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CONSISTENCY OF HIGGS AND W MASS DETERMINATIONS IN A MODEL WITH COMPOSITE SCALARS*

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ABSTRACT

Predictions for the masses M_W and M_H arise in models, where the Higgs boson is a fermion–anti-fermion bound state. One type of prediction stems from Pagels–Stokar relations based on the use of Ward Identities in the calculation of the Goldstone Boson decay constants and expresses M_W and M_H in terms of integrals containing the dynamically generated mass functions $\Sigma_i(p^2)$. Another type of prediction emerges from the renormalization group equations via infrared quasi–fixed–points of the running Yukawa couplings. Especially for a high scale of new physics Λ both methods should become rather precise and it is unclear which method is better. We exemplify for the BHL model of top condensation that these two methods lead to the same predictions for M_W and M_H in lowest loop order.

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1. Introduction

A composite Higgs sector might solve the theoretical problems of scalars in the Standard Model. One possibility is that the Higgs particle is essentially a fermion–anti-fermion bound state explaining naturally why some Yukawa couplings are of order unity. Instead of fundamental scalars one needs therefore suitable interactions capable of forming a scalar condensate and an effective scalar sector which describes the symmetry breaking in analogy to the Ginzburg–Landau description of superconductivity. A nice realization of such ideas is top condensation[1,2], as e.g. realized in the so–called BHL model [1]. We use this model to demonstrate the consistency of different methods to determine the W and Higgs masses. The BHL Lagrangian consists just of the kinetic parts of the ordinary quarks, leptons and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge fields and a new attractive four–fermion interaction. The BHL Lagrangian is

$$\mathcal{L}_{BHL} = \mathcal{L}_{kinetic} + G\overline{L}t_R\overline{t}_R L \ , \tag{1}$$

where $\mathcal{L}_{kinetic}$ contains the kinetic terms for all gauge fields, quarks and leptons. $L^T = (t_L, b_L)$ is the third generation doublet of quarks containing the left–handed top and bottom fields and t_R is the right–handed component of the top quark. The symmetry breaking of the BHL model by loop effects with a high energy cutoff Λ can be studied in the large N_c limit (where N_c is the number of colors) in analogy to Nambu–Jona-Lasinio (NJL) models. For $G > G_{cr} = 8\pi^2/N_c\Lambda^2$ the gap equation is found to be critical and a top condensate emerges. The breakdown of global symmetries by this condensate implies the existence of composite Goldstone Bosons. In the NJL–treatment of the BHL model these Goldstone Bosons are found as massless poles in fermion scattering amplitudes along with a composite Higgs particle of mass $2m_t$. It turns out that this set of composite scalar fields is equivalent to the usual Standard Model Higgs doublet ϕ with a non–vanishing vacuum expectation value (VEV). The symmetry breaking is thus more or less identical to the Standard Model. The composite Goldstone Bosons are therefore eaten exactly as in the Standard Model and produce the usual W and Z mass relations.

When the W mass is calculated in large N_c approximation in terms of the top mass [1] then one finds for $N_c = 3$

$$M_W^2 = m_t^2 \, \frac{3g_2^2}{32\pi^2} \ln\left(\frac{\Lambda^2}{m_t^2}\right) \ . \tag{2}$$

Low values of Λ lead to a phenomenologically unacceptable large top mass and one is thus forced to large Λ which requires however a fine–tuning of $G \to G_{cr}$. Since the ratio M_W/m_t is so crucial for the phenomenological viability one should study improvements of the above large N_c relation. Using Ward Identities one can derive

the so–called Pagels–Stokar formulae which can also be obtained directly by inserting running masses into the derivation of eq. (2). Another possibility is to use immediately the Standard Model as the effective low–energy theory of BHL. This gives constraints on the running of the Standard Model top Yukawa coupling g_t, which determines the top mass. In the following we will discuss how these two approaches are related and we will see that the two methods give the same results.

