A light pseudoscalar in a model with lepton family symmetry $O(2)$

W. G., L. Lavoura, D. Neubauer, arXiv:0805.1175 [hep-ph]

Motivation:

- Extension of the SM with $\nu_R \Rightarrow$ seesaw mechanism for small neutrino masses
- Lepton mixing features imposed by symmetries $\Rightarrow$ more than one Higgs doublet required $\Rightarrow$ multi-Higgs-doublet models (mHDM)
- Electroweak precision tests compatible with mHDMs?
- Attempt to connect properties of lepton sector with those of scalar sector

Program:

1. **A μ–τ symmetric neutrino mass matrix**
2. **A 3HDM for a μ–τ symmetric neutrino mass matrix**
3. **A 3HDM model with family symmetry $O(2)$**
4. **Electroweak precision tests, multi-HDMs and the oblique parameters**
5. **The scalar sector in the $O(2)$ model**
6. **Replacing the reflection symmetry by a CP symmetry**
7. **Conclusions**

1. A μ–τ symmetric neutrino mass matrix

Majorana mass matrix

Basis where charged-lepton mass matrix is diagonal:

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$$

μ–τ interchange symmetry:

$$S\mathcal{M}_\nu S = \mathcal{M}_\nu \quad \text{with} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Permutation matrix S for $2 \leftrightarrow 3$

Theorem of Schur:

$\mathcal{M}_\nu^T = \mathcal{M}_\nu \Rightarrow \exists$ unitary matrix U with

$U^T \mathcal{M}_\nu U = \mathrm{diag}\,(m_1, m_2, m_3)$

PMNS or lepton mixing matrix U

μ–τ-symmetric $\mathcal{M}_\nu \Rightarrow$ real eigenvector $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \Rightarrow$

3rd column in lepton mixing matrix U, $m_3 = |z - w|$

$$U = \mathrm{diag}\,(1, e^{i\alpha}, e^{i\alpha}) \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}\,(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

with $\boxed{\theta_{23} = 45^\circ,\ \theta_{13} = 0^\circ}$

Modest goal: explains near maximal atm. mixing, small reactor mixing angle

6 physical parameters in $\mathcal{M}_\nu \leftrightarrow$
$m_{1,2,3}$, θ_{12}, 2 Majorana phases $\beta_2 - \beta_1$, $\beta_3 - \beta_1$

SUMMARY:
μ–τ-symmetric $\mathcal{M}_\nu$ compatible with all data,
no prediction for $\theta_{12} \equiv \theta$ and neutrino masses

Remark: Tribimaximal mixing (Harrison, Perkins, Scott)
$\sin^2\theta_{12} = 1/3$ or $\theta_{12} = 35.26^\circ$
Compatible with all data!
Much more ambitious to obtain TBM by symmetries
TBM by symmetry or accidental?
(Albright, Rodejohann, arXiv:0804.4581 [hep-ph])

Neutrino masses and mixing from data: 2σ (95% CL)

G.L. Fogli et al., arXiv:0805.2517

$$\begin{aligned}
\Delta m^2_{\odot} &= (7.66 \pm 0.35) \times 10^{-5}\,\mathrm{eV}^2 \\
\Delta m^2_{\mathrm{atm}} &= (2.38 \pm 0.27) \times 10^{-3}\,\mathrm{eV}^2 \\
\sin^2\theta_{12} &= 0.326\ {}^{+0.05}_{-0.04} \\
\sin^2\theta_{23} &= 0.45\ {}^{+0.16}_{-0.09} \\
\sin^2\theta_{13} &< 0.032
\end{aligned}$$

$$\begin{aligned}
\theta_{12} &= 34.8^\circ\ {}^{+3.0^\circ}_{-2.5^\circ} \\
\theta_{23} &= 42.1^\circ\ {}^{+9.3^\circ}_{-5.2^\circ} \\
\theta_{13} &< 10.3^\circ
\end{aligned}$$

2. A 3HDM for a μ–τ symmetric neutrino mass matrix

W.G., L. Lavoura 2001

Multiplets: $D_{\alpha L}$, α_R $\nu_{\alpha R}$ $(\alpha = e, \mu, \tau)$, ϕ_j $(j = 0, 1, 2)$

