GRAVITATIONAL PRODUCTION OF VECTOR DARK MATTER

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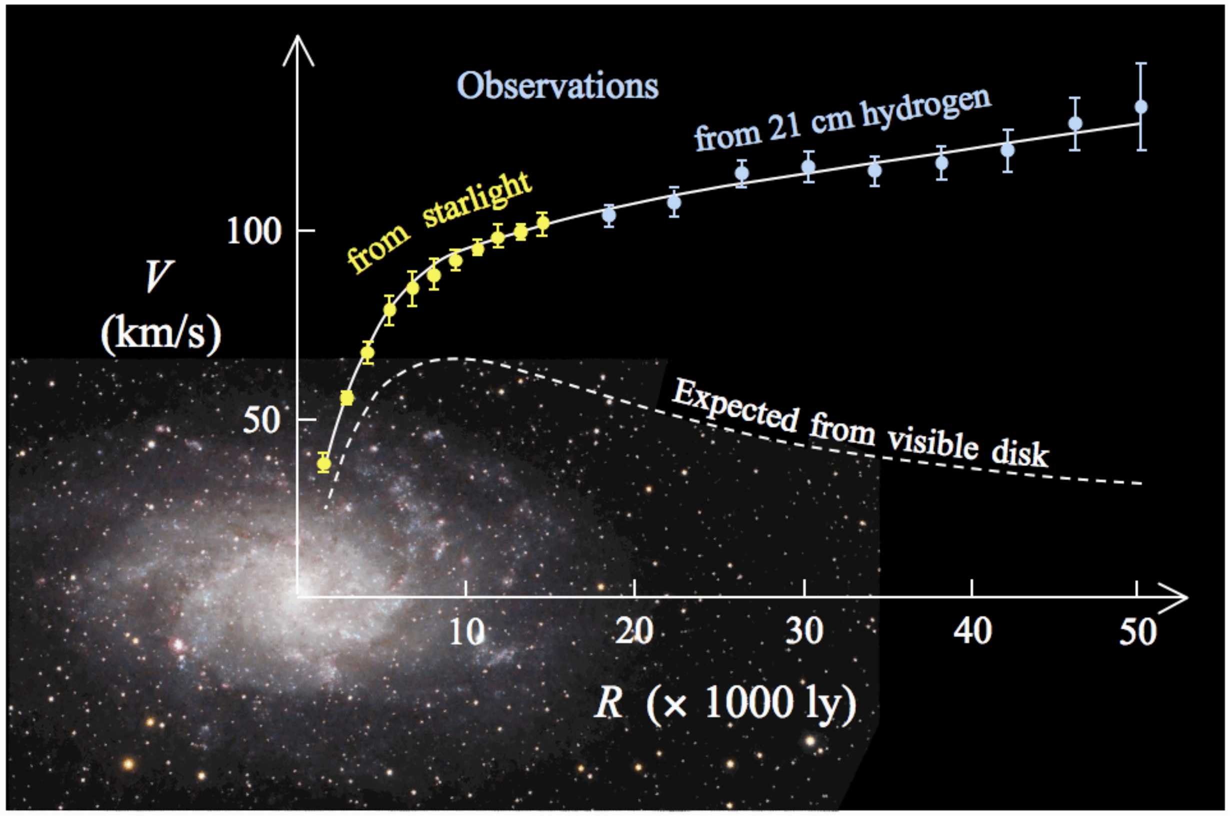
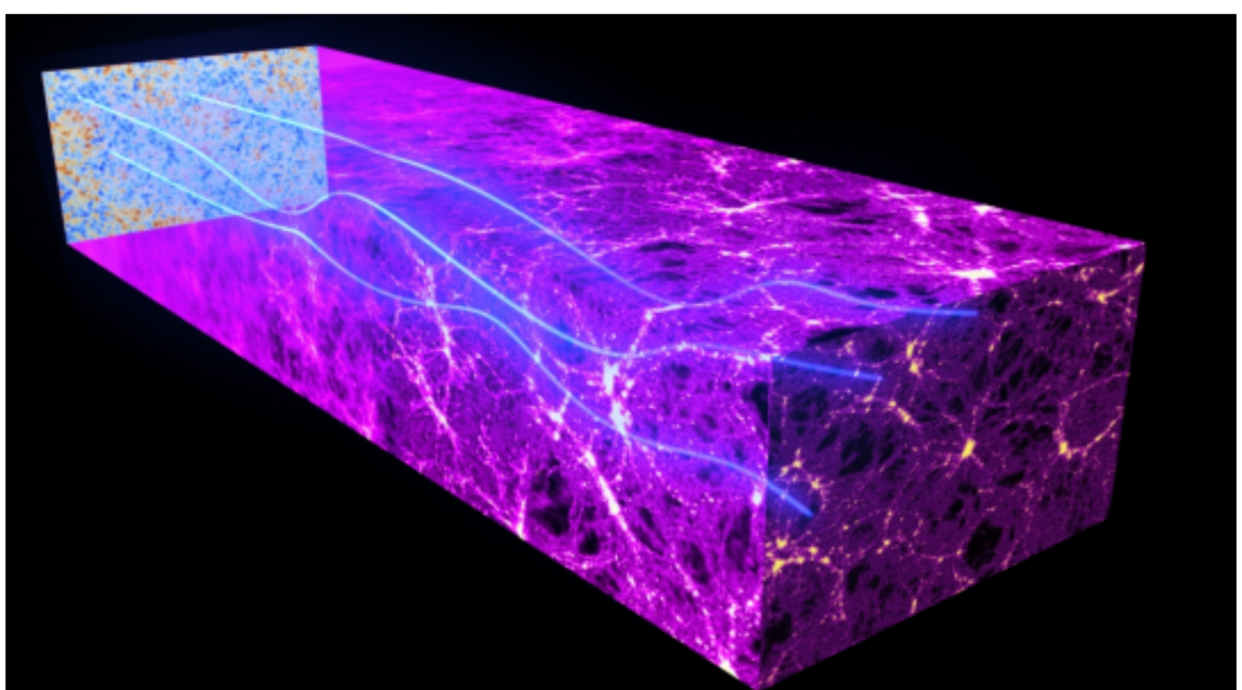
DARK MATTER EVIDENCE

Galatic rotation curves

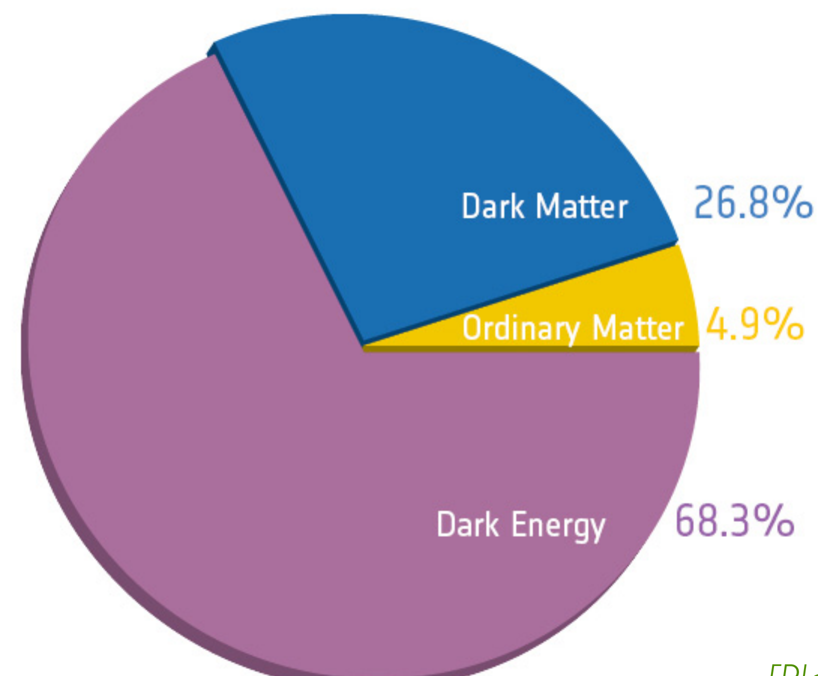


Cluster collisions

Gravitational lensing of CMB



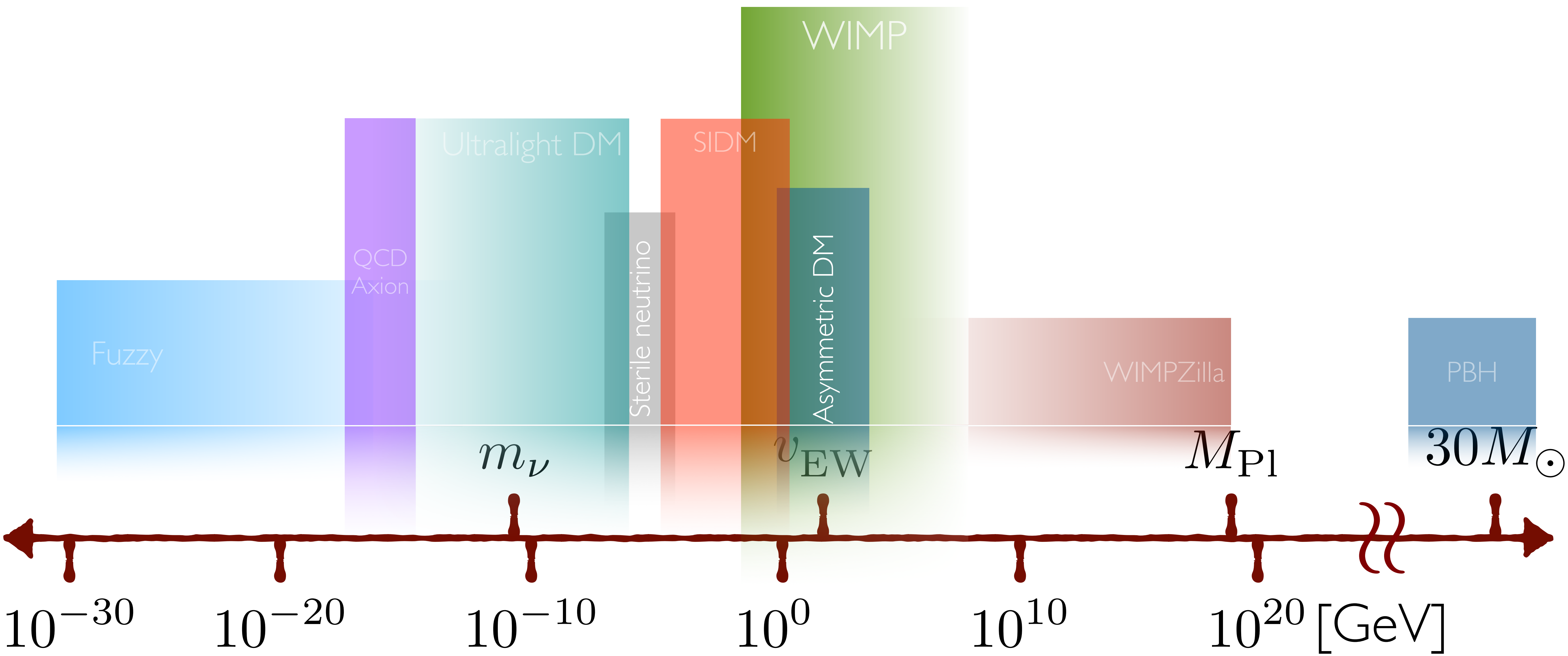
DM ~ 85% of total matter



[Planck Satellite]

DARK MATTER EVIDENCE

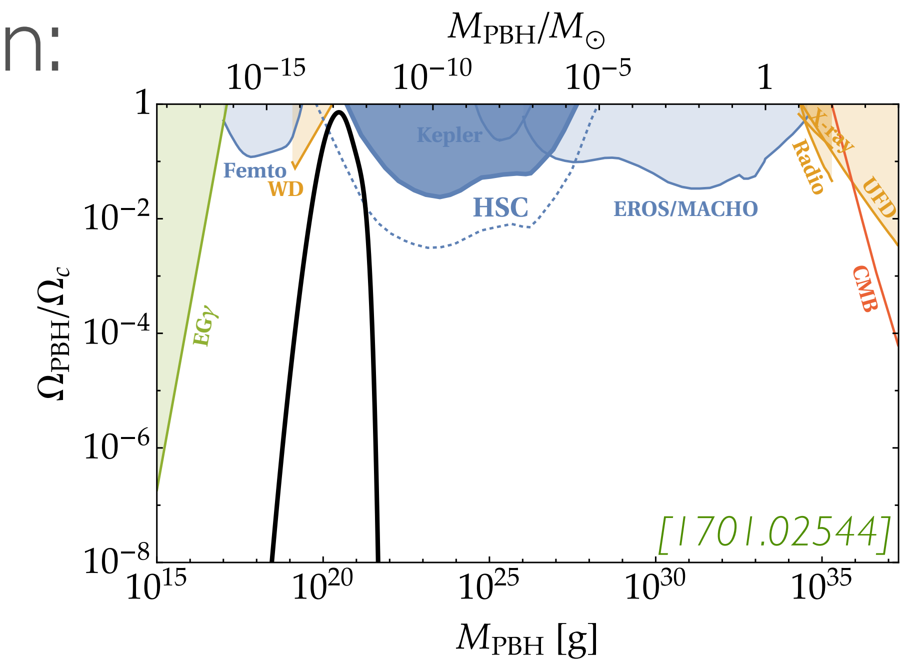
- Dark matter (DM) is massive and stable.
- DM interacts with gravity!
- No (so far) evidence of non-gravitational interactions of DM.



