Matter Effects and CP-Violation at Neutrino Factories *

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Neutrino factories allow precise measurements of neutrino masses, leptonic mixing angles, leptonic CP-violation and matter effects. Some aspects of matter effects and their role in the disentanglement of parameters in very long baseline neutrino oscillation experiments are discussed.

Neutrino oscillation experiments are so far essentially described by vacuum oscillations in a two flavour picture. Increased precision in the future will however require oscillation formulae with all flavours and the mixing matrix U:

$$\begin{aligned} P(\nu_{e_l} \to \nu_{e_m}) = & \left|\langle \nu_m | U e^{-iHt} U^\dagger | \nu_l \rangle\right|^2 \\ = \delta_{lm} - 4\sum_{i>j} & \mathrm{Re} J_{ij}^{e_l e_m} \sin^2 \Delta_{ij} \\ -2\sum_{i>j} & \mathrm{Im} J_{ij}^{e_l e_m} \sin 2\Delta_{ij} \end{aligned} \quad (1)$$

where $J_{ij}^{e_l e_m} := U_{li} U_{lj}^* U_{mi}^* U_{mj}$ and $\Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E}$, with $\Delta m_{ij}^2 = m_i^2 - m_j^2$. It can be easily seen [1] that the oscillation probabilities in eq. (1) depend for three or more non-degenerate neutrinos on CP-violating phases in the mixing matrix U. Additionally matter effects [2,3] (i.e. the matter potential V) must be taken into account in eq. (1) where appropriate, e.g. for beams crossing the earth, where the presence of matter implies to a good approximation a mapping of the vacuum masses and mixings to effective parameters in matter. This leads in the limit where the small solar mass splitting Δm_{21}^2 can be ignored in vacuum to the mappings $\theta_{13} \to \theta_{13,m}$, $m_1^2 \to m_{1,m}^2$, $m_3^2 \to m_{3,m}^2$ such that all Δm_{ij}^2 are affected in matter, while θ_{12}, θ_{23} and the CP-phase δ are unchanged [4]. Note that there are different mappings for neutrinos and antineutrinos due to the opposite sign of the interaction. Matter effects make the general expressions for the oscillation of three or more neutrinos rather lengthy, but it is important to note that this allows tests of matter effects, which are so far experimentally unprobed. Optimal sensitivity is obtained [2] when the resonance condition is fulfilled, i.e. for $2EV \simeq \Delta m_{31}^2 \cos 2\theta_{13}$. For small θ_{13}, $\Delta m_{31}^2 = 3.5 \times 10^{-3}$ eV^2 and typical matter densities in the earth crust one finds the neutrino energies $E_{opt} \simeq 15$ GeV and $E_{opt} \simeq 8$ GeV in the core. Another important point of matter effects is the extraction of the sign of Δm_{31}^2, which is impossible in vacuum oscillations resulting in mass ordering ambiguities [5,2] shown in Fig. 1. Optimal extraction of the sign of Δm^2 via matter effects is again governed by the resonance condition.

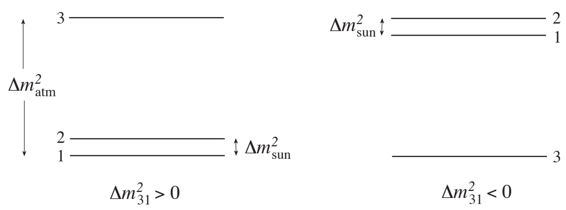

Figure 1. Vacuum oscillation order ambiguities.

Neutrino factories [6] would be ideal to extract neutrino mixings, masses, and via matter effects the sign of Δm_{31}^2, and they would also allow precise tests of matter effects [2,4]. In vacuum the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillation probabilities simplify for $\Delta m_{21}^2 = 0$ a lot and are given by

$$P(\nu_e \leftrightarrow \nu_\mu) = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2(\Delta_{31}) \ . \quad (2)$$

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The relevant matter parameter mappings are

$$\sin^2 2\theta_{13,m} = \frac{\sin^2 2\theta_{13}}{C_\pm^2} \ ; \ \Delta m_{31,m}^2 = \Delta m_{31}^2 C_\pm \quad (3)$$

with

$$C_\pm^2 = \left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta \right)^2 + \sin^2 2\theta \ . \quad (4)$$

The resulting the total appearance rates in the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ appearance channels are shown in fig. 2 for typical neutrino factory parameters as a function of baseline. There are sizable matter effects for sufficiently large baseline. This can be easily understood when the oscillatory term in eq. (2) is expanded in a power series in L and when the matter mappings eq. (4) and eq. (3) are inserted. The matter corrections in the θ_{13} mixing angle compensate in the leading linear term the corrections in Δm_{31}^2. This term scales like L^2 which becomes compensated in the rates by the geometric $1/L^2$ factor of the beam leading to a constant rate at small L. This cancellation works however only for the first term of the expansion and an extra $C_\pm^2$ factor survives already in the next term proportional to L^4. Together with the $1/L^2$ flux factor the rates are thus for small L described by an inverted parabola $\propto 1 - \Delta m_{31,m}^2 L^2/12E$. The changes due to the presence of matter are thus for moderate L simply a consequence of the effective $\Delta m_{31,m}^2$ in matter, which can be clearly seen in figure 2. For resonant channels the effective $\Delta m_{31,m}^2$ is decreased, reducing thus the curvature of the parabola which makes the rates for larger L bigger. The opposite is true for anti-resonant channels. The discussion becomes more complicated for larger L when the power law expansion breaks down.