2. Compositeness and the Renormalization Group

Using the auxiliary field formalism we can rewrite the four–fermion coupling term of eq. (1) with the help of a static, non propagating, scalar doublet $\varphi := -G\overline{t}_R L$ of mass G^{-1} such that the Lagrangian eq. (1) becomes [3]

$$\mathcal{L}_{aux} = \mathcal{L}_{kinetic} - \overline{L}\varphi t_R - \overline{t}_R\varphi^\dagger L - G^{-1}\varphi^\dagger\varphi \ . \tag{3}$$

The dynamics of the original model generates further terms in the Lagrangian which depend on this scalar field φ. For large cutoff Λ only renormalizable terms are allowed in the effective Lagrangian such that we obtain

$$\mathcal{L}_{eff} = \mathcal{L}_{aux} + Z_\varphi \left(D_\mu\varphi\right)^\dagger \left(D^\mu\varphi\right) + \delta M^2\varphi^\dagger\varphi - \frac{\delta\lambda}{2}\left(\varphi^\dagger\varphi\right)^2 - \delta g_t\left(\overline{L}\varphi t_R + \overline{t}_R\varphi^\dagger L\right) + \delta\mathcal{L}_{kinetic} \ . \tag{4}$$

Note that symmetry breaking occurs when $\delta M^2 > G^{-1}$ which is achieved for $G > G_{cr}$. Having the Wilson renormalization group approach in mind we can immediately read off those conditions which express the composite nature of the effective scalar Lagrangian $Z_\varphi \overset{p^2\to\Lambda^2}{\longrightarrow} 0$, $\delta M^2 \overset{p^2\to\Lambda^2}{\longrightarrow} 0$, and $\delta\lambda \overset{p^2\to\Lambda^2}{\longrightarrow} 0$. This expresses simply the fact that all dynamical effects must disappear as the momentum approaches the scale of new physics. Additionally we have the normalization conditions $\delta g_t \overset{p^2\to\Lambda^2}{\longrightarrow} 0$ and $\delta\mathcal{L}_{kinetic} \overset{p^2\to\Lambda^2}{\longrightarrow} 0$. We can now use the freedom to rescale the scalar field φ by defining $\phi := \frac{\varphi}{\sqrt{Z_\varphi}}$ such that the Lagrangian becomes[a]

$$\mathcal{L}_{eff} = \mathcal{L}_{kinetic} + \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) + \frac{\lambda v^2}{2}\phi^\dagger\phi - \frac{\lambda}{2}\left(\phi^\dagger\phi\right)^2 - g_t\left(\overline{L}\phi t_R + \overline{t}_R\phi^\dagger L\right) \ . \tag{5}$$

Here we introduced $\frac{\lambda v^2}{2} = \frac{\delta M^2 - G^{-1}}{Z_\varphi}$, $\lambda = \frac{\delta\lambda}{Z_\varphi^2}$ and $g_t = \frac{1+\delta g_t}{\sqrt{Z_\varphi}}$ and the effective Lagrangian has now become the Standard Model. From the definition of g_t, λ and v we see that the conditions of compositeness are

$$\lim_{p^2\to\Lambda^2} g_t^{-2}(p^2) = 0 \ , \quad \lim_{p^2\to\Lambda^2} \frac{\lambda(p^2)}{g_t^4(p^2)} = 0 \ , \quad \lim_{p^2\to\Lambda^2} \frac{\lambda(p^2)v^2(p^2)}{2g_t^2(p^2)} = -G^{-1} \ , \tag{6}$$

[a] We ignore the fermionic wave function contribution as it does not play any role for the constituent conditions.

where Λ corresponds to the high energy cutoff of the BHL–model.

