Renormalization Group Evolution of Neutrino Mixings and Applications[a]

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ABSTRACT

The renormalization group running of neutrino masses, mixing angles and CP phases is discussed, including analytical formulae for the Standard Model and MSSM. Several applications are presented, in particular consequences for the potential of future precision measurements of θ_{23} as well as the stability of texture zeros against radiative corrections in the see-saw scenario.

1. Introduction

Renormalization group (RG) effects are important for testing models for fermion masses and mixings, since they cause an energy dependence of all quantities of a theory. Thus, model predictions, which are typically obtained at a very high energy scale, cannot be compared to experimental data unless the RG evolution or running is taken into account. We discuss this effect in the lepton sector and study the running of the mixing angles and CP phases as well as the neutrino mass eigenvalues.

2. Analytical Understanding of the Running below the See-Saw Scale

At energies below the scale of new physics that generates neutrino masses, massive neutrinos can be incorporated into the Standard Model (SM) or MSSM in a model-independent way by including an effective dimension 5 mass operator with coupling constant κ, which is related to the Majorana mass matrix of the light neutrinos by $m_\nu = -\frac{v^2}{4}\kappa$. Then the running of neutrino masses and mixings is determined by the RG equation [1–4]

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = C\,(Y_e^\dagger Y_e)^T\,\kappa + C\,\kappa\,(Y_e^\dagger Y_e) + \alpha\,\kappa\;, \tag{1}$$

where $C = -\frac{3}{2}$ in the SM, $C = 1$ in the MSSM, and $t := \ln(\mu/\mu_0)$ with the renormalization scale μ and an arbitrary energy scale μ_0. The first two terms contain the matrix of charged lepton Yukawa couplings Y_e and thus distinguish between the generations. Hence, they are responsible for the running of the mixing parameters. We work in the basis where Y_e is diagonal. The quantity α contains gauge and Yukawa couplings as well as the Higgs self-coupling in the SM. As it consists only of real numbers, it is flavour-blind. Thus, it influences the running of the mass eigenvalues but not that of the mixings.

In order to calculate the evolution of the neutrino mass matrix, Eq. (1) is not sufficient, since all the couplings it contains are running quantities themselves. Hence, one needs to

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solve a rather complex system of coupled differential equations, which can only be done numerically. Nevertheless, it is possible to gain an analytic insight into the running of the neutrino masses and mixings by employing suitable approximations. To this aim, Eq. (1) can be translated into differential equations for these quantities [5–7]. In the limit $y_e, y_\mu \ll y_\tau$ and $\theta_{13} \ll 1$, they become very simple. For example, the RG equations for the mixing angles are [7]

$$\begin{aligned}
\dot{\theta}_{12} &= -\frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2\, \frac{|m_1\, e^{i\varphi_1} + m_2\, e^{i\varphi_2}|^2}{\Delta m_\odot^2} + \mathscr{O}(\theta_{13}) \,, && (2)\\
\dot{\theta}_{13} &= \frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{12}\, \sin 2\theta_{23}\, \frac{m_3}{\Delta m_\mathrm{a}^2\,(1+\zeta)} \times \\
&\quad \times \left[m_1 \cos(\varphi_1 - \delta) - (1+\zeta)\, m_2\, \cos(\varphi_2 - \delta) - \zeta m_3\, \cos\delta\right] + \mathscr{O}(\theta_{13}) \,, && (3)\\
\dot{\theta}_{23} &= -\frac{Cy_\tau^2}{32\pi^2} \sin 2\theta_{23}\, \frac{1}{\Delta m_\mathrm{a}^2} \left[c_{12}^2\, |m_2\, e^{i\varphi_2} + m_3|^2 + s_{12}^2\, \frac{|m_1\, e^{i\varphi_1} + m_3|^2}{1+\zeta}\right] + \mathscr{O}(\theta_{13}) \,. && (4)
\end{aligned}$$

All expressions are multiplied by a numerical factor and the tau Yukawa coupling. Their product is of the order of $10^{-6} \tan^2\beta$ in the MSSM. Hence, the change of the mixing angles is very small in the SM and in the MSSM for a small $\tan\beta$. If $\tan\beta$ is large, a fast running still requires some enhancement from the other terms, which can stem from those involving the mass eigenvalues, provided that the absolute neutrino masses are large enough. In this case, the CP phases become important, since they can damp the running. This happens for non-zero Majorana phases φ_1, φ_2 in the case of θ_{12} and θ_{23}, while the running of θ_{13} is maximally damped if all phases are zero. Generically, the change of the solar angle is largest, since $\dot{\theta}_{12}$ is proportional to $1/\Delta m_\odot^2$, while the larger atmospheric mass squared splitting appears in the denominators of $\dot{\theta}_{13}$ and $\dot{\theta}_{23}$.

The RG evolution of the neutrino masses m_i $(i = 1, 2, 3)$ is determined by

$$\dot{m}_i = \frac{m_i}{16\pi^2} \left(\alpha + Cy_\tau^2\, F_i\right) , \tag{5}$$

where the functions F_i depend on the mass eigenvalues, the mixing angles and the Dirac phase δ. As the flavour-blind term α contains large quantities like the top Yukawa coupling, it usually dominates the running, except in the MSSM with a large $\tan\beta$. This results in a common rescaling of all three masses. They increase by about 10% to 25% in the MSSM, and by up to 80% in the SM [7].

