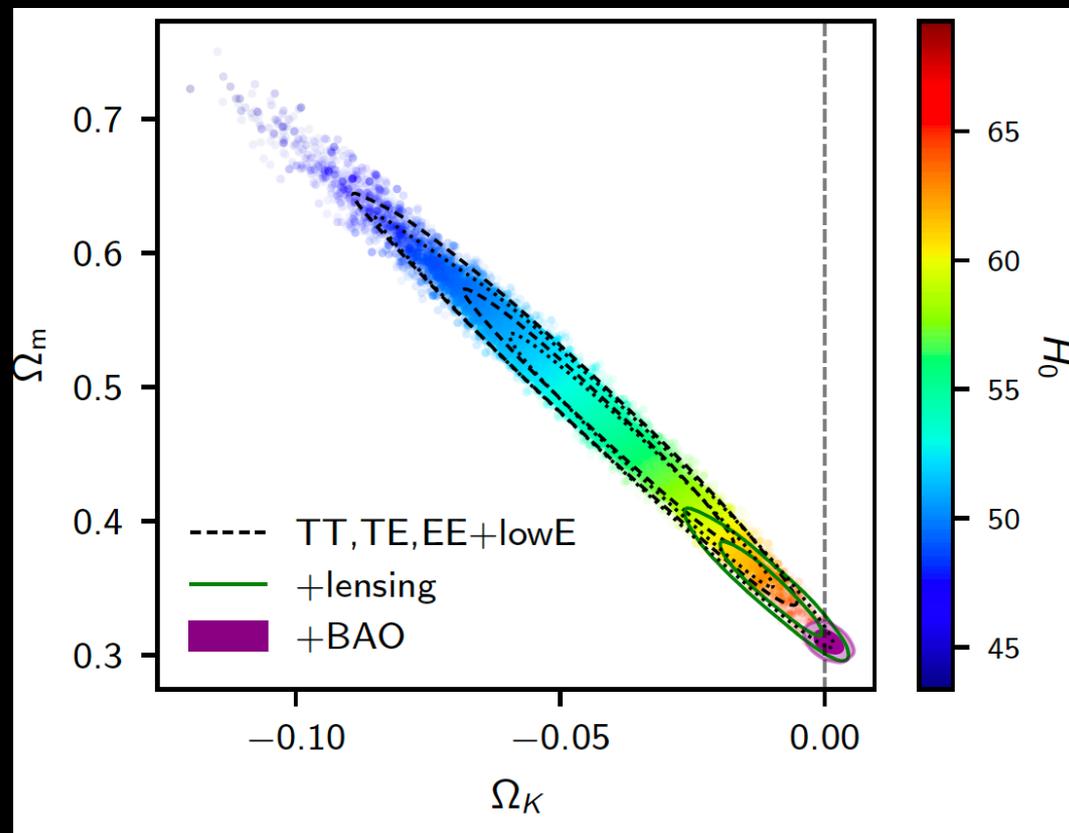
We also compute the **Bayesian evidence ratio by making use of the Savage–Dickey density ratio**. In this case the Bayes factor can be written as

$$B_{01} = \frac{p(\Omega_K | d, M_1)}{\pi(\Omega_K | M_1)} \Big|_{\Omega_K=0}$$

where M_1 denotes the model with curvature, $p(\Omega_K | d, M_1)$ is the posterior for Ω_K in this theoretical framework, computed from a specific dataset d , and $\pi(\Omega_K | M_1)$ is the prior on Ω_K that we assume to be flat in the range $-0.2 \leq \Omega_K \leq 0$.

For Planck we obtain a Bayes ratio of $|\ln B_{01}| = 3.3$, i.e. a strong evidence for a closed universe with respect to a flat one.

What about Planck+BAO?



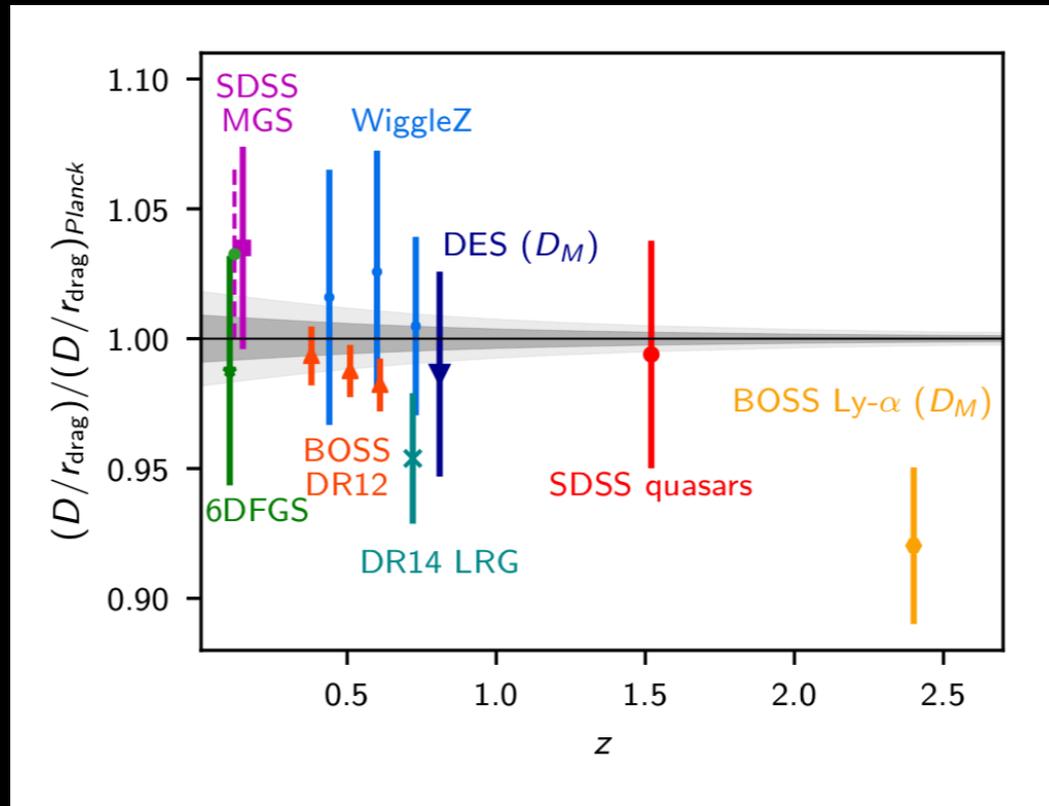
Planck 2018, *Astron.Astrophys.* 641 (2020) A6

Adding BAO data, a joint constraint is very consistent with a flat universe.

$$\Omega_K = 0.0007 \pm 0.0019 \quad (68\%, \text{TT, TE, EE+lowE} \\ \text{+lensing+BAO}).$$

Given the significant change in the conclusions from Planck alone, it is reasonable to **investigate whether they are actually consistent**. In fact, a basic assumption for combining complementary datasets is that these ones must be consistent, i.e. **they must plausibly arise from the same cosmological model**.

BAO tension

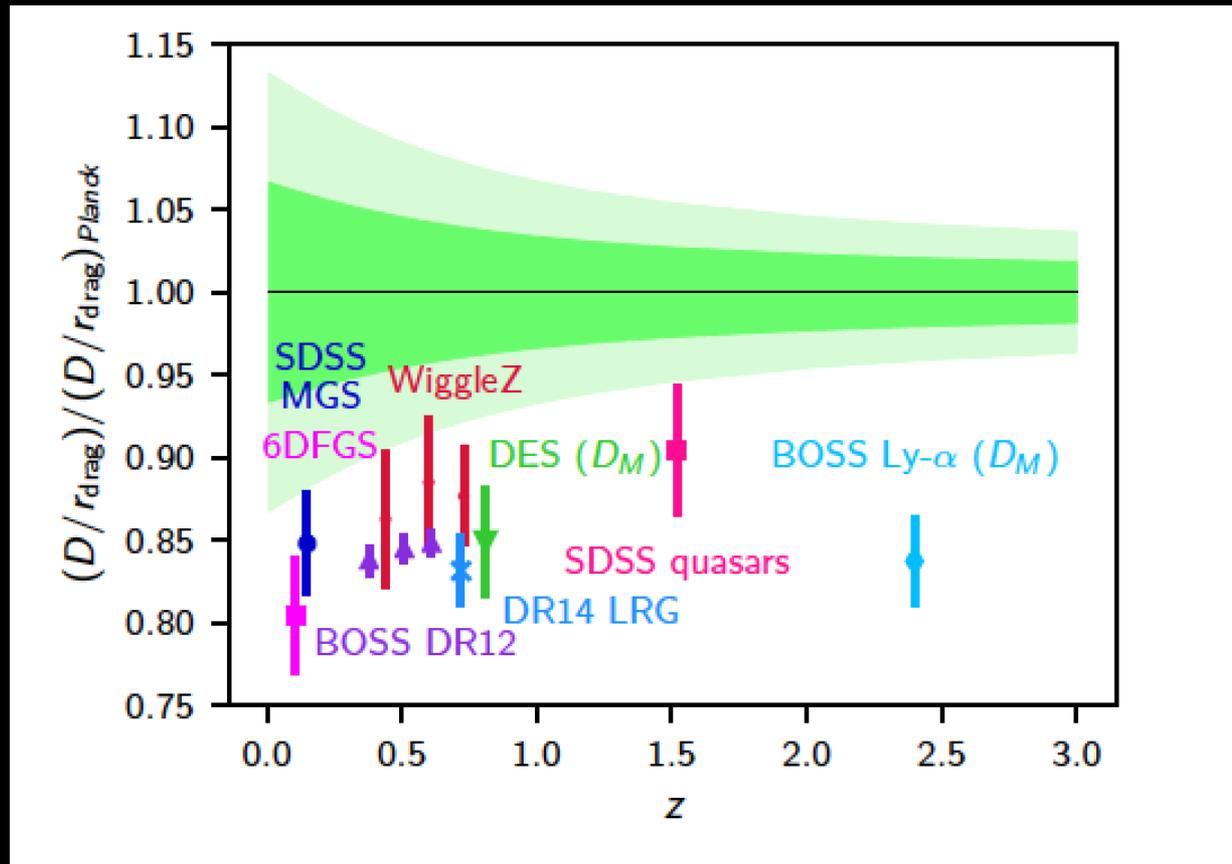


Planck 2018, Astron.Astrophys. 641 (2020) A6

This is a plot of the acoustic-scale distance ratio, $D_V(z)/r_{\text{drag}}$, as a function of redshift, taken from several recent BAO surveys, and divided by the mean acoustic-scale ratio obtained by Planck adopting a model. r_{drag} is the comoving size of the sound horizon at the baryon drag epoch, and D_V , the dilation scale, is a combination of the Hubble parameter $H(z)$ and the comoving angular diameter distance $D_M(z)$.

In a Λ CDM model the BAO data agree really well with the Planck measurements...

BAO tension



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

... but when we let curvature to vary
there is a striking disagreement between Planck spectra and BAO measurements!

BAO tension

Observable	Redshift	BAO (68% CL)	Planck (68% CL)	Tension
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.38	$1,518 \pm 22.8$	$1,843 \pm 100$	2.9σ
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.51	$1,977 \pm 26.9$	$2,361 \pm 115$	3.0σ
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.61	$2,283 \pm 32.3$	$2,726 \pm 130$	3.3σ
$H(r_{d,\text{fid}}/r_d)$ ($\text{km s}^{-1} \text{Mpc}^{-1}$)	0.38	81.5 ± 1.9	71.6 ± 3.3	2.6σ
$H(r_{d,\text{fid}}/r_d)$ ($\text{km s}^{-1} \text{Mpc}^{-1}$)	0.51	90.5 ± 1.97	78.9 ± 3.1	3.1σ
$H(r_{d,\text{fid}}/r_d)$ ($\text{km s}^{-1} \text{Mpc}^{-1}$)	0.61	97.3 ± 2.1	85.0 ± 3.0	3.3σ

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

In the Table we have the constraints on DM and H(z) from the recent analysis of BOSS DR12 data and the corresponding constraints obtained indirectly from Planck, assuming a Λ CDM model with curvature.

Planck is inconsistent with each of the BAO measurements at more than 3σ !

The assumption of a flat universe could therefore mask a cosmological crisis where disparate observed properties of the Universe appear to be mutually inconsistent.

BAO tension

Additional dataset	$\Delta\chi^2_{\text{eff}}$	ΔN_{data}	$\log_{10}\mathcal{I}$
flat Λ CDM			
+ BAO	+6.15	8	0.2
+ CMB lensing	+8.9	9	0.6
Λ CDM + Ω_K			
+ BAO	+16.9	8	-1.8
+ CMB lensing	+16.9	9	-0.84

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

As we can see from the Table, the **Planck χ^2 best fit is worse by $\Delta\chi^2 \approx 16.9$ when the BAO data are included** under the assumption of curvature. This is a significantly larger $\Delta\chi^2$ than obtained for the case of Λ CDM ($\Delta\chi^2 \approx 6.15$).

The BAO dataset that we adopted consists of two independent measurements (6dFGS36 and SDSS-MGS37) with relatively large error bars, and six correlated measurements from BOSS DR12.

BAO tension

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Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

To quantify the discrepancy between two cosmological datasets, D1 and D2, we use the following quantity based on the DIC approach:

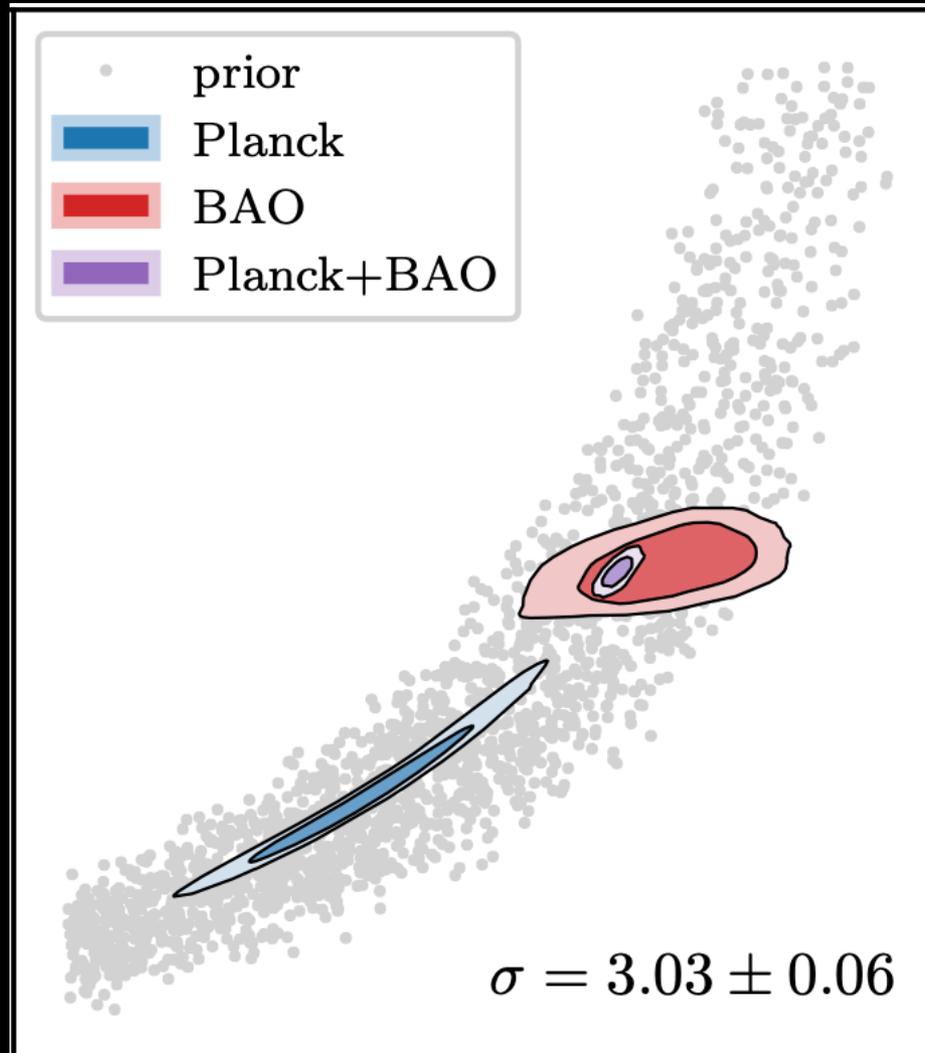
$$\mathcal{I}(D_1, D_2) \equiv \exp\{-\mathcal{F}(D_1, D_2)/2\}$$

where

$$\mathcal{F}(D_1, D_2) = \text{DIC}(D_1 \cup D_2) - \text{DIC}(D_1) - \text{DIC}(D_2)$$

