Strong Phase Transitions from TeV Scale Fermions

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Introduction and Motivation

- Baryon Asymmetry in the Universe (BAU)?
  $\rightarrow$ Sakharov conditions:
  - B-number violation;
  - C and CP violation;
  - interactions out of thermal equilibrium;
- Interactions out of thermal equilibrium?
  $\rightarrow$ Strongly First Order (SFO) Electroweak Phase Transition (EWPT)!
- Solution within the Standard Model (SM)?
  $\rightarrow$ No strong EWPT! (plus not enough $\not{C}\not{P}$) $\Rightarrow$ new physics needed!
- Usually, new bosons $\rightarrow$ $\mathcal{O}(100)$ papers ...

WHAT ABOUT NEW FERMIONS AND PHASE TRANSITIONS?

Extra Dimensions, Composite Higgs, B–anomalies ... $\Rightarrow$ new fermions!

Rather uncharted territory (but: Carena+ '04, Fok+ '08, Davoudiasl+ '12, Fairbairn+ '13, Egana-Ugrinovic '17).

Contents

Overview

A Minimal Vector-Like Lepton (VLL) Model

- Dirac fermion model for strong PTs in the Early Universe?
  $\rightarrow$ need strong couplings to the Higgs!
- However: strong Yukawas $\Rightarrow$ large custodial symmetry breaking!
- Solution $\rightarrow$ a minimal model which can posses (approximate) custodial symmetry:

$$L_{L,R} = \begin{pmatrix} N \\ E \end{pmatrix}_{L,R} \sim (1,2,-1/2), \quad N'_{L,R} \sim (1,1,0), \quad E'_{L,R} \sim (1,1,-1).$$

- VLL masses + Yukawa couplings (assume negligible mixing with the SM):

$$-\mathcal{L}_{VLL} = y_{N_R} \overline{L}_L \tilde{H} N'_R + y_{N_L} \overline{N}'_L \tilde{H}^\dagger L_R + y_{E_R} \overline{L}_L H E'_R + y_{E_L} \overline{E}'_L H^\dagger L_R$$
$$+ m_L \overline{L}_L L_R + m_N \overline{N}'_L N'_R + m_E \overline{E}'_L E'_R + \text{h.c.}.$$

- EW symmetry breaking $\Rightarrow$ mass matrices ($v = 246$ GeV, $v_h = v/\sqrt{2} \simeq 174$ GeV):

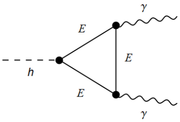
$$\mathcal{M}_N = \begin{pmatrix} m_L & v_h\, y_{N_L} \\ v_h\, y_{N_R} & m_N \end{pmatrix}, \quad \mathcal{M}_E = \begin{pmatrix} m_L & v_h\, y_{E_L} \\ v_h\, y_{E_R} & m_E \end{pmatrix}.$$

- Diagonalization $\Rightarrow$ eigenmasses $m_{N_1} < m_{N_2}, m_{E_1} < m_{E_2}$ and mass basis couplings.

Approach

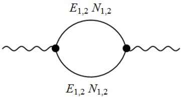
- Calculate the 1–loop finite T effective potential ($V(0,T) \equiv 0$):

$$V(\phi,T) = V^{\rm SM}_{\rm tree}(\phi) + V^{\rm SM}_{1-\rm loop}(\phi,T) + V^{\rm VLL}_{1-\rm loop}(\phi,T) + V_{\rm Daisy}(\phi,T);$$

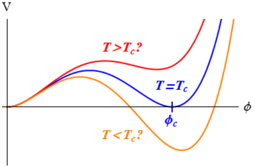


- Seven parameters ⇒ scan approach:

$$m_L, m_N, m_E \in [500, 1500]\ \ {\rm GeV},$$
$$y_{N_{L,R}}, y_{E_{L,R}} \in \left[2, \sqrt{4\pi}\right];$$



- Impose $0.71 \le \mu_{\gamma\gamma} < 1.29$ (ATLAS '18), $\Delta\chi^2\,(S,T) \le 6.18$;
- Calculate PT strength for each point $\longrightarrow$ $\xi \equiv \phi_c/T_c$.



Thermal Evolution of the Effective Potential: Multistep Phase Transition

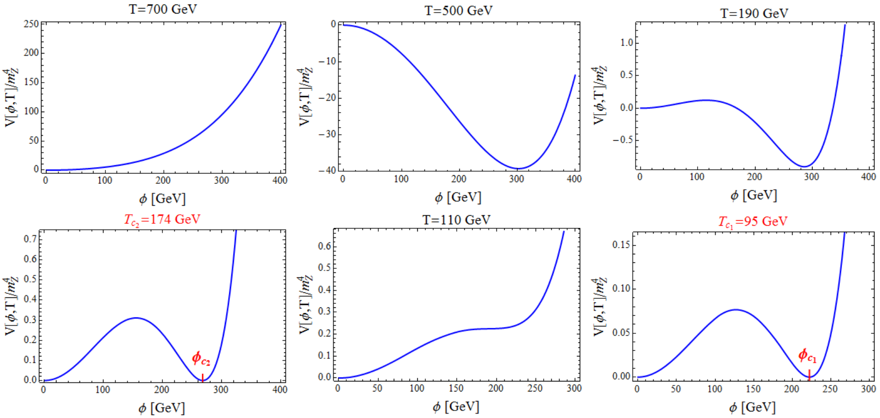
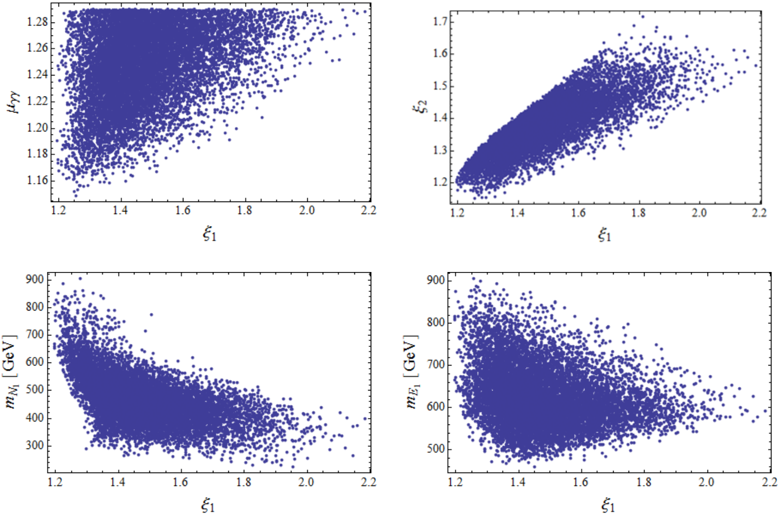


Figure: Typical temperature dependence of the 1–loop effective potential in the VLL model under study.

N.B.: Only the last SFOPT is responsible for generating the BAU! $\Rightarrow \xi_1 \geq 1.3$.

Correlations Between PT Strengths and Observables



Overview

Stochastic Gravitational Backgrounds (Caprini+ 15', '19)

- Strong PTs in the Early Universe ⇒ Gravitational Wave (GW) stochastic background!
- When $T < T_c$, spherical bubbles of the new phase start to nucleate
  ⇒ nucleation rate per unit volume ($S_3(T)$ = bounce action):

$$\mathcal{P}(T) = A(T)\, e^{-\frac{S_3(T)}{T}}$$

- One bubble nucleated per horizon (on average) ⇒ nucleation temperature T_n:

$$\frac{S_3(T_n)}{T_n} \simeq 140 \quad \text{(for electroweak PT's)}$$

- More and more bubbles nucleate → collisions between bubbles
  ⇒ GW production from several sources (radiation–dominated Universe):
  + bubble wall collisions
  + sound waves in the plasma
  + magnetohydrodynamic (MHD) turbulence
- Stochastic GW backgrounds probed with future space–based interferometers:
  * Laser Interferometer Space Antenna (LISA)
  * Deci-hertz Interferometer Gravitational wave Observatory (DECIGO)
  * Big Bang Observatory (BBO)

Space–Based Interferometers: LISA (LISA collaboration '17)

- Equilateral triangle configuration → three identical spacecraft, three arms with six laser links
- Three independent interferometric combinations:

  → two virtual Michelson interferometers ⇒ simultaneous measurement of GWs

  → third combination (largely) insensitive to GWs ⇒ measure instrumental noise background

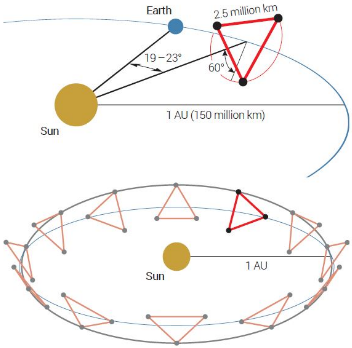


Figure 4: Depiction of the LISA Orbit.