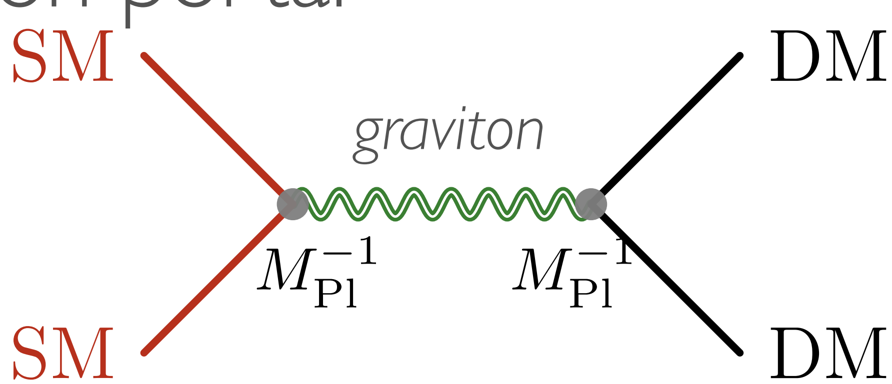
GRAVITATIONAL DARK MATTER

- Nightmare scenario: DM only interacts gravitationally!

- Mechanisms for gravitational DM production:
  - Primordial black holes (PBH) as DM

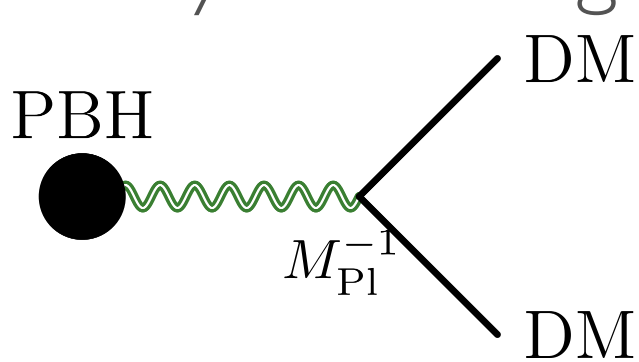


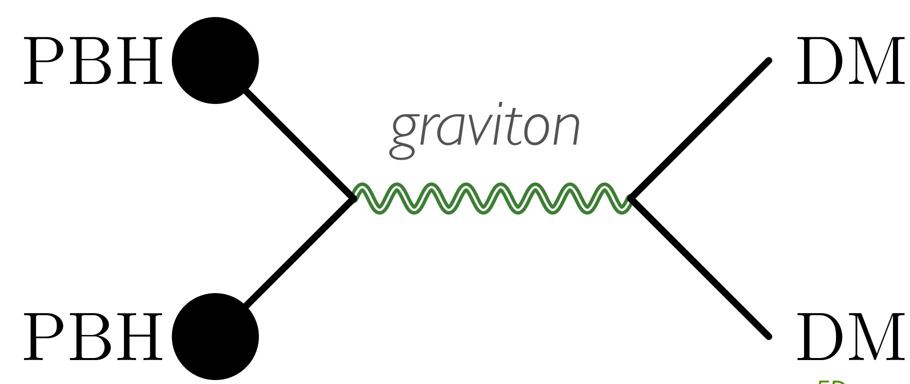
  - Freeze-in via graviton portal



[Garny, Sandora, Sloth 2015; +...]

  - PBH decay/scattering





[Hooper, Krnjaic, McDermott 2019; +...]

[Bernal, Zapata 2020; +...]

GRAVITATIONAL PARTICLE PRODUCTION

- DM from gravitational particle production due to quantum fluctuations

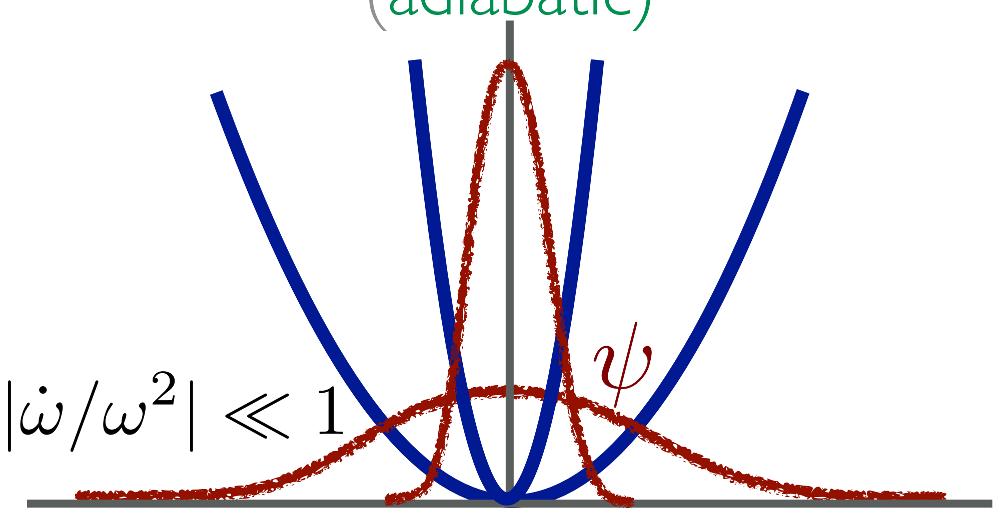
[Schrodinger 1939; Parker 1969 ; Zeldovich, Starobinsky 1972; Ford 1987; +…]

- Rapid expansion in the early universe (during and after inflation) leads to non-adiabatic production of quantum excitations from the background fields.

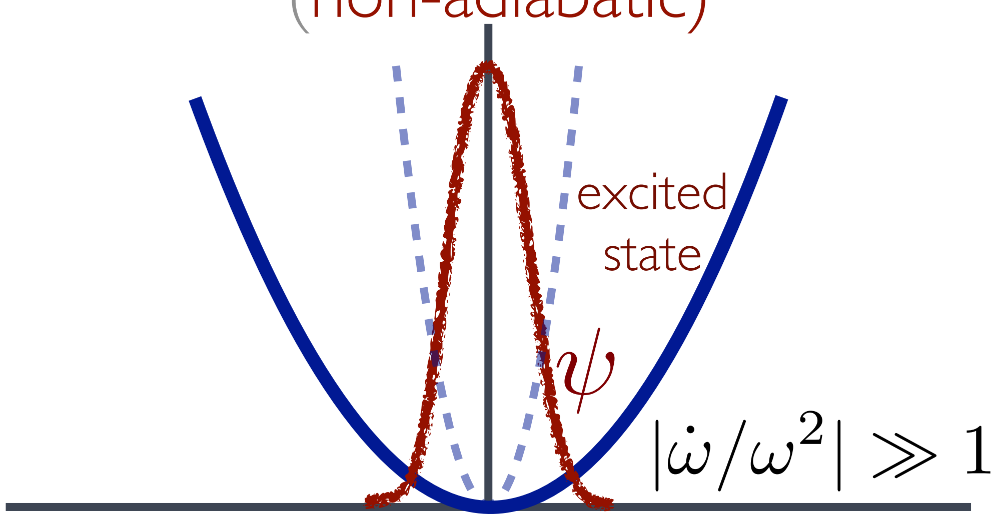
- Analogy: 1D hormonic oscillator with time-dependent frequency

$$\ddot{x}(t) + \omega^2(t)x(t) = 0$$

slow frequency change (adiabatic)



fast frequency change (non-adiabatic)

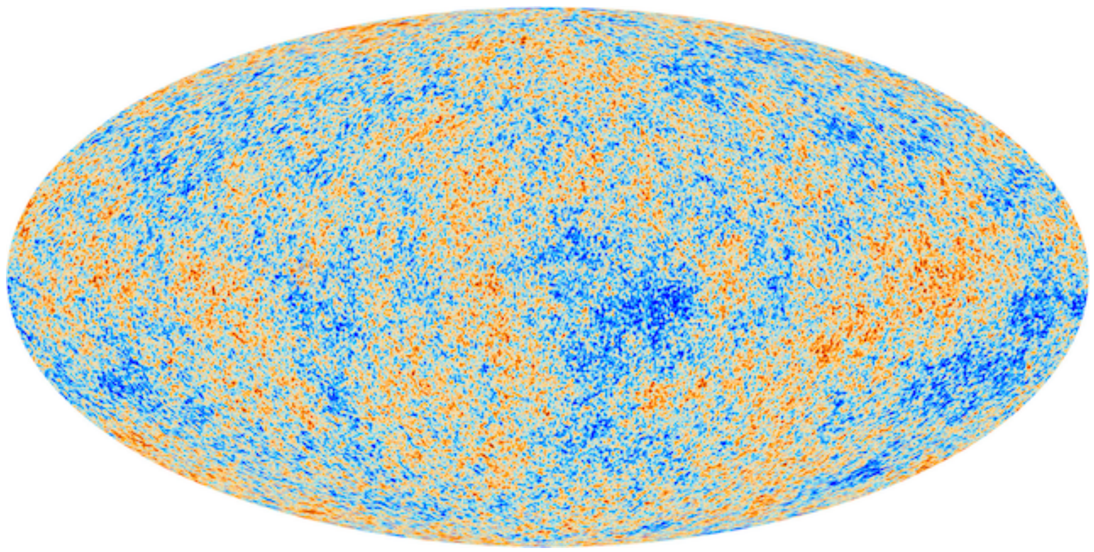


GRAVITATIONAL DARK MATTER

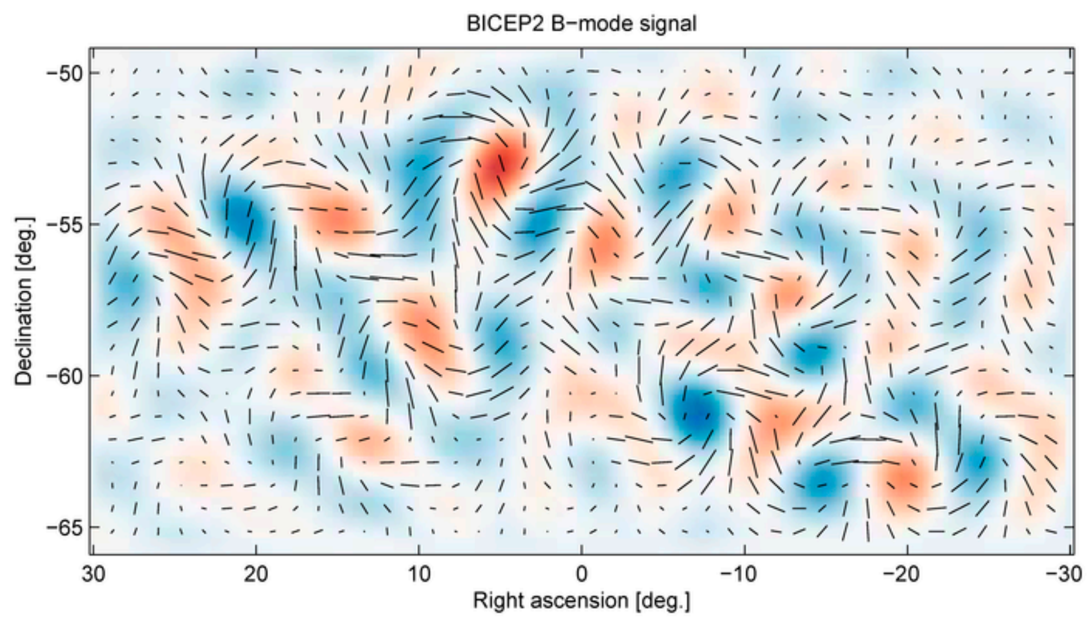
- Why gravitational production of dark matter?
  - Unavoidable consequence of rapid expansion of the Universe
  - DM number density is tied with inflationary observables; Hubble scale H_I, reheating temperature $T_{\rm rh}$, inflaton potential $V(\phi)$, etc.
  - Beside gravity no other interaction is required
  - Spectator field: not relevant directly for inflationary dynamics
  - Inflation dynamics is less important during inflation, however it could be relevant during the reheating phase.