3. Applications

Planned oscillation experiments will yield a significantly more precise measurement of the lepton mixing angles, see e.g. [8–10]. This means that even small radiative corrections to these parameters could be interesting. As an example, let us consider the running of θ_{23} under the assumption that it is exactly maximal at a very high energy scale, possibly due to some symmetry [7, 9]. The resulting change is shown in Fig. 1. The gray-shaded regions mark those parts of the parameter space where the expected experimental uncertainty is smaller than the RG change, with the lightest region requiring the most

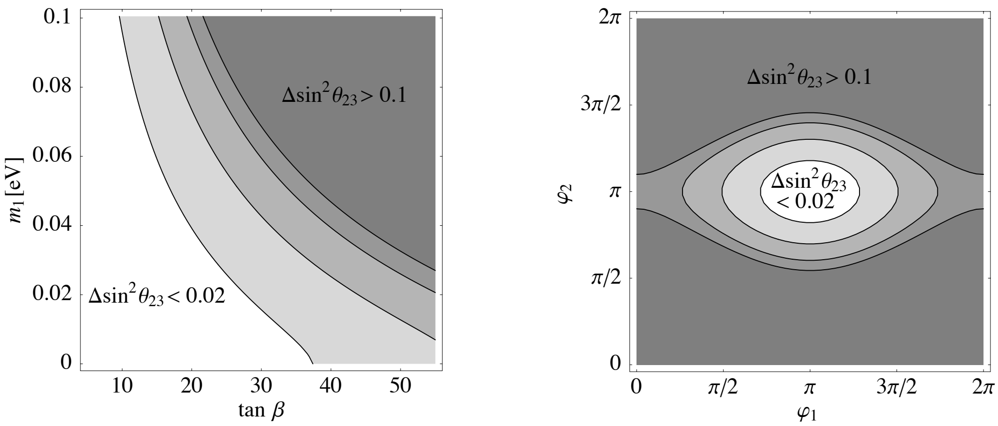

Figure 1: Corrections to $\sin^2\theta_{23}$ caused by the RG evolution from $2 \cdot 10^{16}$ GeV to 100 GeV in the MSSM with a normal mass hierarchy. The initial conditions were $\theta_{23} = \pi/4$, $\theta_{13} = 0$ and the best-fit values for θ_{12} and the mass squared differences. The left diagram shows the dependence on $\tan\beta$ and on the mass m_1 of the lightest neutrino for vanishing Majorana phases. In the right diagram, the dependence on the Majorana phases φ_1 and φ_2 is displayed for $m_1 = 0.075$ eV and $\tan\beta = 50$. The contour lines correspond to the sensitivities of planned experiments as given in [9], $\Delta\sin^2\theta_{23} = 0.02$, 0.05, 0.08 and 0.1.

sensitive experiment. As these regions cover a significant part of the parameter space, we expect precision experiments to have a good potential to discover deviations from maximal atmospheric mixing and to provide valuable input for neutrino mass models.

The evolution of the neutrino mass eigenvalues has to be applied when calculating the size of the baryon asymmetry generated by leptogenesis, for instance. As the relevant processes occur at high energies, the running of the input parameters, which include the neutrino masses, has to be considered [7, 11]. In thermal leptogenesis, there are two competing effects: while the maximal CP asymmetry in the decays of the singlet neutrinos increases with increasing masses, the efficiency factor decreases. In numerical studies, it was found that the second correction is usually larger, so that the upper bound on the mass of the light neutrinos becomes stronger due to RG effects [12, 13].

4. Stability of Texture Zeros in the Type I See-Saw Scenario

In the type I see-saw scenario, the RG evolution exhibits new effects in the energy region above the mass scale of the heavy singlet neutrinos. There the masses and mixings of the light neutrinos are determined by the neutrino Yukawa couplings Y_ν and the singlet mass matrix M via $m_\nu = -\frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$. The RG equation for m_ν has the same form as Eq. (1), but with additional terms proportional to $Y_\nu^\dagger Y_\nu$, which in general are non-diagonal in the basis where Y_e is diagonal. These terms can cause a significant running, since Y_ν may have large entries.

An application of this concerns texture zeros in the neutrino mass matrix. If these are realized at very high energy, they are removed by the RG evolution provided that Y_ν is not diagonal. Therefore, textures which have been classified as incompatible with experimental data [14,15] can be reconciled with the data [16]. A rather extreme example is shown in Fig. 2, where the correct low-energy values of the lepton mixing angles are generated from vanishing mixings, i.e. a diagonal neutrino mass matrix, at high energy.

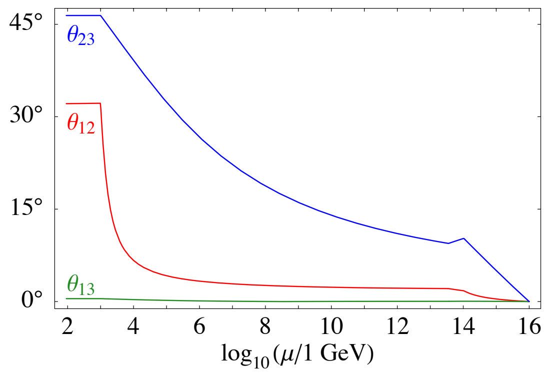

Figure 2: Running from vanishing lepton mixings at 10^{16} GeV to the LMA solution at low energy. We used the MSSM with $\tan\beta = 50$ and a SUSY breaking scale $M_{\mathrm{SUSY}} = 1$ TeV.

However, the usual classification of forbidden and allowed textures is valid for hierarchical neutrino masses and for Majorana phases significantly different from 0 and π.

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