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Neutrino Factory Parameter Correlations

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Neutrino masses, mixing angles and leptonic CP violation enter in a rather complex way into the full neutrino oscillation formulae. The extraction of these parameters at neutrino factories requires thus a careful treatment of parameter correlations. We studied these correlations in the multidimensional parameter space, leading to modifications in the physics reach, which amount in some cases to one order of magnitude. We included the uncertainties of all involved parameters, statistical errors, but no systematic errors. The observed parameter correlations are especially important for the largest possible values of Δm^2_{21}, where the θ_{13} sensitivity limit are considerably reduced.

The physics potential of neutrino factories depends on a number of physics and machine parameters which are not easily overlooked. A common strategy is to discuss only one or two parameters at a time, while the remaining parameters are set to "standard values" (like the muon flux or $\sin^2 2\theta_{23} = 1$). This is acceptable for rough estimates of the physics capabilities of such experiments. Possible correlations with the parameters which are held fixed can, however, lead to quite severe errors, especially close to the sensitivity limits, where a strong interplay between several parameters exists. Similar problems arise in a two step analysis, where the disappearance channel is analyzed with sub-leading corrections ignored, and where the extracted leading parameters are used for the extraction of sub-leading parameters. An analysis, where all the parameter correlations are considered [1] shows these previously ignored correlations in the parameter space and leads to modifications of the physics reach. We discuss here the modification of the θ_{13} sensitivity limit.

The full three neutrino oscillation formulae have in general a rather complex structure. For an analytic understanding one can use simplified expressions by expanding in $\alpha = \Delta m^2_{12}/\Delta m^2_{31} \ll 1$, which is a small quantity due to the hierarchy of the neutrino mass splittings. In a numerical treatment one must, however, be careful since there is a second small quantity, the mixing angle θ_{13}, which is bound by $\sin^2 2\theta_{13} < 0.1$. The expansion parameter α and the angle θ_{13} can thus take values of roughly equal size. All terms in the above expansion in α are proportional to $\alpha^{n_\alpha} \cdot \theta_{13}^{n_\theta}$, such that we have to group terms of similar numerical size by identical combined powers $n = n_\alpha + n_\theta$. The leading $n = 2$ terms are for the $\nu_e \to \nu_\mu$ appearance and $\nu_\mu \to \nu_\mu$ disappearance probabilities in vacuum ("+" for $\nu' s$ and "−" for $\bar{\nu}' s$):

$$\begin{aligned}
P(\nu_e \to \nu_\mu) &\approx \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta \\
&\pm \alpha\ \sin\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\sin^3\Delta \\
&+ \alpha\ \cos\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\Delta\sin^2\Delta \\
&+ \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2\Delta \qquad (1)\\
P(\nu_\mu \to \nu_\mu) &\approx 1 - \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2\Delta \\
&+ 2\alpha\cos^2\theta_{13}\cos^2\theta_{12}\sin^2 2\theta_{23}\Delta\cos\Delta \qquad (2)
\end{aligned}$$

The smallness of α and θ_{13} allows to classify the parameter space in the scheme below, but note that all results are based on a full numerical calculation.

• **Leading parameters:**
For $\Delta m^2_{21} = 0$ and $\theta_{13} = 0$, there are no transitions in the $\nu_e \to \nu_\mu$ appearance channel. The disappearance probability reduces to the two neutrino case $P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23}\sin^2\Delta$ being controlled by θ_{23}, Δm^2_{31}, which are *leading parameters.* These parameters have already been roughly determined and a neutrino factory will allow precision measurements by fitting the differential event rate spectrum. The measurement of these parameters will be dominated by the $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ disappearance channels and will be limited mostly by systematical errors only.

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• **Sub-leading parameters:**
For $\theta_{13} \neq 0$ and $\Delta m^2_{21} = 0$, the first term in the appearance probabilities eq. (1) becomes non-zero: $P(\nu_e \to \nu_\mu) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta$. The probability depends in matter also on the sign of Δm^2_{31} [1–3, 5]. In addition to the leading parameters, the analysis depends at this level thus also on θ_{13} and sgnΔm^2_{31}, which are the *sub-leading parameters*. The possibility to determine these sub-leading parameters depends crucially on the value of θ_{13}. Sufficiently large θ_{13} will allow experimental test of MSW-effects, which have so far not been measured. The measurement of θ_{13} and the search for matter effects [2, 3, 6, 7] are important topics for the physics program of a neutrino factory, which do not depend on Δm^2_{21} and θ_{12} being in the LMA-MSW range.