Gravitational Wave Signature

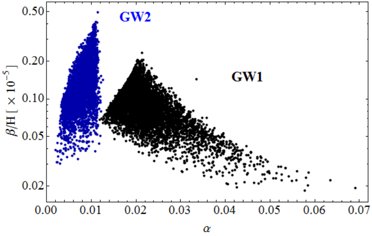
- GW amplitude and spectrum controlled (mostly) by two parameters:

$$\alpha = \frac{\text{vacuum energy}}{\text{radiation energy}}, \quad \beta = \frac{d}{dt}\log\mathcal{P}(t) = \text{“inverse PT duration”}$$

- Main GW sources → sound waves ($\Omega_{\rm sw}$), MHD turbulence ($\Omega_{\rm turb}$), bubble collisions ($\Omega_{\rm col}$):

$$h^2\Omega_{\{\rm sw,turb,col\}}(f) \propto \left(\frac{\beta}{H_n}\right)^{-\{1,1,2\}} \left(\frac{\alpha}{1+\alpha}\right)^{\{2,\frac{3}{2},2\}} S_{\{\rm sw,turb,col\}}(f;\beta/H_n)$$

- Typical values of α and β for our VLL model:



⇒ SW contribution dominant for $f \in [10^{-3}, 1]$ Hz (LISA/DECIGO/BBO max sensitivity).

GW Spectrum Calculation and Detection Prospects

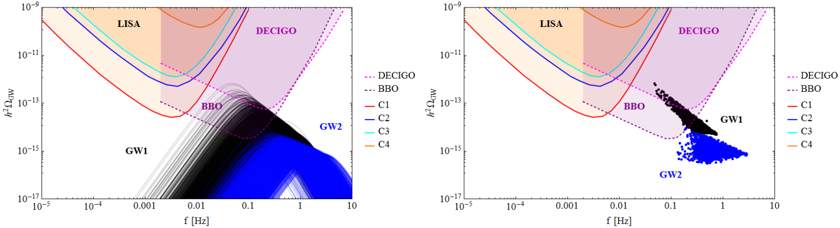
- Compute the bounce action $S_3(T)$, find the nucleation temperature T_n:

$$\frac{S_3(T_n)}{T_n} \simeq 140;$$

- Calculate α and β for the two SFOPTs:

$$\alpha = \frac{|V(\phi_{\rm true}, T_n)| + T_n \frac{\partial V(\phi_{\rm true}, T)}{\partial T}\Big|_{T_n}}{\rho_{\rm rad}(T_n)}, \qquad \frac{\beta}{H_n} = T_n \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T_n}.$$

- Compute GW spectrum → GW detectable by DECIGO/BBO:



Collider Predictions

Benchmark point $\rightarrow$ strongest PT:

$$y_{N_L} \simeq 3.4,\ y_{N_R} \simeq 3.49,\ y_{E_L} \simeq 3.34,\ y_{E_R} \simeq 3.46,$$
$$m_L \simeq 1.06\ \mathrm{TeV},\ m_N \simeq 0.94\ \mathrm{TeV},\ m_E \simeq 1.34\ \mathrm{TeV}.$$

- N_1 not a suitable Dark Matter candidate $\Rightarrow$ SM-VLL mixing should be present!
- Measurements: $W\tau\nu$ and $Z\tau\tau$ couplings $\Rightarrow$ take $y_{\tau E} \simeq 0.05$;
- For simplicity, $y_{\nu N} \simeq 0 \Rightarrow \mathrm{BR}(N_1 \rightarrow W\tau) = 1$;
- Predictions for the benchmark $\rightarrow m_{N_1} \simeq 400$ GeV, $m_{E_1} \simeq 600$ GeV, and:

$$\mathrm{BR}(E_1 \rightarrow N_1 W) \simeq 1, \quad \sigma(pp \rightarrow \psi_{\mathrm{NP}}\psi_{\mathrm{NP}}) \simeq 0.3\ \mathrm{fb}, \quad \sigma(pp \rightarrow \psi_{\mathrm{NP}}\psi_{\mathrm{SM}}) \simeq \mathcal{O}(10^{-4})\ \mathrm{fb}$$

Direct production of VLLs suppressed ...

More promising search avenue? $\longrightarrow \mu_{\gamma\gamma}$!

Summary

- Studied the impact of new VLLs on the phase structure of the Universe;

  $\rightarrow$ TeV-scale VLLs with strong Yukawas can induce SFOEWPTs!

- Interestingly, such a simple model predicts a complex PT structure:

  $\rightarrow$ First example of single-field multistep SFOPT!

- GW signature $\rightarrow$ multiple peaks, possibly detectable by DECIGO or BBO;

- $\mu_{\gamma\gamma}$ = most promising collider signature $\rightarrow$ 5% precision @ HL-LHC!

  (CMS 1307.7135, ATLAS 1307.7292)

  $\Rightarrow$ HL-LHC can fully test our model for VLL-induced SFOEWPTs!

Overview

Bridging Heavy BSM to Experiment

$$E = \Lambda_{\rm BSM} : \mathcal{L}_{\rm UV}[\varphi_H, \varphi_L]$$

$\downarrow$ $\int$ out φ_H

$$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(\Lambda_{\rm BSM})\mathcal{O}_i$$

$\downarrow$ RGE (done once and for all)

$$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(m_{\rm EW})\mathcal{O}_i \xrightarrow{\text{map (done once and for all)}} E = m_{\rm EW} : \text{observables}$$

Bridging Heavy BSM to Experiment

$E = \Lambda_{\mathrm{BSM}} : \mathcal{L}_{\mathrm{UV}}[\varphi_H, \varphi_L]$ —Feynman diagrams→ $\langle \varphi_L \varphi_L \cdots \varphi_L \rangle_{\mathrm{UV}}$

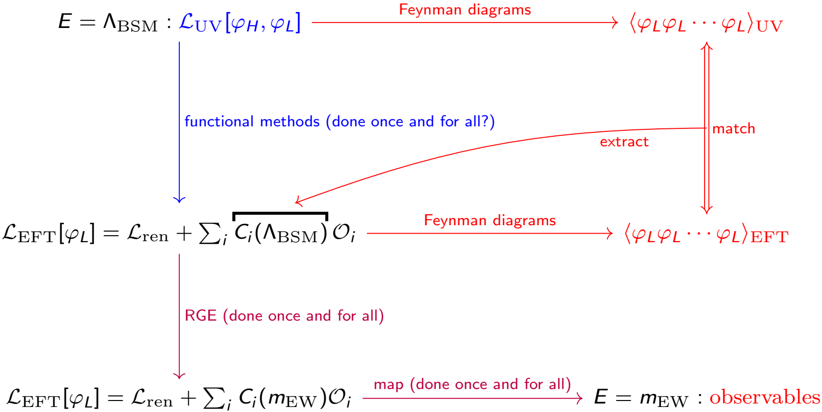
$\Updownarrow$ match (extract)

$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i \overbrace{C_i(\Lambda_{\mathrm{BSM}})}\, \mathcal{O}_i$ —Feynman diagrams→ $\langle \varphi_L \varphi_L \cdots \varphi_L \rangle_{\mathrm{EFT}}$

↓ RGE (done once and for all)

$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i C_i(m_{\mathrm{EW}})\mathcal{O}_i$ —map (done once and for all)→ $E = m_{\mathrm{EW}}$: observables

Bridging Heavy BSM to Experiment



The Effective Action Through Functional Methods

- Path integral definition of the effective action:

$$e^{i\,S_{eff}[\varphi_L]} = \int [\mathcal{D}\varphi_H] e^{i\,S_{UV}[\varphi_H,\varphi_L]}$$