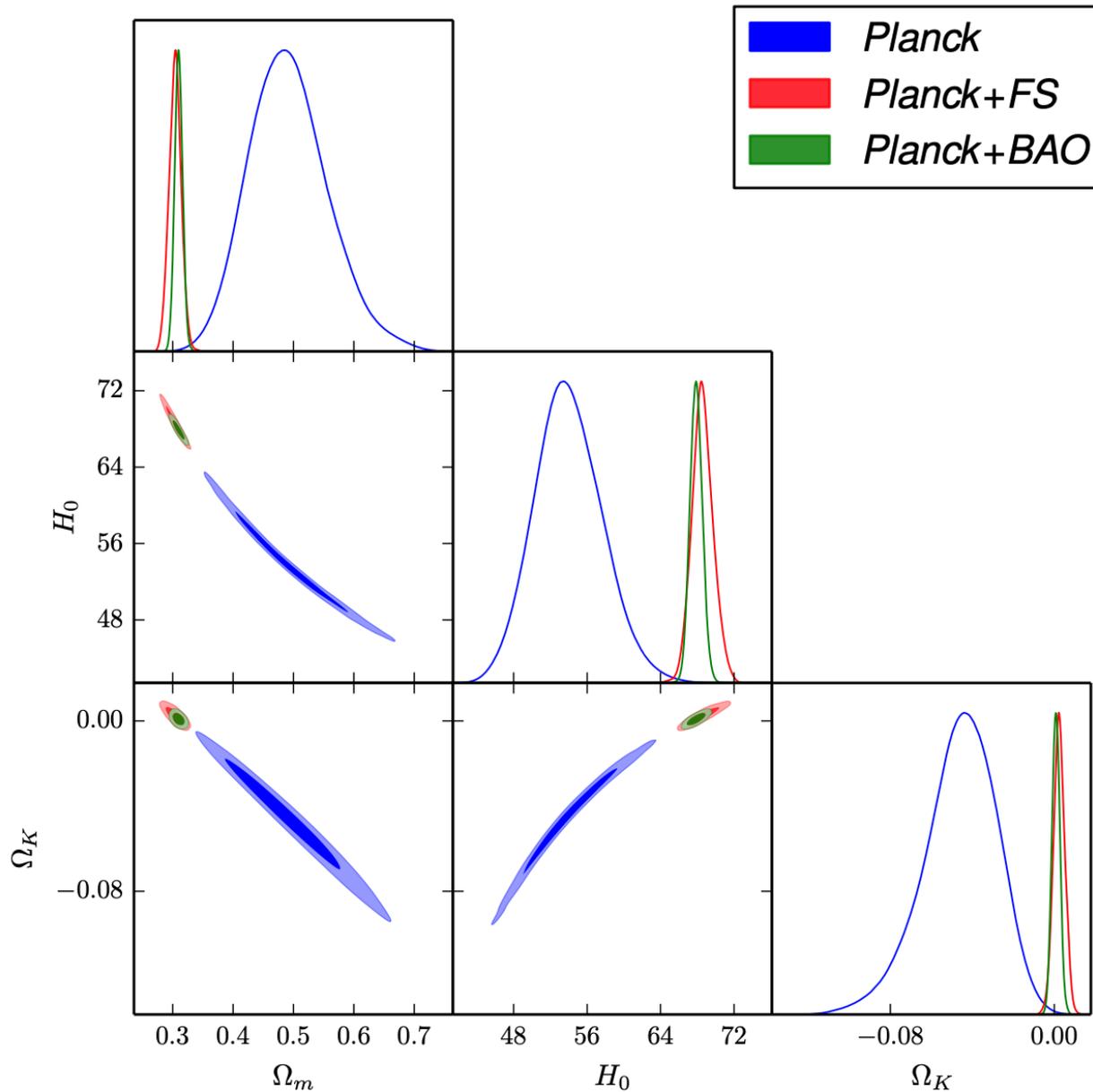
Following the Jeffreys scale the agreement/disagreement is considered ‘substantial’ if $|\log_{10} \mathcal{I}| > 0.5$, ‘strong’ if $|\log_{10} \mathcal{I}| > 1.0$ and ‘decisive’ if $|\log_{10} \mathcal{I}| > 2.0$. When is positive, then two datasets are in agreement, whereas they are in tension if this parameter is negative. We find a **strong disagreement between Planck and BAO**.

BAO tension



In agreement with
Handley, Phys.Rev.D 103 (2021) 4, L041301

What about Planck+FS?



The strong disagreement between Planck and BAO is evident in this triangular plot, as well as that with the full-shape (FS) galaxy power spectrum measurements from the BOSS DR12 CMASS sample, at an effective redshift $z_{\text{eff}} = 0.57$.

For Planck and FS we find $\log_{10} I \sim -2.5$, i.e. a decisive disagreement on the Jeffreys-like scale.

CMB lensing tension

Additional dataset	$\Delta\chi^2_{\text{eff}}$	ΔN_{data}	$\log_{10}\mathcal{I}$
flat Λ CDM			
+ BAO	+6.15	8	0.2
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+ BAO	+16.9	8	-1.8
+ CMB lensing	+16.9	9	-0.84

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

Another tension is present between Planck power spectra and the constraints on the lensing potential derived from the four-point correlation function of Planck CMB maps.

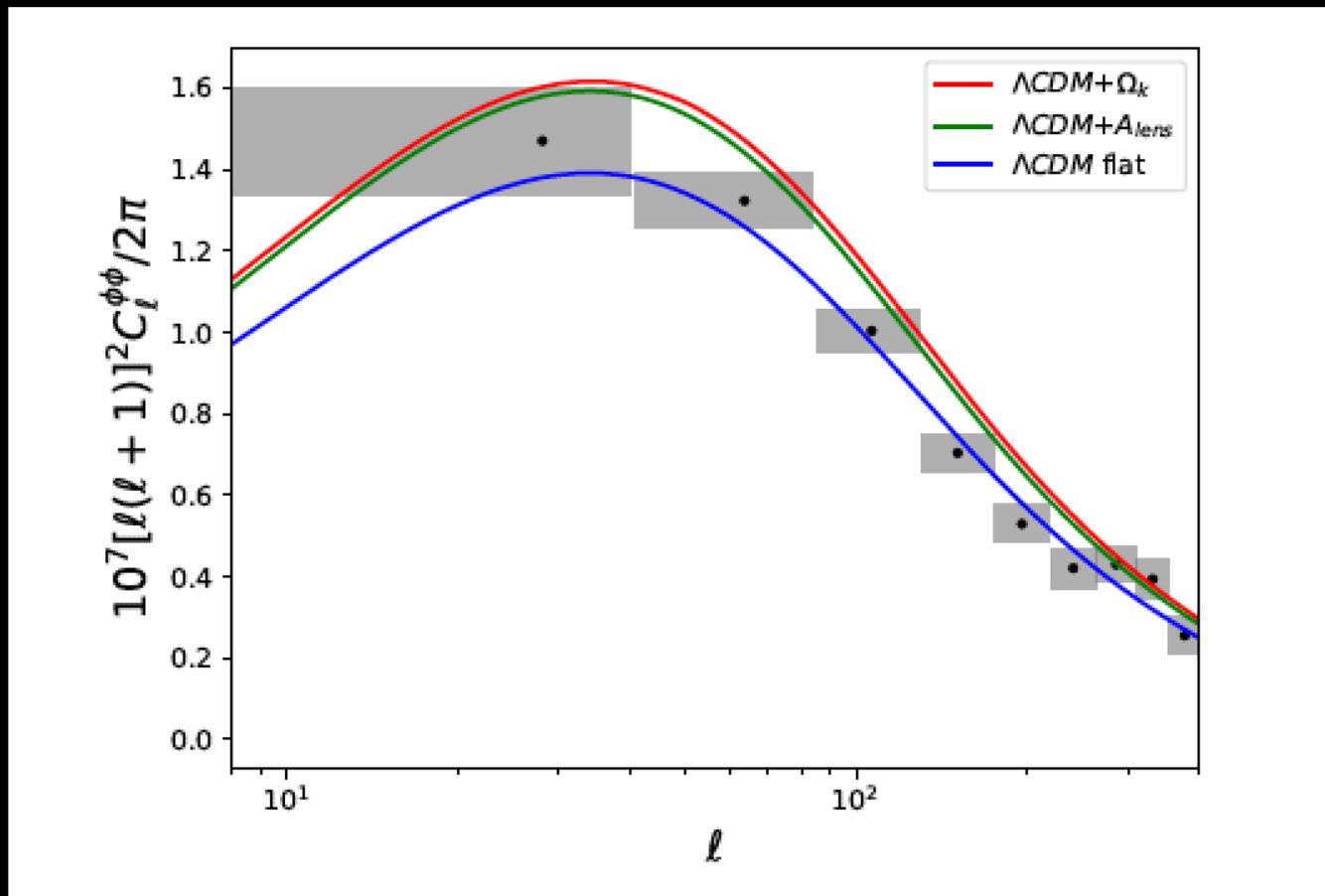
The inclusion of CMB lensing in Planck increases the best-fit $\Delta\chi^2 = 16.9$ in the case of Λ CDM + Ω_K (while in the case of the Λ CDM model, we have $\Delta\chi^2 = 8.9$). The CMB lensing dataset consists of nine correlated data points.

We identify **substantial discordance between Planck and CMB lensing.**

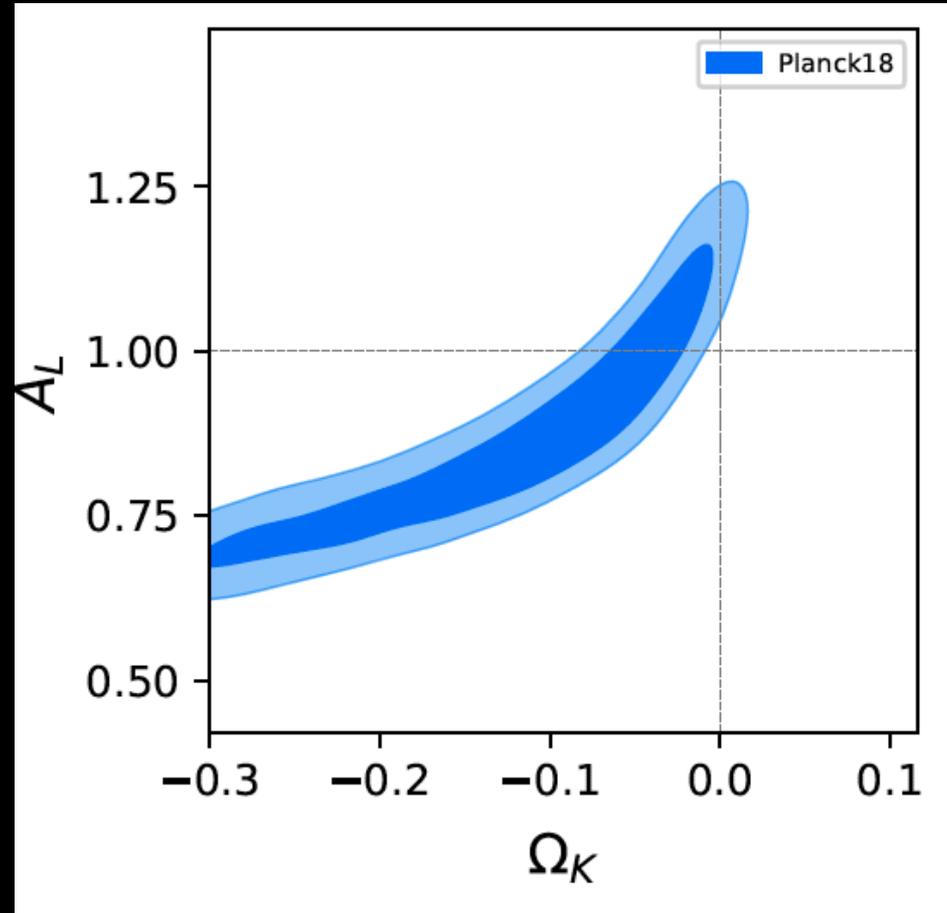
The combination of Planck with external datasets should be, therefore, considered with caution when working within a non-flat Universe.

What about CMB lensing?

Closed models predict substantially higher lensing amplitudes than in Λ CDM, because the dark matter content can be greater, leading to a larger lensing signal. The reasons for the pull towards negative values of Ω_K are essentially the same as those that lead to the preference for $A_L > 1$.



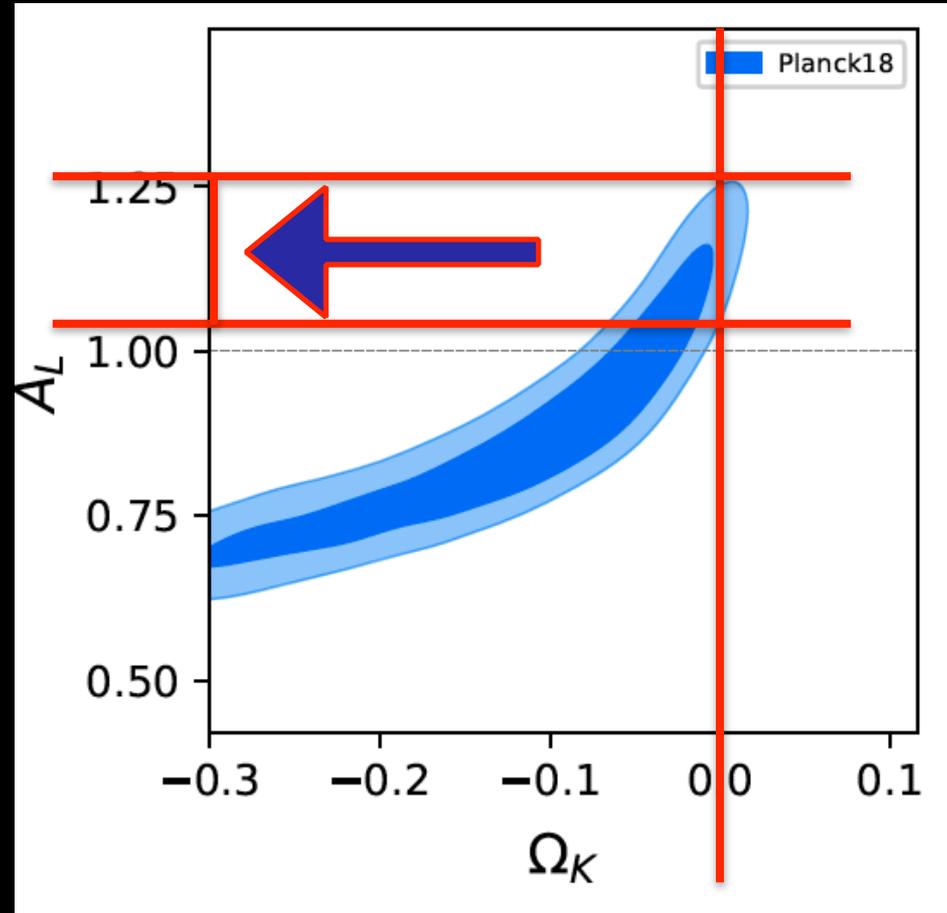
A closed universe (Friedmann 1922) can explain A_L !



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

A degeneracy between curvature and the A_L parameter is clearly present. A closed universe can provide a robust physical explanation to the enhancement of the lensing amplitude. In fact, the curvature of the Universe is not new physics beyond the standard model, but it is predicted by the General Relativity, and depends on the energy content of the Universe.

