

Correlations in Neutrino Factory Parameter Space

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Abstract

The extraction of neutrino masses, mixing angles and leptonic CP violation at neutrino factories requires a careful treatment of parameter correlations. Including the uncertainties of all involved parameters and statistical errors we find previously ignored correlations in the multidimensional parameter space, leading to modifications in the physics reach, which amount in some cases to one order of magnitude. The parameter correlations are especially important for the largest possible values of Δm_{21}^2 , where the θ_{13} sensitivity limit are considerably reduced.

Neutrino factory studies depend on a number of physics and machine parameters which are not easily overlooked and discussed at the same time. A common strategy to deal with this situation is to discuss only one or two parameters simultaneously, while the remaining parameters are set to “standard values” (like the muon flux or $\sin^2 2\theta_{23} = 1$). This is acceptable for rough estimates of the physics capabilities of such experiments. Possible correlations with the parameters which are held fixed can, however, lead to quite severe errors, especially close to the sensitivity limits, where a strong interplay between several parameters exists. Similar problems arise in a two step analysis, where the disappearance channel is analyzed with sub-leading corrections ignored, and where the extracted leading parameters are used for the extraction of sub-leading parameters. An analysis, where all the parameter correlations are considered [1] shows these previously ignored correlations in the parameter space and leads to modifications of the physics reach. We discuss here the modification of the θ_{13} sensitivity limit.

The full three neutrino oscillation formulae have in general a rather complex structure. For an analytic understanding one can use simplified expressions by expanding in $\alpha = \Delta m_{12}^2 / \Delta m_{31}^2 \ll 1$, which is a small quantity due to the

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hierarchy of the neutrino mass splittings. In a numerical treatment one must, however, be careful since there is a second small quantity, the mixing angle θ_{13} , which is bound by $\sin^2 2\theta_{13} < 0.1$. The expansion parameter α and the angle θ_{13} can thus take values of roughly equal size. All terms in the above expansion in α are proportional to $\alpha^{n_\alpha} \cdot \theta_{13}^{n_\theta}$, such that we have to group terms of similar numerical size by identical combined powers $n = n_\alpha + n_\theta$. The leading $n = 2$ terms are for the $\nu_e \rightarrow \nu_\mu$ appearance and $\nu_\mu \rightarrow \nu_\mu$ disappearance probabilities in vacuum (“+” for ν' s and “−” for $\bar{\nu}'$ s):

$$\begin{aligned}
P(\nu_e \rightarrow \nu_\mu) \approx & \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\
& \pm \alpha \sin \delta_{\text{CP}} \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin^3 \Delta \\
& + \alpha \cos \delta_{\text{CP}} \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \Delta \sin^2 \Delta \\
& + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2 \Delta
\end{aligned} \tag{1}$$

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_\mu) \approx & 1 - \cos^2 \theta_{13} \sin^2 2\theta_{23} \sin^2 \Delta \\
& + 2\alpha \cos^2 \theta_{13} \cos^2 \theta_{12} \sin^2 2\theta_{23} \Delta \cos \Delta
\end{aligned} \tag{2}$$

The smallness of α and θ_{13} allows to classify the parameter space in the scheme below, but note that all results are based on a full numerical calculation.

• **Leading parameters:**

For $\Delta m_{21}^2 = 0$ and $\theta_{13} = 0$, there are no transitions in the $\nu_e \rightarrow \nu_\mu$ appearance channel. The disappearance probability reduces to the two neutrino case $P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 \Delta$ being controlled by θ_{23} , Δm_{31}^2 , which are *leading parameters*. These parameters have already been roughly determined and a neutrino factory will allow precision measurements by fitting the differential event rate spectrum. The measurement of these parameters will be dominated by the $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ disappearance channels and will be limited mostly by systematical errors only.

• **Sub-leading parameters:**

For $\theta_{13} \neq 0$ and $\Delta m_{21}^2 = 0$, the first term in the appearance probabilities eq. (1) becomes non-zero: $P(\nu_e \rightarrow \nu_\mu) = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta$. The probability depends in matter also on the sign of Δm_{31}^2 [1,4,3,2]. In addition to the leading parameters, the analysis depends at this level thus also on θ_{13} and $\text{sgn} \Delta m_{31}^2$, which are the *sub-leading parameters*. The possibility to determine these sub-leading parameters depends crucially on the value of θ_{13} . Sufficiently large θ_{13} will allow experimental test of MSW-effects, which have so far not been measured. The measurement of θ_{13} and the search for matter effects [3,2,5,6] are important topics for the physics program of a neutrino factory, which do not depend on Δm_{21}^2 and θ_{12} being in the LMA-MSW range.