INFLATIONARY FLUCTUATIONS

- Scalar → inflaton fluctuation → adiabatic density perturbations



- Tensor → metric fluctuation → primordial gravity waves



- Vector → cold relic → DM

OUTLINE

I. The vector dark matter model

II. Cosmological evolution

III. Evolution of the vector mode functions

IV. Vector DM relic abundance

V. Next to minimal gravitational vector DM model

VI. Reheating dynamics and role of inflaton

MINIMAL VECTOR DM

- Massive Abelian vector X_μ

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} - \frac{1}{2}m_X^2 g^{\mu\nu}X_\mu X_\nu\right)$$

[Graham, Mardon, Rajendran 2015]

- FLRW metric $ds^2 = a^2(\tau)\left(d\tau^2 - d\vec{x}^2\right)$

Non-minimal coupling with gravity see [Alonso-Alvarez, Jaeckel, Hugle 2020]

- Mode equations

$$X_\mu(t,\mathbf{x}) = \sum_{\lambda=\pm,L}\int\frac{d^3k}{(2\pi)^{3/2}}\epsilon_\mu^\lambda(\mathbf{k})\mathcal{X}_\mu^\lambda(t,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\mathcal{X}_\pm'' + \omega_\pm^2(\tau)\mathcal{X}_\pm = 0, \qquad \widetilde{\mathcal{X}}_L'' + \omega_L^2(\tau)\widetilde{\mathcal{X}}_L = 0$$

$$\widetilde{\mathcal{X}}_L(\tau,\mathbf{k}) \equiv \frac{a(\tau)m_X}{\sqrt{k^2 + a^2(\tau)m_X^2}}\mathcal{X}_L(\tau,\mathbf{k})$$

$$\omega_\pm^2(\tau) = k^2 + a^2m_X^2$$

$$\omega_L^2(\tau) = k^2 + a^2m_X^2 - \frac{k^2}{k^2 + a^2m_X^2}\left(\frac{a''}{a} - \frac{3a^2m_X^2}{k^2 + a^2m_X^2}\frac{a'^2}{a^2}\right)$$

tachyonic enhancement

PROCEDURE FOR CALCULATING DM DENSITY

- Solve mode equations with Bunch-Davies initial condition

$$\lim_{\tau \to -\infty} \widetilde{\mathcal{X}}_L(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

- Energy density of the longitudinal component of vector DM

$$\langle \rho_L \rangle \equiv \langle 0| : \hat{T}_{00} : |0\rangle$$

$$\frac{d\langle \rho_L \rangle}{d\ln k} = \frac{k^3}{2\pi^2 a^4} \Bigg\{ |\mathcal{X}_L'|^2 - (\mathcal{X}_L' \mathcal{X}_L^* + \mathcal{X}_L'^* \mathcal{X}_L) \frac{k^2}{k^2 + a^2 m_X^2} \frac{a'}{a}$$

$$+ \left(\frac{k^4}{(k^2 + a^2 m_X^2)^2} \left(\frac{a'}{a}\right)^2 + k^2 + a^2 m_X^2 \right) |\mathcal{X}_L|^2 \Bigg\}$$

- Number density at $H(a_\star) = m_X$

$$\frac{d\langle n_L(a_\star)\rangle}{d\ln k} = \frac{1}{\sqrt{m_X^2 + k^2/a^2(\tau_\star)}} \frac{d\langle \rho_L(a_\star)\rangle}{d\ln k}$$

- Present relic density

$$\Omega_X h^2 = \frac{\rho_X}{\rho_c} h^2 = \frac{m_X\, n_X(T_0)}{\rho_c} h^2$$

COSMOLOGICAL EVOLUTION

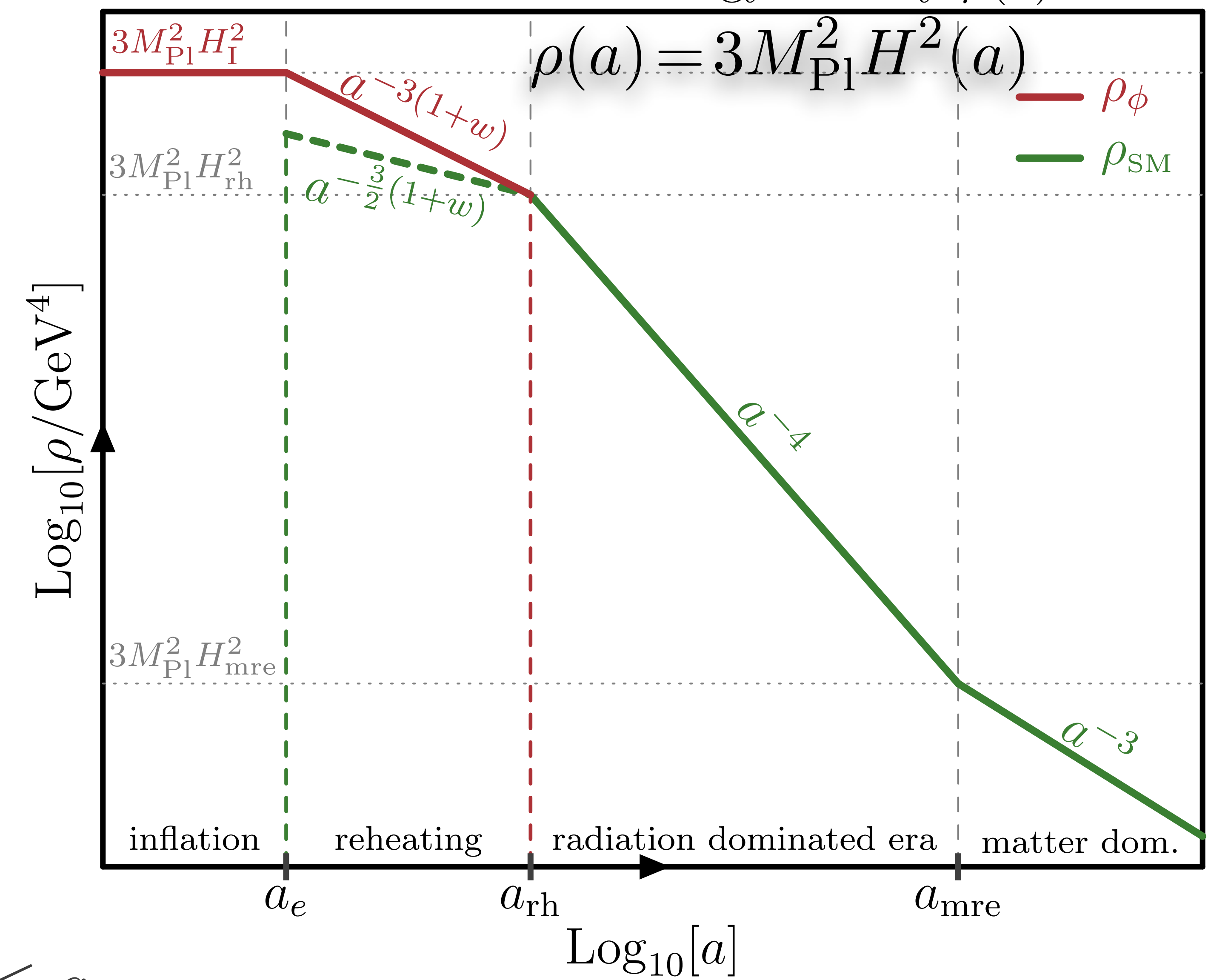
- Inflation: de Sitter solution
- Reheating: non-standard cosmology

  $$w = \rho_\phi / p_\phi : [-1/3, 1]$$

- Standard cosmological evolution after reheating

$$H(a) = \begin{cases} H_{\rm I}, & a \leq a_e \\ H_{\rm I}\left(\frac{a_e}{a}\right)^{\frac{3(1+w)}{2}}, & a_e < a \leq a_{\rm rh} \\ H_{\rm rh}\left(\frac{a_{\rm rh}}{a}\right)^2, & a_{\rm rh} < a \end{cases}$$

Evolution of the energy density $\rho(a)$



$$\gamma \equiv \sqrt{\frac{H_{\rm rh}}{H_{\rm I}}}$$

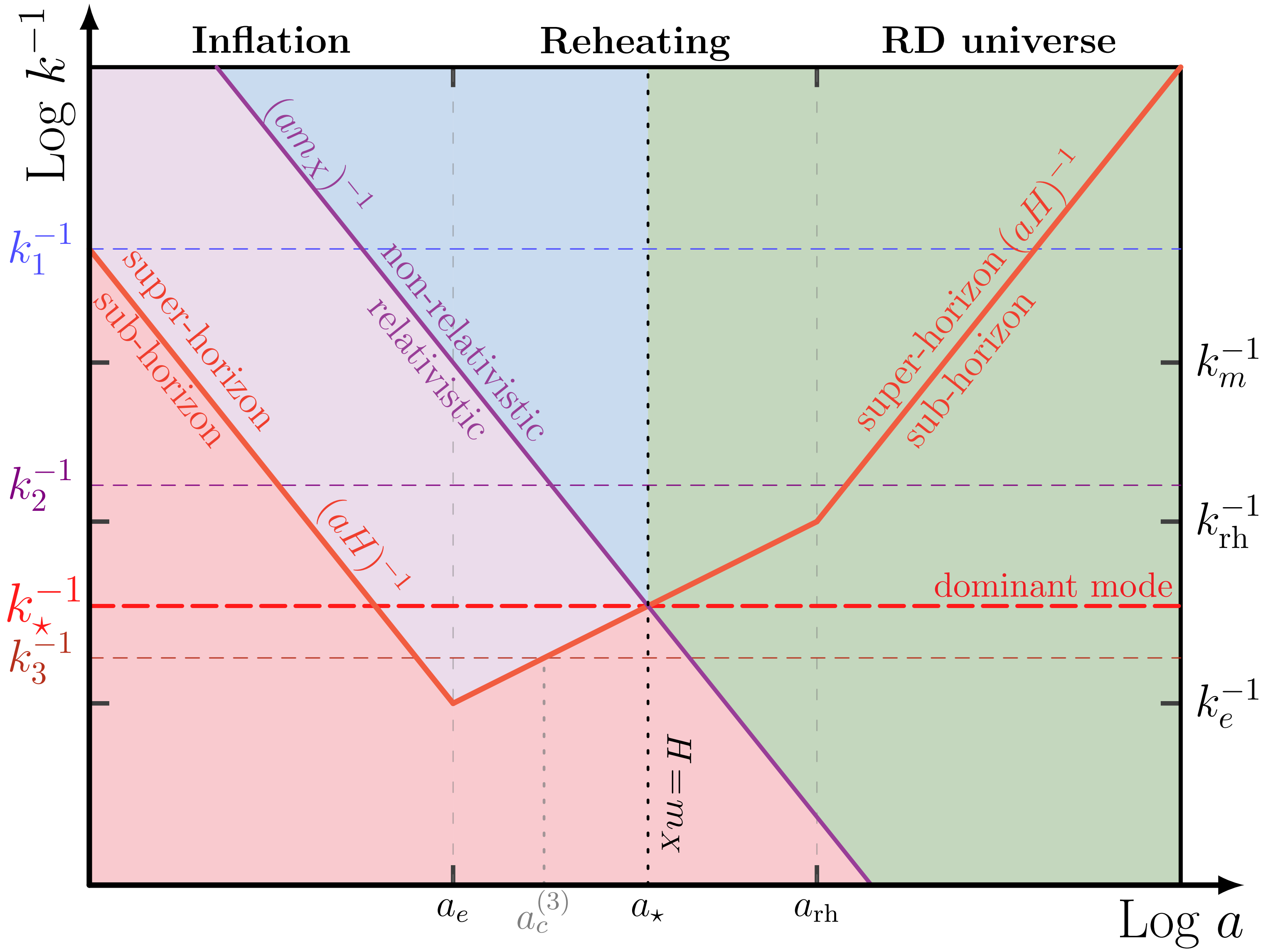
Reheating efficiency

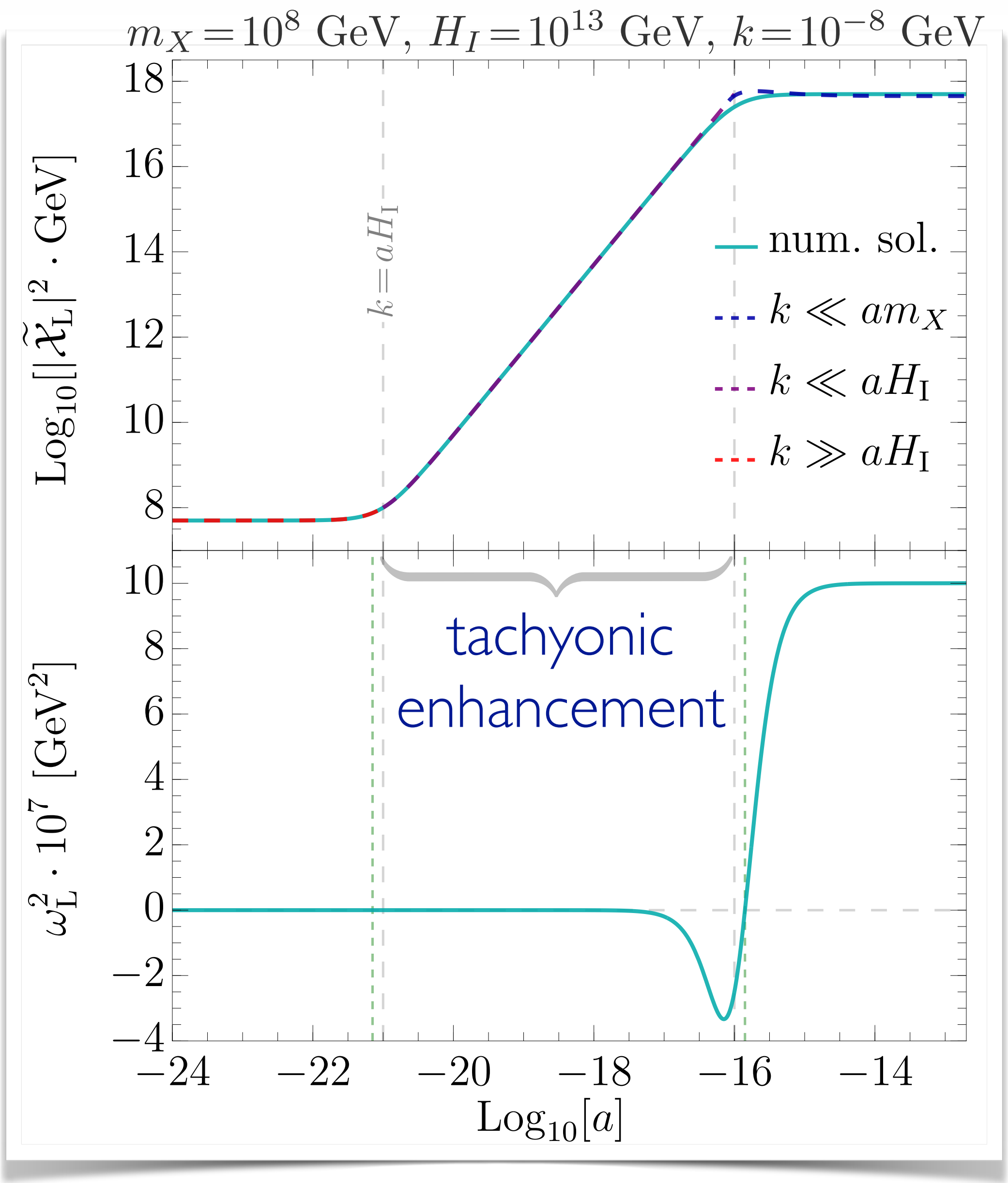
$$10^{-18} \leq \gamma \leq 1$$

$T_{\rm rh} \gtrsim 10\,{\rm MeV}$

Non-instantaneous reheating with w=0 also discussed in [Ema, Nakayama, Tang 2019, Kolb, Long 2020]