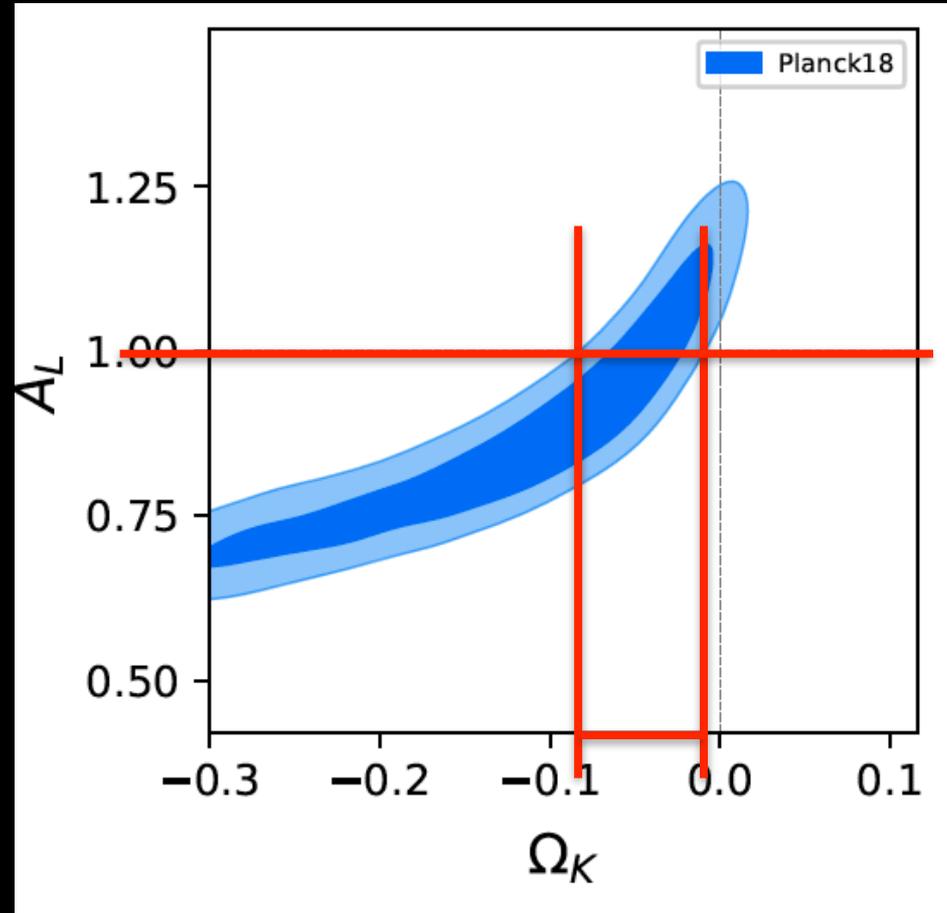
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The evolution over time of the geometry of the universe is described by
Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

which relate the purely geometric properties of space-time, with the distribution of energy of the universe. For this it is sufficient to know the energy content of the Universe to determine its geometry and vice-versa.

Adopting a 4-dimensional coordinate system for the space-time and the Cosmological Principle, i.e. a universe homogeneous and isotropic at large scales, the resulting metric is the **Friedmann-Lemaitre-Robertson-Walker (FLRW)**, that describes the distance between two events in space-time.

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \right]$$

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The curvature parameter k can be positive, null or negative, depending on the value of the curvature of the universe: positive, flat or negative.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

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$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Combining together the **FLRW metric** and **Einstein's equations** we obtain the **Friedmann equations** that describe the **expansion history of the universe**:

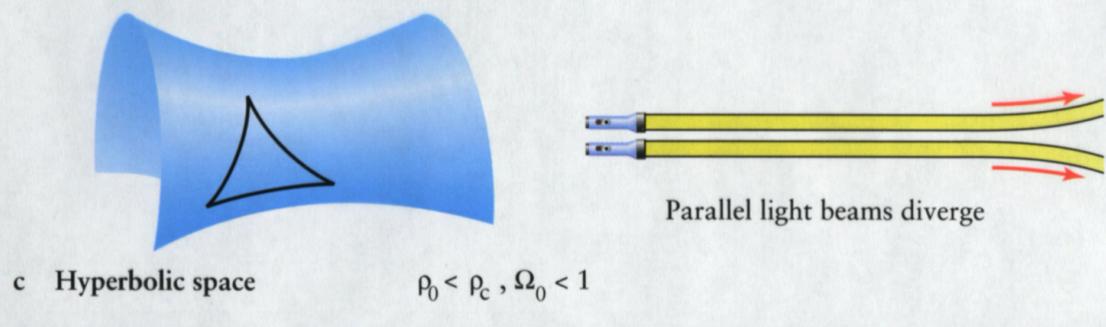
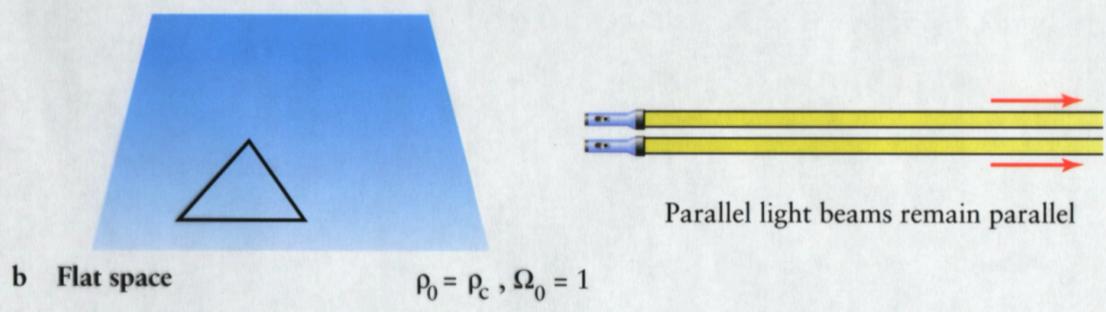
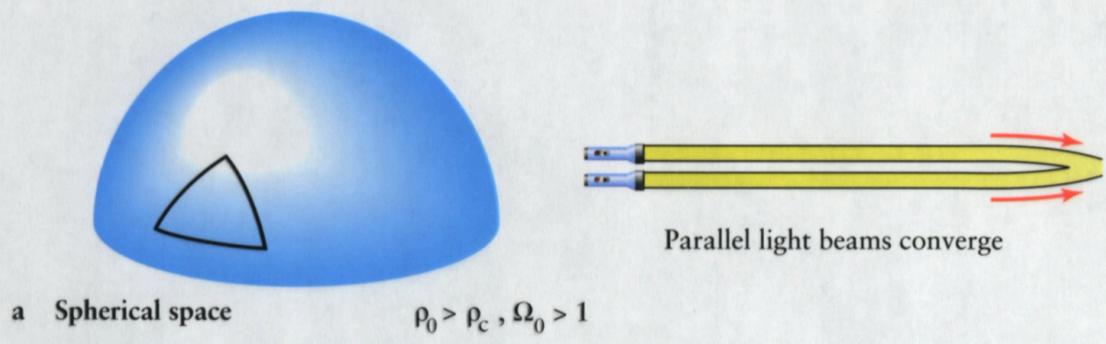
1st

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

2ⁿ

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

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If we divide the **1st Friedmann equation**, for the critical density (density of a flat universe), we obtain today:

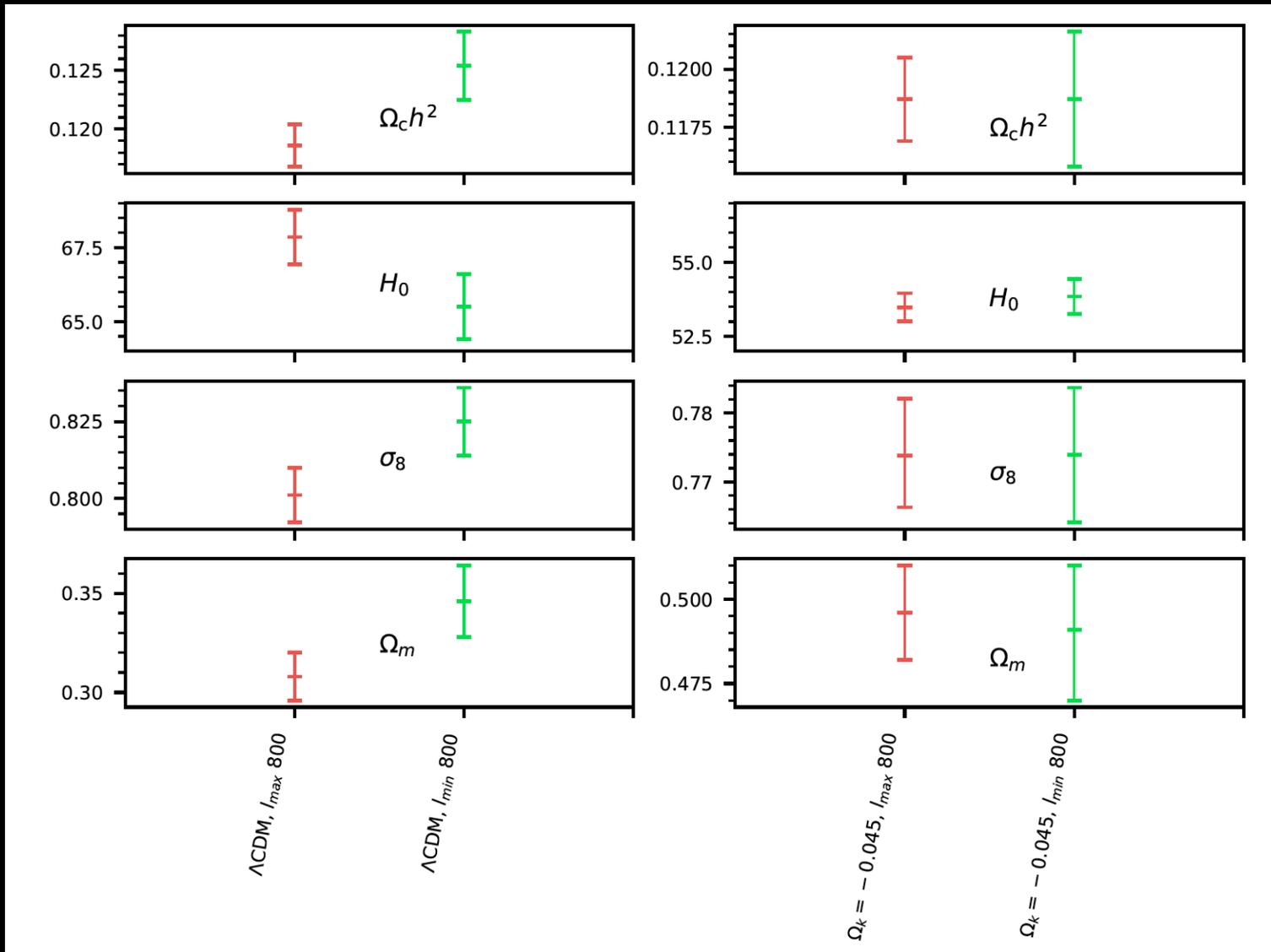
$$\Omega = \sum_i \Omega_i = \Omega_m + \Omega_\Lambda + \Omega_r = 1 - \Omega_k$$

From this equation it is possible to estimate the curvature of the universe, independently measuring the various contributions to the total density parameter Ω .

Figure: <http://w3.phys.nthu.edu.tw>

{	$\Omega > 1 \quad \Omega_k < 0$	$k > 0$: closed Universe
	$\Omega = 1 \quad \Omega_k = 0$	$k = 0$: flat Universe
	$\Omega < 1 \quad \Omega_k > 0$	$k < 0$: open Universe

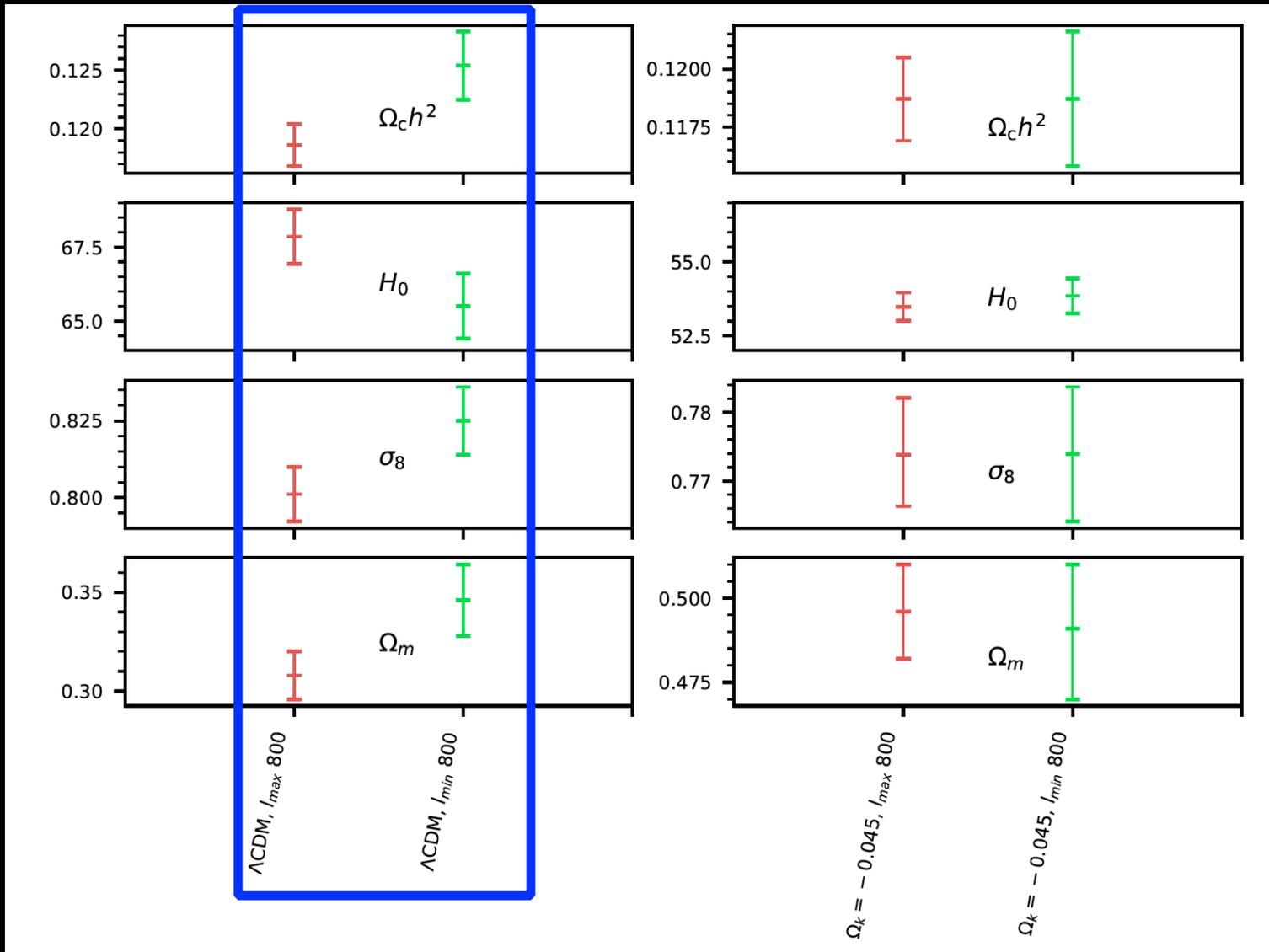
Curvature can explain internal tension



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

In a closed Universe with $\Omega_k = -0.045$, the cosmological parameters derived in the two different multipole ranges are now fully compatible.