These compositeness conditions must obviously be fulfilled in any sensible non–perturbative treatment of the dynamics like the bubble approximation. Especially for large Λ one may impose the compositeness conditions directly on the running parameters of the effective Lagrangian (the Standard Model). Conditions (6) lead thus to constraints on the renormalization group equations of the Standard Model . Using the full one–loop β–functions [4] these conditions can be studied numerically, but if we restrict ourself to lowest order $1/N_c$ including the QCD–running we can study the boundary conditions (6) analytically. The β–functions for $N_c = 3$ are in this approximation

$$\frac{d}{dt}g_t^2 = \frac{1}{(4\pi)^2}\left(6g_t^2 - 16g_3^2\right)g_t^2 \,, \tag{7}$$

$$\frac{d}{dt}g_3^2 = -\frac{14}{(4\pi)^2}g_3^4 \,, \tag{8}$$

and using eq. (6) as boundary condition we find the solutions

$$g_t^2(p^2) = \frac{g_3^{16/7}(p^2)}{3\left[g_3^{2/7}(p^2) - g_3^{2/7}(\Lambda^2)\right]} \,, \tag{9}$$

$$g_3^2(p^2) = \frac{g_3^2(M_Z^2)}{1 + \frac{14 g_3^2(M_Z^2)}{(4\pi)^2}t} \,, \tag{10}$$

where $t = \frac{1}{2}\ln\frac{p^2}{M_Z^2}$. Thus $m_t = g_t(M_Z^2)v/\sqrt{2}$ with $v = 246\ GeV$ is predicted by eq. (9) in terms of Λ and $g_3(\Lambda^2)$. In addition eq. (10) must be used to express $g_3^2(\Lambda^2)$ by Λ and the known experimental input $\alpha_3(M_Z^2) = g_3^2(M_Z^2)/4\pi = [0.115 - 0.125]$.

The predicted top mass is the so called "infrared quasi–fixed–point". This means that the resulting top mass depends for large Λ only extremely mildly on the precise boundary condition at Λ. This mild sensitivity can be seen explicitly by demanding $g_t^{-2} \stackrel{p^2\to\Lambda^2}{\longrightarrow} \delta$ with e.g. $|\delta| \leq \pi^{-1}$ instead of $g_t^{-2} \stackrel{p^2\to\Lambda^2}{\longrightarrow} 0$. Alternatively one can see that the prediction for the top mass depends only extremely weakly on Λ when $g_3^2(\Lambda^2)$ in eq. (9) is expressed by $\alpha_3(M_Z^2)$ via eq. (10).

The predictions from this renormalization group method should become more precise as Λ becomes larger. The reason is that the infrared quasi–fixed–point is more attractive for large scales and that other effects like thresholds etc. should become less important compared to the renormalization group running. When using this renormalization group techniques one should however keep in mind that one

assumes quietly that there are no further bound states in the spectrum.[b]

3. Predictions from Pagels–Stokar Formulae

The dynamically generated W–mass is determined by the eaten composite Goldstone modes. The Goldstone Boson decay constants are essentially given by the fermion–loop self energies of the condensing fermion while the other contributions are strongly suppressed. Using Ward Identities the W–mass can be calculated in terms of the momentum dependent mass function of the top quark $\Sigma_t(p^2)$ with Euclidean momentum p. This leads to the well known Pagels–Stokar formula [5,6]

$$M_W^2 = \frac{3g_2^2}{2(4\pi)^2}\int_0^{\Lambda^2} dp^2 \frac{\Sigma_t^2(p^2)}{p^2+\Sigma_t^2(p^2)} \ , \tag{11}$$

where $\Sigma_t(p^2)$ is the solution of the gap (Schwinger–Dyson) equation which contains the dynamics of the new interaction at the scale Λ .We define the top mass at an Euclidean scale M_Z by

$$m_t = \Sigma_t(M_Z^2) \ . \tag{12}$$

Now, interpreting the Standard Model as an effective description, we identify the masses of the top condensation model with their Standard Model definitions. For the W–mass we have $M_W = g_2 v/2$, the running of Σ_t should be connected to the corresponding renormalization group of the effective top–Yukawa coupling. A naive approach for the top mass function therefore leads to

$$\Sigma_t(p^2) = \frac{g_t(p^2)v}{\sqrt{2}} \ , \tag{13}$$

and we get from the following constituent condition from eq. (11):

$$1 = \frac{3}{(4\pi)^2}\int_0^{\Lambda^2} dp^2 \frac{g_t^2(p^2)}{p^2+g_t^2(p^2)\frac{v^2}{2}} \ . \tag{14}$$