Symmetries:

Family lepton number symmetries $U(1)_\alpha$

$s:$ $D_{\mu L} \leftrightarrow D_{\tau L}$, $\mu_R \leftrightarrow \tau_R$, $\nu_{\mu R} \leftrightarrow \nu_{\tau R}$, $\phi_1 \leftrightarrow \phi_2$

$\mathbb{Z}_2:$ ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$, e_R, ϕ_0 change sign

Symmetry breaking:

Soft breaking:

$U(1)_\alpha$ in mass term of ν_R (mass matrix M_R)

Spontaneous breaking:

μ–τ interchange symmetry s, $\mathbb{Z}_2$

Consequences:

- $U(1)_\alpha \Rightarrow$ Yukawa couplings diagonal
- $\mathbb{Z}_2 \Rightarrow \nu_R$ couples only to ϕ_0
- $s \Rightarrow \mu$–τ symmetry of M_D, M_R

$$\begin{aligned}
\mathcal{L}_Y &= -y_1\, \bar{D}_{eL} \tilde{\phi}_0 \nu_{eR} - y_2 \left(\bar{D}_{\mu L} \tilde{\phi}_0 \nu_{\mu R} + \bar{D}_{\tau L} \tilde{\phi}_0 \nu_{\tau R} \right) \\
&\quad -y_3\, \bar{D}_{eL} \phi_0 e_R - y_4 \left(\bar{D}_{\mu L} \phi_1 \mu_R + \bar{D}_{\tau L} \phi_2 \tau_R \right) \\
&\quad -y_5 \left(\bar{D}_{\mu L} \phi_2 \mu_R + \bar{D}_{\tau L} \phi_1 \tau_R \right) + \text{H.c.}
\end{aligned}$$

- $M_D = \mathrm{diag}\,(a,\, b,\, b)$
- M_R μ–τ symmetric
- $\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D$ μ–τ symmetric

$$\frac{m_\mu}{m_\tau} = \left| \frac{y_4 v_1 + y_5 v_2}{y_4 v_2 + y_5 v_1} \right|$$

m_μ / m_τ small by finetuning

Mechanism for μ–τ symmetry in $\mathcal{M}_\nu$:

G_f family symmetry generated by $U(1)_\alpha$, $s \Rightarrow$

$$G_f \xrightarrow{\text{soft}} \mathbb{Z}_2^{\mu\text{-}\tau} \xrightarrow{\text{SSB}} \{e\}$$

At tree level, $\mathbb{Z}_2^{\mu\text{-}\tau}$ unbroken in neutrino sector

Blum, Hagedorn, Lindner, arXiv:0709.3450 [hep-ph]

Interpreting the model in terms of the group $O(2)$:

Abstract characterization of $O(2)$:

$$g(\theta + 2\pi) = g(\theta) \qquad g(\theta_1)\, g(\theta_2) = g(\theta_1 + \theta_2)$$
$$s^2 = e \qquad s\, g(\theta)\, s = g(-\theta)$$

Irreducible representations:

1-dimensional:

$$\underline{1} : \; g(\theta) \to 1, \; s \to 1 \quad \text{and} \quad \underline{1}' : \; g(\theta) \to 1, \; s \to -1$$

2-dimensional: countably infinite set numbered by $n \in \mathbb{N}$

$$\underline{2}^{(n)} : \; g(\theta) \to \begin{pmatrix} e^{in\theta} & 0 \\ 0 & e^{-in\theta} \end{pmatrix}, \; s \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Tensor products:

$$m > n: \qquad \underline{\mathbf{2}}^{(m)} \otimes \underline{\mathbf{2}}^{(n)} = \underline{\mathbf{2}}^{(m+n)} \oplus \underline{\mathbf{2}}^{(m-n)}$$

$$m = n: \qquad \underline{\mathbf{2}}^{(n)} \otimes \underline{\mathbf{2}}^{(n)} = \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{2}}^{(2n)}$$

$$(D_{\mu L}, D_{\tau L}),\ (\mu_R, \tau_R),\ (\nu_{\mu R}, \nu_{\tau R}) \in \underline{\mathbf{2}}^{(1)}$$