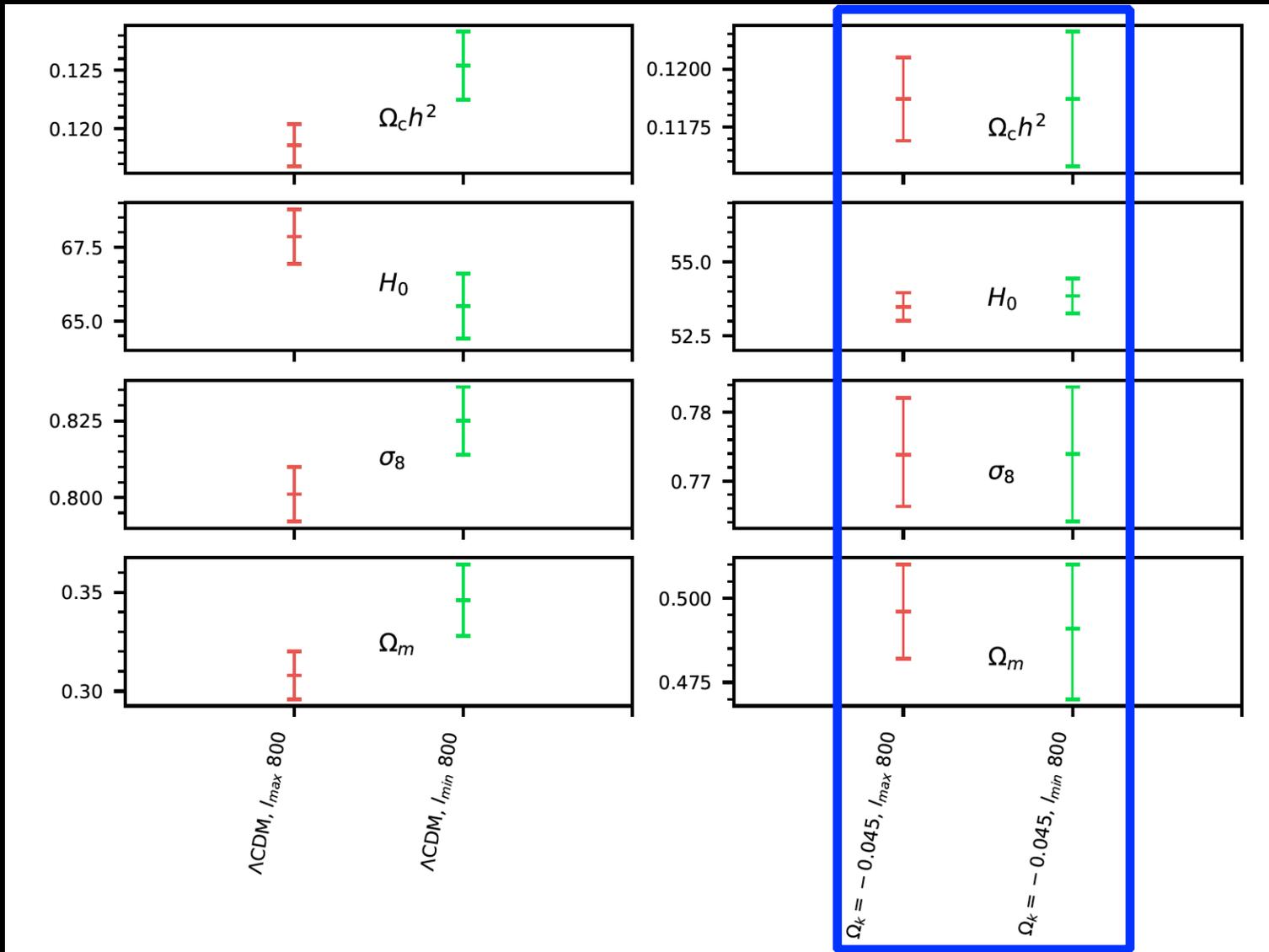
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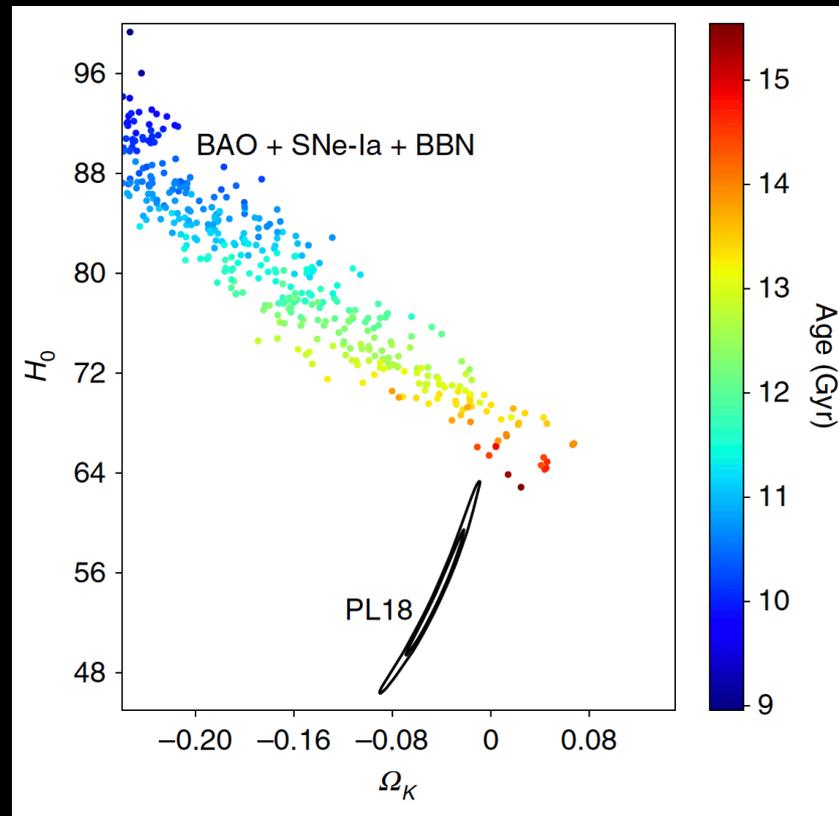
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What about non-CMB data?



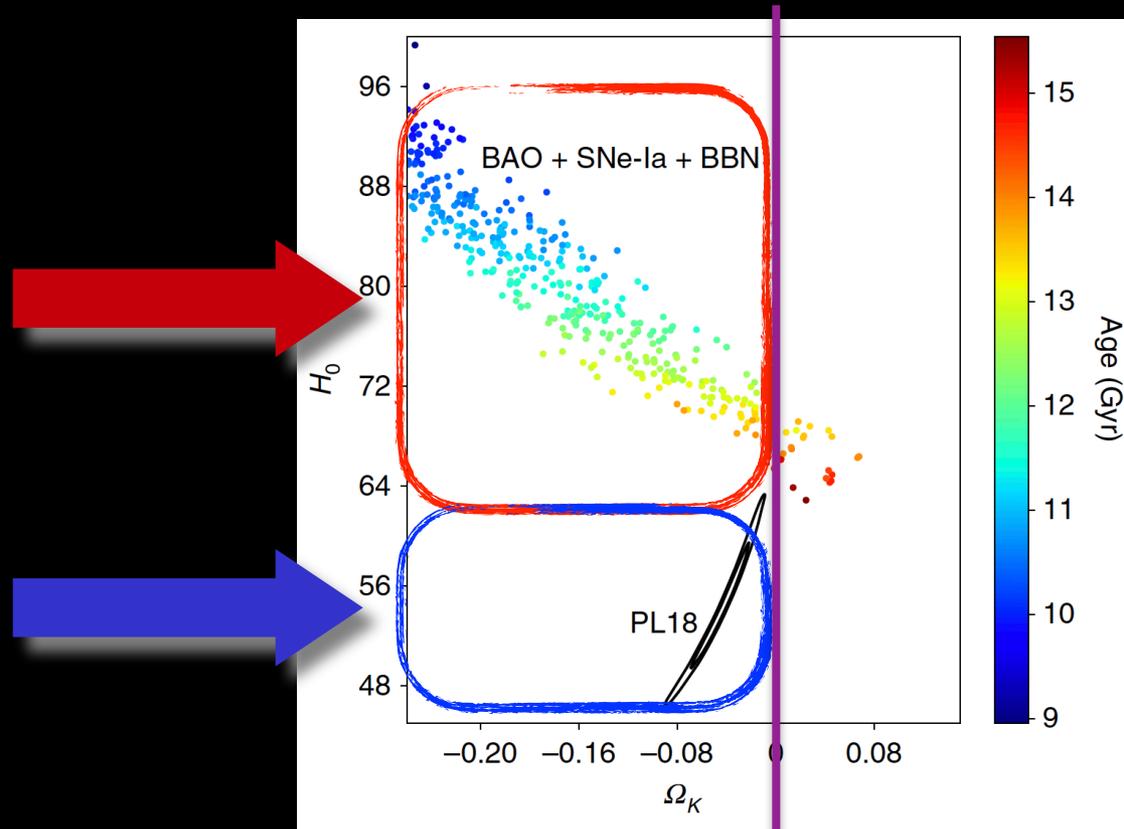
Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

It is now interesting to address the **compatibility of Planck with combined datasets**, like BAO + type-Ia supernovae + big bang nucleosynthesis data.

In principle, **each dataset prefers a closed universe**,

but **BAO+SN-Ia+BBN gives $H_0 = 79.6 \pm 6.8$ km/s/Mpc** at 68%cl, perfectly consistent with R20, but at 3.4σ tension with Planck.

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Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

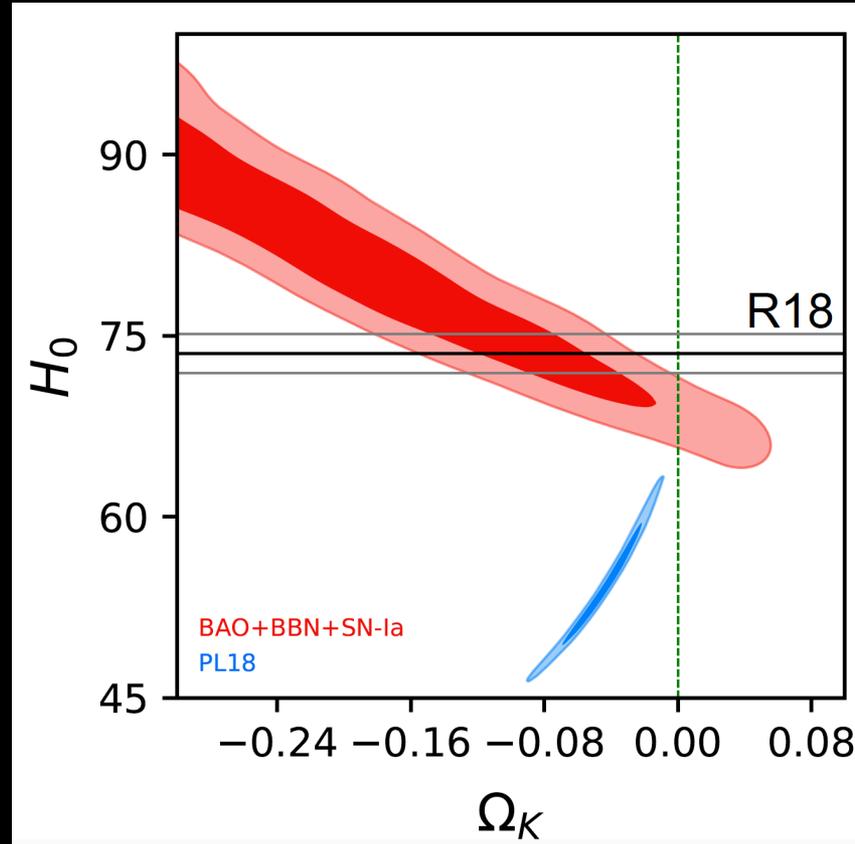
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BAO+SNIa+BBN+R18 gives $\Omega_K = -0.091 \pm 0.037$ at 68%cl.

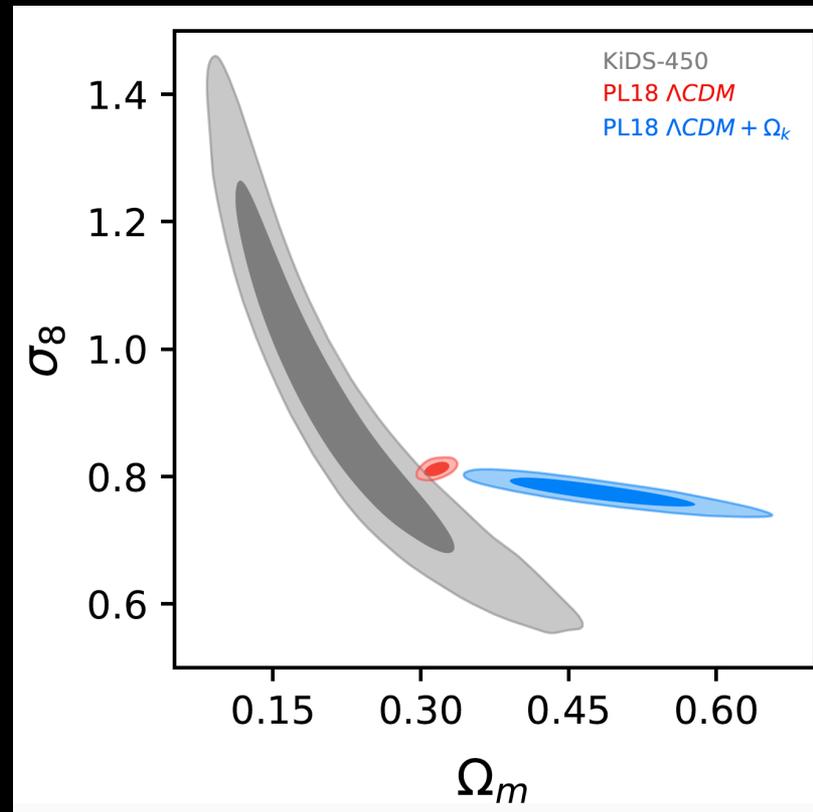
Curvature can't explain external tensions



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

Varying Ω_K , both the well know tensions on H_0 and S_8 are exacerbated.
In a Λ CDM + Ω_K model, Planck gives $H_0 = 54.4^{+3.3}_{-4.0}$ km/s/Mpc at 68% cl., increasing the tension with R20 at 5.4σ .

Curvature can't explain external tensions



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

Varying Ω_k , both the well know tensions on **H0** and **S8** are exacerbated.

In a Λ CDM + Ω_k model, Planck gives S8 in disagreement at about 3.8σ with KiDS-450, and more than 3.5σ with DES.

Major objections

- Uniform prior on Ω_k instead of a prior peaked in zero, as predicted by inflation.

The inflation is a model that needs to be tested against the data, not the contrary.

The prior is flat and uniform on Ω_k because we are looking for a constraint independent from any underlying theoretical model.

We should use the CMB data to derive observational constraints Ω_k , therefore an inflationary prior that strongly prefers a flat Universe could bias our results.

Major objections

- Use of the low multipoles ($l < 30$) data showing an amplitude suppression as predicted by a closed universe.

For a curved universe the primordial power spectrum used by the Boltzmann code to analyse the data is parametrised as:

$$\Delta(k) = \frac{(q^2 - 4K)^2}{q(q^2 - K)} k^{n_s - 1} \quad q = \sqrt{k^2 + K}$$

where K is the curvature parameter (+1 = closed, 0 = flat, -1 = open).

This form ensures that potential fluctuations are constant per logarithmic interval in wavenumber k . This is a strong assumption about how primordial fluctuations behave to scales larger than the curvature scale, and wants to generalize the concept of scale-invariant fluctuations to scales close to it.

This has not a theoretical motivation, so the χ^2 shouldn't be over-interpreted.

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- Use of the low multipoles ($l < 30$) data showing an amplitude suppression as predicted by a closed universe.

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$$q = \sqrt{k^2 + K}$$

where K is the curvature parameter (+1 = closed, 0 = flat, -1 = open).

A more accurate predictions for the primordial power spectrum in a curved Universe can be found in
Handley, Phys. Rev. D100 (2019) 123517,
and this increases the evidence for a closed universe from Planck.

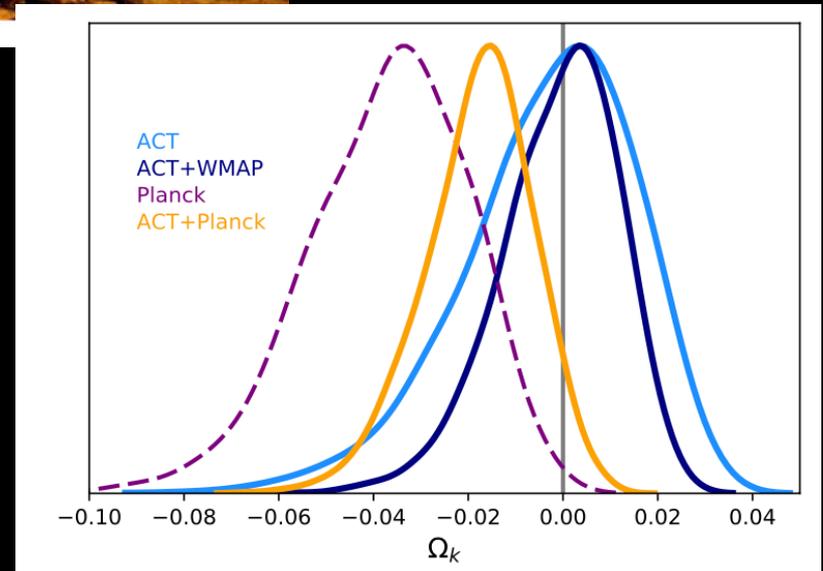
What about different CMB experiments?