- Stationary phase approx. $\Rightarrow$ classical solution:

$$\frac{\delta S_{UV}[\varphi_H,\varphi_L]}{\delta \varphi_H} = 0 \quad \Rightarrow \quad \varphi_H^{cl} = \varphi_H^{cl}(\varphi_L)$$

- Taylor expansion around the minimum (η = quantum fluctuations, assume no φ_L in loops):

$$S_{UV}[\varphi_H^{cl} + \eta, \varphi_L] = S_{UV}[\varphi_H^{cl}(\varphi_L), \varphi_L] + \frac{1}{2} \frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta \varphi_H^2}\bigg|_{\varphi_H^{cl}} \eta^2 + \mathcal{O}(\eta^3)$$

- Integrate over $\eta \Rightarrow$ Gaussian integral ($c_s = \frac{1}{2}$ for real scalars, $-\frac{1}{2}$ for Weyl fermions):

$$e^{i\,S_{eff}[\varphi_L]} = e^{i\,S_{UV}[\varphi_H^{cl}(\varphi_L),\varphi_L]} \left[\det\left(-\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta \varphi_H^2}\bigg|_{\varphi_H^{cl}}\right)\right]^{-c_s} \xrightarrow{\det(A)=\exp(\mathrm{Tr}\log A)}$$

$$S_{eff}[\varphi_L] = \underbrace{S_{UV}[\varphi_H^{cl}(\varphi_L), \varphi_L]}_{=\text{ tree level}} + \underbrace{i c_s \mathrm{Tr}\log\left(-\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta \varphi_H^2}\bigg|_{\varphi_H^{cl}}\right)}_{=\text{ 1–loop}}$$

The Effective Action at One Loop

$$\mathcal{L}_{UV}[\varphi_H, \varphi_L] = \mathcal{L}_{IR}[\varphi_L] + \left(\varphi_H^\dagger T[\varphi_L] + \text{h.c.}\right) + \varphi_H^\dagger \Big(\underbrace{-D^2 - M^2}_{\text{diagonal}} - U[\varphi_H^{cl}(\varphi_L), \varphi_L] \Big) \varphi_H$$

- Assume only heavy fields in the loop $\Rightarrow$ "heavy–only" contribution:

$$S_{\text{eff}}^{\text{1-loop}} = i c_s \mathrm{Tr} \log \left(D^2 + M^2 + U[\varphi_H^{cl}(\varphi_L), \varphi_L] \right) \equiv i c_s \mathrm{Tr} \log \mathcal{Q}_H$$

- Tr includes coordinate space trace, tr over internal indices (spin, gauge, flavour):

$$\begin{aligned} S_{\text{eff}}^{\text{1-loop}} &= i c_s \,\mathrm{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \langle p | \log \mathcal{Q}_H | x \rangle \langle x | p \rangle \\ &= i c_s \,\mathrm{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \log \left[-(iD_\mu - p_\mu)^2 + M^2 + U \right] \\ &= \frac{c_s}{16\pi^2} \,\mathrm{tr} \int d^d x \int [d^d p] \sum_{n=1}^{\infty} \frac{1}{n} \left[\Delta \left(2 i p_\mu D^\mu + D^2 + U \right) \right]^n, \quad \Delta = (p^2 - M^2)^{-1} \end{aligned}$$

- $\Rightarrow$ power series in $\frac{1}{M}$, momentum integrals factorize
- Fine print $\rightarrow$ assume no open covariant derivatives in U
- Never separate $D_\mu = \partial_\mu + i g G_\mu^a T^a$

  $\Rightarrow$ Covariant Derivative Expansion (CDE) (in contrast to Feynman diagrams)

The Universal One–Loop Effective Action (UOLEA) (Murayama+ '14, Ellis+ '15)

- The (heavy–only) Universal One Loop Effective Action looks like ($F_{\mu\nu} \equiv -i[D_\mu, D_\nu]$):

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{\text{1-loop}} = -ic_s \,\mathrm{tr}\Big\{ & f_2^i\, U_{ii} + f_3^i\, F_i^{\mu\nu} F_{\mu\nu,i} + f_4^{ij}\, U_{ij} U_{ji} \\
& + f_5^i\, [D^\mu, F_{\mu\nu,i}][D_\rho, F_i^{\rho\nu}] + f_6^i\, F^\mu{}_{\nu,i} F^\nu{}_{\rho,i} F^\rho{}_{\mu,i} \\
& + f_7^{ij}\, [D^\mu, U_{ij}][D_\mu, U_{ji}] + f_8^{ijk}\, U_{ij} U_{jk} U_{ki} + f_9^i\, U_{ii} F_i^{\mu\nu} F_{\mu\nu,i} \\
& + f_{10}^{ijkl}\, U_{ij} U_{jk} U_{kl} U_{li} + f_{11}^{ijk}\, U_{ij} [D^\mu, U_{jk}][D_\mu, U_{ki}] \\
& + f_{12}^{ij}\, \big[D^\mu, [D_\mu, U_{ij}]\big] \big[D^\nu, [D_\nu, U_{ji}]\big] + f_{13}^{ij}\, U_{ij} U_{ji} F_i^{\mu\nu} F_{\mu\nu,i} \\
& + f_{14}^{ij}\, [D^\mu, U_{ij}][D^\nu, U_{ji}] F_{\nu\mu,i} + f_{15}^{ij}\, \big(U_{ij}[D^\mu, U_{ji}] - [D^\mu, U_{ij}]U_{ji}\big)[D^\nu, F_{\nu\mu,i}] \\
& + f_{16}^{ijklm}\, U_{ij} U_{jk} U_{kl} U_{lm} U_{mi} + f_{17}^{ijkl}\, U_{ij} U_{jk} [D^\mu, U_{kl}][D_\mu, U_{li}] \\
& + f_{18}^{ijkl}\, U_{ij} [D^\mu, U_{jk}] U_{kl} [D_\mu, U_{li}] + f_{19}^{ijklmn}\, U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni} \Big\}
\end{aligned}
$$

- U_{ij}, $F_i^{\mu\nu}$ depend on the UV theory
- $f_N^{ijk\cdots} \equiv f_N(m_i, m_j, m_k, \ldots)$ are model independent $\Rightarrow$ universality!
- Assumptions: no open covariant derivatives in U + no light fields in the loops!

The Fermionic Functional Determinant

- Start with a generic fermionic Lagrangian (heavy–only):

$$\mathcal{L}_{\rm VLF} = \overline{\Psi}\,(i\gamma_\mu D^\mu - M - W)\,\Psi$$

- D_μ $\rightarrow$ interactions with massless gauge fields (unbroken generators)
- W $\rightarrow$ interactions with scalars and/or massive gauge fields (broken generators)
- Can the previous master formula be used?
  $\rightarrow$ Yes! Square the quadratic operator, use $\mathrm{Tr}\log A + \mathrm{Tr}\log B = \mathrm{Tr}\log AB$:

$$\begin{aligned} S^{\text{1-loop}}_{\rm ferm} &= \frac{i}{2} c_f \left[\mathrm{Tr}\log\left(i\gamma_\mu D^\mu - M - W\right) + \mathrm{Tr}\log\left(-i\gamma_\mu D^\mu - M - W\right)\right] \\ &\equiv \frac{i}{2} c_f \,\mathrm{Tr}\log\left(D^2 + M^2 + U_{\rm ferm}\right), \\ U_{\rm ferm} &\equiv -\frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu] - i\gamma^\mu[D_\mu, W] - i[\gamma^\mu, W]D_\mu + \{M, W\} + W^2 \end{aligned}$$

- $[\gamma^\mu, W] \neq 0 \Rightarrow$ open covariant derivative $\Rightarrow$ UOLEA not applicable!
- Even if $[\gamma^\mu, W] = 0$, $U_{\rm ferm}$ complicated $\Rightarrow$ applying UOLEA not straightforward!
- What if the fermionic quadratic operator is not "squared"?

