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Top Condensation: The Idea and its Phenomenological Viability*

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Abstract

We discuss Dynamical Symmetry Breaking via top condensation models and argue that phenomenologically viable scenarios are possible. First model independent symmetry and order of magnitude arguments in favor of such ideas are given. Then models which realize these features are listed with comments on their advantages and problems. Finally it is argued that scenarios with a scale of new physics $\Lambda = \mathcal{O}(TeV)$ may produce a low enough top mass and may be consistent with available electro–weak precision data.

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1 Introduction

The last five years of particle physics resulted in a spectacular confirmation of the Standard Model including by now even genuine electro–weak radiative corrections. This may be viewed as the success story of renormalizable gauge theories starting from QED to non–abelian QCD based on $SU(3)_c$ and cumulating in the full Standard Model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The Standard Model has however the property that the ground state of the theory does not respect the symmetry of the Lagrangian. A technically consistent way to realize this symmetry breaking is given by the Higgs mechanism which requires to introduce a fundamental scalar doublet into the theory. There is however so far no evidence for the existence of such a scalar Higgs particle. Moreover there are the well known theoretical arguments against the Standard Model Higgs sector, most important the hierarchy problem. Thus it is widely believed that despite its success the symmetry breaking sector of the Standard Model cannot be the final word.

There appear two ways to avoid the problems of the scalar sector: First scalars can become "natural" as superpartners of supersymmetric (SUSY) extensions of the Standard Model. This very attractive possibility solves the hierarchy problem and leads even to models which nicely fit into a GUT picture. The second possibility is that fundamental scalars do not exist at all. In this case the Higgs mechanism is just an effective description of some dynamical symmetry breaking (DSB) mechanism. A nice analogy is the Ginsburg–Landau formulation of superconductivity, where the Meissner effect is nicely understood via an effective photon mass inside the superconductor. DSB is an attractive possibility since it is a mechanism which arises automatically in field theories if the corresponding rearrangement of the vacuum leads to an energetically favored situation. This is well known in solid state physics and it would be very natural if it explained the breaking of the electro–weak gauge group in particle physics. If the Higgs mechanism is really such an effective description of electro–weak symmetry breaking like the Ginsburg–Landau description of superconductivity, then a microscopic model leading to DSB in analogy to the BCS theory must exist. There are in principle many possibilities and similar to superconductivity it is very likely that the correct mechanism is only recognized with the help of experimental input. We will argue that models which lead to the formation of a top quark condensate have systematically nice features and are good good candidates for DSB models of electro–weak symmetry breaking.

Note that a gauge invariant Higgs mechanism does not imply the existence of a scalar Higgs particle. I.e. composite Goldstone Bosons required for a gauge invariant Higgs mechanism arise from the broken global symmetries and the remaining composite spectrum depends strongly on the dynamics involved. But if the resulting effective model has to be valid over large scale ranges then at least one extra composite scalar (Higgs) or vector (rho) particle must show up to unitarize otherwise unboundedly growing Goldstone Boson scattering amplitudes. The type of particle which shows up to unitarize amplitudes is a dynamical property of the DSB model under consideration. Technicolor (TC) models [1], for example, are constructed in analogy to QCD and contain a "Techni–rho". The BHL model [2] is an example where only a single composite Higgs scalar shows up. This is an important difference since it is often said these days that DSB models are inconsistent with precision data since TC models typically give deviations from the Standard Model at the percent level. One has to realize that this is only an argument against models with relatively light (composite or fundamental) vector–like states. In DSB models with composite Higgses, however, one typically gets deviations from the Standard Model at an acceptable permille level like in non–composite models with scalars. Thus the smallness of deviations from the Standard Model is an argument for the absence of vector like states and not for or against DSB. Since most models of top condensation systematically lead to composite Higgses we think that these models are not endangered by to large deviations from the Standard Model, even though it is unfortunately very hard or impossible to calculate radiative corrections precise enough.

2 Model Independent Features of Top Condensation

Before we consider explicit models of top condensation we would like to discuss the expected model independent features following from considerations about symmetry, order of magnitude and Flavor Changing Neutral Currents (FCNC). Due to these features we think that there are good reasons to hope that some top condensation model may be relevant in nature. These model independent features are:

i) Global symmetries: If the scalar field of the Standard Model is discarded then there are only kinetic terms of known fermions and gauge bosons left. The remaining Lagrangian is however phenomenologically unacceptable since the electro–weak symmetry is unbroken. If we call the left–handed doublet of top and bottom $L = (t_L, b_L)$ and the right–handed top quark singlet t_R then the remaining Lagrangian possesses a global $U(2)_L \times U(1)_{tR}$ symmetry. To trigger electro–weak symmetry breaking via top condensation we must add some new attractive force to the Lagrangian which we choose to respect the original global symmetry. If this force is strong enough in order to produce a top condensate $\langle 0| \overline{L} t_R |0\rangle = (c_t \Lambda^2, 0)$ with $c_t \neq 0$ then the global symmetry is broken to $U(1)_{tV} \times U(1)_{bL}$. Three Goldstone Bosons corresponding to the three broken generators emerge as a consequence of the Goldstone theorem. The Goldstone Bosons have the correct quantum numbers to be "eaten" by the $SU(2)_L$ gauge bosons such as to give mass. The relation of the Goldstone Boson decay constants $F_\pm$ and F_0 to the weak boson masses is as usual given by

$$m_W^2 = g_2^2 F_\pm^2 \; ; \quad m_Z^2 = (g_1^2 + g_2^2) F_0^2 \; . \tag{1}$$

Note that this dynamical Higgs mechanism is merely a consequence of the global symmetries and the assumed top condensate. Thus many forces which respect the above symmetries and which produce the desired top condensate should work.

ii) Order of magnitude: If the symmetry breaking is triggered by a fermionic condensate, then the Goldstone Boson decay constants $F_\pm$ and F_0 must be homogeneous functions of the dynamically generated fermion mass(es) $\Sigma(k^2)$. This follows simply from the previous symmetry considerations which require that the Goldstone Bosons must disappear in the limit where the condensate vanishes. Such homogeneous relations can be seen explicitly in the Pagels–Stokar [3] formulas arising from the transition of Goldstone Bosons to the $SU(2)_L$ gauge bosons via a fermion loops. Equivalent results are obtained by looking at the difference between broken and unbroken phase of the gauge boson – gauge boson transition [4] (see also Section 4.).