$O(2)$ multiplets of the model:

$$D_{eL},\ e_R,\ \nu_{eR},\ \frac{1}{\sqrt{2}}(\phi_1 + \phi_2) \in \underline{\mathbf{1}}, \quad \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \in \underline{\mathbf{1}}'$$

$$G_f = U(1)_{L_e} \times U(1)_{(L_\mu + L_\tau)/2} \times O(2)_{(L_\mu - L_\tau)/2} \times \mathbb{Z}_2$$

3. A 3HDM model with family symmetry $O(2)$

Multiplets: $D_{\alpha L},\ \alpha_R\ (\alpha = e, \mu, \tau),\ \phi_j\ (j = 0, 1, 2)$

Symmetries: $s,\ \mathbb{Z}_2$

$$U(1):\ \left\{\begin{array}{rcl} (D_{\mu L},\, \tau_R,\, \nu_{\mu R}) & \rightarrow & e^{+i\theta}\,(D_{\mu L},\, \tau_R,\, \nu_{\mu R})\,, \\ (D_{\tau L},\, \mu_R,\, \nu_{\tau R}) & \rightarrow & e^{-i\theta}\,(D_{\tau L},\, \mu_R,\, \nu_{\tau R})\,, \\ \phi_1 & \rightarrow & e^{+2i\theta}\,\phi_1, \\ \phi_2 & \rightarrow & e^{-2i\theta}\,\phi_2. \end{array}\right.$$

$$\begin{aligned} \mathcal{L}_Y \;=\; & -y_1\,\bar{D}_{eL}\tilde{\phi}_0\nu_{eR} - y_2\left(\bar{D}_{\mu L}\tilde{\phi}_0\nu_{\mu R} + \bar{D}_{\tau L}\tilde{\phi}_0\nu_{\tau R}\right) \\ & -y_3\,\bar{D}_{eL}\phi_0 e_R - y_4\left(\bar{D}_{\mu L}\phi_1\mu_R + \bar{D}_{\tau L}\phi_2\tau_R\right) + \text{H.c.} \end{aligned}$$

Symmetry breaking: $U(1)$ softly by dim ≤ 3, s, $\mathbb{Z}_2$ spont.

Consequences: $\mathcal{M}_\nu$ μ–τ symmetric, $\dfrac{m_\mu}{m_\tau} = \left|\dfrac{v_1}{v_2}\right|$

Family symmetry: $O(2) \times \mathbb{Z}_2 \qquad (U(1),\ s \rightsquigarrow O(2))$

$O(2)$ assignments:

$$
\begin{aligned}
\underline{\mathbf{1}}: &\quad D_{eL},\ \nu_{eR},\ e_R,\ \phi_0 \\
\underline{\mathbf{2}}^{(1)}: &\quad (D_{\mu L},\ D_{\tau L}),\ (\tau_R,\ \mu_R),\ (\nu_{\mu R},\ \nu_{\tau R}) \\
\underline{\mathbf{2}}^{(2)}: &\quad (\phi_1,\ \phi_2)
\end{aligned}
$$

4. Electroweak precision tests, mHDMs the oblique parameters

Peskin, Takeuchi; Altarelli, Barbieri;
Maksymyk, Burgess, London

Oblique = indirect: effects of new physics in 1-loop corrections of vector boson selfenergies

1. The electroweak gauge group is the SM group $SU(2) \times U(1)$.
2. New-physics particles have suppressed couplings to the light fermions with which experiments are performed; they couple mainly to the SM gauge bosons γ, Z^0 and $W^{\pm}$.
3. The relevant ew measurements are those made at the energy scales $q^2 \approx 0$, $q^2 = m_Z^2$ and $q^2 = m_W^2$.