For large Λ we can neglect the low energy behavior of $g_t(p^2)$ and replace Σ_t in the denominator by an infrared cutoff of the electro–weak scale:

$$\frac{3}{(4\pi)^2}\int_{M_Z^2}^{\Lambda^2} \frac{g_t^2(p^2)}{p^2} dp^2 = 1 \ . \tag{15}$$

If the running of g_t is fixed by the renormalization group then eq. (15) constitutes a top mass prediction since it can be used to determine the single free boundary

[b] Or equivalently that all other states have mass Λ such that they do not contribute to the running.

condition. As both the infrared quasi–fixed–point prediction eq. (9) and our new prediction eq. (15) should become more precise for large Λ one should expect that both results are consistent or at least approximately consistent. But inserting eqs. (9) and (10) into eq. (15) leads to

$$\frac{3}{(4\pi)^2}\int_{M_Z^2}^{\Lambda^2}\frac{g_t^2(p^2)}{p^2}dp^2 = \frac{3}{7}\int_{g_3^2(\Lambda^2)}^{g_3^2(M_Z^2)} g_t^2\frac{dg_3^2}{g_3^4} \to \infty\ , \tag{16}$$

i.e. this consistency of the two predictions appears badly violated. The solution to this problem can be found in the scale dependence of Σ_t. The renormalization group equation describes the change of the couplings corresponding to a simultaneous rescaling of all the outer momenta. But the contribution to g_t which stems from the Higgs line cannot contribute to Σ_t because there is no momentum flow into the Higgs leg which ends in the vacuum. To describe this situation correctly we introduce a new running coupling constant $\tilde{g}_t$ which is defined by

$$\Sigma_t = \tilde{g}_t\frac{v}{\sqrt{2}}\ , \tag{17}$$

and does not contain wave function corrections of the Higgs (see fig. 1). The

Fig. 1. Quantum corrections to g_t and $\tilde{g}_t$.

connection between g_t and $\tilde{g}_t$ can be seen more clearly if we keep in mind that g_t is a three point function with two independent momenta. Let $\mathrm{g}_t(p^2, q^2)$ be the Yukawa coupling for parallel[c]incoming top momentum p and Higgs momentum q.Then g_t and $\tilde{g}_t$ describe the renormalization group running with respect to different momentum arguments:

$$g_t(p^2) = \mathrm{g}_t(p^2, p^2)\ , \tag{18}$$
$$\tilde{g}_t(p^2) = \mathrm{g}_t(p^2, M_Z^2)\ . \tag{19}$$

This naturally implies $\tilde{g}_t(M_Z^2) = g_t(M_Z^2)$ such that the wave function of the Higgs field is normalized to one at the scale of the Higgs mass.

[c]Angles between p and q will not be important for our purposes.

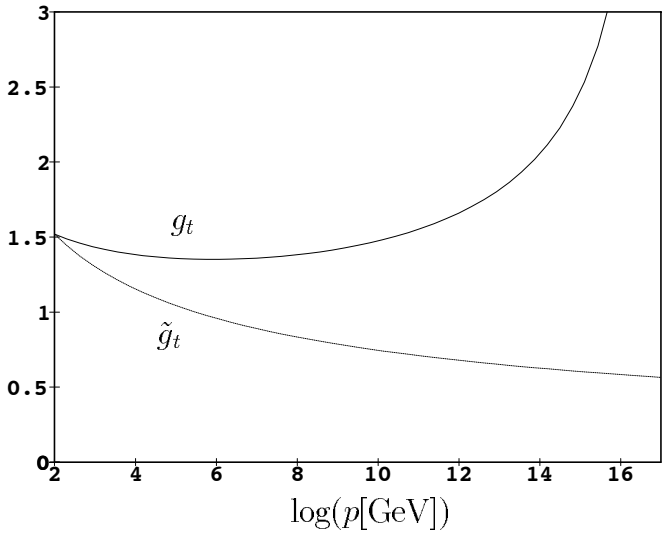

Fig. 2. g_t and $\tilde{g}_t$ for $g_3^2(M_Z^2) = 1.52$ and $\Lambda = 10^{17}$GeV.