The Fermionic Functional Determinant

- Same manipulations as before, but no “squaring”:

$$\mathcal{L}^{\text{1-loop}}_{\text{ferm}} = \frac{-c_f}{16\pi^2}\,\mathrm{tr}\int [d^d p]\, \log\left[\not{p} - M - (-i\gamma_\mu D^\mu + W)\right]$$
$$= \frac{c_f}{16\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\int [d^d p]\,\mathrm{tr}\left\{\left[\not{\Delta}\,(-i\gamma_\mu D^\mu + W)\right]^n\right\}, \quad \not{\Delta} \equiv (\not{p} - M)^{-1}$$

- Transparent power counting:

$$D_\mu,\, W \sim \Lambda^0, \quad \not{\Delta} \sim \Lambda^{-1}$$

  $\Rightarrow$ all operators at dim–n given by n–th term in log expasion!

- The most general form of W (Quevillon+ '20):

$$W_{ij} = S_{ij} + i\gamma^5 P_{ij} + \gamma_\mu V^\mu_{ij} + \gamma_\mu\gamma^5 A^\mu_{ij} + \sigma_{\mu\nu} T^{\mu\nu}_{ij}$$

- Tensor term $\sigma_{\mu\nu} T^{\mu\nu}_{ij}$ non–renormalizable (at least dim–5) $\rightarrow$ not considered here

- Vector–Like Fermions + SMEFT in mind $\rightarrow$ work in the unbroken phase (no scalar VEVs)

  $\Rightarrow$ The interaction term W has a simpler form:

$$W = S + i\gamma^5 P$$

Example #1: Dimension–2 Terms (A.A., P. Huang '20)

$$\mathcal{L}_{\rm ferm}^{\text{1-loop}} = \frac{c_f}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \, \mathrm{tr} \left\{ \left[\Delta\!\!\!/ \left(-i\gamma_\mu D^\mu + S + i\gamma^5 P \right) \right]^n \right\}$$

- Work out the simple example of $n = 2 \to$ write explicitly the flavour indices $(i, j, k, \ldots)$:

$$\mathcal{L}_{\rm ferm}^{n=2} = \frac{c_f}{16\pi^2} \frac{1}{2} \int [d^d p] \, \mathrm{tr} \left[\Delta\!\!\!/_i \left(S_{ij} + i\gamma^5 P_{ij} \right) \Delta\!\!\!/_j \left(S_{ji} + i\gamma^5 P_{ji} \right) \right] \qquad \left(\Delta\!\!\!/_i = \frac{p\!\!\!/ + m_i}{p^2 - m_i^2} \right)$$

$$= \frac{c_f n_D}{16\pi^2} \frac{1}{2} \left\{ \int [d^d p] \frac{\mathrm{tr_s}(\Delta\!\!\!/_i \Delta\!\!\!/_j)}{n_D} \mathrm{tr_g}(S_{ij} S_{ji}) + \int [d^d p] \frac{\mathrm{tr_s} \left[\Delta\!\!\!/_i (i\gamma^5) \Delta\!\!\!/_j (i\gamma^5) \right]}{n_D} \mathrm{tr_g} \left(P_{ij} P_{ji} \right) \right\}$$

$$\equiv \frac{c_f n_D}{16\pi^2} \left\{ \frac{1}{2} g_2^{ij} \, \mathrm{tr_g}(S_{ij} S_{ji}) + \frac{1}{2} \left(g_2^{i(j)} + \cdots \right) \mathrm{tr_g} \left(P_{ij} P_{ji} \right) \right\}, \qquad n_D \equiv \mathrm{tr_s} \mathbb{1} = 4$$

- Cyclic property of the trace $\Rightarrow$ $\mathbb{Z}_2$–symmetry for $\mathrm{tr_g}(S_{ij} S_{ji})$, $\mathrm{tr_g}(P_{ij} P_{ji})$
  $\Rightarrow$ symmetry factor of $\frac{1}{2}$
- Universal coefficients:

$$g_2^{ij} \equiv g_2(m_i, m_j), \quad i\gamma^5 (p\!\!\!/ + m) i\gamma^5 = (p\!\!\!/ - m_j) + \cdots \;\Rightarrow\; g_2^{i(j)} \equiv g_2(m_i, -m_j)$$

- $\mathcal{O}(S^2)$ and $\mathcal{O}(P^2)$ universal coefficients related!
  $\Rightarrow$ Is it enough to compute just the operators containing only S?

Example #2: $\mathcal{O}(X^4 D^2)$ Terms (A.A., P. Huang '20)

- Specify all gauge invariant independent operators at $\mathcal{O}(S^4 D^2)$, expand commutators, group terms:

$$16\pi^2 \mathcal{L}_{S^4 D^2} = c_f\, n_D \left[g_{12}^{ijkl} \mathrm{tr_g} \left(S_{ij} S_{jk} [D_\mu, S]_{kl} [D^\mu, S]_{li} \right) + \frac{1}{2} g_{13}^{ijkl} \mathrm{tr_g} \left(S_{ij} [D_\mu, S]_{jk} S_{kl} [D^\mu, S]_{li} \right) \right]$$
$$= c_f\, n_D\, \mathrm{tr_g} \left[-g_{12}^{ijkl} \left(S_{ij} S_{jk} S_{kl} D^2 S_{li} \right) + \left(g_{12}^{ijkl} + g_{12}^{jkli} - g_{13}^{jkli} \right) \left(S_{ij} S_{jk} S_{kl} D^\mu S_{li} D_\mu \right) \right.$$
$$\left. + \frac{1}{2} \left(g_{13}^{ijkl} + g_{13}^{jkli} - g_{12}^{ijkl} - g_{12}^{klij} \right) \left(S_{ij} S_{jk} D^\mu S_{kl} S_{li} D_\mu \right) \right]$$

- Write down terms from CDE (log expansion):

$$16\pi^2 \mathcal{L}_{S^4 D^2} = -c_f \left[\int [d^d p]\, \mathrm{tr_s} \left(\not{\Delta}_i \not{\Delta}_j \not{\Delta}_k \not{\Delta}_l \gamma_\mu \not{\Delta}_l \gamma_\nu \not{\Delta}_l \right) \mathrm{tr_g} \left(S_{ij} S_{jk} S_{kl} D^\mu D^\nu S_{li} \right) \right.$$
$$+ \int [d^d p]\, \mathrm{tr_s} \left(\not{\Delta}_i \not{\Delta}_j \not{\Delta}_k \not{\Delta}_l \gamma_\mu \not{\Delta}_l \not{\Delta}_i \gamma_\nu \right) \mathrm{tr_g} \left(S_{ij} S_{jk} S_{kl} D^\mu S_{li} D^\nu \right)$$
$$\left. + \frac{1}{2} \int [d^d p]\, \mathrm{tr_s} \left(\not{\Delta}_i \not{\Delta}_j \not{\Delta}_k \gamma_\mu \not{\Delta}_k \not{\Delta}_l \not{\Delta}_i \gamma_\nu \right) \mathrm{tr_g} \left(S_{ij} S_{jk} D^\mu S_{kl} S_{li} D^\nu \right) \right]$$

- Equate the two expressions $\Rightarrow g_{12}^{ijkl}, g_{13}^{ijkl} = \ldots$ (note redundancy $\rightarrow$ serves as cross–check!)

Example #2: $\mathcal{O}(X^4 D^2)$ Terms (A.A., P. Huang '20)

- $\mathcal{O}(X^4 D^2)$ universal coefficients for even powers of P? Use γ^5 identities:

$$i\gamma^5(\not{p} + m_i)i\gamma^5 = \not{p} - m_i$$
$$i\gamma^5(\not{p} + m_i)(\not{p} + m_j)i\gamma^5 = -(\not{p} - m_i)(\not{p} - m_j)$$

- $\Rightarrow$ flip mass signs, adjust symmetry factors:

$$\frac{1}{2} g_{13}^{ijkl} \mathrm{tr_g} \left(S_{ij} \left[D_\mu, S\right]_{jk} S_{kl} \left[D^\mu, S\right]_{li} \right) \to g_{13}^{ijk(l)} \mathrm{tr_g} \left(S_{ij} \left[D_\mu, S\right]_{jk} P_{kl} \left[D^\mu, P\right]_{li} \right)$$
$$\to \frac{1}{2} g_{13}^{i(j)k(l)} \mathrm{tr_g} \left(P_{ij} \left[D_\mu, P\right]_{jk} P_{kl} \left[D^\mu, P\right]_{li} \right)$$
$$g_{12}^{ijkl} \mathrm{tr_g} \left(S_{ij} S_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, S\right]_{li} \right) \to g_{12}^{(i)jkl} \mathrm{tr_g} \left(P_{ij} S_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, P\right]_{li} \right)$$
$$\to -g_{12}^{i(j)(k)l} \mathrm{tr_g} \left(P_{ij} S_{jk} \left[D_\mu, P\right]_{kl} \left[D^\mu, S\right]_{li} \right)$$