Vector boson selfenergies:

$$\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}\left(q^2\right) + q^\mu q^\nu B_{VV'}\left(q^2\right),$$

$VV' = \gamma\gamma,\ \gamma Z,\ ZZ,\ WW$

Selfenergy functions:

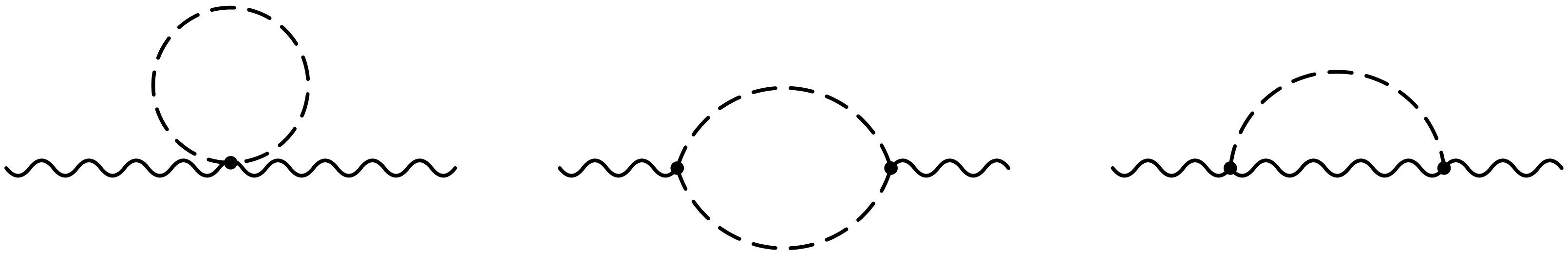
$A_{VV'}|_{\mathrm{NP}}$ computed with new physics

$A_{VV'}|_{\mathrm{SM}}$ computed with SM

Difference physically meaningful:

$$A_{VV'}\left(q^2\right) = A_{VV'}\left(q^2\right)\big|_{\mathrm{NP}} - A_{VV'}\left(q^2\right)\big|_{\mathrm{SM}}$$

New Physics: mHDM



(a) (b) (c)

$$\frac{\alpha}{4 s_W^2 c_W^2} S = \frac{A_{ZZ}\left(m_Z^2\right) - A_{ZZ}(0)}{m_Z^2} - \left.\frac{\partial A_{\gamma\gamma}\left(q^2\right)}{\partial q^2}\right|_{q^2=0} + \frac{c_W^2 - s_W^2}{c_W s_W} \left.\frac{\partial A_{\gamma Z}\left(q^2\right)}{\partial q^2}\right|_{q^2=0}$$

$$\alpha T = \frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}$$

$$\frac{\alpha}{4 s_W^2} U = \frac{A_{WW}\left(m_W^2\right) - A_{WW}(0)}{m_W^2} - c_W^2 \frac{A_{ZZ}\left(m_Z^2\right) - A_{ZZ}(0)}{m_Z^2} - s_W^2 \left.\frac{\partial A_{\gamma\gamma}\left(q^2\right)}{\partial q^2}\right|_{q^2=0} + 2 c_W s_W \left.\frac{\partial A_{\gamma Z}\left(q^2\right)}{\partial q^2}\right|_{q^2=0}$$

Remarks:

Vector-boson propagator in a general 't Hooft gauge

$$\frac{-g^{\mu\nu} + k^\mu k^\nu/m_V^2}{k^2 - m_V^2} - \frac{k^\mu k^\nu/m_V^2}{k^2 - m_G^2}$$

SM subtraction guarantees independence of m_G and finiteness of T (S, U are finite anyway)

In general 6 oblique parameters $S, \ldots, X$

Example:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2 s_W^2 (1 - \Delta r)} \quad \text{with} \quad c_W^2 = \frac{m_W^2}{m_Z^2},\ s_W^2 = 1 - c_W^2$$

1-loop corrections: $\Delta r = \Delta r|_{\mathrm{SM}} + \Delta r'$

$$\Delta r' = \frac{\alpha}{s_W^2}\left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U\right)$$