The renormalization group equation describes the running of g_t. Eq. (7) may in fact be derived by calculating

$$\frac{d}{dt} g_t^2 = \sigma \frac{d}{d\sigma}\ \mathrm{g}_t^2((\sigma p)^2, (\sigma p)^2) \tag{20}$$

from the relevant diagrams. Similarly we may formally define a corresponding renormalization group equation for the newly defined quantity $\tilde{g}_t$ and calculate the corresponding β–function in large N_c–limit

$$\frac{d}{dt} \tilde{g}_t^2 = \sigma \frac{d}{d\sigma}\ \tilde{\mathrm{g}}_t^2((\sigma p)^2, M_Z^2) = \frac{1}{(4\pi)^2} \left(-16 g_3^2\right) \tilde{g}_t^2\ . \tag{21}$$

We solve eq. (21) using the boundary condition $\tilde{g}_t(M_Z^2) = g_t(M_Z^2)$ with $g_t(M_Z^2)$ defined by eq. (9) and obtain

$$\tilde{g}_t^2(p^2) \ = \ \frac{g_3^{16/7}(p^2)}{3\left[g_3^{2/7}(M_Z^2) - g_3^{2/7}(\Lambda^2)\right]}\ . \tag{22}$$

If we insert this result instead of g_t into the left hand side of eq. (15) we find

$$\frac{3}{(4\pi)^2} \int\limits_{M_Z^2}^{\Lambda^2} \frac{\tilde{g}_t^2(p^2)}{p^2} dp^2 = \frac{3}{7} \int\limits_{g_3^2(\Lambda^2)}^{g_3^2(M_Z^2)} \tilde{g}_t^2 \frac{dg_3^2}{g_3^4} = 1\ . \tag{23}$$

Hence the Pagels–Stokar condition is fulfilled if we use correctly $\tilde{g}_t$ instead of g_t. Eq. (15) must be written therefore correctly as

$$\frac{3}{(4\pi)^2} \int\limits_{M_Z^2}^{\Lambda^2} \frac{\tilde{g}_t^2(p^2)}{p^2} dp^2 = 1\ , \tag{24}$$

and the Pagels–Stokar formula is then in perfect agreement with the corresponding renormalization group running.

There is a nice way to see this consistency of eq. (24) without explicitly inserting eq. (22). Consider the evolution of $\tilde{g}_t^2/g_t^2$:

$$\frac{d}{dt}\frac{\tilde{g}_t^2}{g_t^2} = \left(\frac{\frac{d}{dt}\tilde{g}_t^2}{\tilde{g}_t^2} - \frac{\frac{d}{dt}g_t^2}{g_t^2}\right)\frac{\tilde{g}_t^2}{g_t^2} \ . \tag{25}$$

If we insert then eq. (7) and eq. (21) into eq. (25), we obtain immediately

$$\frac{d}{dt}\frac{\tilde{g}_t^2}{g_t^2} = -\frac{6}{(4\pi)^2}\tilde{g}_t^2 \ . \tag{26}$$

According to fig. (1) this equation involves only the wave function correction of the Higgs sector and at one loop level this is just the single diagram in fig. (3).

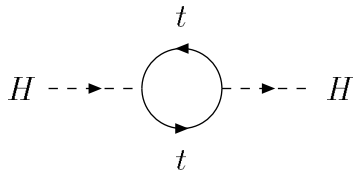

Fig. 3. Higgs self energy diagram.

Integrating eq. (26) leads to:

$$\left.\frac{\tilde{g}_t^2}{g_t^2}\right|_{M_Z^2}^{\Lambda^2} = -\frac{3}{(4\pi)^2}\int_{M_Z^2}^{\Lambda^2}\tilde{g}_t^2\frac{dp^2}{p^2} \ . \tag{27}$$

Since the compositeness condition requires a pole for g_t while eq. (21) implies that $\tilde{g}_t$ tends to zero we find that $\tilde{g}_t^2/g_t^2$ goes to zero at Λ. Together with the normalization condition $\tilde{g}_t(M_Z) = g_t(M_Z)$ one obtains therefore exactly the result eq. (23) without making use of explicit solutions of the renormalization group equations. In this way we can even add other terms or new interactions which contribute to g_t and $\tilde{g}_t$ but do not change the Higgs wave function.