• **Sub-sub-leading parameters:**
Finally, for $\Delta m^2_{21} \neq 0$ (with $\theta_{13} \neq 0$), effects due to the small solar mass squared splitting are added and the remaining three parameters appear in the oscillation formulae: Δm^2_{21}, θ_{12} and the CP phase $\delta_{\rm CP}$. Measuring leptonic CP violation is an exciting possibility for neutrino factories. In order to obtain sufficient rates, this requires, however, that the LMA-MSW region is the correct solution to the solar neutrino problem. One can see immediately from eq. (1) that Δm^2_{21} (i.e. α) and θ_{13} are the crucial parameters with determine the absolute and relative strength of the CP-violating effects. The foreseeable experiments are often not too far from the sensitivity limit, where it is important to understanding how to search optimally for CP violation and how $\delta_{\rm CP}$ can be extracted. θ_{12} and Δm^2_{21} can be measured only very poorly in neutrino factory experiments. To a good approximation only the product $\Delta m^2_{21}\sin 2\theta_{21}$ can be determined, which can be seen from eq. (1). Thus Δm^2_{21} and θ_{12} are highly correlated.

The study peformed in [1] is based on proper statistical quantities which are based on event rates and which include the full oscillation formulae in matter. The above classification allows, however, a qualitative understanding of these results and inspection of eq. (1) makes clear that sizable parameter correlations exist for the currently favoured LMA-MSW case. An example for the dependence of sensitivity limits on parameter correlations can be seen in fig. 1, which shows the sensitivity reach for measurements of $\sin^2 2\theta_{13}$. The left plot shows the limits for a two parameter fit of $\sin^2 2\theta_{13}$ and Δm^2_{31}, where all the other parameters are held fixed. This means that these parameters are assumed to be known without any error, which is definitively not the case for e.g. the CP phase. The right plot shows for comparison the result when all parameters and correlations taken into account (for details see [1]). One can see that the sensitivity limit for θ_{13} becomes substancially weaker for larger values of Δm^2_{21} due to the correlations with other parameters. The main reason for the weaker limits in the right plot of fig. 1 can, however, already be seen in eq. (1), by noting that the α^2 term (which does not depend on θ_{13}) makes it for large α more difficult to extract θ_{13}. This weaker sensitivity limit depends therefore not much on how unknown parameters (like the CP phase) are treated.

In conclusion we demonstrated how sensitivity limits depend on the treatment (or inclusion) of unknown or partially known parameters. We showed that the sensitivity limit for θ_{13} can deteriorate by up to one order of magnitue if Δm^2_{21} were 10^{-4} eV^2. The overall precision of the rate mesasurements does not deteriorate, which means that a certain parameter combination is better known than the individual parameters. The reduced sensitivity limit might thus partially be overcome by combining different experimental setups which allow to disentangle these correations.

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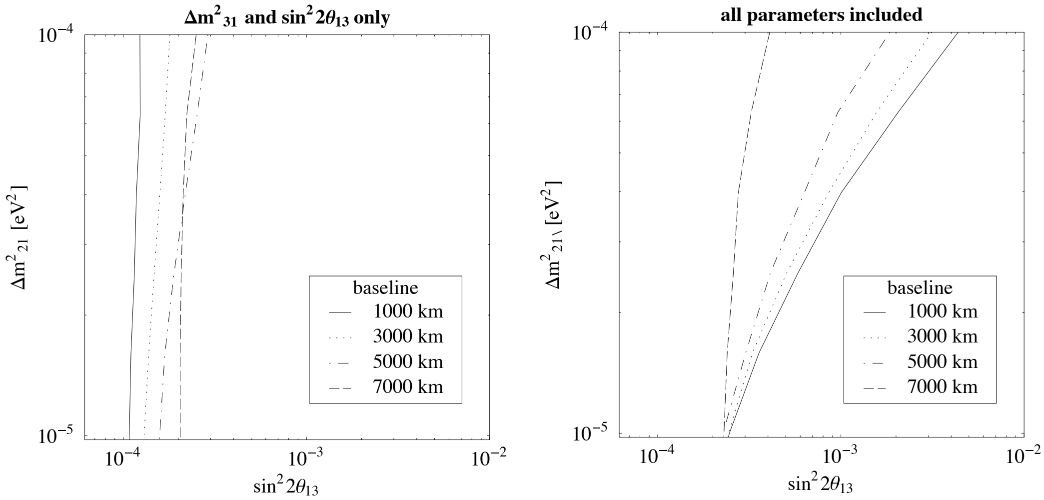

FIG. 1: Sensitivity reach for measurements of $\sin^2 2\theta_{13}$ without and with parameter correlations included. The area to the left of the lines indicates the parameter range where measurements are compatible with $\sin^2 2\theta_{13} = 0$ at 99% C.L.. The beam energy is 50 GeV with different baselines as given in the legend.