MODE EVOLUTION DURING INFLATION

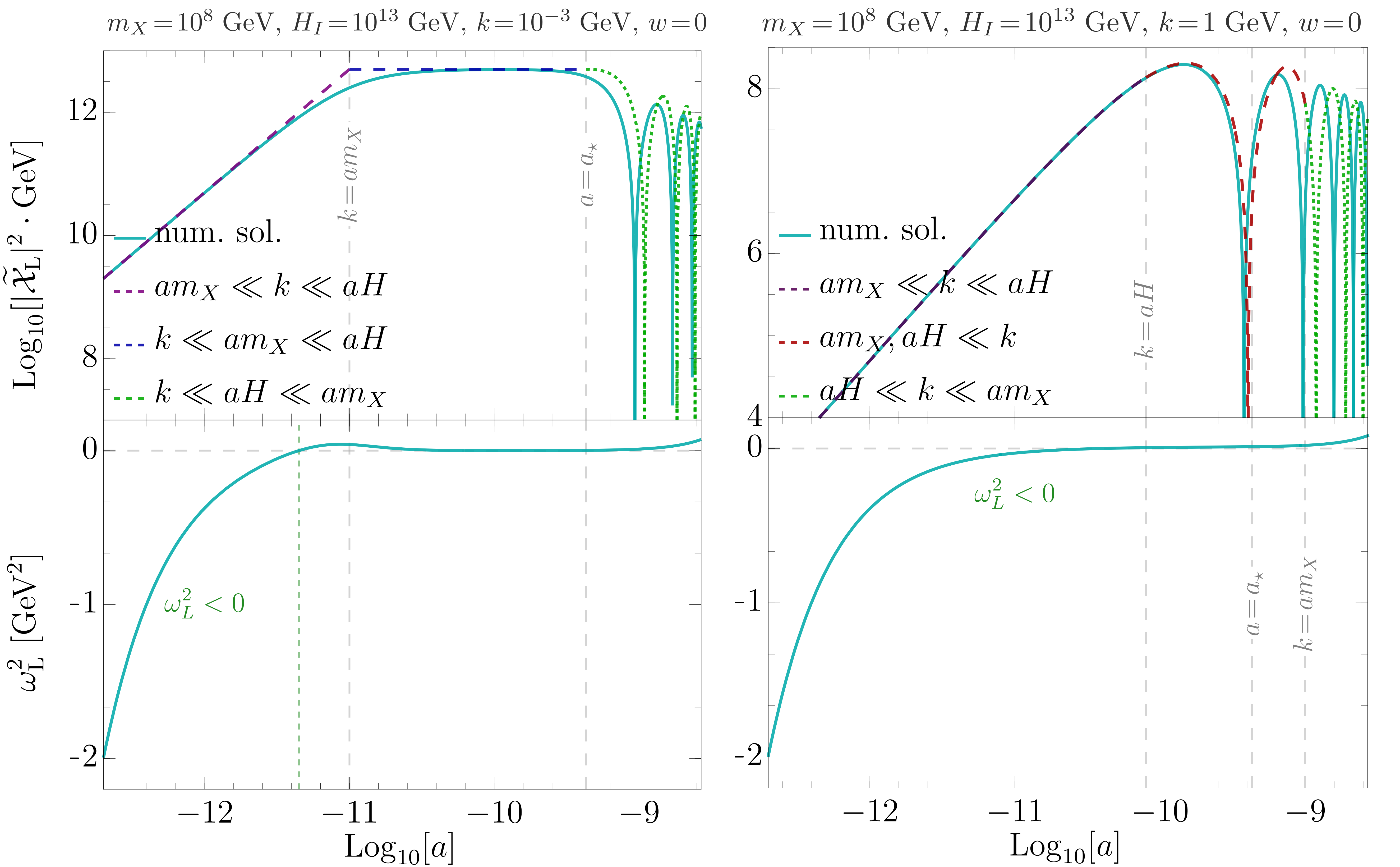




$$\omega^2(\tau \leq \tau_e) = k^2 + a^2 m_X^2 - \frac{k^2(2k^2 - a^2 m_X^2)}{(k^2 + a^2 m_X^2)^2} a^2 H_{\rm I}^2$$

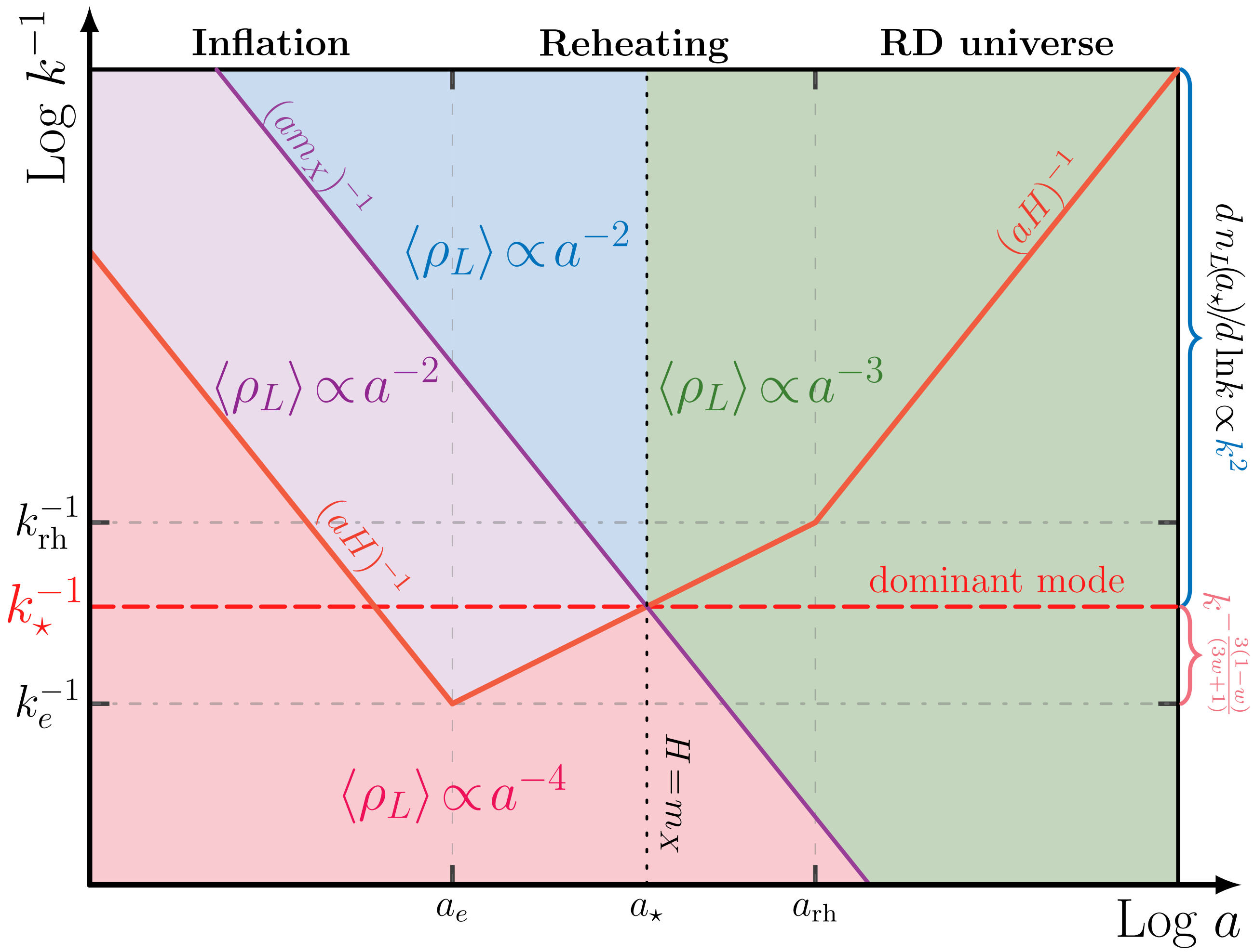
$$\approx \begin{cases} k^2, & k \gg a_e H_{\rm I} \text{ (red)} \\ k^2 - 2a^2 H_{\rm I}^2, & a m_X \ll k \ll a_e H_{\rm I} \text{ (purple)} \\ a^2 m_X^2 + \frac{k^2}{a^2 m_X^2} a^2 H_{\rm I}^2, & k \ll a m_X \ll a_e H_{\rm I} \text{ (blue)} \end{cases}$$