It is interesting that the W–mass generated by a single top–loop is consistent with the renormalization group method using only the graph in fig. (3). This can be understood by considering that the W–mass is connected to the Goldstone Boson wave function by gauge symmetry. Since custodial $SU(2)$ symmetry is just broken by finite terms, the logarithmically divergent pieces of the Goldstone Boson wave function and the Higgs wave function must be identical. Only these logarithmically divergent terms contribute to the renormalization group equation.

4. The Quartic Higgs Coupling

The techniques of the last sections can also be applied to the Higgs mass and the quartic Higgs coupling λ. The renormalization group equation for λ is in large N_c

approximation

$$\frac{d}{dt}\lambda = \frac{12}{(4\pi)^2} g_t^2 (\lambda - g_t^2) \ . \tag{28}$$

Note that the term proportional to λ^2 is missing because it is suppressed by $1/N_c$. Using the compositeness conditions eq. (6) and the above result for g_t the running λ is

$$\lambda(p^2) = \frac{2\left[g_3^{18/7}(p^2) - g_3^{18/7}(\Lambda^2)\right]}{27\left[g_3^{2/7}(p^2) - g_3^{2/7}(\Lambda^2)\right]^2} \ . \tag{29}$$

As in the case of the top Yukawa coupling we define a coupling $\tilde{\lambda}$ which does not contain any Higgs wave function corrections (see fig. (4)):

$$\tilde{\lambda} = \lambda \frac{\tilde{g}_t^4}{g_t^4} \ , \tag{30}$$

so that the renormalization group equation for $\tilde{\lambda}$ reads

$$\frac{d}{dt}\tilde{\lambda} = -\frac{12}{(4\pi)^2}\tilde{g}_t^4 \ . \tag{31}$$

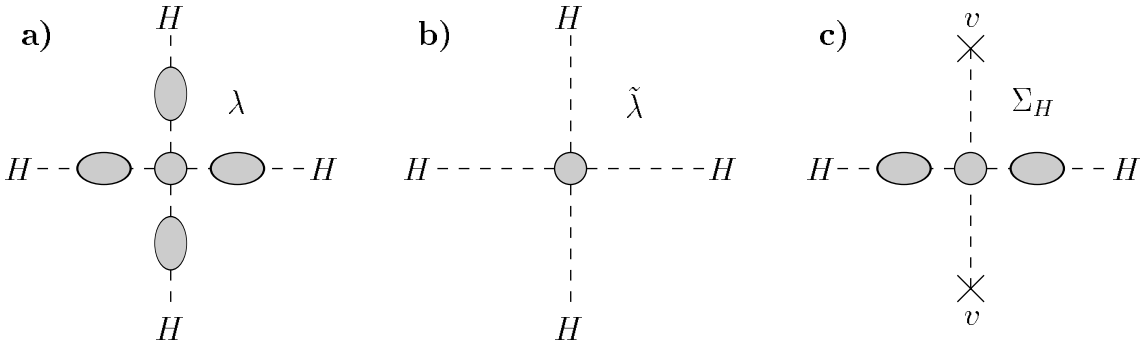

Fig. 4. Quantum corrections to λ, $\tilde{\lambda}$ and Σ_H.