• **Sub-sub-leading parameters:**

Finally, for $\Delta m_{21}^2 \neq 0$ (with $\theta_{13} \neq 0$), effects due to the small solar mass squared splitting are added and the remaining three parameters appear in the

oscillation formulae: Δm_{21}^2 , θ_{12} and the CP phase δ_{CP} . Measuring leptonic CP violation is an exciting possibility for neutrino factories. In order to obtain sufficient rates, this requires, however, that the LMA-MSW region is the correct solution to the solar neutrino problem. One can see immediately from eq. (1) that Δm_{21}^2 (i.e. α) and θ_{13} are the crucial parameters with determine the absolute and relative strength of the CP-violating effects. The foreseeable experiments are often not too far from the sensitivity limit, where it is important to understanding how to search optimally for CP violation and how δ_{CP} can be extracted. θ_{12} and Δm_{21}^2 can be measured only very poorly in neutrino factory experiments. To a good approximation only the product $\Delta m_{21}^2 \sin 2\theta_{21}$ can be determined, which can be seen from eq. (1). Thus Δm_{21}^2 and θ_{12} are highly correlated.

The above classification and inspection of eq. (1) makes clear that sizable parameter correlations exist for the currently favoured LMA-MSW case. An example for the dependence of sensitivity limits on parameter correlations can be seen in fig. 1, which shows the sensitivity reach for measurements of $\sin^2 2\theta_{13}$. The left plot shows the limits for a two parameter fit of $\sin^2 2\theta_{13}$ and Δm_{31}^2 only. The right plot shows the result when all parameters and correlations taken into account (for details see [1]). One can see that the sensitivity limit for θ_{13} becomes weaker for larger values of Δm_{21}^2 due to the correlations with other parameters. The main reason for the weaker limits in the right plot of fig. 1 can, however, already be seen in eq. (1), by noting that the α^2 term (which does not depend on θ_{13}) makes it for large α more difficult to extract θ_{13} .

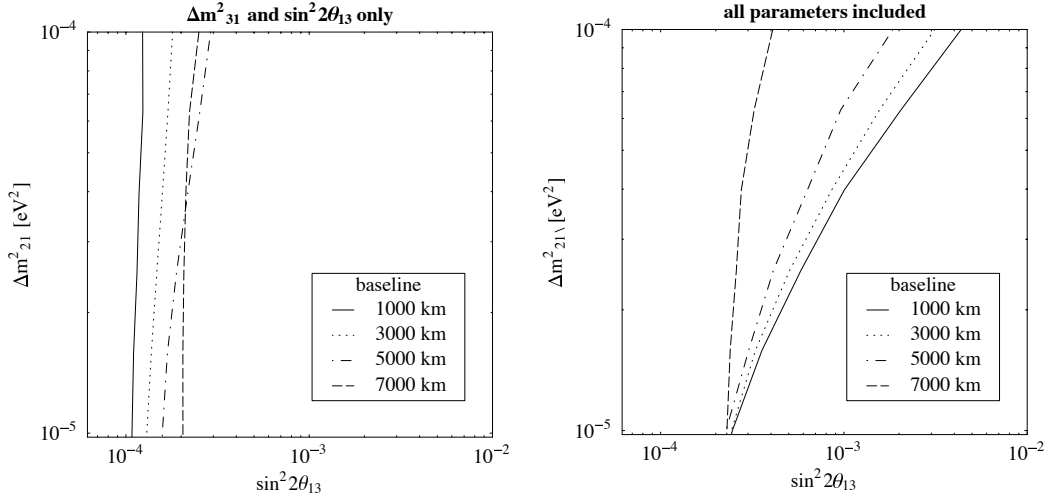


Fig. 1. Sensitivity reach for measurements of $\sin^2 2\theta_{13}$ without and with parameter correlations included. The area to the left of the lines indicates the parameter range where measurements are compatible with $\sin^2 2\theta_{13} = 0$ at 99% C.L.. The beam energy is 50 GeV with different baselines as given in the legend.

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