MATTER EFFECTS AND CP-VIOLATION IN NEUTRINO OSCILLATION*

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The potential of very long baseline neutrino oscillation experiments to measure precisely leptonic mixing angles, CP-violation and matter effects is discussed.

Almost all neutrino oscillation results are so far described by vacuum oscillations in a two flavour picture. Increased precision in the future will however require oscillation formulae with all flavours and the mixing matrix \( U \):

\[
P(\nu_e \rightarrow \nu_m) = \left| \langle \nu_m | U^* e^{-iHBt} U | \nu_e \rangle \right|^2
= \delta_{jm} - 4 \sum_{i>j} \text{Re} J_{ij}^{em} \sin^2 \Delta_{ij}
- 2 \sum_{i>j} \text{Im} J_{ij}^{em} \sin \Delta_{ij}
\]

where \( J_{ij}^{em} \) := \( U_{e i} U_{m j}^* \frac{m_j^2 - m_i^2}{16 \pi^2} \), and \( \Delta_{ij} := \frac{\Delta m^2_{ij}}{16 \pi^2} \), with \( \Delta m^2_{21} = m_2^2 - m_1^2 \). It can be easily seen that the oscillation probabilities in eq. (1) depend for three or more non-degenerate neutrinos on CP-violating phases in the mixing matrix \( U \). Additionally matter effects\(^2\) must be taken into account in eq. (1) where appropriate, e.g. for beams crossing the earth. The presence of matter implies a mapping of the vacuum masses and mixings to effective parameters in matter. This leads in the limit where the small solar mass splitting \( \Delta m^2_{21} \) is ignored to the mappings \( \theta_{13} \rightarrow \theta_{33, m} \), \( m_1^2 \rightarrow m_{13}^2 \), \( m_2^2 \rightarrow m_{33}^2 \) such that all \( \Delta m^2_{ij} \) are affected, while \( \theta_{12}, \theta_{23} \) and the CP-phase \( \delta \) are unchanged\(^4\). Note that there are different mappings for neutrinos and antineutrinos due to the opposite sign of the interaction. Matter effects make the general expressions for the oscillation of three or more neutrinos rather lengthy, but is important to note that this allows tests of matter effects, which are so far experimentally unprobed. Optimal sensitivity is obtained\(^2\) when the resonance condition is fulfilled, i.e. for \( 2E \sim \Delta m^2_{23} \cos \theta_{23} \). For negligible small \( \theta_{13} \), \( \Delta m^2_{23} = 3.5 \times 10^{-3} \text{ eV}^2 \) and typical densities in the earth crust one finds \( E_{\text{opt}} \approx 15 \text{ GeV} \) and \( E_{\text{opt}} \approx 8 \text{ GeV} \) in the core. Another important point of matter effects is the extraction of the sign of \( \Delta m^2_{23} \), which is undetermined in vacuum oscillations resulting in mass ordering ambiguities\(^5\) as shown in Fig. 1. The extraction of the sign of \( \Delta m^2 \) via matter effects is again connected to the resonance condition.

Neutrino factories\(^6\) would be an ideal tool to probe the above effects, but it will take some time to be built and it is interesting what could be achieved meanwhile with conventional neutrino beams. The potential of several experimental setups is therefore compared\(^7\) by showing the precision of the leading oscillation parameters (\( \Delta m^2_{21} \) and \( \theta_{23} \)) as well as the sensitivity to the sub-leading parameters (\( \theta_{12} \) and the sign of \( \Delta m^2_{31} \)). The energy thresholds, resolutions and the dependence on these parameters is also shown. The considered experimental setups include two beams, namely (I) conventional wide band beams and (II) neu-


neutrino factory beams which are assumed to point at three different types of detectors, specifically (a) magnetized iron detectors, (b) large water or ice Cherenkov detectors ("neutrino-telescopes") and (c) ring imaging water Cherenkov detectors. Due to their high threshold neutrino-telescopes are usually not considered for neutrino oscillation, but it has been shown recently that the effective threshold can be considerably lower in a high rate oscillation experiment\textsuperscript{3,7}. Figs. 2 and 3 show

![Figure 2](image1.png)

Figure 2. Sensitivity to $\theta_{13}$.

in comparison the sensitivity to $\theta_{13}$ and the $\theta_{13}$ range where the sign of $\Delta m^2$ can be extracted\textsuperscript{1}. The figures show that conventional beams offer quite some improvements until a neutrino factory is built. The possibility to extract the sign of $\Delta m^2$ implies precise tests of matter effects \textsuperscript{2}, CP-violating effects can not be measured in disappearance channels and are most likely measured with neutrino factories. Examples of CP-violating effects are shown\textsuperscript{1} in Fig. 4. In summary there is a very promising future for "precision neutrino oscillation physics", resulting in very precise masses, mixings and maybe CP-violation in the lepton sector. Neutrino factories, which will take some time to be realized, would be the best option. Meanwhile the remarkable potential of conventional setups should be exploited.

![Figure 3](image2.png)

Figure 3. $\theta_{13}$ sensitivity range for the sign of $\Delta m^2$.

![Figure 4](image3.png)

Figure 4. Example of CP violating effects at a neutrino factory.

**References**