We find therefore

$$\tilde{\lambda}(M_Z^2) = -\lambda \frac{\tilde{g}_t^4}{g_t^4}\bigg|_{M_Z^2}^{\Lambda^2} = \int_{M_Z^2}^{\Lambda^2} \frac{d}{dt}(-\tilde{\lambda})\frac{dp^2}{2p^2} = \frac{6}{(4\pi)^2}\int_{M_Z^2}^{\Lambda^2} \tilde{g}_t^4 \frac{dp^2}{p^2} \ . \tag{32}$$

Now we have to introduce the Standard Model Higgs mass definition. The Higgs mass function $\Sigma_H(p^2)$ which corresponds to fig. (4c) involves twice the coupling g_t and twice $\tilde{g}_t$. Expressed in λ this means:

$$\Sigma_H^2(p^2) = \lambda \frac{\tilde{g}_t^2(p^2)}{g_t^2(p^2)} v^2 \ . \tag{33}$$

With our normalization condition $g_t(M_Z^2) = \tilde{g}_t(M_Z^2)$ insertion of this mass function into eq. (32) leads to:

$$M_H^2 = \Sigma_H^2(M_Z^2) = \frac{12}{(4\pi)^2} \int\limits_{M_Z^2}^{\Lambda^2} \tilde{g}_t^2 \Sigma_t^2(p^2) \frac{dp^2}{p^2} \ . \tag{34}$$

This is a corresponding Pagels–Stokar formula for M_H which was also found by V. N. Gribov [7] for a specific case. One can directly evaluate this formula in analogy to the Pagels–Stokar calculations of the W–mass using the top–loop diagram with four outer Higgs lines. The outer momenta are kept at the infrared cutoff so that the Yukawa couplings can be replaced by the running $\tilde{g}_t$ to improve the diagram. In this way one arrives at eq. (34).

We have demonstrated that the renormalization group equations lead to the same result for M_W and M_H as the direct calculation via Pagels–Stokar formulae. Moreover the behavior of λ and g_t at the scale Λ shows the real power of the renormalization group analysis for top condensation models in the following calculation. The ratio of the running Higgs and top mass squared is defined by

$$\frac{\Sigma_H^2(p^2)}{\Sigma_t^2(p^2)} = \frac{\lambda(p^2) \frac{\tilde{g}_t^2(p^2)}{g_t^2(p^2)} v^2}{\tilde{g}_t^2(p^2) \frac{v^2}{2}} = \frac{2\lambda(p^2)}{g_t^2(p^2)} = \frac{4\left[g_3^{18/7}(p^2) - g_3^{18/7}(\Lambda^2)\right]}{9 g_3^{16/7}(p^2)\left[g_3^{2/7}(p^2) - g_3^{2/7}(\Lambda^2)\right]} \ . \tag{35}$$

One can easily find the square root of this ratio in the limit $p^2 \to \Lambda^2$:

$$\lim_{p^2 \to \Lambda^2} \frac{\Sigma_H(p^2)}{\Sigma_t(p^2)} = 2 \ , \tag{36}$$

which is precisely what we expect: The binding energy goes to zero at the condensation scale so that the bound states do not exist above Λ. This result requires the $1/N_c$–expansion and does not hold in the one–loop Standard Model, which does not respect that expansion.

5. Conclusions

We exemplified in this paper for the BHL model connections between two techniques which predict mass ratios m_t/M_W and M_H/M_W in composite Higgs models with an effective scalar sector. Since the techniques which are applied in this paper are rather general we believe that our results are much more generally valid in composite Higgs models with effective Yukawa and Higgs couplings. For a large scale of new physics Λ both predictions which arise from infrared quasi–fixed–points of the renormalization group running of the effective Yukawa coupling and from using Pagels–Stokar relations which are based on Ward Identities should become rather precise. This leads to the question if these two methods are related or if one of the

methods fails. The insertion of the running Standard Model top Yukawa coupling g_t into the Pagels–Stokar formula leads somewhat surprisingly to an infinite W–mass. This apparent contradiction between the two approaches comes from the fact that the top mass function Σ_t of the Standard Model is not controlled by the running coupling g_t, but by a modified running coupling $\tilde{g}_t$. Taking this into account we show at one loop that the Pagels–Stokar relation and the renormalization group method are equivalent. This equivalence was then used to generally construct an "analogous Pagels–Stokar formula" for the Higgs mass by imposing corresponding boundary conditions on the renormalization group flow of the quartic Higgs coupling λ. This relation was also obtained recently by Gribov [7] in a specific model. Additionally we find the expected mass relation $\Sigma_H = 2\Sigma_t$ at the compositeness scale Λ.

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