MODE EVOLUTION AFTER INFLATION

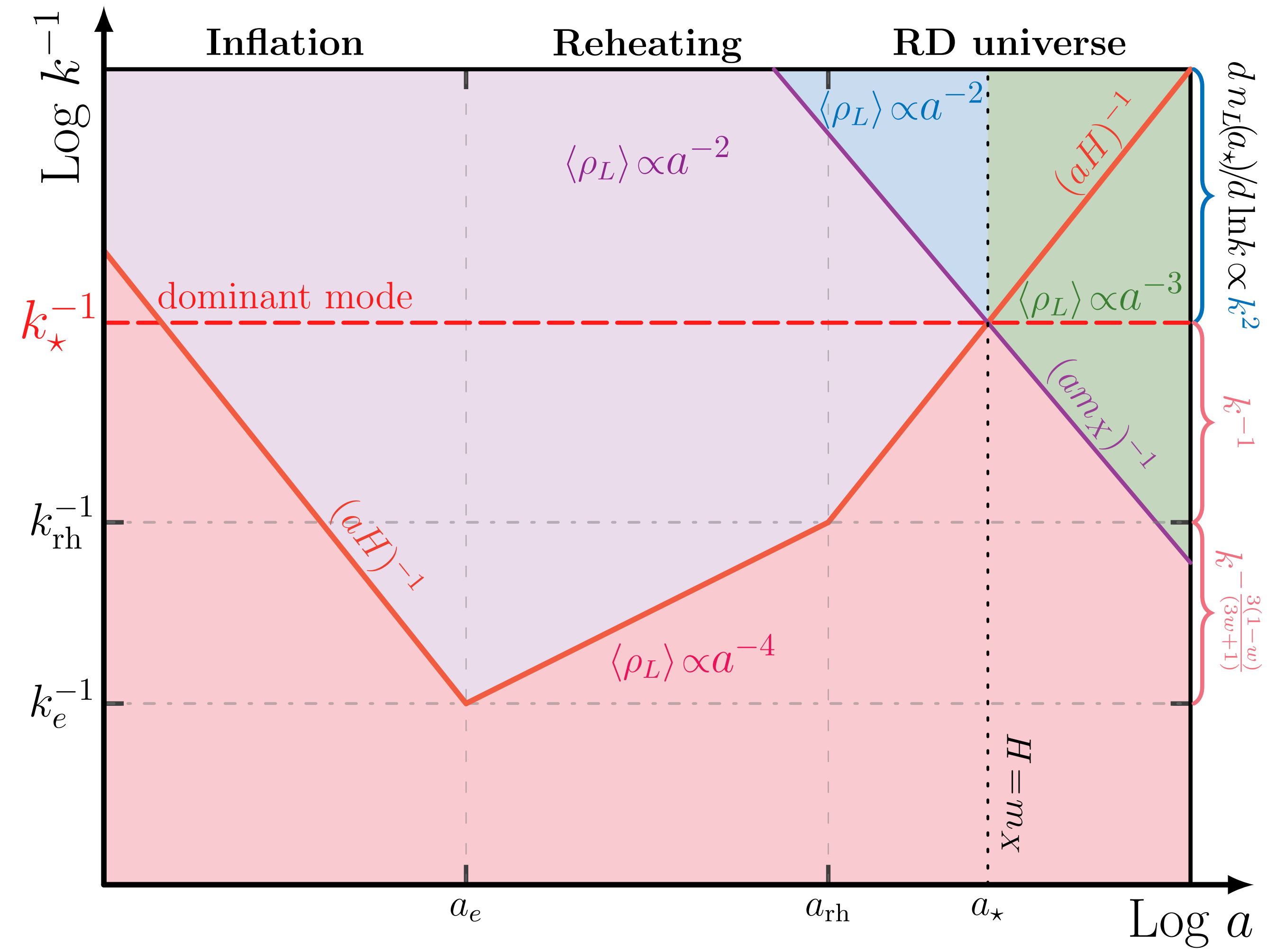


ENERGY DENSITY SCALING

$H_{\rm rh} \leq m_X < H_I$

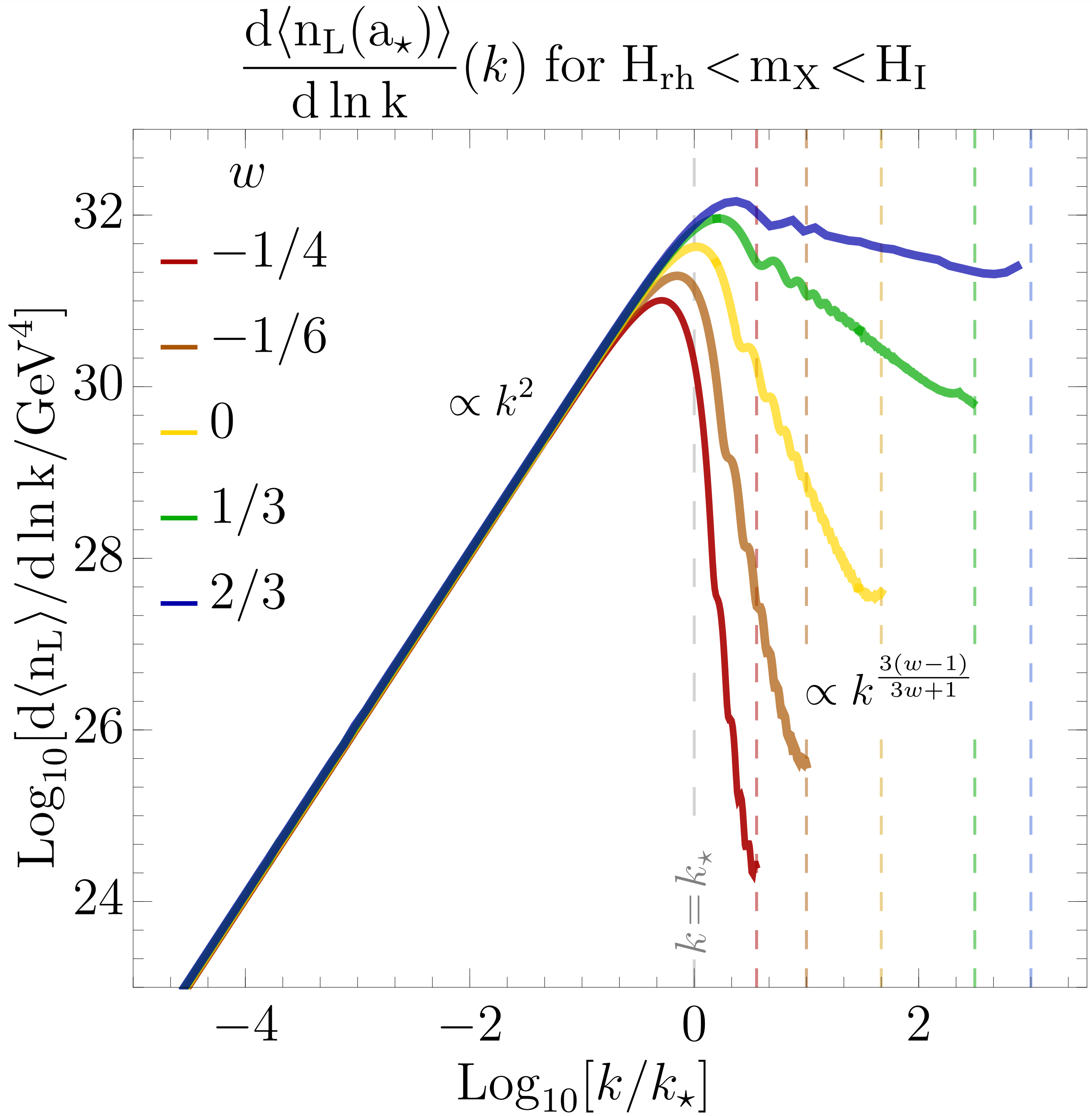


$m_X < H_{\rm rh}$



$$\frac{d\langle\rho_L\rangle}{d\ln k} \propto \begin{cases} k^{-\frac{2(1-3w)}{1+3w}}\, a^{-4}, & a(\tau)m_X,\, a(\tau)H(\tau) \ll k \ \text{(red)}, \\ k^2 a^{-2}, & a(\tau)m_X,\, k \ll a(\tau)H(\tau) \ \text{(purple, blue)}, \\ k^2 a^{-3}, & a(\tau)H(\tau),\, k \ll a(\tau)m_X \ \text{(green)}, \end{cases}$$

HEAVY DM NUMBER DENSITY



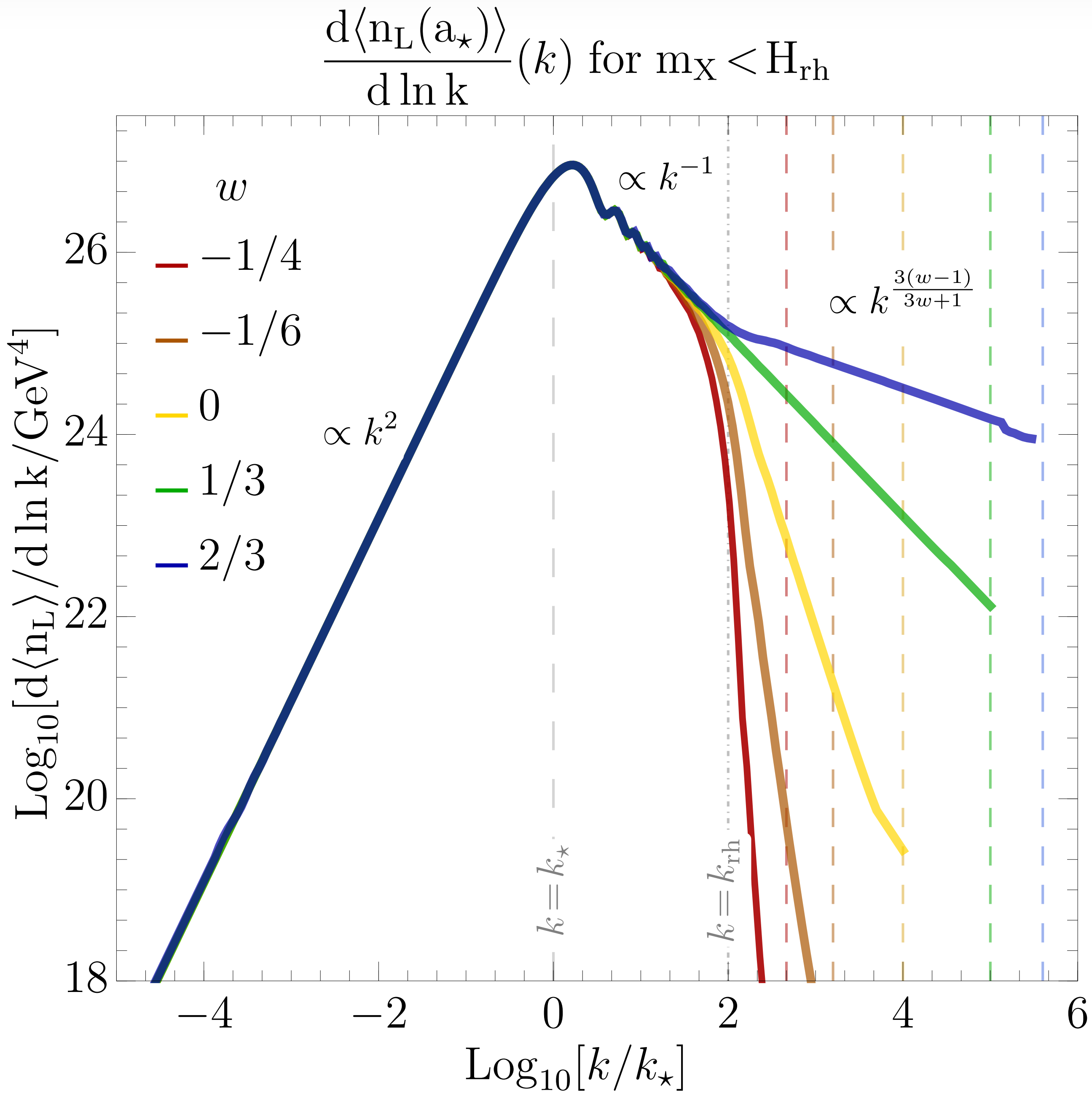
$m_X = 10^8$ GeV

$H_I = 10^{13}$ GeV

$\gamma = 10^{-3}$

$$\frac{d\langle n_L(a_\star)\rangle}{d\ln k} = \frac{1}{8\pi^2}\begin{cases} H_I^{\frac{2\left(3w^2+3w+2\right)}{(1+w)(1+3w)}}\, m_X^{\frac{2}{1+w}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{(1+3w)}}, & k_\star < k < k_e, \\ H_I^{\frac{2(1+3w)}{3(1+w)}}\, m_X^{\frac{1-3w}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & k < k_\star. \end{cases}$$

LIGHT DM NUMBER DENSITY



$m_X = 10^3$ GeV

$H_I = 10^{13}$ GeV

$\gamma = 10^{-3}$

$$\frac{d\langle n_L(a_\star)\rangle}{d\ln k} = \frac{1}{8\pi^2}\begin{cases} m_X^{3/2} H_I^{\frac{3(3+w)}{2(1+3w)}} \gamma^{\frac{1-3w}{1+w}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{1+3w}}, & k_{rh} < k < k_e, \\ m_X^{3/2} H_I^{5/2} \gamma^{\frac{-1+3w}{3(1+w)}} \left(\frac{a_e}{k}\right), & k_\star < k < k_{rh}, \\ H_I \gamma^{\frac{2(1-3w)}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & k < k_\star. \end{cases}$$

ISOCURVATURE DENSITY PERTURBATIONS

- Gravitational coupled vector DM generates isocurvature density fluctuations, which are severely constrained by the CMB data.

- However, the vector DM isocurvature density perturbations falls as k^3 for the long-wavelength modes $k < k_\star$. [Graham, Mardon, Rajendran 2015]

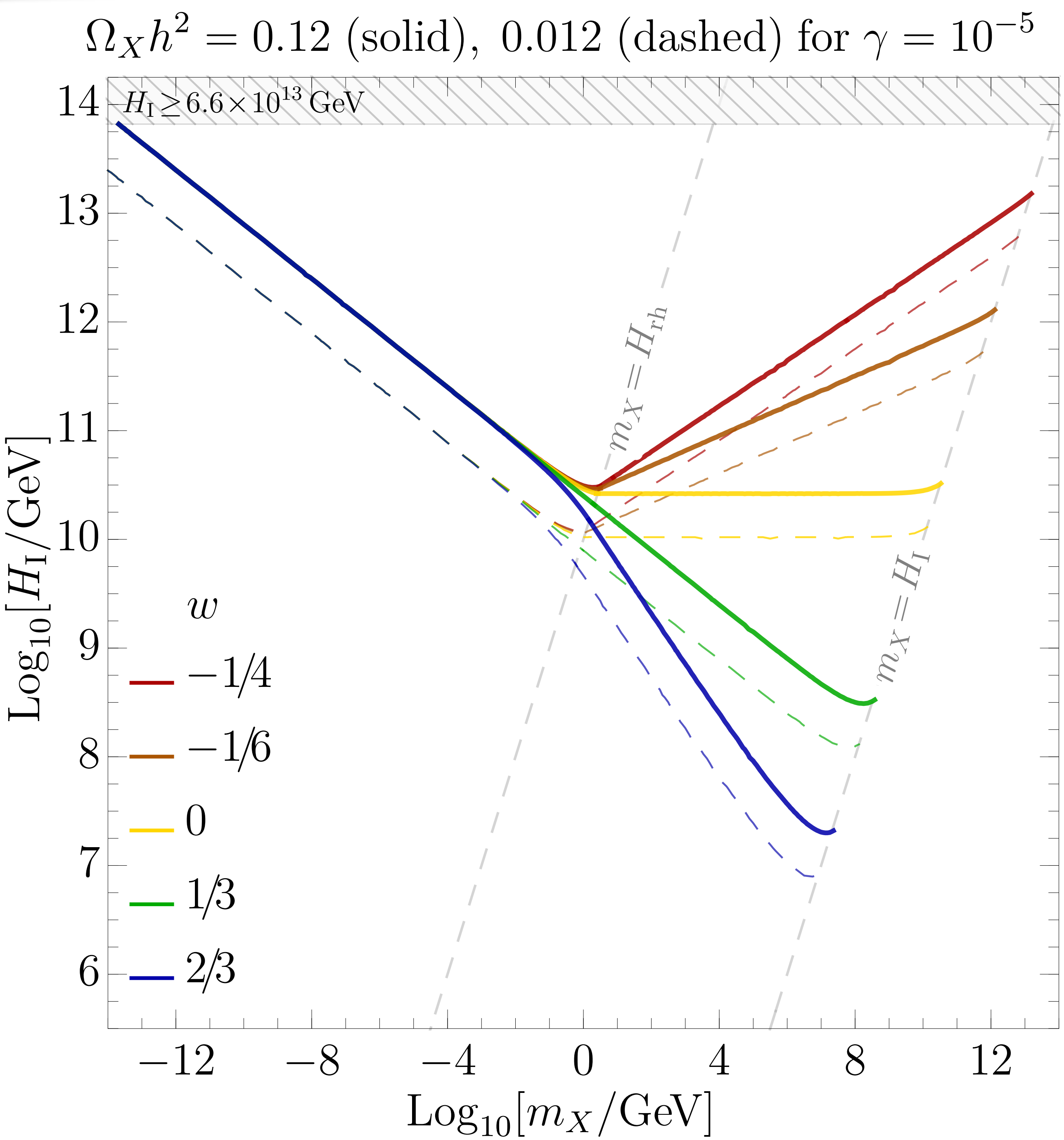
- If $k_\star$ correspond to cosmological scales much smaller than the CMB scale, i.e. $k_\star \gg k_{\rm CMB} \approx 0.05\,{\rm Mpc}^{-1}$, then vector modes would not generate dangerous isocurvature perturbations.