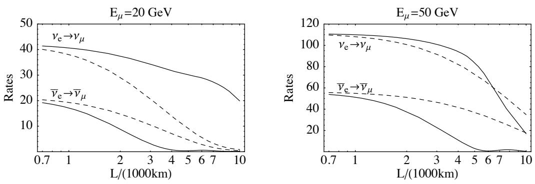

Figure 2. Matter effects in the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ channels for for $E_\mu = 20$ GeV (left plot) and $E_\mu = 50$ GeV (right plot) at a neutrino factory. Dashed lines correspond to vacuum oscillation, solid lines include matter effects (see [2]).

The presence of matter leads also to interesting effects in the $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ disappearance channels. The disappearance probability starts however with one and all matter effects are obviously negligible until the oscillatory terms become in the resonance regime comparable to one for large enough baseline. Interestingly matter effects in the appearance channel (resulting in "wrong sign muon events") go in the same direction as matter effects in the disappearance channel ("same sign muon events"). It is thus possible to combine all muon events and get an even bigger matter signal [2]. This can be seen in figure 3, where the combined differential event rate spectrum is shown for a baseline of 7332 km, and where both channels contribute for this large baseline approximately half of the matter effects. This may become important if the challenging charge identification (CID) requirements in the wrong sign muon channel can not be achieved, since it allows an analysis even without CID. The information in the disappearance channel is however also important with CID, since this allows a global analysis in combination with the appearance channels [2].

Long baseline oscillation experiments at neutrino factories allow in given approximation altogether a remarkable precision for the leading oscillation parameters $|\Delta m_{31}^2|$, θ_{23} and very good limits or measurements for θ_{13} and the sign of Δm_{31}^2. This could be done to a very good approximation in two steps by combining the two channels shown in figure 3 (with appropriate weights). Matter effects cancel and a precise determination of $|\Delta m_{31}^2|$, θ_{23} is possible. With the leading parameters known it becomes possible to extract via matter effects the sign of Δm_{31}^2 and θ_{13} by fitting the individual channels in figure 3.

This is unfortunately not the full story, since CP-violating effects become relevant when the

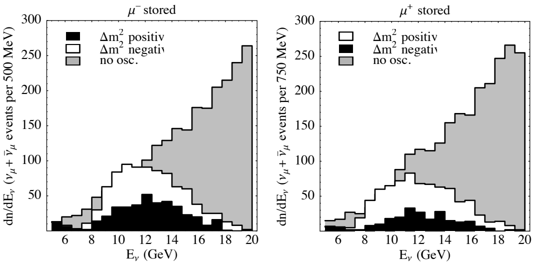

Figure 3. Spectral matter effects at a neutrino factory for $E_\mu = 20$ GeV, $L = 7332$ km and $\sin^2 2\Theta_{13} = 0.1$ (see [4]).

small, but non vanishing Δm^2_{21} is turned on. CP-violating effects exist only in the appearance channels and figure 4 shows how the total rates are modified for different CP-phases for $\Delta m^2_{21} = 10^{-4}$ eV^2, $\sin^2 2\theta_{13} = 0.01$ and $E_\mu = 20$ GeV. Note for comparison that the solid line in the left (right) plot of figure 4 is identical to the upper (lower) solid line in the left plot of figure 2. Since

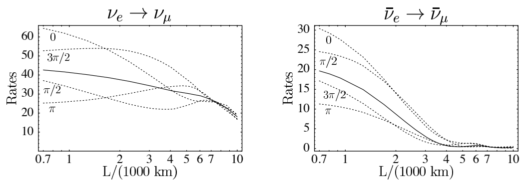

Figure 4. Example of CP violating effects at a neutrino factory. For details see text and [2].

the event rates depend for a given baseline L now on the CP-phase δ and on θ_{13} it becomes immediately clear that more information is required to separate these parameters. There is in principle more information in the differential event rate spectrum and studies have been performed where the separation with the help of the spectrum is demonstrated [8]. It is however important to keep in mind that these studies assume that Δm^2_{21} is known without any error, which will probably not be true. There are also further systematic effects (e.g. deviation of θ_{12} from maximal mixing, matter density uncertainties, CID limitations) which make the parameter disentanglement at one baseline much more complicated. The situation is considerably improved by combining suitably information from different baselines [9].

Neutrino factories are in summary very promising tools for "precision neutrino oscillation physics", resulting in very precise determinations of $|\Delta m^2_{31}|$, θ_{23}, matter effects, θ_{13}, the sign of Δm^2_{31} and the leptonic CP-violation phase. The disentanglement of the small mixing angle θ_{13}, the CP-phase δ and matter effects is definitively possible, but further studies are required to clarify some open details.

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