Example #2: $\mathcal{O}(X^4 D^2)$ Terms (A.A., P. Huang '20)

- Full $\mathcal{O}(X^4 D^2)$ contribution (including P insertions):

$$
\begin{aligned}
16\pi^2 \mathcal{L}_{X^4 D^2} = c_f\, n_D \Big\{ \mathrm{tr_g} \Big[& g_{12}^{ijkl} \left(S_{ij} S_{jk} [D_\mu, S]_{kl} [D^\mu, S]_{li} \right) + g_{12}^{ijk(l)} \left(S_{ij} S_{jk} [D_\mu, P]_{kl} [D^\mu, P]_{li} \right) \\
& + g_{12}^{ij(k)l} \left(S_{ij} P_{jk} [D_\mu, P]_{kl} [D^\mu, S]_{li} \right) + g_{12}^{i(j)kl} \left(P_{ij} P_{jk} [D_\mu, S]_{kl} [D^\mu, S]_{li} \right) \\
& + g_{12}^{(i)jkl} \left(P_{ij} S_{jk} [D_\mu, S]_{kl} [D^\mu, P]_{li} \right) - g_{12}^{ij(k)(l)} \left(S_{ij} P_{jk} [D_\mu, S]_{kl} [D^\mu, P]_{li} \right) \\
& - g_{12}^{i(j)(k)l} \left(P_{ij} S_{jk} [D_\mu, P]_{kl} [D^\mu, S]_{li} \right) + g_{12}^{i(j)k(l)} \left(P_{ij} P_{jk} [D_\mu, P]_{kl} [D^\mu, P]_{li} \right) \Big] \\
& + \mathrm{tr_g} \left[\frac{1}{2} g_{13}^{ijkl} \left(S_{ij} [D_\mu, S]_{jk} S_{kl} [D^\mu, S]_{li} \right) + g_{13}^{ijk(l)} \left(S_{ij} [D_\mu, S]_{jk} P_{kl} [D^\mu, P]_{li} \right) \right. \\
& + g_{13}^{ij(k)l} \left(S_{ij} [D_\mu, P]_{jk} P_{kl} [D^\mu, S]_{li} \right) - \frac{1}{2} g_{13}^{ij(k)(l)} \left(S_{ij} [D_\mu, P]_{jk} S_{kl} [D^\mu, P]_{li} \right) \\
& \left. - \frac{1}{2} g_{13}^{i(j)(k)l} \left(P_{ij} [D_\mu, S]_{jk} P_{kl} [D^\mu, S]_{li} \right) + \frac{1}{2} g_{13}^{i(j)k(l)} \left(P_{ij} [D_\mu, P]_{jk} P_{kl} [D^\mu, P]_{li} \right) \right] \Big\}
\end{aligned}
$$

- γ^5 identities + symmetry factor adjustments $\Rightarrow$ only two terms to compute instead of 14!
- Streamlined computation $\Rightarrow$ automation! (Chakrabortty+ '18)
- Straightforward (although complex) generalization to dim–8!

Overview

Model Description (AA, P. Huang '18)

- Consider a vector–like lepton model that can accommodate a strong EW phase transition:

$$L_{L,R} = \begin{pmatrix} N \\ E \end{pmatrix}_{L,R} \sim (1, 2, Y), \quad N'_{L,R} \sim (1, 1, Y + \frac{1}{2}), \quad E'_{L,R} \sim (1, 1, Y - \frac{1}{2})$$

- VLL Lagrangian (assume negligible mixing with the SM):

$$\begin{aligned} \mathcal{L}_{VLL} = \overline{L}(i\gamma_\mu D_L^\mu - m_L)L + \overline{N}'(i\gamma_\mu D_N^\mu - m_N)N' + \overline{E}'(i\gamma_\mu D_E^\mu - m_E)E' \\ - \left(y_{N_R} \overline{L}_L \tilde{H} N'_R + y_{N_L} \overline{L}_R \tilde{H} N'_L + y_{E_R} \overline{L}_L H E'_R + y_{E_L} \overline{L}_R H E'_L + \text{h.c.} \right) \end{aligned}$$

- M matrix in the $(L,\ E',\ N')$ basis:

$$M = \begin{pmatrix} m_L \, \mathbb{1}_{2\times 2} & 0_{2\times 1} & 0_{2\times 1} \\ 0_{1\times 2} & m_E & 0 \\ 0_{1\times 2} & 0 & m_N \end{pmatrix}$$

Model Description (AA, P. Huang '18)

- Define Yukawa linear combinations:

$$y_A \equiv \frac{y_{A_L} + y_{A_R}}{2}, \quad z_A \equiv \frac{y_{A_L} - y_{A_R}}{2}, \quad A = N, E$$

- $\Rightarrow$ S and P matrices:

$$S = \begin{pmatrix} 0_{2\times 2} & y_E\, H_{2\times 1} & y_N\, \widetilde{H}_{2\times 1} \\ y_E^*\, H^\dagger_{1\times 2} & 0 & 0 \\ y_N^*\, \widetilde{H}^\dagger_{1\times 2} & 0 & 0 \end{pmatrix}, \quad P = i \begin{pmatrix} 0_{2\times 2} & z_E\, H_{2\times 1} & z_N\, \widetilde{H}_{2\times 1} \\ -z_E^*\, H^\dagger_{1\times 2} & 0 & 0 \\ -z_N^*\, \widetilde{H}^\dagger_{1\times 2} & 0 & 0 \end{pmatrix}$$

- $[D_\mu, S]$ and $[D_\mu, P]$ matrices $\rightarrow$ replace $H, \widetilde{H}$ by $(D_\mu H), (D_\mu \widetilde{H})$
- Field strength matrix:

$$F_{\mu\nu} \equiv -i\,[D_\mu, D_\nu] = \begin{pmatrix} g_1 Y\, B_{\mu\nu}\, \mathbb{1}_{2\times 2} + \frac{g_2}{2} W^a_{\mu\nu}\sigma^a & 0_{2\times 1} & 0_{2\times 1} \\ 0_{1\times 2} & g_1 \left(Y - \frac{1}{2}\right) B_{\mu\nu} & 0 \\ 0_{1\times 2} & 0 & g_1 \left(Y + \frac{1}{2}\right) B_{\mu\nu} \end{pmatrix}$$

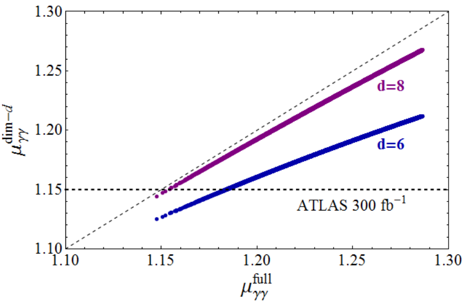
Loop–Induced Higgs Couplings: $h \to \gamma\gamma$

- Where to expect better sensitivities?

  $\to$ Higgs couplings induced at 1 loop in the SM: $h \to \gamma\gamma$!
- Scan approach:

$$m_L, m_N, m_E \in [500, 1500] \ \text{GeV},$$
$$y_{N_{L,R}}, y_{E_{L,R}} \in \left[2, \sqrt{4\pi}\right];$$

- Constraints $\to$ strong 1st order phase transition, $h \to \gamma\gamma$, EW precision tests

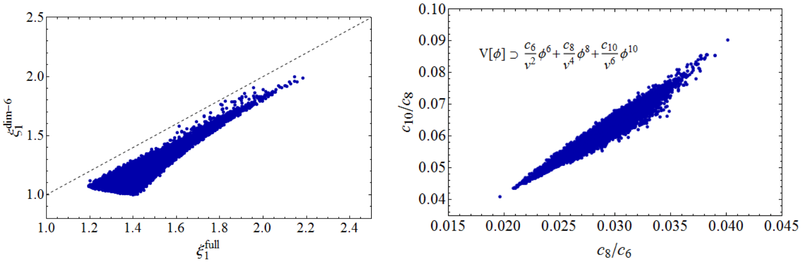


Strong First Order Electroweak Phase Transition (SFOPT)

- How much of the PT dynamics can be captured by $|H|^6$?