T in mHDM: n_d Higgs doublets

Grimus, Lavoura, Ogreid, Osland,
arXiv:0711.4022, 0802.4353 [hep-ph]

$$
\begin{aligned}
T = & \frac{1}{16\pi s_W^2 m_W^2} \left\{ \sum_{a=2}^{n_d} \sum_{b=2}^{2n_d} \left| \left(\mathcal{U}^\dagger \mathcal{V} \right)_{ab} \right|^2 F\left(m_a^2, \mu_b^2 \right) \right. \\
& - \sum_{2 \leq b < b' \leq 2n_d} \left| \left(\mathcal{V}^\dagger \mathcal{V} \right)_{bb'} \right|^2 F\left(\mu_b^2, \mu_{b'}^2 \right) \\
& + 3 \sum_{b=2}^{2n_d} \left| \left(\mathcal{V}^\dagger \mathcal{V} \right)_{1b} \right|^2 \left[F\left(m_Z^2, \mu_b^2 \right) - F\left(m_W^2, \mu_b^2 \right) \right] \\
& \left. - 3 \left[F\left(m_Z^2, m_h^2 \right) - F\left(m_W^2, m_h^2 \right) \right] \right\}
\end{aligned}
$$

Notation: $a = 1$, $b = 1 \rightarrow$ Goldstone bosons, m_h SM Higgs mass, m_a charged-scalar masses, μ_b neutral-scalar masses, $\mathcal{U}$ diagonalization matrix for charged scalars,

$\tilde{\mathcal{V}} \equiv \begin{pmatrix} \mathrm{Re}\,\mathcal{V} \\ \mathrm{Im}\,\mathcal{V} \end{pmatrix}$ diagonalization matrix for neutral scalars

$$F\,(x,\;y) = \frac{x+y}{2} - \frac{xy}{x-y}\,\ln\frac{x}{y}$$

Properties of F:

$F(x,y) = F(y,x), \quad F(x,y) = 0 \Leftrightarrow x = y, \quad F(x,y) \geq 0 \,\forall\, x,y$

Often T more important for ew precision tests than S, U:

- **Quadratic dependence of F on scalar masses**
- **Prefactor of T large**

Remark: ρ parameter = strength of NC vs. CC (Veltman)
At tree level $\rho = 1$ in SM

$$\Delta\rho|_{\mathrm{NP}} = \alpha T$$

$\Delta\rho$ defined at $q^2 = 0$

5. The scalar sector in the $O(2)$ model

Scalar potential: 1 complex (a_4), 9 real parameters

$$
\begin{aligned}
V \;=\;& \mu_0\, \phi_0^\dagger \phi_0 + \mu_{12} \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \right) + \mu_\mathrm{m} \left(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \right) \\
& + a_1 \left(\phi_0^\dagger \phi_0 \right)^2 + a_2\, \phi_0^\dagger \phi_0 \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \right) \\
& + a_3 \left(\phi_0^\dagger \phi_1\, \phi_1^\dagger \phi_0 + \phi_0^\dagger \phi_2\, \phi_2^\dagger \phi_0 \right) \\
& + a_4\, \phi_0^\dagger \phi_1\, \phi_0^\dagger \phi_2 + a_4^*\, \phi_1^\dagger \phi_0\, \phi_2^\dagger \phi_0 + a_5 \left[\left(\phi_1^\dagger \phi_1 \right)^2 + \left(\phi_2^\dagger \phi_2 \right)^2 \right] \\
& + a_6\, \phi_1^\dagger \phi_1\, \phi_2^\dagger \phi_2 + a_7\, \phi_1^\dagger \phi_2\, \phi_2^\dagger \phi_1
\end{aligned}
$$

Soft breaking of $U(1) \subset O(2)$: $\mu_m \neq 0$

Minimum of V with respect to phases:

$$
\mu_\mathrm{m} = -|\mu_\mathrm{m}|,\; a_4 = -|a_4|,\; v_0 > 0,\; v_1 > 0,\; v_2 > 0
$$

Conditions on VEVs:

$$\frac{v_1}{v_2} = \frac{m_\mu}{m_\tau}, \qquad v \equiv \sqrt{v_0^2 + v_1^2 + v_2^2} \simeq 246 \text{ GeV}$$

8 real parameters determine scalar sector!