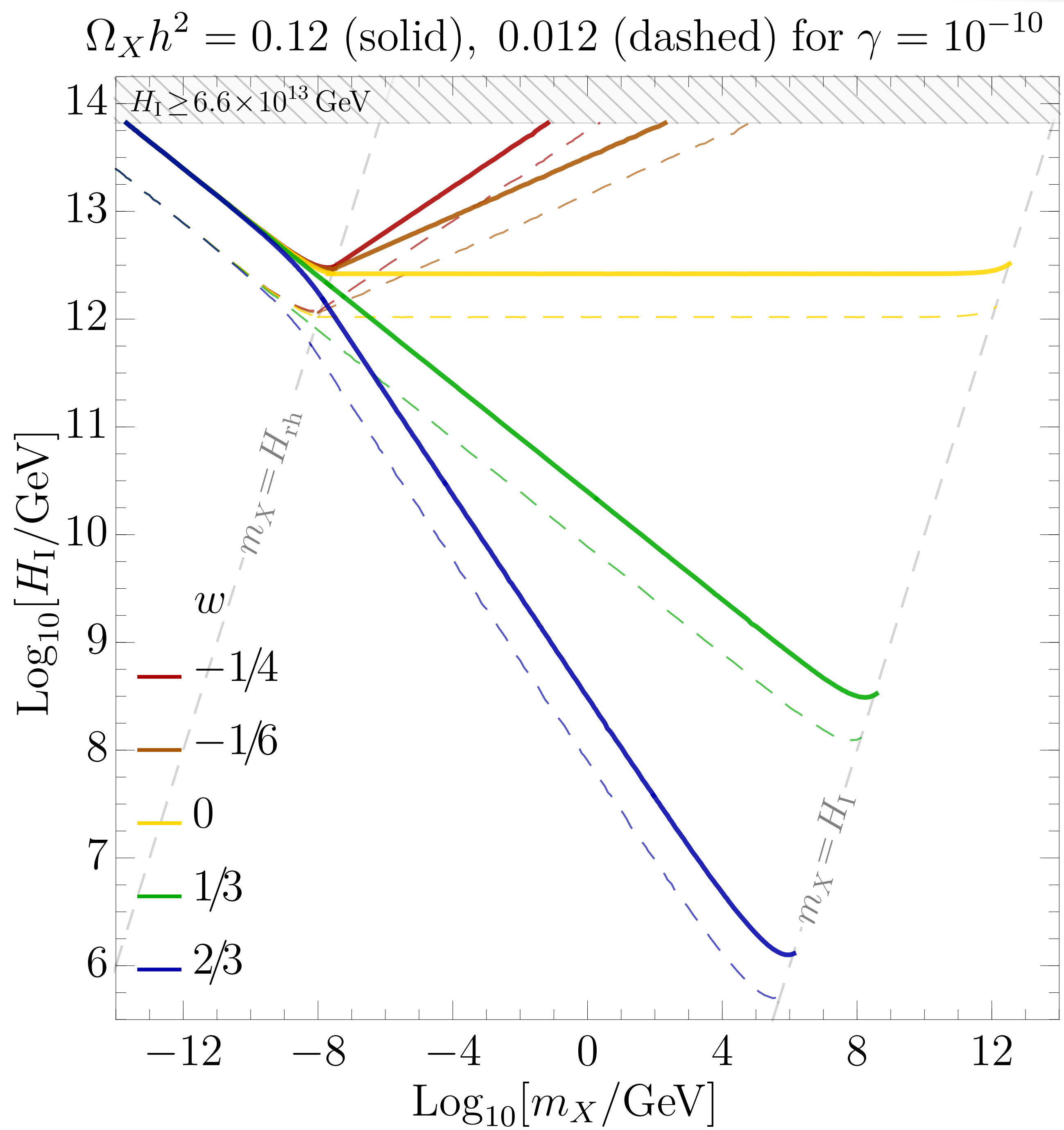
$$k_\star \approx 1400\,{\rm pc}^{-1}\sqrt{\frac{m_X}{10^{-14}\ {\rm GeV}}}\begin{cases}\left(\frac{H_{\rm rh}}{m_X}\right)^{\frac{1-3w}{6(1+w)}}, & H_{\rm rh} \le m_X < H_{\rm I}\,, \\ 1\,, & H_{\rm mre} \le m_X < H_{\rm rh}\end{cases}$$

- Requiring $k_\star \gg k_{\rm CMB}$ places constraint on $m_X \ge H_{\rm rh} \ge 10^{-14}\,{\rm GeV}$, corresponds to $T_{\rm rh} \gtrsim 100\,{\rm GeV}$ for $w = (-1/3, 1/3)$.

DM RELIC ABUNDANCE

$$\Omega_X h^2 \approx 0.12 \times \begin{cases} \left(\frac{m_X}{0.33\text{GeV}}\right)^{\frac{2w}{1+w}} \left(\frac{H_\text{I}}{3.3\times 10^{10}\text{GeV}}\right)^{\frac{5+w}{2(1+w)}} \left(\frac{10^{-5}}{\gamma}\right)^{\frac{3w-1}{1+w}}, & H_\text{rh} \leq m_X < H_\text{I} \\ \left(\frac{m_X}{2\times 10^{-14}\text{GeV}}\right)^{1/2} \left(\frac{H_\text{I}}{6.6\times 10^{13}\text{GeV}}\right)^{2}, & m_X < H_\text{rh} \end{cases}$$





NEXT TO MINIMAL VECTOR DM

- Vector DM model which interacts with the SM through gravity and higher-dimensional effective operators suppressed by the Planck scale!

$$S = \int \mathrm{d}^4x \sqrt{-\overline{g}} \left[\frac{M_{\mathrm{Pl}}^2}{2} \overline{R} + \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{DM}} + \mathcal{L}_{\mathrm{inf}} + \mathcal{L}_{\mathrm{int}} \right]$$

- Abelian massive vector DM

$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu$$

- Interaction Lagrangian

$$\mathcal{L}_{\mathrm{int}} = \frac{h^{\mu\nu}}{2M_{\mathrm{Pl}}} \left[T_{\mu\nu}^{\mathrm{SM}} + T_{\mu\nu}^{\mathrm{DM}} \right] + g_{\mathcal{H}} m_\phi \phi |\mathcal{H}|^2 + \frac{\mathcal{C}_X^\phi}{2M_{\mathrm{Pl}}} m_X^2 \phi X_\mu X^\mu + \frac{\mathcal{C}_X^{\mathcal{H}}}{2M_{\mathrm{Pl}}^2} m_X^2 X_\mu X^\mu |\mathcal{H}|^2$$

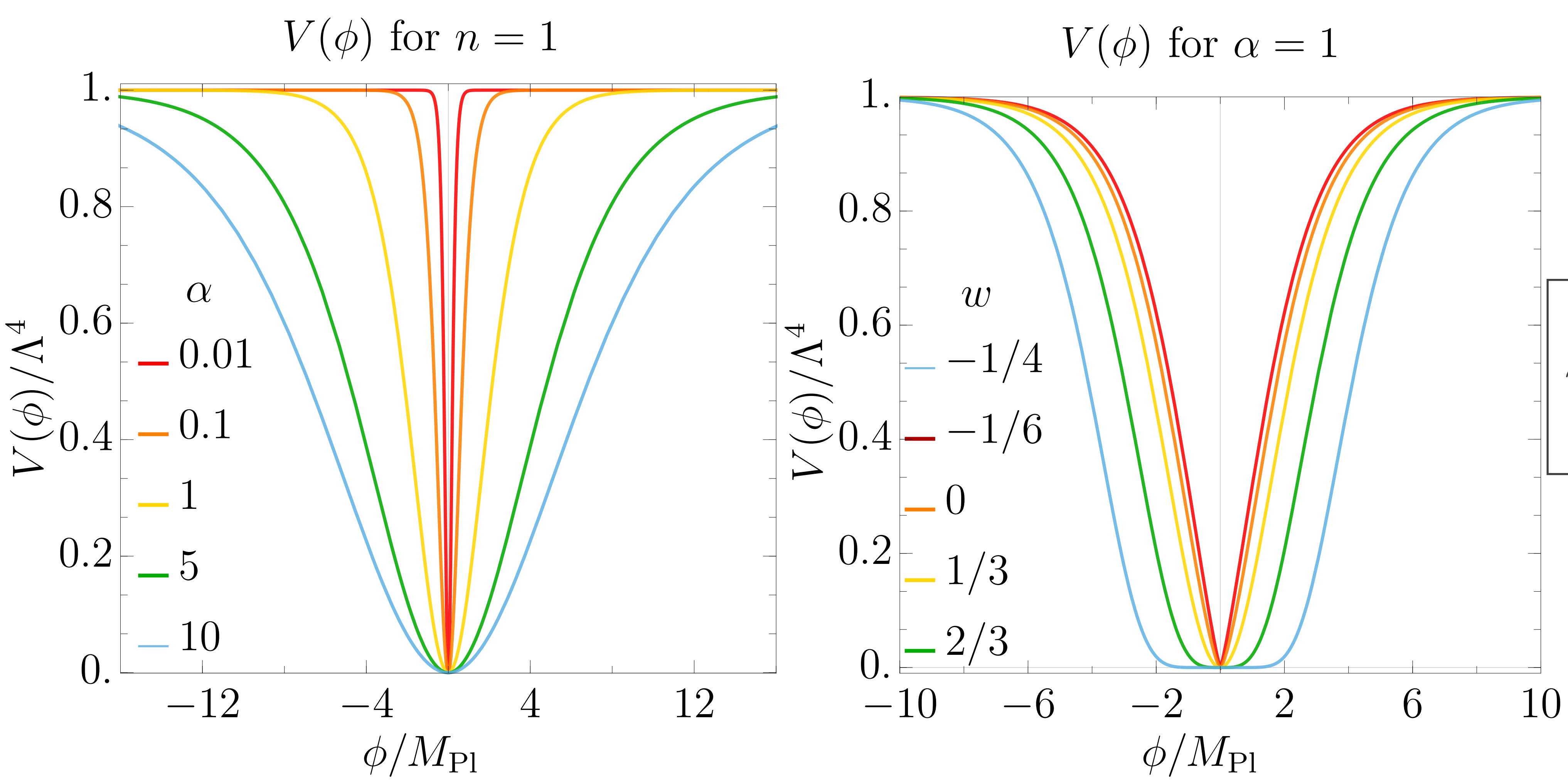
INFLATON DYNAMICS

- We consider α-attractor T-model of inflation

$$\mathcal{L}_{\rm inf} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{M}\right) = \begin{cases} \Lambda^4 & |\phi| \gg M \\ \Lambda^4 \left|\frac{\phi}{M}\right|^{2n} & |\phi| \ll M \end{cases}$$

$$M = \sqrt{6\alpha}\,M_{\rm Pl}$$



$$w = \frac{\langle p_\phi \rangle}{\langle \rho_\phi \rangle} = \frac{n-1}{n+1}$$

INFLATON DYNAMICS

- Boltzmann equation for inflaton

$$\dot{\rho}_\phi + 3H(1+w)\rho_\phi = -\rho_\phi \Gamma_\phi$$

$$w = \frac{\langle p_\phi \rangle}{\langle \rho_\phi \rangle} = \frac{n-1}{n+1}$$

- Using envelop approximation

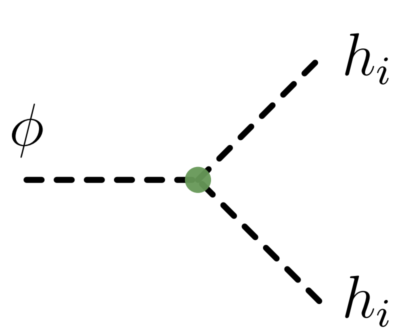
[Shtanov, Traschen, Brandenberger 1994]

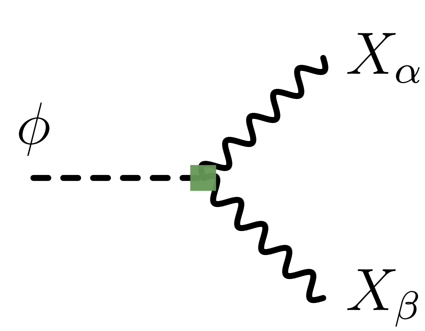
$$\phi(t) = \varphi(t)\mathcal{P}(t) = \varphi(t) \sum_{j=-\infty}^{\infty} \mathcal{P}_j e^{-ij\omega t}$$