  → compare PT strength from full 1–loop potential vs $|H|^6$–only ⇒ good agreement!

  → check EFT validity ⇒ convergent expansion!



- Advantages?

  ⇒ simpler numerics (PT strength, GW calculation) with polynomial potential!

  ⇒ is $|H|^6$ enough when other scalars are integrated out?

Overview

Summary and Conclusions

- Reviewed integration of heavy NP at 1–loop through path integral (functional) methods
- Functional methods for EFT matching $\rightarrow$ solid alternative to Feynman diagrams
  - $+$ universal coefficients done once and for all $\Rightarrow$ perhaps more model–independent?
  - $+$ no tricky minus signs or symmetry factors!
- Used functional methods to integrate out VLFs at 1–loop
  $\rightarrow$ Fermionic universal coefficients for nondegenerate masses computed for the 1st time!
- Common origin of terms involving even powers of γ_5 and terms with no γ_5's
  $\Rightarrow$ a more streamlined calculation!
- Applied the results to a fermionic model for strong phase transitions
  $\rightarrow$ PT strength well captured by dim–6 operators!
- Outlook $\rightarrow$ dim–6 SMEFT enough for capturing EWPT dynamics in the multi–scalar case?

THANK YOU FOR YOUR ATTENTION!

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THANK YOU FOR YOUR ATTENTION!

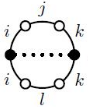
Backup

Covariant Diagrams (Zhang '16)

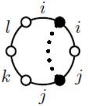
$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = \frac{c_s}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \, \text{tr}\left\{ \left[\Delta \left(2ip_\mu D^\mu + D^2 + U \right) \right]^n \right\}$$

- Three possible insertions: D^2, $2ip_\mu D^\mu$, U.
- Redundancy $\Rightarrow$ no need to consider D^2 or contracted adjacent $2ip_\mu D^\mu$ $\Rightarrow$ less diagrams! Notation: $p \leftrightarrow q$, $P_\mu \equiv iD_\mu$

Element of diagram	Symbol	Expression
heavy propagator (bosonic)	i	1
P insertion (bosonic, heavy)	i j	$2P_\mu \delta_{ij}$
U insertion (heavy-heavy)	i j	$U_{H\,ij}$



$$= -ic_s \frac{2^2}{2} \mathcal{I}[q^2]^{2121}_{ijkl} \, \text{tr}(P^\mu U_{ij} U_{jk} P_\mu U_{kl} U_{li})$$



$$= -ic_s 2^2 \mathcal{I}[q^2]^{2211}_{ijkl} \, \text{tr}(P^\mu U_{ij} P_\mu U_{jk} U_{kl} U_{li})$$

- Extract universal coefficients by matching onto:

$$f_{17}^{ijkl} \text{tr}\left(U_{ij} U_{jk} [P_\mu, U]_{kl} [P^\mu, U]_{li} \right) + f_{18}^{ijkl} \text{tr}\left(U_{ij} [P_\mu, U]_{jk} U_{kl} [P^\mu, U]_{li} \right)$$

$$\supset \left(-f_{17}^{ijkl} + f_{18}^{ijkl} + f_{18}^{jkli} \right) \text{tr}\left(P^\mu U_{ij} U_{jk} P_\mu U_{kl} U_{li} \right) + \left(f_{17}^{jkli} + f_{17}^{klij} - f_{18}^{ijkl} - f_{18}^{klij} \right) \text{tr}\left(P^\mu U_{ij} P_\mu U_{jk} U_{kl} U_{li} \right)$$

Heavy–Light Loops (Murayama+ '16, Ellis+ '16, Portoles+ '16, Zhang '16)

- What about both heavy and light fields running in the same loop?
  → consider quantum fluctuations for light fields too!

$$S_{eff}^{\text{1-loop}} \stackrel{?}{=} ic_s \log\det \begin{pmatrix} -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H^2}\right|_{\varphi_H^{cl}} & -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H\delta\varphi_L}\right|_{\varphi_H^{cl}} \\ -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_L\delta\varphi_H}\right|_{\varphi_H^{cl}} & -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_L^2}\right|_{\varphi_H^{cl}} \end{pmatrix}$$

$$\equiv ic_s \log\det \begin{pmatrix} \mathcal{Q}_H & X_{HL} \\ X_{LH} & \mathcal{Q}_L \end{pmatrix}$$

- No, b/c it includes long–distance contributions, i.e. $p \sim m_{\varphi_L}$
  ⇒ 1PI effective action, not S_{eff}^{1-loop} !

- To get S_{eff}^{1-loop}, need to somehow isolate the short–distance $p \sim m_{\varphi_H}$ contributions...

Heavy–Light Loops (Portoles+ '16, Zhang '16)

- Method of "expansion by regions"
  $\rightarrow$ identity holds (in dim. reg.) order by order in $\frac{1}{M}$ ($M \gg m$):

$$\int [d^d p] \frac{1}{(p^2 - M^2)(p^2 - m^2)^2} = \underbrace{\int [d^d p] \frac{1}{(p^2 - M^2)p^4}}_{\text{hard region: } p \sim M \gg m} + \underbrace{\int [d^d p] \frac{1}{(-M^2)(p^2 - m^2)^2}}_{\text{soft region: } p \sim m \ll M} + \mathcal{O}(M^{-4})$$

- Diagonalize the fluctuation matrix:

$$V \begin{pmatrix} \mathcal{Q}_H & X_{HL} \\ X_{LH} & \mathcal{Q}_L \end{pmatrix} V^\dagger = \begin{pmatrix} \mathcal{Q}_H - X_{HL}\mathcal{Q}_L^{-1}X_{LH} & 0 \\ 0 & \mathcal{Q}_L \end{pmatrix}$$

- The one–loop effective action is therefore:

$$S_{eff}^{\text{1-loop}} = i c_s \mathrm{Tr} \log \left(\mathcal{Q}_H - X_{HL}\mathcal{Q}_L^{-1}X_{LH} \right) \Big|_{\text{hard region}}$$

- The rest of the computation (e.g. momentum shift) proceeds as in the heavy–only case

CP–Even Dimension–6 Operators (Huo '15)

- Set all masses equal to m and $y_{E_L} = y_{E_R} \equiv y_E$, $y_{N_L} = y_{N_R} \equiv y_N$:

$$
\begin{aligned}
16\pi^2 \mathcal{L}^{\rm CP}_{\rm dim-6} = & \frac{2\left(y_E^6 + y_N^6\right)}{15m^2} |H|^6 - \frac{2(y_E^2 + y_N^2)^2}{5m^2} |H|^2 \Box |H|^2 - \frac{4(y_E^2 - y_N^2)^2}{5m^2} \left|H^\dagger D_\mu H\right|^2 \\
& - \frac{y_E^4 - 4y_E^2 y_N^2 + y_N^4}{5m^2} |H|^2 \left(H^\dagger D^2 H + \text{h.c.}\right) - \frac{7\left(y_E^2 + y_N^2\right)}{120m^2} g_2^2 |H|^2 W^i_{\mu\nu} W^i_{\mu\nu} \\
& - \frac{\left(7 - 40Y + 80Y^2\right) y_E^2 + \left(7 + 40Y + 80Y^2\right) y_N^2}{120m^2} g_1^2 |H|^2 B_{\mu\nu} B_{\mu\nu} \\
& + \frac{(3 - 20Y) y_E^2 + (3 + 20Y) y_N^2}{60m^2} g_1 g_2 \left(H^\dagger \sigma^i H\right) W^i_{\mu\nu} B_{\mu\nu} + \frac{y_E^2 + y_N^2}{5m^2} \left|D^2 H\right|^2 \\
& + \frac{2\left(y_E^2 + y_N^2\right)}{15m^2} g_1 \left(H^\dagger i \overleftrightarrow{D}_\mu H\right) \partial_\nu B_{\nu\mu} + \frac{2\left(y_E^2 + y_N^2\right)}{15m^2} g_2 \left(H^\dagger i \overleftrightarrow{D}^i_\mu H\right) D_\nu W^i_{\nu\mu} \\
& - \frac{1 + 8Y^2}{15m^2} g_1^2 (\partial_\mu B_{\mu\nu})^2 - \frac{1}{15m^2} g_2^2 (D_\mu W^i_{\mu\nu})^2 - \frac{1}{30m^2} \frac{g_2^3}{3!} \epsilon_{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}
\end{aligned}
$$

- Also, renormalization of EW gauge kinetic terms + Higgs kinetic, mass, and quartic terms.