To thicken the mystery we have the new ACT results:

ACT-DR4 + WMAP gives at 68% CL

$$\Omega_k = -0.001 \pm 0.012$$



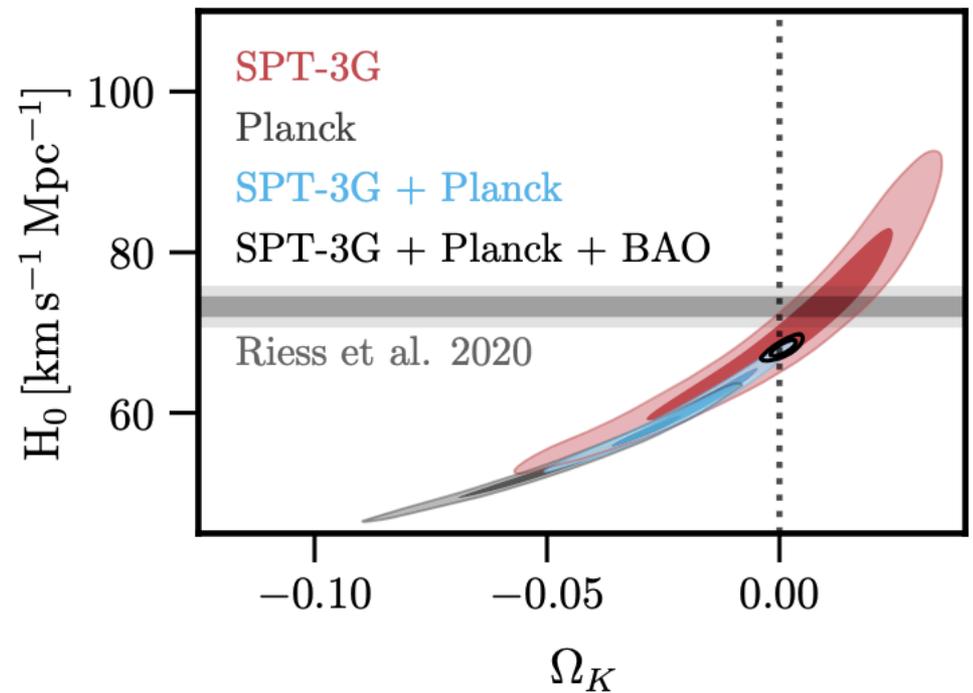
What about different CMB experiments?

CMB Polarization
Measurements
with SPTpol

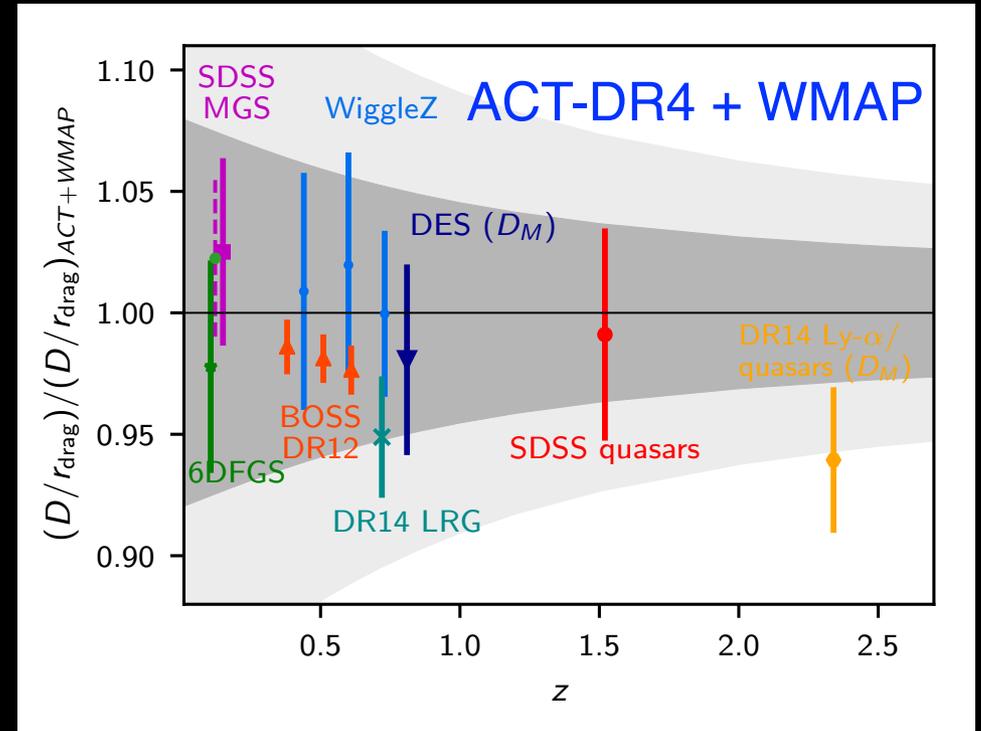
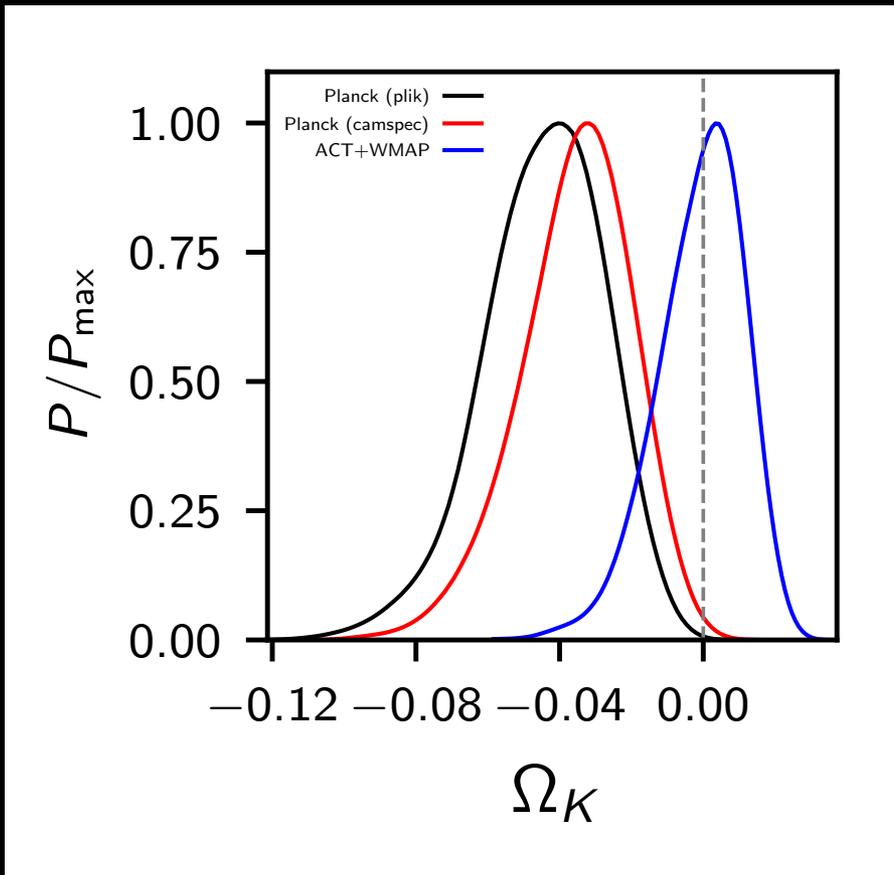
Nicholas Harrington
UC Berkeley

SPT-3G gives at 68% CL:

$$\Omega_K = 0.001^{+0.018}_{-0.019}$$



ACT-DR4

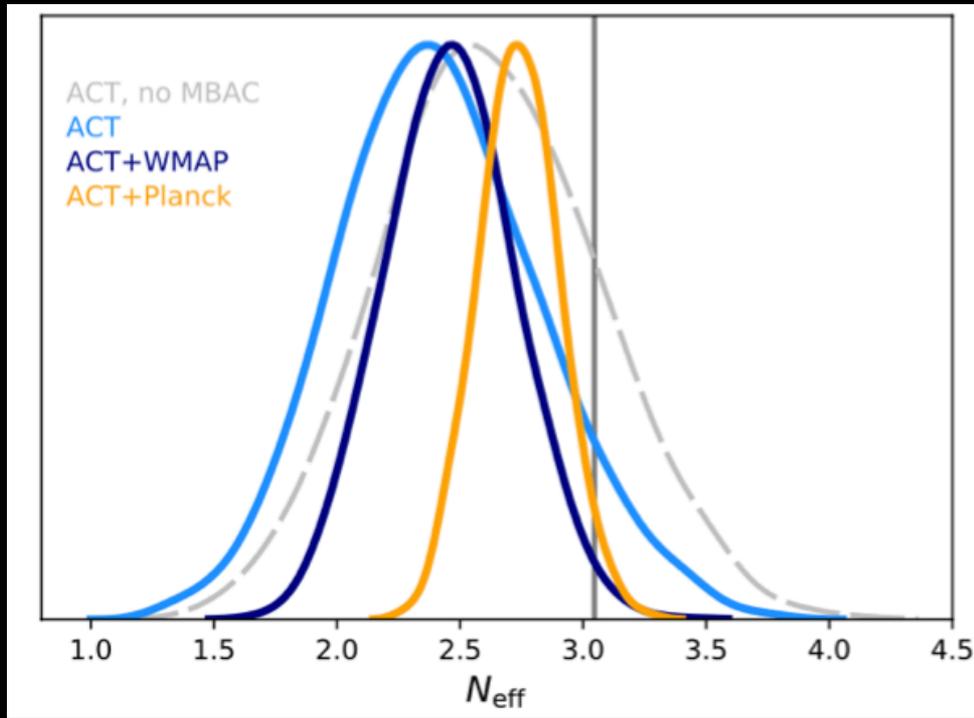


Di Valentino et al. in preparation

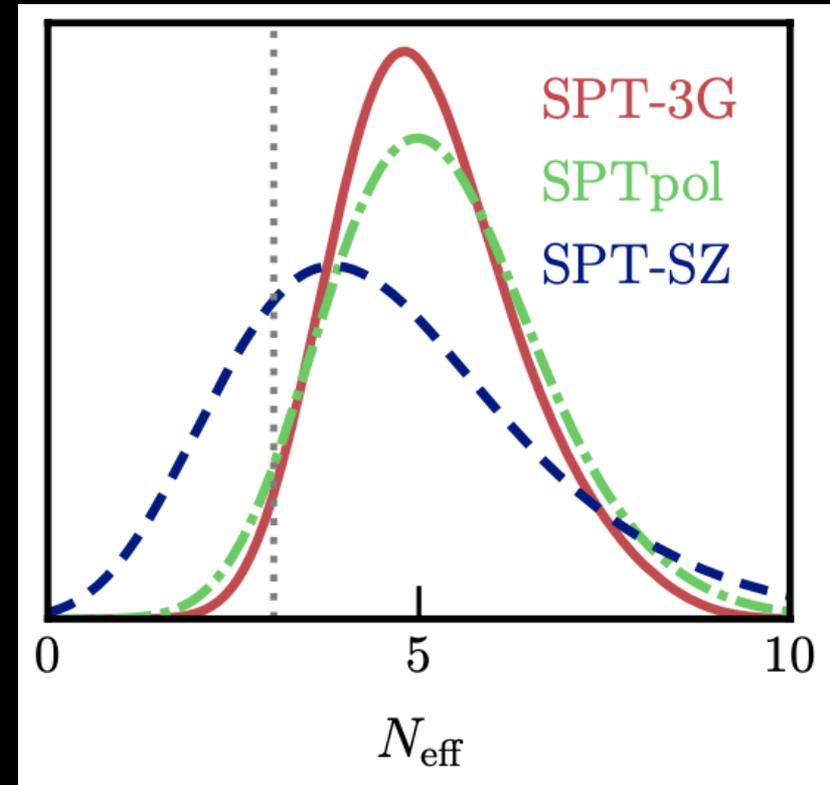
Confirmation of our result!

When precise CMB measurements at arc-minute angular scales are included, since gravitational lensing depends on the matter density, its detection **breaks the geometrical degeneracy**.

ACT-DR4 vs SPT-3G

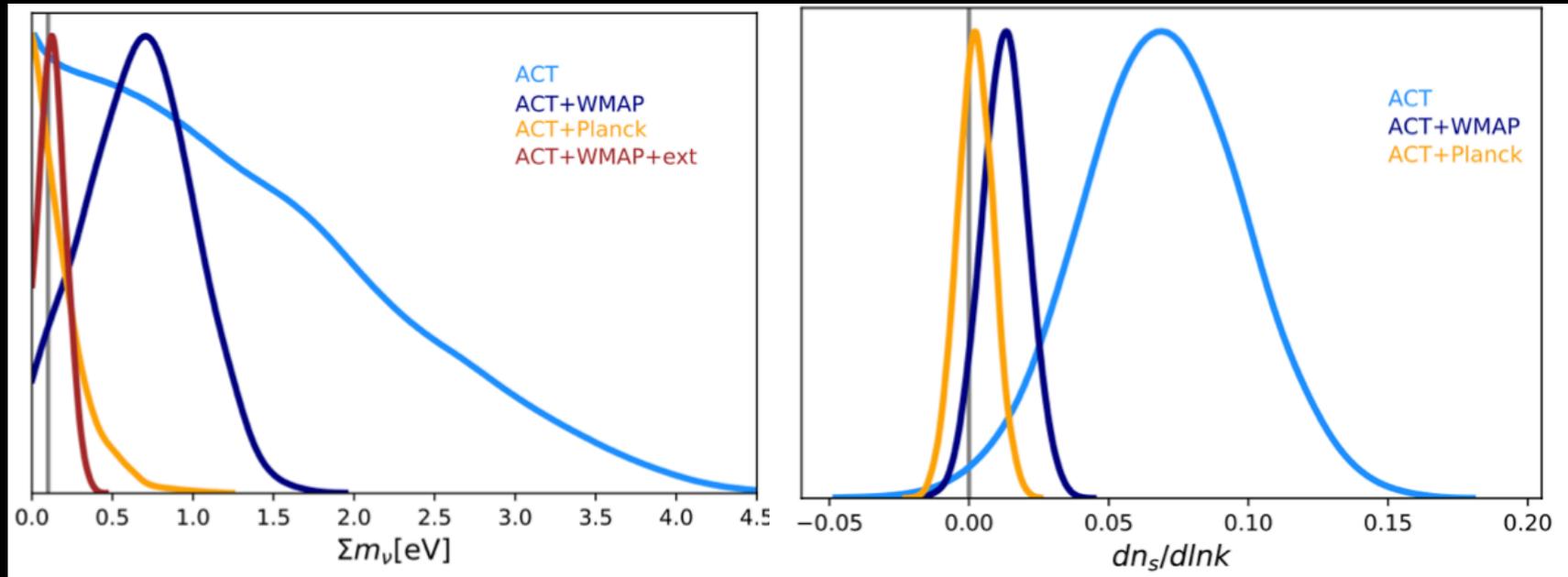


ACT-DR4 2020, Aiola et al., arXiv:2007.07288 [astro-ph.CO]

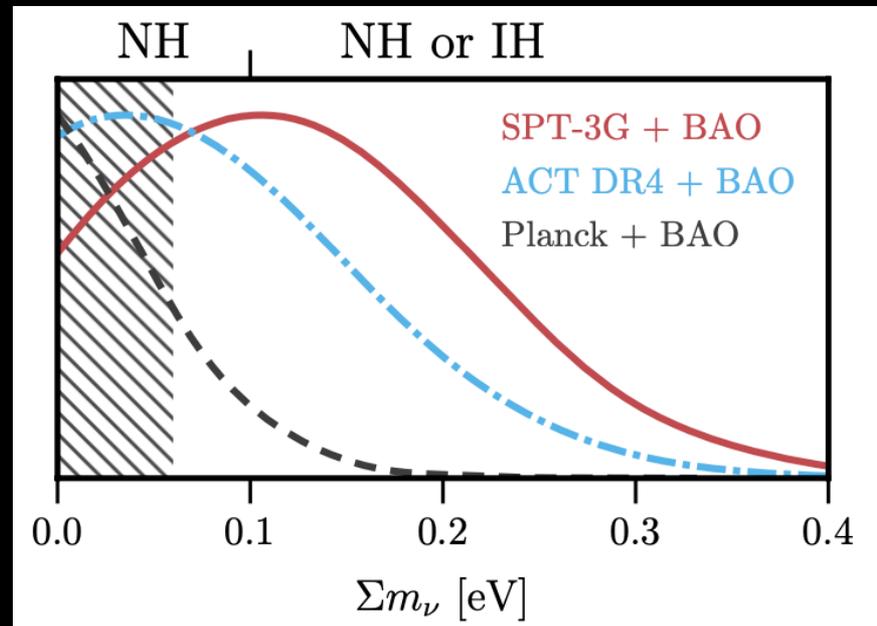


SPT-3G, arXiv:2103.13618 [astro-ph.CO]