Existence of a light pseudoscalar:

$$|\mu_m| = \frac{A}{2} v_1 v_2 - \frac{|a_4|}{2} v_0^2 \quad \text{with} \quad A \equiv a_6 + a_7 - 2a_5 > 0$$

$v_1/v_2 \simeq 0.06$ **small!**

$$v_1 \to 0 \Rightarrow \mu_m \to 0 \Rightarrow$$

spontaneous breaking of $U(1) \subset O(2)$

$\Rightarrow$ **Goldstone boson**

$v_1 \neq 0 \Rightarrow$ **light pseudoscalar** S_4^0

Physical scalars: 2 charged scalars ($m_{1,2}$),
3 (neutral) scalars ($\mu_{1,2,3}$), 2 pseudoscalars ($\mu_{4,5}$)

$$\boxed{\mu_4 \ll \mu_5} \tag{1}$$

Constraints:

Higgs strahlung: $e^+e^- \to Z^* \to Z + \text{scalar}$

Associated production:

$e^+e^- \to Z^* \to \text{scalar} + \text{pseudoscalar}$

Electroweak precision tests at LEP

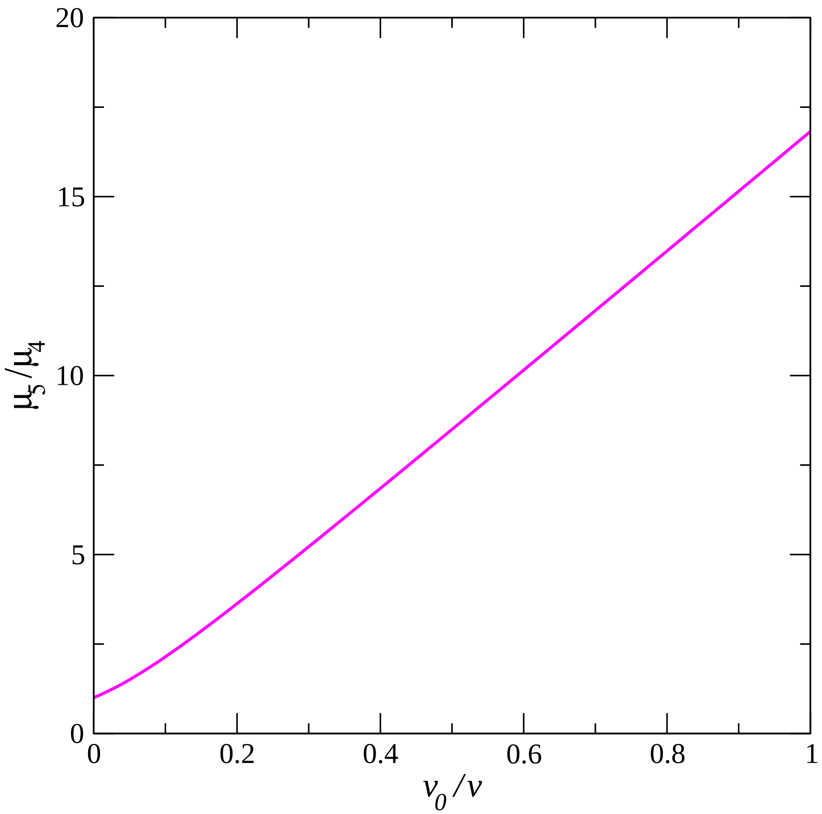


Figure 1: Lower bound on μ_5/μ_4 as function of v_0/v

Note: $v_0 \Rightarrow \mu_4 = \mu_5$, no light pseudoscalar!
Couple ϕ_0 to quarks $\Rightarrow$
$v_0 \gtrsim 100$ GeV to avoid large top Yukawa coupling

Reference scenario:

v_0/v	a_1	a_2	a_3	$\lvert a_4\rvert/\lvert a_4\rvert_{\max}$	a_5	a_6	a_7
$1/\sqrt{2}$	2.5	3	-5	0.4	1.5	2	3

$$
\begin{aligned}
m_1 &= 389.2\ \text{GeV}, & m_2 &= 245.8\ \text{GeV}, \\
\mu_1 &= 434.8\ \text{GeV}, & \mu_2 &= 171.9\ \text{GeV}, \\
\mu_3 &= 231.8\ \text{GeV}, & & \\
\mu_4 &= 14.3\ \text{GeV}, & \mu_5 &= 173.8\ \text{GeV}.
\end{aligned}
$$