CP–Odd Dimension–6 Operators

- Set all masses equal to m, but $y_{E_L} \neq y_{E_R}$, $y_{N_L} \neq y_{N_R}$ (otherwise no $\not{CP}$):

$$16\pi^2 \mathcal{L}^{\not{CP}}_{\mathrm{dim}-6} = -\frac{(|y_{N_L}|^2 + |y_{N_R}|^2)\mathrm{Im}(y_{N_L} y^*_{N_R}) - (|y_{E_L}|^2 + |y_{E_R}|^2)\mathrm{Im}(y_{E_L} y^*_{E_R})}{3m^2}\widetilde{\mathcal{O}}_f$$
$$-\frac{(1+6Y+12Y^2)\,\mathrm{Im}(y_{N_L} y^*_{N_R}) + (1-6Y+12Y^2)\,\mathrm{Im}(z_{E_L} y^*_{E_R})}{12m^2} g_1^2 |H|^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$$
$$+\frac{(1+6Y)\,\mathrm{Im}(y_{N_L} y^*_{N_R}) + (1-6Y)\,\mathrm{Im}(y_{E_L} y^*_{E_R})}{12m^2} g_1 g_2 (H^\dagger \sigma^a H) \widetilde{W}^a_{\mu\nu} B^{\mu\nu}$$
$$-\frac{\mathrm{Im}(y_{N_L} y^*_{N_R} + y_{E_L} y^*_{E_R})}{12m^2} g_2^2 |H|^2 \widetilde{W}^a_{\mu\nu} W^{\mu\nu,a}, \quad \widetilde{\mathcal{O}}_f \equiv \frac{i}{2}|H|^2\left[(D^2H)^\dagger H - H^\dagger D^2 H\right]$$

- $\widetilde{\mathcal{O}}_f$ breaks custodial symmetry $\Rightarrow$ leading source of $\not{CP}$ at $\mathcal{O}(y^2_{E,N})$, not $\mathcal{O}(y^4_{E,N})$

Re–deriving Higgs Low Energy Theorems

- Goal → reproduce the Low Energy Theorem (LET) expression for $h \to \gamma\gamma$
- Broken phase → consider $E_{i=1,2}$ mass eigenstates, assume N_i's decoupled:

$$\mathcal{L}_{\rm VLL} \supset \overline{E}_i \left[i\gamma^\mu D_\mu \delta_{ij} - m_i \delta_{ij} - y_{ij} h - \tilde{y}_{ij} h (i\gamma^5) \right] E_j$$

$$D_\mu = \partial_\mu + i\, Q_E e\, A_\mu \;\Rightarrow\; F_{\mu\nu} = Q_E e\, A_{\mu\nu}, \quad S_{ij} = y_{ij} h, \quad P_{ij} = \tilde{y}_{ij} h$$

- Relevant universal terms for $h \to \gamma\gamma$ in the broken phase:

$$\mathcal{L}^{\text{1-loop}}_{\rm VLF} \supset \frac{c_f n_D}{16\pi^2} {\rm tr_g} \left[g_9^i \left(S_{ii} F_i^{\mu\nu} F_{\mu\nu,i} \right) + g_{10}^i \left(P_{ii} \widetilde{F}_i^{\mu\nu} F_{\mu\nu,i} \right) \right]$$

$$= \frac{Q_E^2 e^2}{24\pi^2} \left(\sum_i \frac{y_{ii}}{m_i} \right) h\, A_{\mu\nu} A^{\mu\nu} - \frac{Q_E^2 e^2}{16\pi^2} \left(\sum_i \frac{\tilde{y}_{ii}}{m_i} \right) h\, \widetilde{A}_{\mu\nu} A^{\mu\nu}$$

$$= \frac{Q_E^2 e^2}{24\pi^2} \left(\frac{\partial \log \left(|\det \mathcal{M}_E| \right)}{\partial v} \right) h\, A_{\mu\nu} A^{\mu\nu} - \frac{Q_E^2 e^2}{16\pi^2} \left(\frac{\partial \arg \left(\det \mathcal{M}_E \right)}{\partial v} \right) h\, \widetilde{A}_{\mu\nu} A^{\mu\nu}$$

- Agreement with previous results in the literature!
- Agreement with symmetric phase calculation of $|H|^2 A_{\mu\nu} A_{\mu\nu}$ and $|H|^2 \widetilde{A}_{\mu\nu} A_{\mu\nu}$!

The Fermionic Universal Master Formula

- Building blocks: S, $[D_\mu, S]$, $[D_\mu, [D^\mu, S]]$, $F_{\mu\nu}$, $[D^\mu, F_{\mu\nu}]$, $\widetilde{F}_{\mu\nu}$ (and $[S \to P]$)

$$
\begin{aligned}
\mathcal{L}^{\text{1-loop}}_{\text{VLF}} \supset \frac{c_f n_D}{16\pi^2} \mathrm{tr_g} \Bigg[& g_1^i\, S_{ii} + \frac{1}{2} g_2^{ij} \left(S_{ij} S_{ji}\right) + \frac{1}{3} g_3^{ijk} \left(S_{ij} S_{jk} S_{ki}\right) + \frac{1}{4} g_4^{ijkl} \left(S_{ij} S_{jk} S_{kl} S_{li}\right) \\
& + \frac{1}{2} g_5^{ij} \left([D_\mu, S]_{ij} [D^\mu, S]_{ji}\right) + \frac{1}{2} g_6^i \left(F_{\mu\nu,i} F_i^{\mu\nu}\right) + \frac{1}{5} g_7^{ijklm} \left(S_{ij} S_{jk} S_{kl} S_{lm} S_{mi}\right) \\
& + g_8^{ijk} \left(S_{ij} [D_\mu, S]_{jk} [D^\mu, S]_{ki}\right) + g_9^i \left(S_{ii} F_i^{\mu\nu} F_{\mu\nu,i}\right) + g_{10}^i \left(P_{ii} \widetilde{F}_i^{\mu\nu} F_{\mu\nu,i}\right) \\
& + \frac{1}{6} g_{11}^{ijklmn} \left(S_{ij} S_{jk} S_{kl} S_{lm} S_{mn} S_{ni}\right) + g_{12}^{ijkl} \left(S_{ij} S_{jk} [D_\mu, S]_{kl} [D^\mu, S]_{li}\right) \\
& + \frac{1}{2} g_{13}^{ijkl} \left(S_{ij} [D_\mu, S]_{jk} S_{kl} [D^\mu, S]_{li}\right) + \frac{1}{2} g_{14}^{ij} [D_\mu, [D^\mu, S]]_{ij} [D_\nu, [D^\nu, S]]_{ji} \\
& + g_{15}^{ij} \left(S_{ij} S_{ji} F_i^{\mu\nu} F_{\mu\nu,i}\right) + \frac{1}{2} g_{16}^{ij} \left(S_{ij} F_j^{\mu\nu} S_{ji} F_{\mu\nu,i}\right) + i\, g_{17}^{ij} \left(S_{ij} [D_\mu, S]_{ji} [D_\nu, F^{\nu\mu}]_i\right) \\
& + g_{18}^{ij} \left((S_{ij} P_{ji} + P_{ij} S_{ji}) \widetilde{F}_i^{\mu\nu} F_{\mu\nu,i}\right) + g_{19}^{ij} \left(S_{ij} \widetilde{F}_j^{\mu\nu} P_{ji} F_{\mu\nu,i}\right) \\
& + g_{20}^i\, [D^\mu, F_{\mu\nu}]_i\, [D_\rho, F^{\rho\nu}]_i + \frac{i}{3} g_{21}^i \left(F^\mu{}_\nu F^\nu{}_\rho F^\rho{}_\mu\right) \Bigg]
\end{aligned}
$$

The Universal One–Loop Effective Action (UOLEA) (Murayama+ '14, Ellis+ '15)

$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = i c_s \,\mathrm{tr} \int \frac{d^d p}{(2\pi)^d} \log\left[-(iD_\mu - p_\mu)^2 + M^2 + U\right]$$