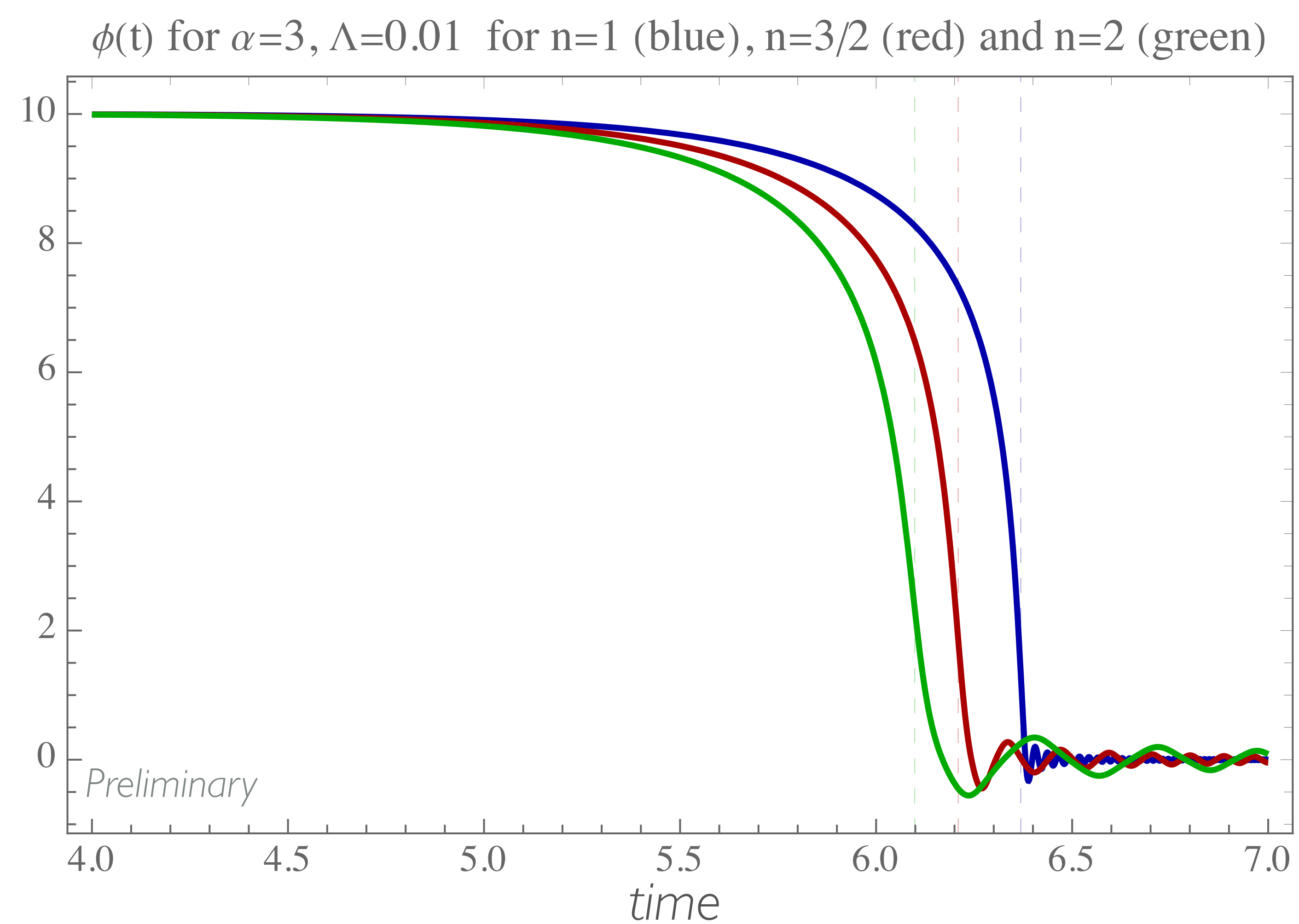
- Coherent inflaton decay

$$\Gamma_\phi \simeq \frac{\varphi(t)^2}{8\pi\rho_\phi} \sum_j j\omega \left|\mathcal{P}_j\right|^2 \left|\mathcal{M}_{0\to f}\right|^2$$

[Ichikawa, Suyama, Takahashi, Yamaguchi 2008]







BOLTZMANN EQUATIONS

- We solve coupled Boltzmann equations for inflaton, radiation and DM

$$\dot{\rho}_\phi + 3\left(\frac{2n}{n+1}\right) H\rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\dot{\rho}_\mathcal{R} + 4H\rho_\mathcal{R} = (1 - B_X)\Gamma_\phi\rho_\phi + 2\langle E_X\rangle\langle\sigma|v|\rangle(n_X^2 - \bar{n}_X^2)$$

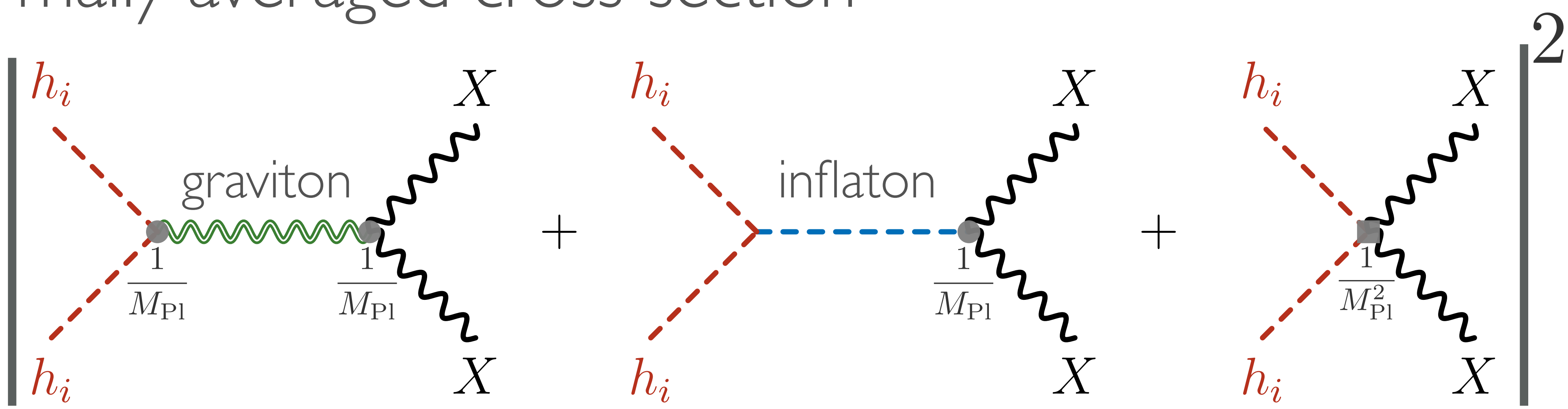
$$\dot{n}_X + 3Hn_X = B_X\Gamma_\phi\frac{\rho_\phi}{m_\phi} - \langle\sigma|v|\rangle(n_X^2 - \bar{n}_X^2),$$

$$H^2 = \frac{1}{3m_{\mathrm{Pl}}^2}(\rho_\phi + \rho_\mathcal{R} + \rho_X)$$

$$B_X = \frac{\Gamma_{\phi\to XX}}{\Gamma_\phi}$$

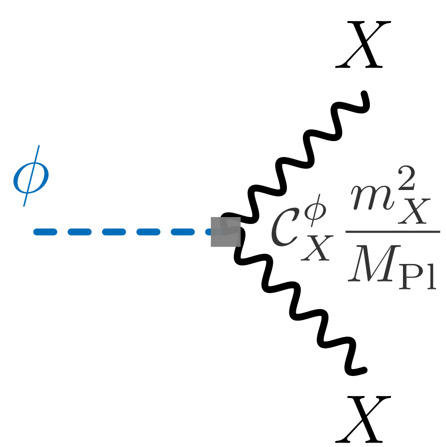
- $\langle\sigma|v|\rangle$ is thermally averaged cross-section