Note: m_1 large to avoid problem with $b \to s\gamma$

$\mu_4 > 10$ GeV $\Rightarrow \Upsilon(4S) \not\to S_4^0 + \gamma$

2HDM type II with light (pseudo)scalar:

Chankowski, Krawczyk, Żochowski

$$
\begin{aligned}
T \;=\; & \frac{1}{16\pi s_W^2 m_W^2} \left\{ \sum_{a=1}^{2} \sum_{b=1}^{5} (Y_a \cdot X_b)^2 \, F\left(m_a^2,\, \mu_b^2\right) \right. \\
& - \sum_{b=1}^{3} \sum_{b'=4}^{5} (X_b \cdot X_{b'})^2 \, F\left(\mu_b^2,\, \mu_{b'}^2\right) \\
& + 3 \sum_{b=1}^{3} (X_0 \cdot X_b)^2 \left[F\left(m_Z^2,\, \mu_b^2\right) - F\left(m_W^2,\, \mu_b^2\right) \right] \\
& \left. - 3 \left[F\left(m_Z^2,\, m_h^2\right) - F\left(m_W^2,\, m_h^2\right) \right] \right\}
\end{aligned}
$$

Y_a (X_b) mass eigenvectors for charged (neutral) scalars

Constraint from EWPT: $T \sim -0.1 \pm 0.1$, similarly S, U

Note: m_h, S, T, U cannot be fitted simultaneously!

(Erler, Langacker (RPP))

Fit mass of SM Higgs boson $m_h = 87$ GeV (LEP-EWWG)