- Gaillard '86, Cheyette '88: momentum shift → insert $e^{\pm i D_\mu \partial/\partial p_\mu}$:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{1-loop}} &= i c_s \,\mathrm{tr} \int \frac{d^d p}{(2\pi)^d} e^{i D_\mu \partial/\partial p_\mu} \log\left[-(iD_\mu - p_\mu)^2 + M^2 + U\right] e^{-i D_\mu \partial/\partial p_\mu} \\ &= i c_s \,\mathrm{tr} \int \frac{d^d p}{(2\pi)^d} \log\left[-\left(\tilde{G}_{\nu\mu} \frac{\partial}{\partial p_\nu} + p_\mu\right)^2 + M^2 + \tilde{U}\right]\end{aligned}$$

- Gauge invariance preserved during all intermediate steps
  ⇒ D_μ's appearing only in commutators:

$$\begin{aligned}\tilde{G}_{\nu\mu} &= \sum_{n=0}^{\infty} i^n \frac{n+1}{(n+2)!} [D_{\mu_1}, [\ldots [D_{\mu_n}, [D_\nu, D_\mu]]]] \frac{\partial^n}{\partial p_{\mu_1} \ldots \partial p_{\mu_n}}, \\ \tilde{U} &= \sum_{n=0}^{\infty} i^n \frac{1}{n!} [D_{\mu_1}, [\ldots [D_{\mu_n}, U]]] \frac{\partial^n}{\partial p_{\mu_1} \ldots \partial p_{\mu_n}}\end{aligned}$$

- Momentum derivatives → cumbersome...

Systematic Treatment of γ^5

- Useful γ^5 identity ($d = 4 - 2\epsilon$ dimensions):

$$i\gamma^5(\not{p} + m)i\gamma^5 = (\not{p} - m) - 2\hat{g}_{\mu\nu}p^\mu\gamma^\nu, \quad g_{\mu\nu}\hat{g}^{\mu\nu} = \hat{g}_{\mu\nu}\hat{g}^{\mu\nu} = -2\epsilon$$

- "Breitenlohner–Maison–'t Hooft–Veltman" (BMHV) scheme for γ^5 in $d = 4 - 2\epsilon$ dimensions
  $\Rightarrow$ evanescent part $\propto \hat{g}_{\mu\nu} \Rightarrow$ origin of δg_2^{ij}:

$$\frac{1}{n_D}\int [d^dp]\,\mathrm{tr_s}(\Delta_i\Delta_j) = \frac{1}{n_D}\int [d^dp]\frac{\mathrm{tr_s}\left[(\not{p}+m_i)(\not{p}+m_j)\right]}{(p^2-m_i^2)(p^2-m_j^2)} \qquad \Rightarrow g_2^{ij}$$

$$\frac{1}{n_D}\int [d^dp]\,\mathrm{tr_s}\left[\Delta_i(i\gamma^5)\Delta_j(i\gamma^5)\right] = \frac{1}{n_D}\int [d^dp]\frac{\mathrm{tr_s}\left[(\not{p}+m_i)(\not{p}-m_j)\right]}{(p^2-m_i^2)(p^2-m_j^2)} \qquad \Rightarrow g_2^{i(j)}$$

$$-\frac{2}{n_D}\int [d^dp]\frac{4\hat{g}_{\mu\nu}p^\mu p^\nu}{(p^2-m_i^2)(p^2-m_j^2)} \qquad \Rightarrow \delta g_2^{ij}$$

- Compute operators containing only $S \Rightarrow$ even powers of P follow immediately:
  - $+$ flip sign of certain masses
  - $+$ adjust symmetry factors
- Bonus $\rightarrow$ no evansesccent part for finite loops
  $\Rightarrow$ simpler computation for dim–5 and dim–6 operators!

Choice of Bubble Wall Velocity

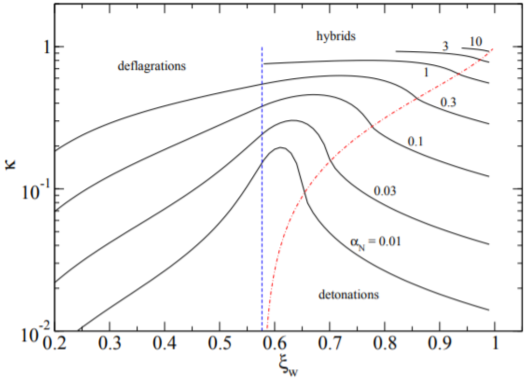


Figure: Ratio of bulk kinetic energy over to vacuum energy κ (efficiency factor) as a function of the bubble wall velocity, ξ_w, for various values of $\alpha_N \equiv \alpha$ (result from 1004.4187).

Our choice: $\xi_w = 0.6 \Rightarrow \kappa \simeq 0.4$ (for typical values of $\alpha \simeq 10^{-1}$).

GW Spectrum Formulae

$$h^2\Omega_{\rm col}(f) = 1.67\times 10^{-5}\left(\frac{0.11\xi_w^3}{0.42+\xi_w^2}\right)\left(\frac{\beta}{H_{\rm PT}}\right)^{-2}\left(\frac{\kappa_{\rm col}\alpha}{1+\alpha}\right)^2\left(\frac{g_{\rm eff}}{100}\right)^{-1/3}\frac{3.8\,(f/f_{\rm col})^{2.8}}{1+2.8\,(f/f_{\rm col})^{3.8}},$$

$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\xi_w\left(\frac{\beta}{H_{\rm PT}}\right)^{-1}\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{g_{\rm eff}}{100}\right)^{-1/3}\frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}\,(1+8\pi f/h_*)},$$

$$h^2\Omega_{\rm sw}(f) = 2.62\times 10^{-6}\xi_w\left(\frac{\beta}{H_{\rm PT}}\right)^{-1}\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{g_{\rm eff}}{100}\right)^{-1/3}\frac{7^{3.5}\,(f/f_{\rm sw})^3}{\left(4+3\,(f/f_{\rm sw})^2\right)^{3.5}}.$$

$$\kappa = 0.4,\ \epsilon = 0.05\ \Rightarrow\ \kappa_{\rm turb} = \epsilon\,\kappa = 0.02.$$

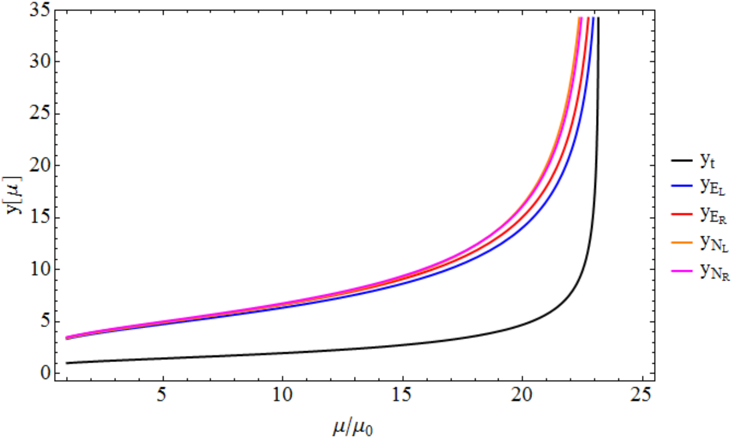
$$f_{\rm col} = (1.65\times 10^{-5}\,{\rm Hz})\left(\frac{0.62}{1.8-0.1\xi_w+\xi_w^2}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\,{\rm GeV}}\right)\left(\frac{g_{\rm eff}}{100}\right)^{1/6},$$

$$f_{\rm turb} = (2.7\times 10^{-5}\,{\rm Hz})\left(\frac{1}{\xi_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\,{\rm GeV}}\right)\left(\frac{g_{\rm eff}}{100}\right)^{1/6},$$

$$h_* = (1.65\times 10^{-5}\,{\rm Hz})\left(\frac{T_{\rm PT}}{100\,{\rm GeV}}\right)\left(\frac{g_{\rm eff}}{100}\right)^{1/6},$$

$$f_{\rm sw} = (1.9\times 10^{-5}\,{\rm Hz})\left(\frac{1}{\xi_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\,{\rm GeV}}\right)\left(\frac{g_{\rm eff}}{100}\right)^{1/6}.$$

Yukawa Running



Other Correlations