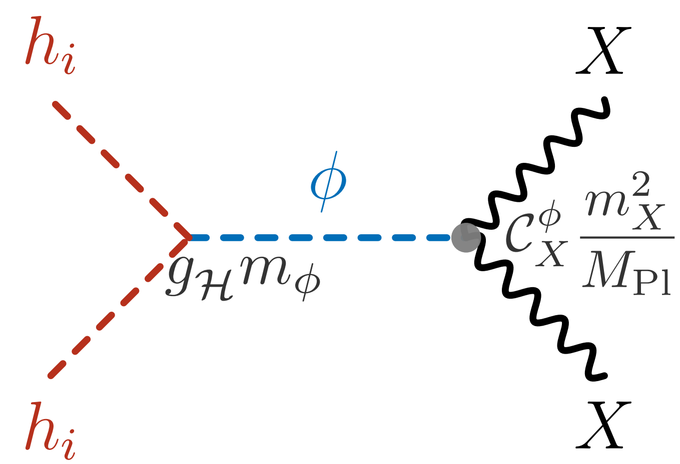
$$\langle\sigma|v|\rangle \propto$$

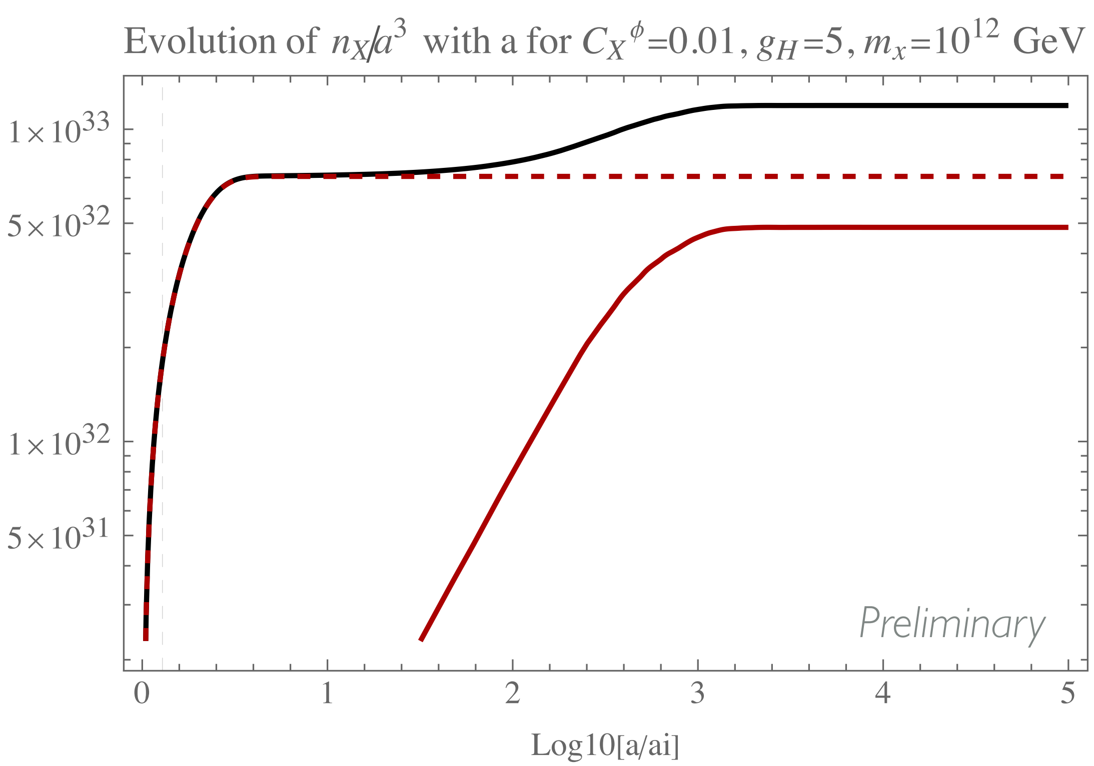


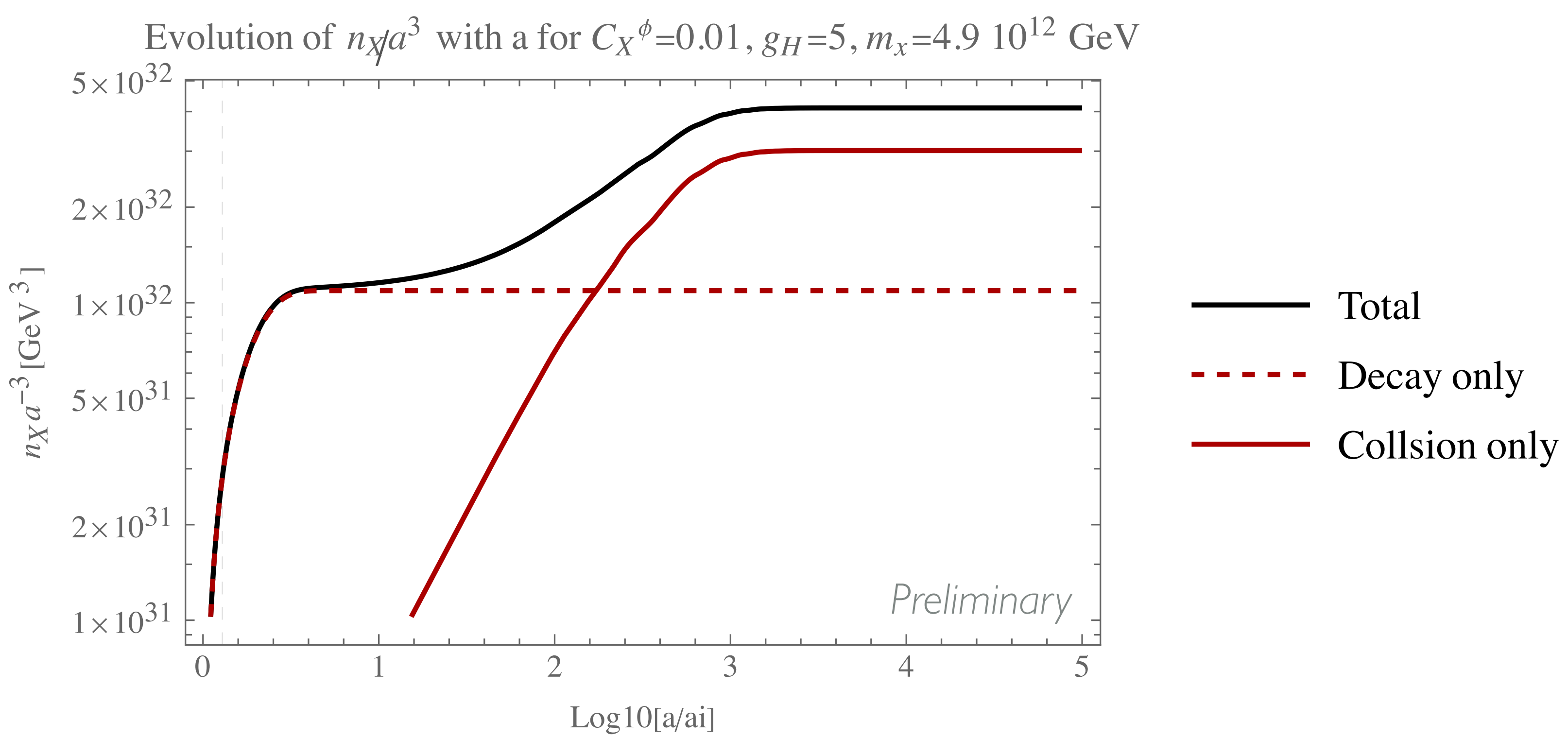
INFLATON DECAY AND EXCHANGE

- Relic density only through inflaton decay and exchange







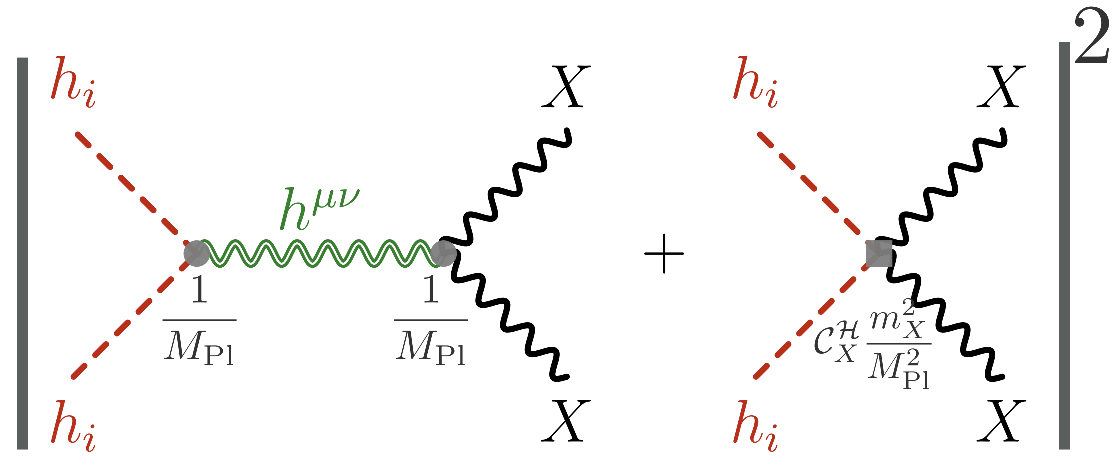


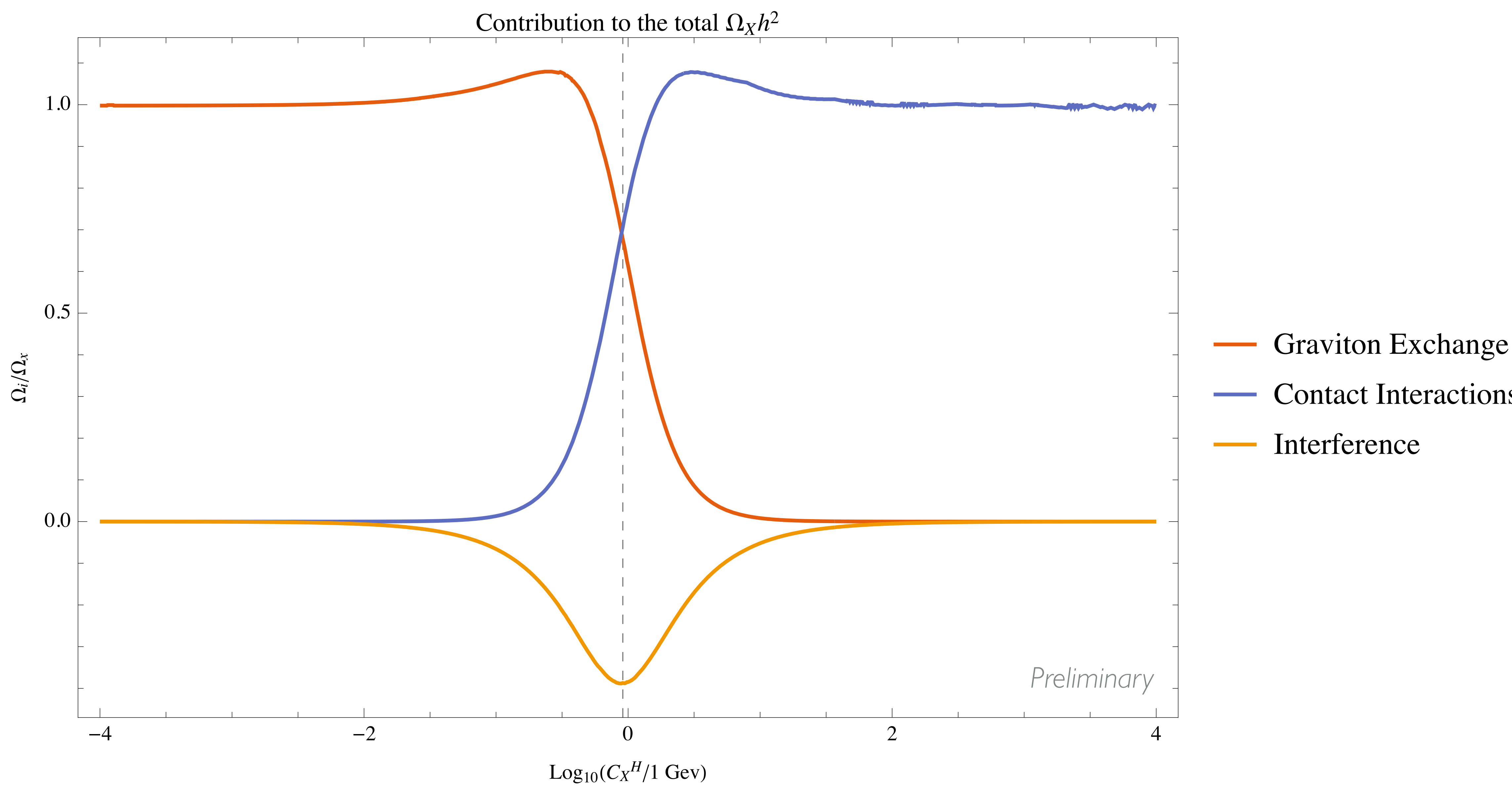
Only inflaton decay discussed in [Takahashi 2007, Moroi, Yin 2020,+...]

GRAVITON EXCHANGE AND EFFECTIVE OPERATOR

- Assuming $\mathcal{C}_X^{\phi} \approx 0$

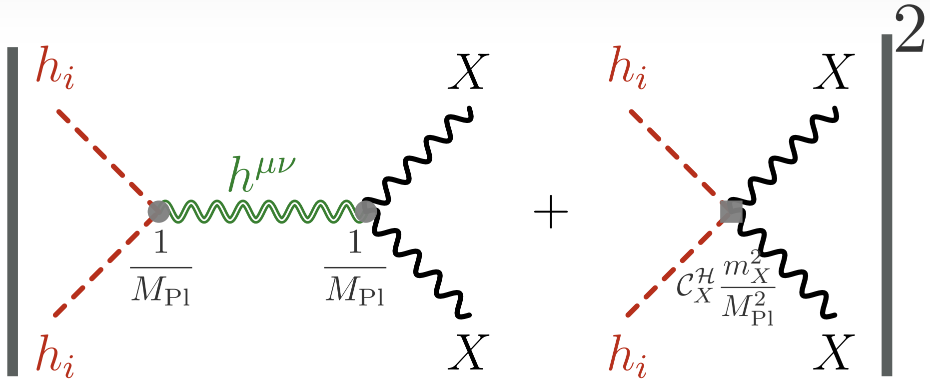
$$\langle \sigma |v| \rangle \propto$$

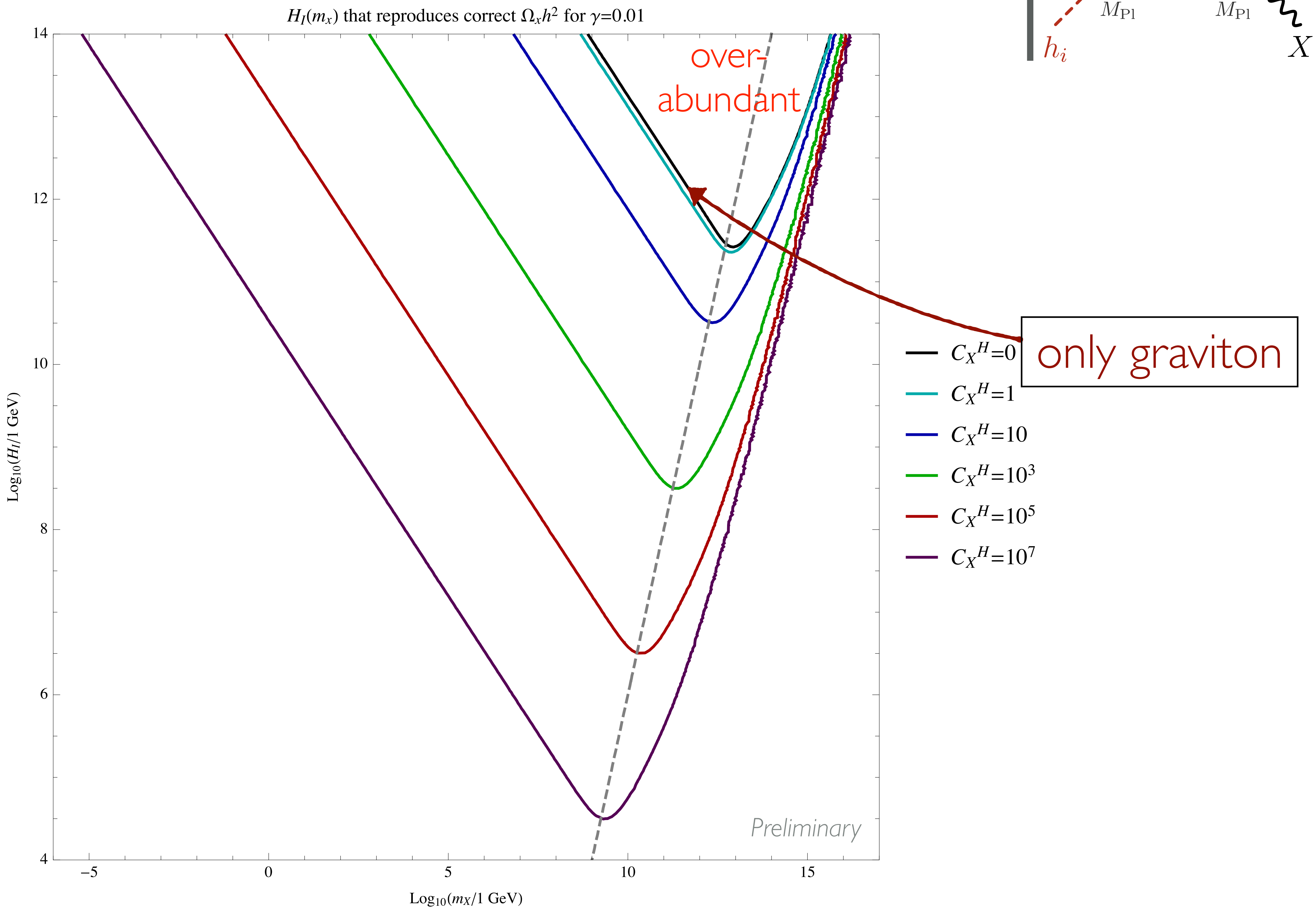




- Assuming $\mathcal{C}_X^\phi \approx 0$

$\langle \sigma |v| \rangle \propto$





CONCLUSIONS

- All the evidences (so far) of DM existence are through gravity.
- Pure gravitational DM is produced in the early universe due to quantum fluctuations.
- We discussed gravitational particle production of massive vector DM with non-standard cosmology during the reheating phase.
- Non-minimal coupling between DM and SM are introduced through higher dimensional operators suppered by the Planck scale.
- DM production through the inflaton decays/exchange and effective operators are potentially important!

Thank you!