ACT-DR4 vs SPT-3G



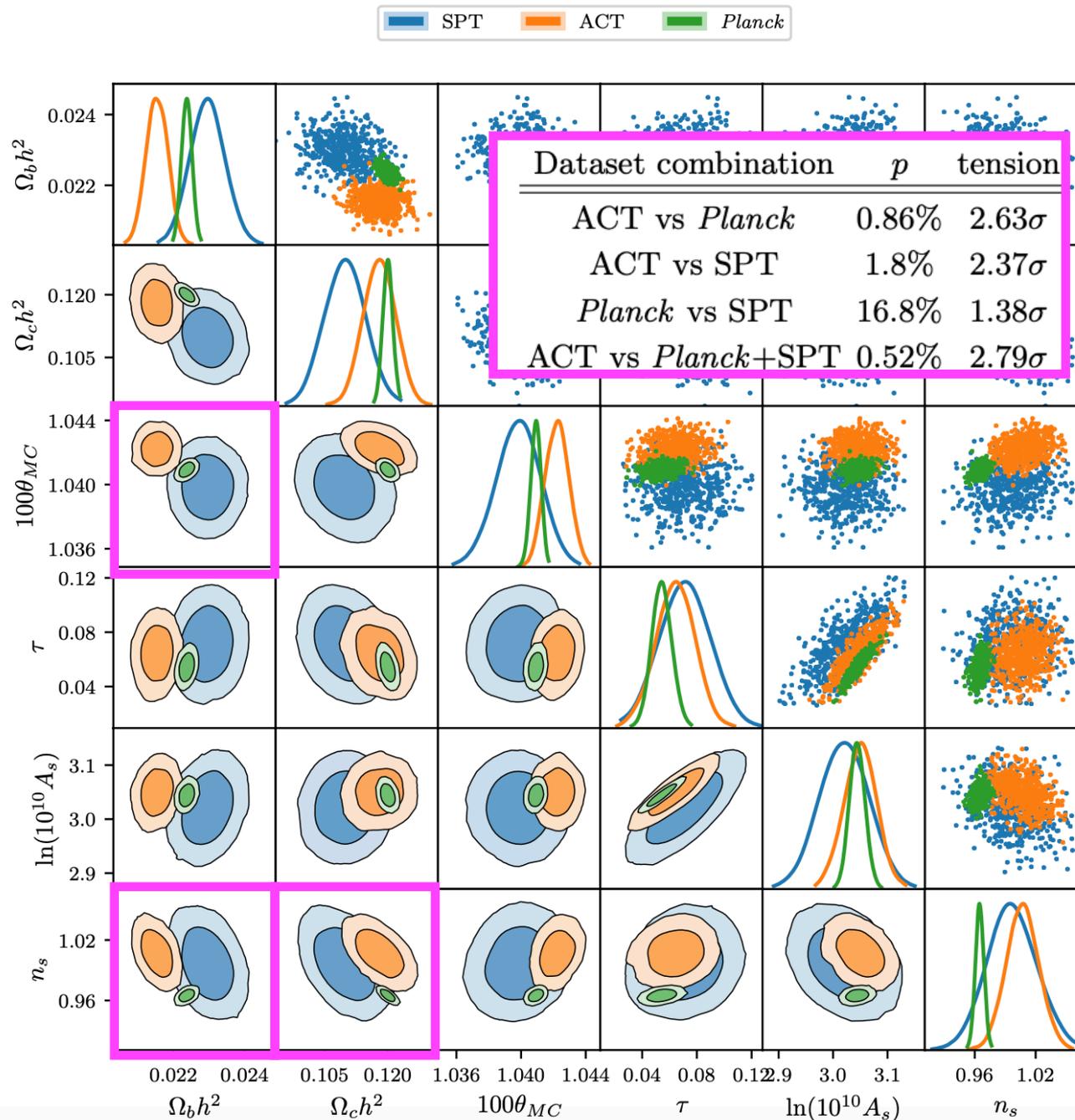
ACT-DR4 2020, Aiola et al., arXiv:2007.07288 [astro-ph.CO]



SPT-3G, arXiv:2103.13618 [astro-ph.CO]

ACT-DR4

Handley and Lemos, arXiv:2007.08496 [astro-ph.CO]



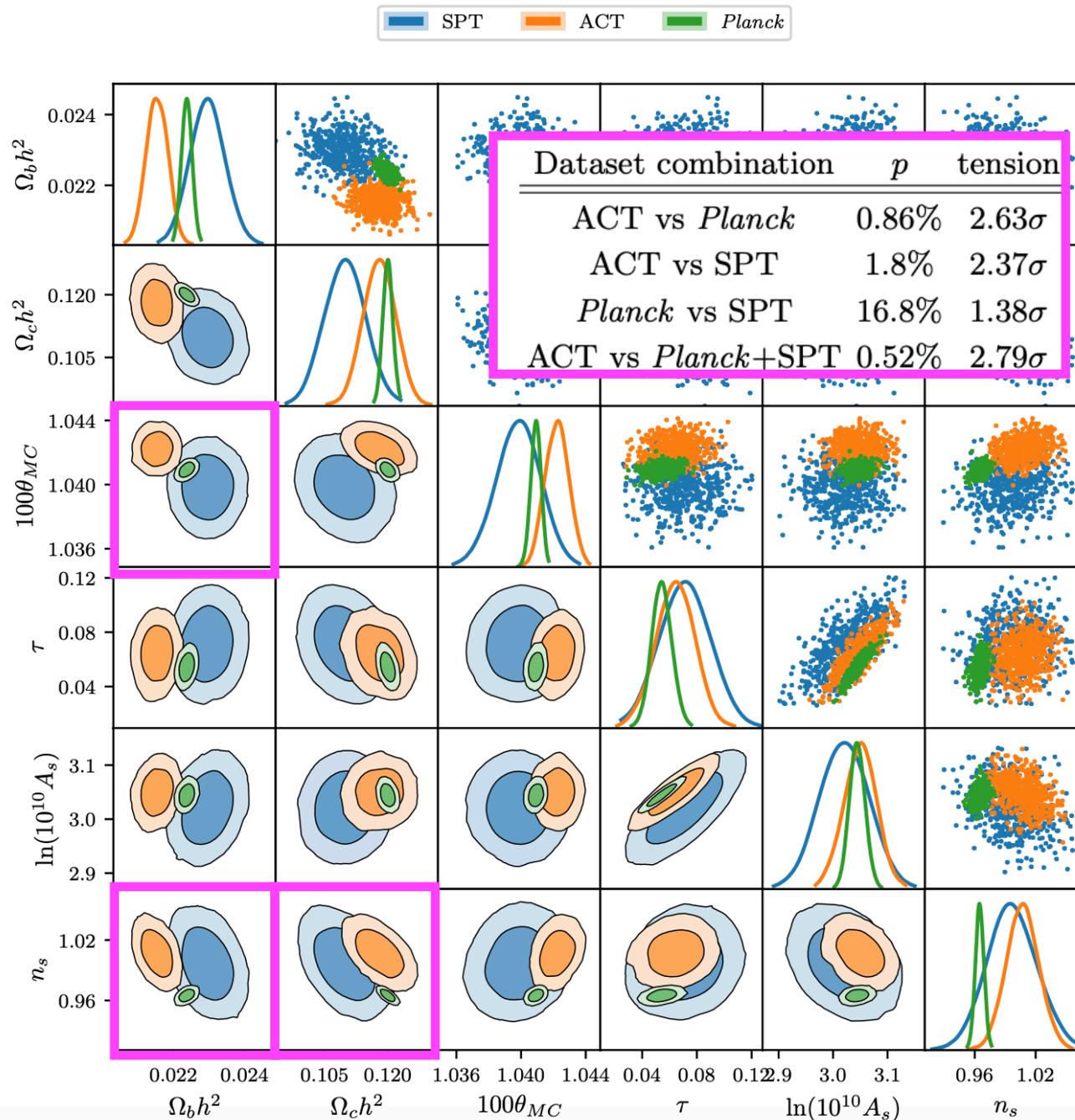
Global tensions between CMB datasets.

For each pairing of datasets this is the tension probability p that such datasets would be this discordant by (Bayesian) chance, as well as a conversion into a Gaussian-equivalent tension.

Between *Planck* and ACT there is a 2.6σ tension.

ACT-DR4

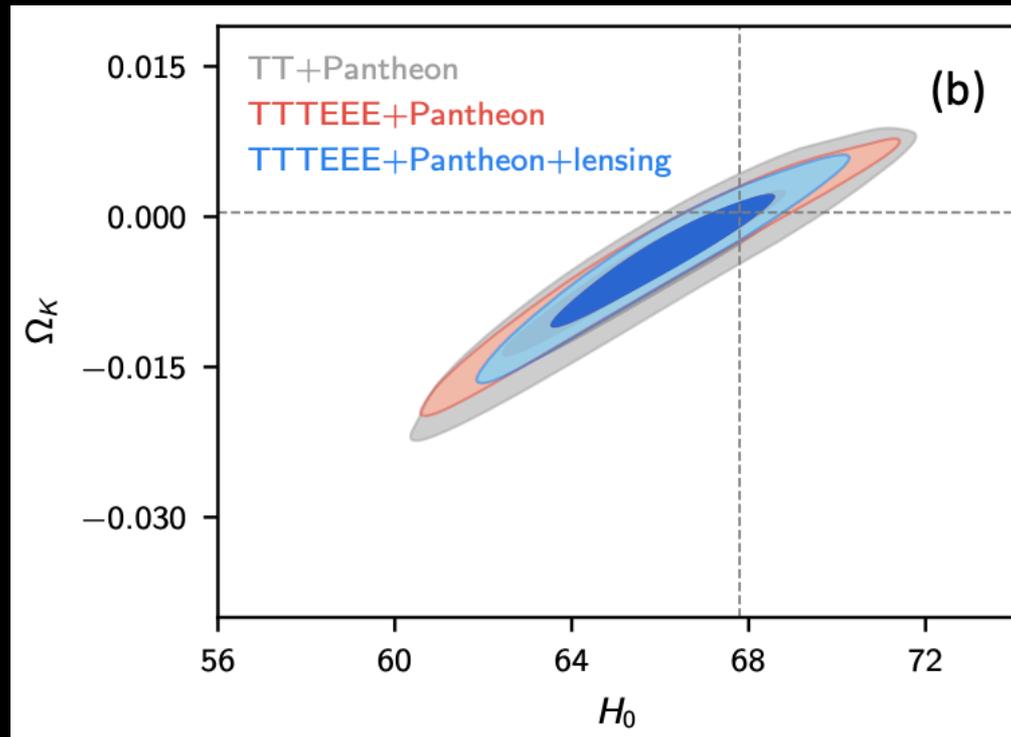
Handley and Lemos, arXiv:2007.08496 [astro-ph.CO]



At this point, given the quality of all the analyses, it is more likely that these **discrepancies** are indicating a problem with the underlying cosmology and our understanding of the **Universe**, rather than the presence of systematic effects.

And this suspect is corroborated by the many other tensions we saw emerging between the other cosmological probes.

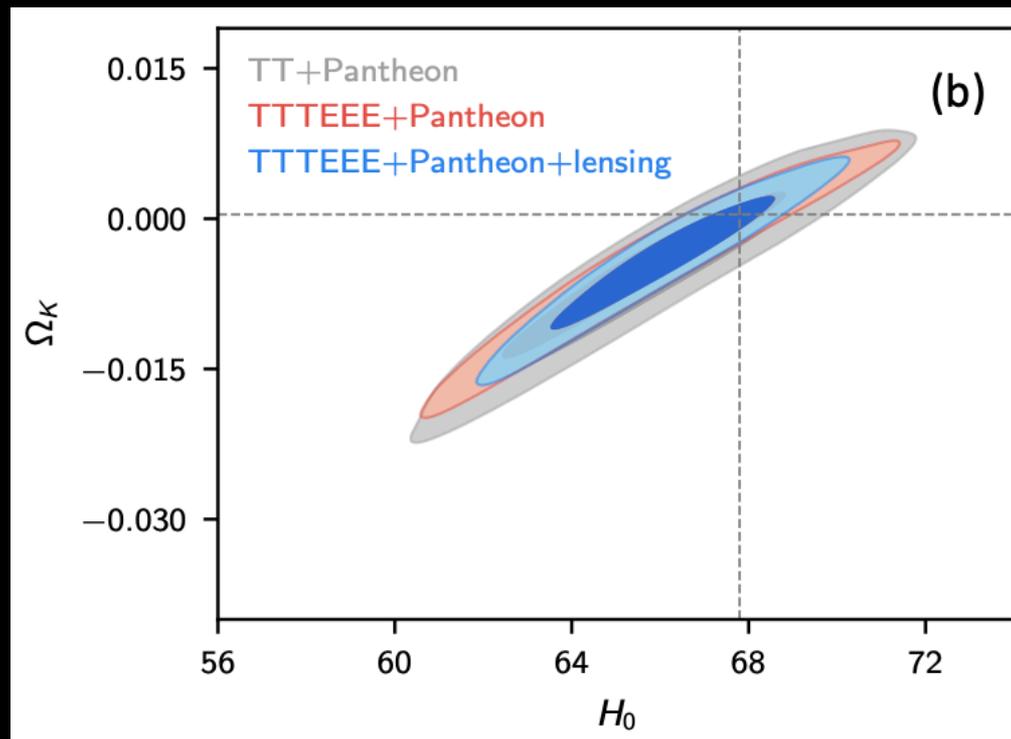
What about Planck + Pantheon?



Efstathiou and Gratton, *Mon.Not.Roy.Astron.Soc.* 496 (2020) 1, L91-L95

Adding Pantheon data, a joint constraint is very consistent with a flat universe.

What about Planck + Pantheon?



Efstathiou and Gratton, *Mon.Not.Roy.Astron.Soc.* 496 (2020) 1, L91-L95

Adding Pantheon data, a joint constraint is very consistent with a flat universe.

Again, what happens if we vary all the parameters together?