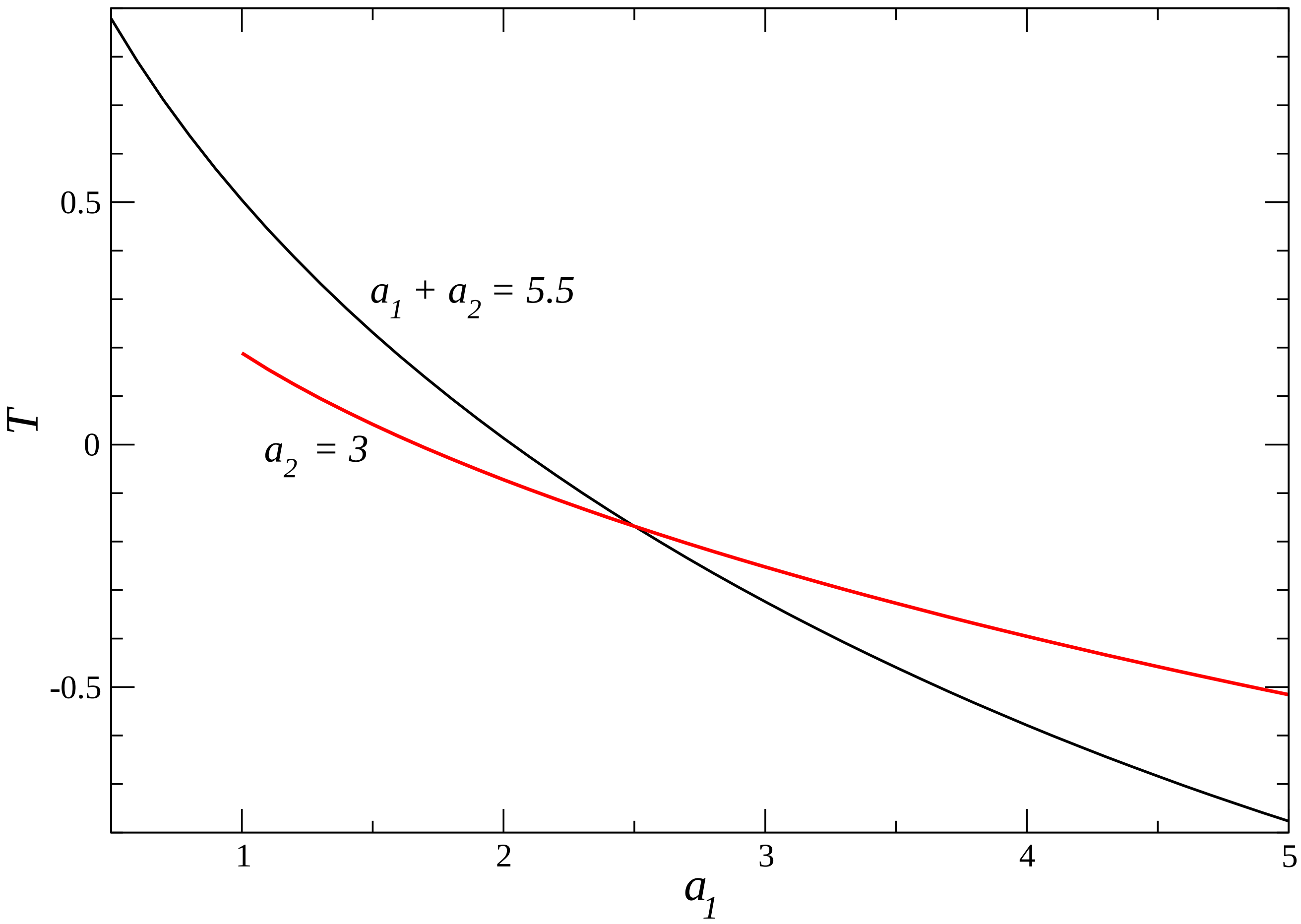


Figure 2: The oblique parameter T as a function of a_1. The input parameters not shown in the plot are as in table.

No drastic finetuning for small T!

6. Replacing the reflection symmetry by a CP symmetry

Replace μ–τ interchange symmetry s by non-trivial CP symmetry

$$CP: \begin{cases} D_{\alpha L} & \longrightarrow & i S_{\alpha\beta} \gamma^0 C \bar{D}^T_{\beta L} \\ \alpha_R & \longrightarrow & i S_{\alpha\beta} \gamma^0 C \bar{\beta}^T_R \\ \nu_{\alpha R} & \longrightarrow & i S_{\alpha\beta} \gamma^0 C \bar{\nu}^T_{\beta R} \\ \phi_0 & \longrightarrow & \phi_0^* \\ \phi_1 & \longrightarrow & \phi_2^* \\ \phi_2 & \longrightarrow & \phi_1^* \end{cases}$$

Symmetry $CP \times U(1) \times \mathbb{Z}_2$

Condition on $\mathcal{M}_\nu$: $\boxed{S\mathcal{M}_\nu S = \mathcal{M}_\nu^*}$

Solution

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y^* \\ y & z & w \\ y^* & w & z^* \end{pmatrix} \quad \text{with} \quad x,\, w \in \mathbb{R}$$

This $\mathcal{M}_\nu$ was first found by Babu, Ma, Valle, hep-ph/0206292 in a SUSY A_4 model

$$\boxed{\theta_{23} = 45^\circ,\ \delta = \pm\pi/2}$$

Maximal atm. mixing, maximal CP violation!

$\theta_{13} = 0^\circ$ possible, but $\theta_{13} \neq 0^\circ$ plus max. CP violation more general case

7. Conclusions

1. **Enforcing lepton mixing properties by symmetries $\Rightarrow$ mHDM**
2. **Ew. precision tests on mHDM by oblique parameter T (also S, U, ...)**

Specific example: $O(2)$ model

- $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$, **3HDM**
- **Connection between lepton and scalar sectors**
  - **Light pseudoscalar S_4^0, $\mu_2 \simeq \mu_5$**
  - **Some mixing angles suppressed by m_μ/m_τ**
- **Smallness of $m_\mu/m_\tau \Rightarrow$ light pseudoscalar**
- $M_{\mathrm{LPS}} \gtrsim 10$ **GeV compatible with all data**