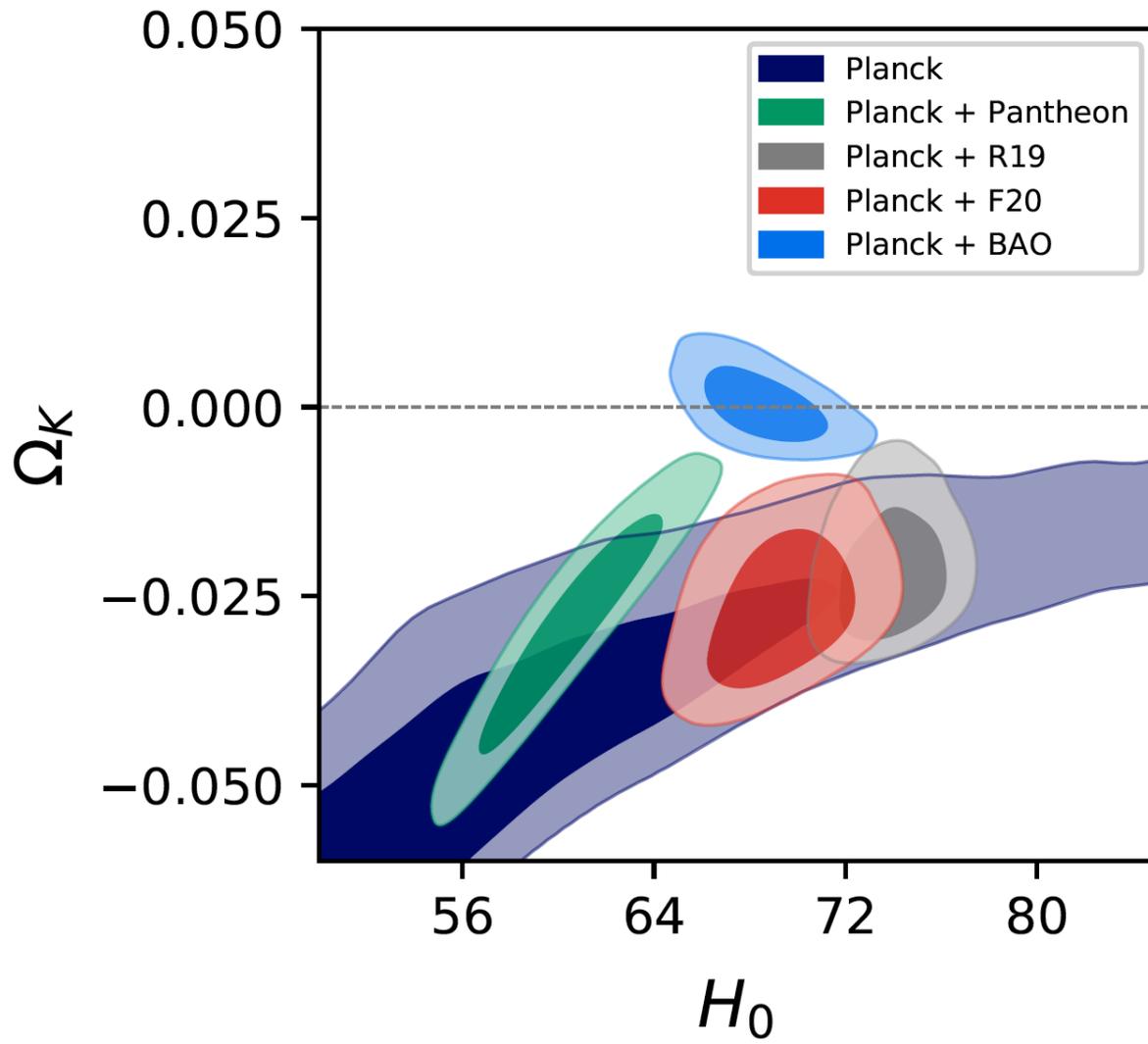
10 parameters: replacing Alens with curvature

Parameters	Planck	Planck +R19	Planck +F20	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02253 ± 0.00019	$0.02253^{+0.00020}_{-0.00016}$	$0.02255^{+0.00019}_{-0.00017}$	0.02243 ± 0.00016	0.02255 ± 0.00018
$\Omega_c h^2$	0.1183 ± 0.0016	$0.1187^{+0.0015}_{-0.0018}$	0.1184 ± 0.0015	0.1198 ± 0.0014	0.1186 ± 0.0015
$100\theta_{MC}$	1.04099 ± 0.00035	$1.04103^{+0.00034}_{-0.00031}$	1.04105 ± 0.00034	1.04095 ± 0.00032	1.04107 ± 0.00034
τ	0.0473 ± 0.0083	$0.052^{+0.009}_{-0.011}$	0.0491 ± 0.0079	0.0563 ± 0.0081	0.0506 ± 0.0082
Σm_ν [eV]	$0.43^{+0.16}_{-0.27}$	< 0.513	$0.28^{+0.11}_{-0.23}$	< 0.194	< 0.420
w	$-1.6^{+1.0}_{-0.8}$	$-2.11^{+0.35}_{-0.77}$	-2.14 ± 0.46	$-1.038^{+0.098}_{-0.088}$	$-1.27^{+0.14}_{-0.09}$
Ω_k	$-0.074^{+0.058}_{-0.025}$	$-0.0192^{+0.0036}_{-0.0099}$	$-0.0263^{+0.0060}_{-0.0077}$	$0.0003^{+0.0027}_{-0.0037}$	$-0.029^{+0.011}_{-0.010}$
$\ln(10^{10} A_s)$	3.025 ± 0.018	$3.037^{+0.016}_{-0.026}$	3.030 ± 0.017	3.049 ± 0.017	3.034 ± 0.017
n_s	0.9689 ± 0.0054	$0.9686^{+0.0056}_{-0.0050}$	0.9693 ± 0.0051	0.9648 ± 0.0048	0.9685 ± 0.0051
α_S	-0.0005 ± 0.0067	-0.0012 ± 0.0066	-0.0010 ± 0.0068	-0.0054 ± 0.0068	-0.0023 ± 0.0065
H_0 [km/s/Mpc]	53^{+6}_{-16}	73.8 ± 1.4	69.3 ± 2.0	$68.6^{+1.5}_{-1.8}$	60.5 ± 2.5
σ_8	$0.74^{+0.08}_{-0.16}$	0.932 ± 0.040	0.900 ± 0.039	0.821 ± 0.027	$0.812^{+0.031}_{-0.018}$
S_8	$0.989^{+0.095}_{-0.063}$	0.874 ± 0.032	$0.900^{+0.034}_{-0.031}$	0.826 ± 0.016	0.927 ± 0.037
Age [Gyr]	$16.10^{+0.92}_{-0.80}$	$14.90^{+0.72}_{-0.32}$	$15.22^{+0.054}_{-0.038}$	13.77 ± 0.10	14.98 ± 0.39
Ω_m	$0.61^{+0.21}_{-0.34}$	$0.264^{+0.010}_{-0.013}$	$0.300^{+0.017}_{-0.020}$	0.305 ± 0.016	$0.393^{+0.030}_{-0.036}$
$\Delta\chi^2_{bestfit}$	0.0	0.62	0.88	14.77	1037.82

Therefore, now we want to check the **robustness** of these results further increasing the number of parameters, in addition to curvature.

10 parameters

Parameters	
$\Omega_b h^2$	0.0
$\Omega_c h^2$	0
$100\theta_{MC}$	1.0
τ	0
Σm_ν [eV]	0
w	0
Ω_k	0
$\ln(10^{10} A_s)$	0
n_s	0
α_s	0
H_0 [km/s/Mpc]	0
σ_8	0
S_8	0
Age [Gyr]	0
Ω_m	0
$\Delta\chi^2_{bestfit}$	0



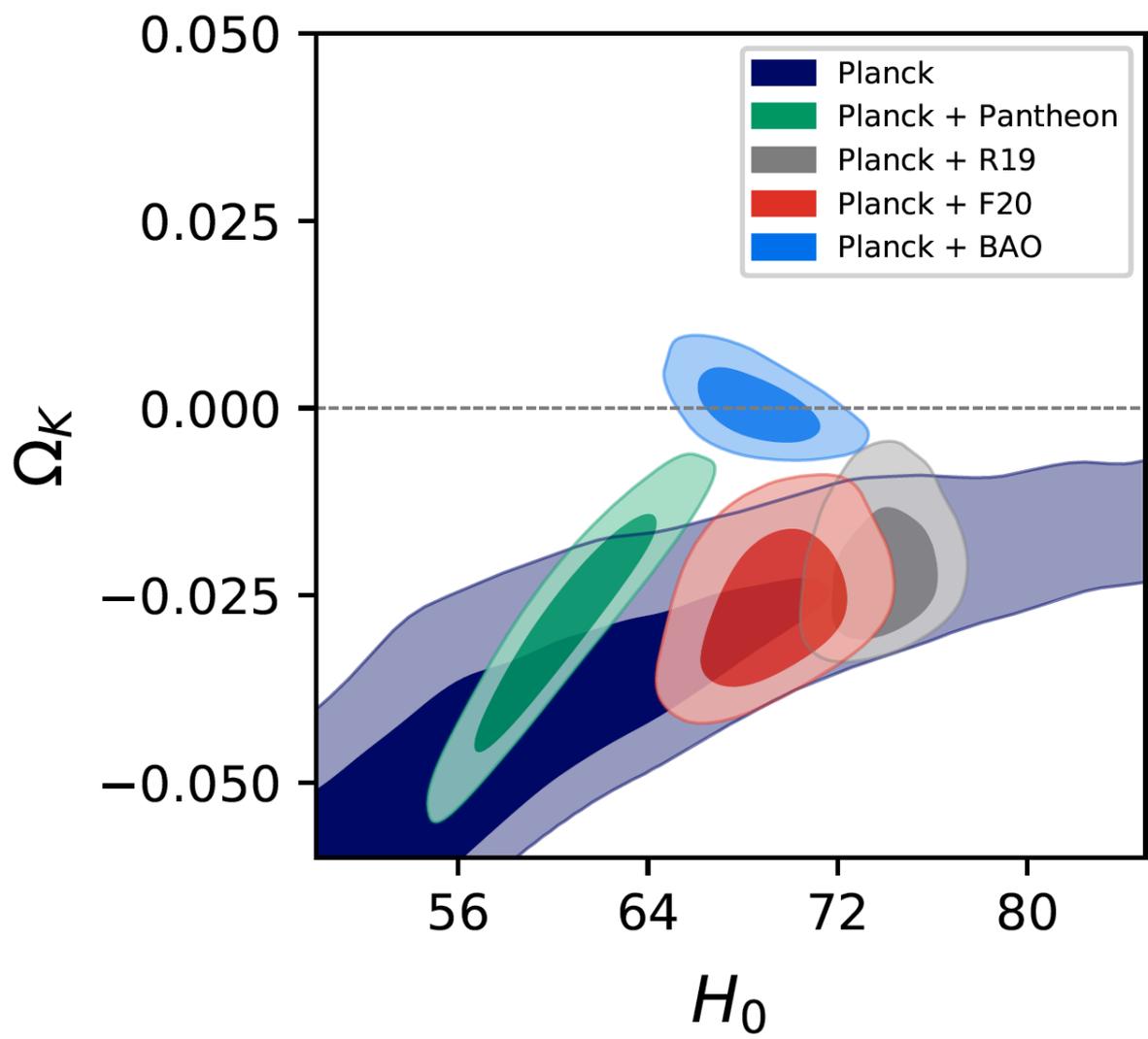
curvature

Planck + Pantheon	
016	0.02255 ± 0.00018
014	0.1186 ± 0.0015
032	1.04107 ± 0.00034
081	0.0506 ± 0.0082
	< 0.420
8	$-1.27^{+0.14}_{-0.09}$
8	$-0.029^{+0.011}_{-0.010}$
7	3.034 ± 0.017
48	0.9685 ± 0.0051
068	-0.0023 ± 0.0065
	60.5 ± 2.5
7	$0.812^{+0.031}_{-0.018}$
6	0.927 ± 0.037
0	14.98 ± 0.39
6	$0.393^{+0.030}_{-0.036}$
	1037.82

The confidence levels from Planck are clearly below the $\Omega_k = 0$ line that describes a flat universe. On the other hand, the Planck data are now in **perfect agreement with the Pantheon, R19, and F20** (Freedman et al. arXiv:2002.01550) measurements, while they are still in strong tension with the BAO measurements, so their combination should be considered with some caution.

10 parameters

Parameters	
$\Omega_b h^2$	0.0
$\Omega_c h^2$	0
$100\theta_{MC}$	1.0
τ	0
Σm_ν [eV]	0
w	0
Ω_k	0
$\ln(10^{10} A_s)$	0
n_s	0
α_s	0
H_0 [km/s/Mpc]	0
σ_8	0
S_8	0
Age [Gyr]	0
Ω_m	0
$\Delta\chi^2_{bestfit}$	0

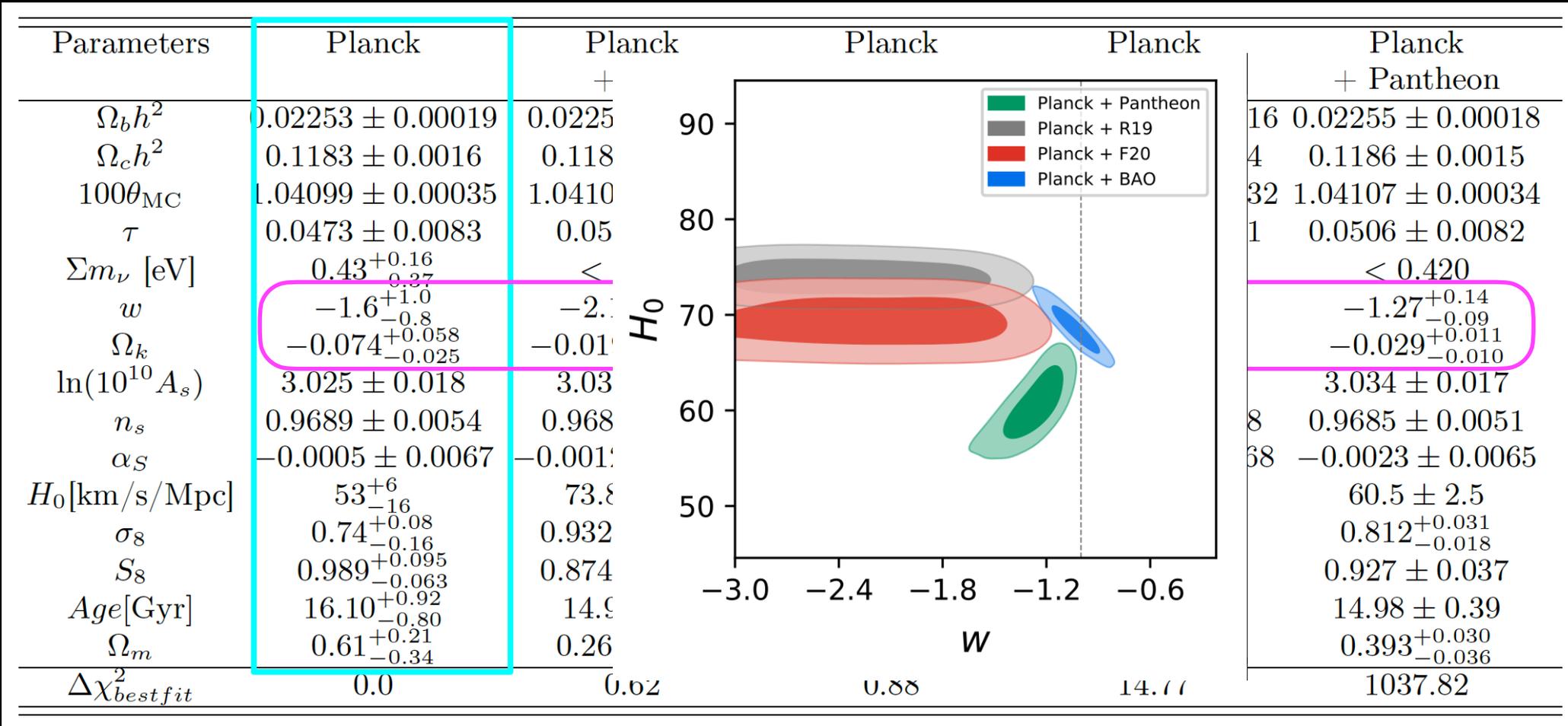


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	1037.82

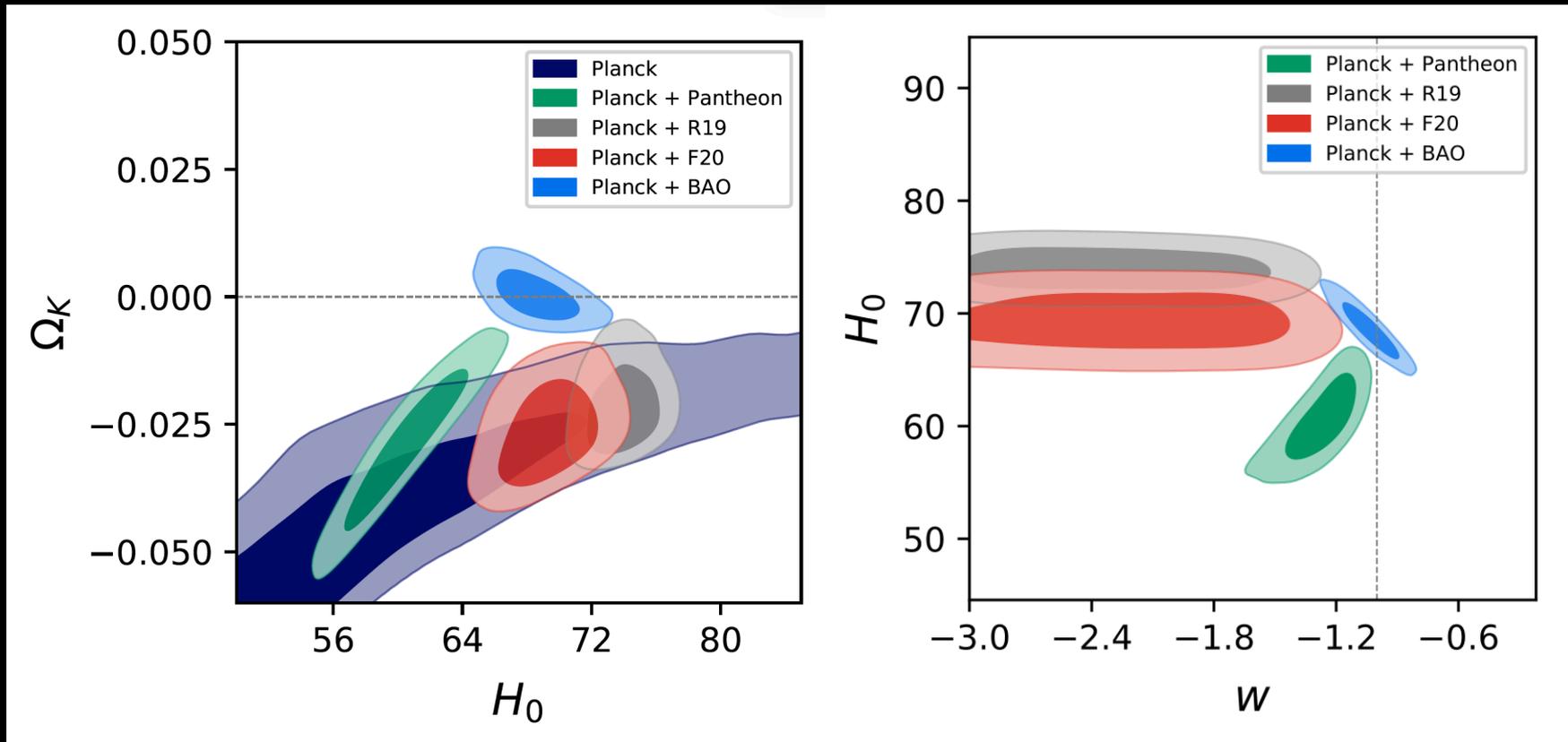
Moreover, all the 95% confidence regions from the Planck+Pantheon, Planck+F20, and Planck+R19 datasets are well below the $\Omega_k = 0$ line. This clearly shows that the recent claims of a closed universe as being incompatible with luminosity distance measurements are simply due to the assumption of a cosmological constant.

10 parameters: replacing Alens with curvature



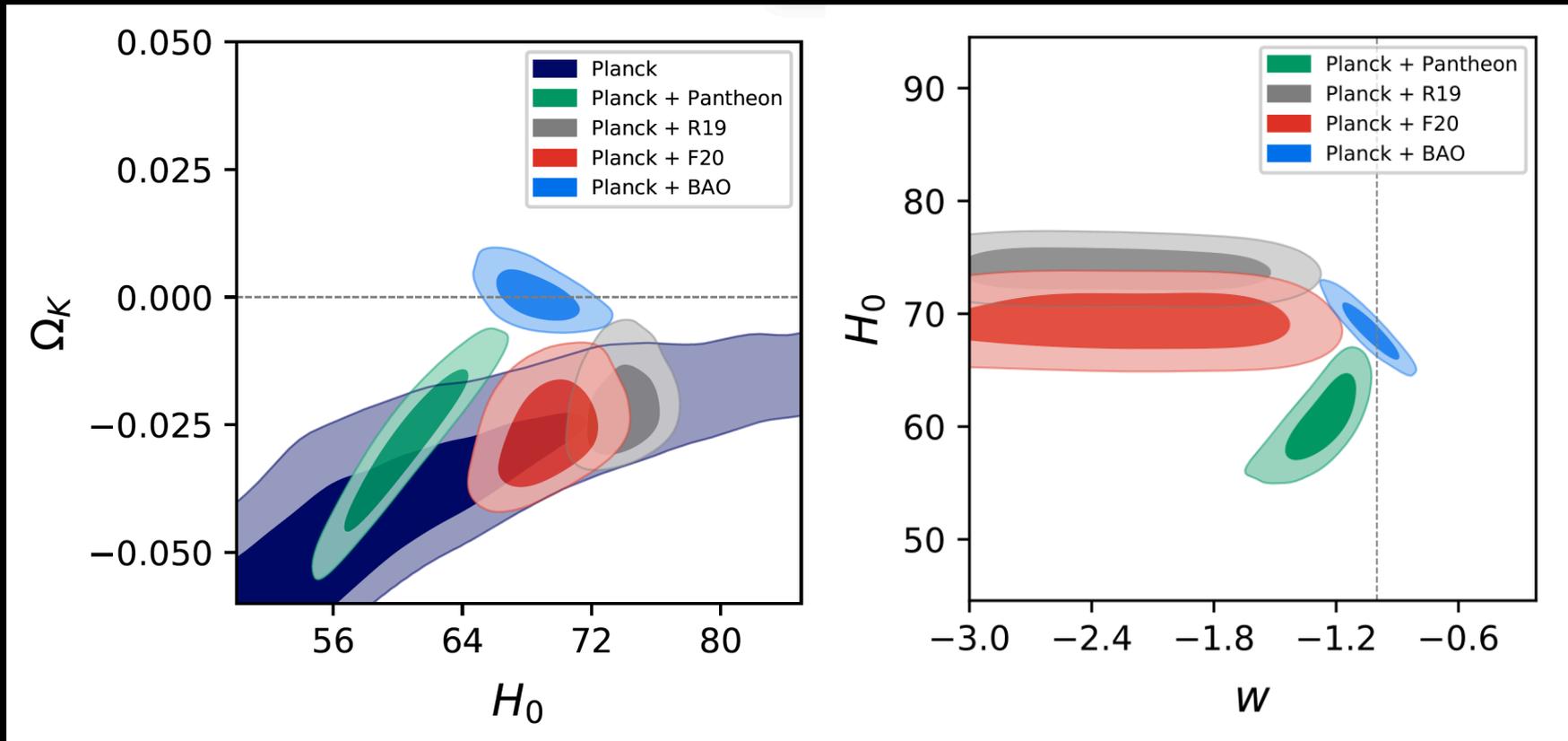
Indeed, all the three datasets, combined with Planck, exclude a cosmological constant, clearly preferring a value of $w < -1$.

Cosmic Discordance



In practice, Planck+Pantheon, Planck+R19, and Planck+F20
all exclude both
a cosmological constant and a flat universe at more than 99% C.L.

Cosmic Discordance



Evidence for a **phantom closed** Universe at more than 99% CL!!

It is interesting to note that if a closed universe increases the fine-tuning of the theory, the removal of a cosmological constant reduces it. It is, therefore, difficult to decide whether a **phantom closed** model is less or more theoretically convoluted than Λ CDM.

IDE + Ω_k

Assuming a closed Universe, can we improve the agreement with H_0 introducing IDE instead?

Parameters	Planck	Planck +BAO	Planck + Pantheon	Planck + R19	Planck + all
$\Omega_b h^2$	0.02261 ± 0.00017	0.02241 ± 0.00016	0.02258 ± 0.00016	0.02247 ± 0.00016	0.02239 ± 0.00015
$\Omega_c h^2$	$0.077^{+0.035}_{-0.019}$	$0.082^{+0.033}_{-0.015}$	$0.068^{+0.013}_{-0.018}$	< 0.0253	$0.093^{+0.013}_{-0.011}$
$100\theta_{MC}$	$1.0437^{+0.0012}_{-0.0023}$	$1.04327^{+0.00009}_{-0.00022}$	$1.0442^{+0.0012}_{-0.0010}$	$1.0480^{+0.0020}_{-0.0008}$	$1.04249^{+0.00074}_{-0.00086}$
τ	$0.0481^{+0.0085}_{-0.0076}$	0.0541 ± 0.0081	0.0495 ± 0.0080	0.0534 ± 0.0079	0.0542 ± 0.0079
n_s	0.9708 ± 0.0047	0.9662 ± 0.0047	0.9701 ± 0.0046	0.9679 ± 0.0046	0.9653 ± 0.0047
$\ln(10^{10} A_s)$	$3.027^{+0.017}_{-0.016}$	3.043 ± 0.016	3.031 ± 0.017	3.040 ± 0.016	3.045 ± 0.016
ξ	< -0.385	$-0.32^{+0.31}_{-0.09}$	$-0.62^{+0.19}_{-0.25}$	$-0.75^{+0.06}_{-0.16}$	-0.23 ± 0.10
Ω_k	$-0.036^{+0.017}_{-0.013}$	-0.0016 ± 0.0024	-0.0261 ± 0.0087	-0.0038 ± 0.0034	0.0006 ± 0.0021
$H_0[(\text{km/s})/\text{Mpc}]$	$58.7^{+4.1}_{-5.2}$	$69.7^{+1.2}_{-1.6}$	$61.6^{+2.0}_{-2.4}$	72.9 ± 1.4	69.93 ± 0.75
σ_8	$1.31^{+0.10}_{-0.54}$	$1.27^{+0.04}_{-0.46}$	$1.36^{+0.20}_{-0.31}$	$3.4^{+1.2}_{-1.4}$	$1.04^{+0.08}_{-0.15}$
Ω_m	0.30 ± 0.11	$0.219^{+0.076}_{-0.040}$	0.240 ± 0.038	$0.084^{+0.010}_{-0.039}$	0.239 ± 0.028
S_8	$1.20^{+0.10}_{-0.22}$	$1.01^{+0.04}_{-0.18}$	$1.20^{+0.14}_{-0.16}$	$1.64^{+0.41}_{-0.27}$	$0.921^{+0.043}_{-0.069}$
$\ln B_{ij}$	0.2	-1.0	3.2	5.8	-0.4

From Planck alone there is still the indication for a **closed universe**,
but **without interaction**.

IDE + Ω_k

Assuming a closed Universe, can we improve the agreement with H_0 introducing IDE instead?

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$\Omega_b h^2$	0.02261 ± 0.00017	0.02241 ± 0.00016	0.02258 ± 0.00016	0.02247 ± 0.00016	0.02239 ± 0.00015
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S_8	$1.20^{+0.10}_{-0.22}$	$1.01^{+0.04}_{-0.18}$	$1.20^{+0.14}_{-0.16}$	$1.64^{+0.41}_{-0.27}$	$0.921^{+0.043}_{-0.069}$
$\ln B_{ij}$	0.2	-1.0	3.2	5.8	-0.4

IDE can't increase the H_0 value enough to solve the tension with R20.

In a closed universe for Planck alone they are still at 3.4σ tension.

IDE + Ω_k

Assuming a closed Universe, can we improve the agreement with H_0 introducing IDE instead?

Parameters	Planck	Planck +BAO	Planck + Pantheon	Planck + R19	Planck + all
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$\ln B_{ij}$	0.2	-1.0	3.2	5.8	-0.4

Planck+Pantheon prefers an **interacting closed universe** at more than 3σ ,

IDE + Ω_k

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$\ln B_{ij}$	0.2	-1.0	3.2	5.8	-0.4

Planck+Pantheon prefers an **interacting closed universe** at more than 3σ ,
but in **disagreement with R20** at 4.8σ .

Concluding...

Most of the **anomalies and tensions** are involving the Planck data:

- H_0 tension
- S_8 tension
- $A_L > 1$ or $\Omega_K < 0$



presenting a serious limitation to the **precision cosmology**.

Are we sure that
the 2018 Planck results are still a confirmation of
the flat standard Λ CDM cosmological model?

Watch out for the elephant in the room!

These **cosmic discordances**
call for new observations and stimulate the investigation of
alternative theoretical models and solutions.

Thank you!

eleonora.di-valentino@durham.ac.uk

References

- Planck 2018, *Astron.Astrophys.* 641 (2020) A6
- Borstnik et al., *hep-ph/0401043*
- Alam et al., *arXiv:2007.08991 [astro-ph.CO]*
- Arendse et al., *arXiv:1909.07986 [astro-ph.CO]*
- Bennett et al. [WMAP collaboration], *arXiv:1212.5225 [astro-ph.CO]*;
- Burns et al. , *Astrophys.J.* 869 (2018) no.1, 56
- Calabrese et al., *Phys. Rev. D*, 77, 123531;
- Dhawan et al. *Astron.Astrophys.* 609 (2018) A72
- Di Valentino and Bridle, *Symmetry* 10 (2018) no.11, 585
- Di Valentino et al., *Phys.Dark Univ.* 30 (2020) 100666
- Di Valentino, *arXiv:2011.00246 [astro-ph.CO]*
- Freedman et al. *arXiv:2002.01550*
- Riess et al., *arXiv:2012.08534 [astro-ph.CO]*
- Soltis et al., *arXiv:2012.09196 [astro-ph.CO]*
- Henning et al., *arXiv:1707.09353 [astro-ph.CO]*
- Heymans et al., *arXiv:2007.15632 [astro-ph.CO]*
- Hildebrandt et al., *arXiv:1606.05338 [astro-ph.CO]*;
- Huang et al., *arXiv:1908.10883 [astro-ph.CO]*
- Jang et al., *arXiv:1702.01118 [astro-ph.CO]*
- Joudaki et al, *arXiv:1601.05786*;
- Knox and Millea, *arXiv:1908.03663 [astro-ph.CO]*
- Mangano et al., *Nucl. Phys. B* 729, 221 (2005) [*hep-ph/0506164*];
- Reid et al., *arXiv:1908.05625 [astro-ph.CO]*
- Riess et al. 2018, *ApJ*, 861, 126
- Riess et al. *arXiv:1903.07603 [astro-ph.CO]*;
- Ross et al., *Mon. Not. Roy. Astron. Soc.* 449, no. 1, 835 (2015);
- Shajib et al. *arXiv:1910.06306*
- Troster et al., *arXiv:1909.11006 [astro-ph.CO]*
- Yuan et al. *arXiv:1908.00993*.
- Vagnozzi, Di Valentino, et al., *arXiv:2010.02230 [astro-ph.CO]*
- Wong et al. *arXiv:1907.04869v1*
- Abbott et al. *arXiv:1710.05835 [astro-ph.CO]*
- Adam et al. [Planck Collaboration], *arXiv:1502.01582 [astro-ph.CO]*;
- Aghanim et al. [Planck Collaboration], *arXiv:1507.02704 [astro-ph.CO]*;
- Aiola et al., *arXiv:2007.07288 [astro-ph.CO]*;
- Anderson et al. [BOSS Collaboration], *Mon. Not. Roy. Astron. Soc.* 441, no. 1, 24 (2014);
- Asgari et al., *arXiv:1910.05336 [astro-ph.CO]*
- Betoule et al. [SDSS Collaboration] *Astron. Astrophys* 568, A22 (2014);
- Beutler et al., *Mon. Not. Roy. Astron. Soc.* 416, 3017 (2011);
- Birrer et al. *arXiv:2007.02941 [astro-ph.CO]*
- Di Valentino et al., *arXiv:1908.01391 [astro-ph.CO]*
- Di Valentino et al. *arXiv:2008.11283 [astro-ph.CO]*
- Di Valentino et al. *arXiv:2008.11284 [astro-ph.CO]*
- Di Valentino et al. *arXiv:2008.11285 [astro-ph.CO]*
- Di Valentino et al. *arXiv:2008.11286 [astro-ph.CO]*
- Di Valentino & Mena, *MNRAS Letters* (2020)
- Di Valentino et al. *Phys. Rev. D* 101, 063502
- Di Valentino et al. *Phys.Rev. D*98 (2018) no.8, 083523
- Di Valentino et al., *arXiv:2003.04935 [astro-ph.CO]*
- Di Valentino et al. *arXiv: 2005.12587 [astro-ph.CO]*
- Di Valentino et al., accepted *MNRASL*, *arXiv:2011.00283*
- Efstathiou and Gratton, *arXiv:2002.06892*
- Erben et al., *Mon. Not. Roy. Astron. Soc.* 433, 2545 (2013) ;
- Gavela et al. *J. Cosmol. Astropart. Phys.* 07 (2009) 034;
- Handley, *arXiv:1908.09139 [astro-ph.CO]*
- Handley and Lemos, *arXiv:2007.08496 [astro-ph.CO]*.
- Heymans et al., *Mon. Not. Roy. Astron. Soc.* 427, 146 (2012);
- Huang et al. *arXiv:1908.10883*
- Khetan et al. *arXiv:2008.07754 [astro-ph.CO]*
- Pesce et al. *arXiv:2001.09213 [astro-ph.CO]*
- Riess, *Nature Reviews Physics* (2019)
- Verde et al. *Nat. Astr.* 3, 891-895 (2019)
- Yang et al., *PHYS. REV. D* 99, 043543 (2019)