For the charged Goldstone Boson decay constant one obtains for example for the contribution of a single fermion

$$F_{\pm}^2 = \frac{N_c}{32\pi^2} \int k^2 dk^2 \frac{\Sigma^2(k^2)}{\left(k^2 + \Sigma^2(k^2)\right) k^2} \; . \tag{2}$$

As expected $F_{\pm}$ goes to zero when the dynamical fermion mass function Σ vanishes. When $\Sigma(k^2)$ behaves smoothly then one finds furthermore that m_t and F_i are of the same order of magnitude. This can be seen by inserting the test–function

$$\Sigma_t(p^2) = m_t \, \Theta(\Lambda^2 - p^2) \; , \tag{3}$$

for which one obtains

$$F_{\pm}^2 \simeq \frac{N_c m_t^2}{32\pi^2} \ln\left(\frac{\Lambda^2}{m_t^2}\right) \; . \tag{4}$$

The Goldstone Boson decay constants F_i are therefore naturally linked to the magnitude of the dynamically created fermion mass(es), i.e. $F_i = \mathcal{O}(m_t)$. A nice example where the above argument can be used is QCD. With appropriately modified coefficients similar relations arise and explain why the dynamically generated quark masses (i.e. the difference between current and constituent quark masses) are naturally linked to the order of the magnitude of the pion decay constant f_π

$$f_\pi = \mathcal{O}(\Delta m_f) = \mathcal{O}(\Lambda_{QCD}) \; . \tag{5}$$

In practise this order of magnitude statements are often rather insensitive to details of the underlying force and are mostly a consequence of the smoothness of Σ or the absence of widely different mass scales.

In the electro–weak interactions the above order of magnitude statements fit nicely to the experimental fact that there is a heavy top mass of the order of the electro–weak Goldstone Boson decay constant $F_i \simeq 123 \; GeV$. This suggest that the electro–weak symmetry could be broken dynamically by a top condensate such that the electro–weak Goldstone Bosons are composite objects made of $\bar{t} - t$. To achieve this we just have to drop the Higgs scalar and invent new forces which trigger top condensation. The relation $m_t = \mathcal{O}(v)$ becomes then a natural consequence together with three Goldstone Bosons from global symmetry considerations. This agrees nicely with phenomenological facts while on the other side more condensates would typically lead to more (unwanted) Goldstone Bosons and would also require more (unobserved) heavy fermions. Thus one single heavy top quark seems ideal from order of magnitude and symmetry considerations.

iii) FCNC: The absence of Flavor Changing Neutral Currents (FCNC) in the limit where light fermions are mass-less is a further advantage of most top condensation models. This must be compared with the difficulties of other approaches. E.g. extended (horizontal) interactions must be added to Technicolor models (Extended Technicolor=ETC) in order to produce ordinary fermion masses. There is initially no top mass and the quark masses are then typically given by

$$m_{ij} \sim g_{ETC}^2 \frac{\left\langle \overline{Q}Q \right\rangle_{TC}}{\Lambda_{ETC}^2} \overset{\text{typ.}}{\simeq} g_{ETC}^2 \frac{\Lambda_{TC}^3}{\Lambda_{ETC}^2} \ . \tag{6}$$

The diagonalization of such a quark mass matrix does however not diagonalize the induced four–fermion operators in flavor space. This leads to experimentally unobserved FCNC's and $\Delta S = 2$ interactions which produce a too large $K_L - K_S$ mass difference. To be compatible with experimental limits it becomes necessary to choose $\Lambda_{ETC} > 500\ TeV$ [5]. Such a big value of Λ_{ETC} in eq. (6) does however imply very tiny quark masses unless the techniquark condensate $\langle 0| \overline{Q}Q |0\rangle_{TC}$ is much bigger than its natural value, Λ_{TC}^3. Such an enhancement of the condensate is achieved in "walking technicolor" ideas [6] in order to cope with the phenomenologically required value of Λ_{ETC}. Nevertheless even then it seems hardly possible to generate radiatively a heavy top mass $\mathcal{O}(v)$ [7].

In contrast it is easily seen that top condensation models based on new flavor diagonal interactions amongst the known fermions can produce *naturally* $m_t = \mathcal{O}(v)$ without FCNC problems. The main reason is that the heavy top mass and the gauge boson masses are produced together by the symmetry breaking. Thus there is no need to generate a large top mass radiatively which can easily generate large FCNC's [8]. If we consider the top quark to be the only massive fermion at this level then FCNC's are exactly absent since the mass matrix is diagonal with a (3,3) entry only. The explanation of light quark and lepton masses would require further interactions, probably at even higher scales, which should also be helpful to explain fermion mass hierarchies. Such extra interactions would feed the top condensate via see–saw diagrams into other places of the mass matrices such that all masses and a non–trivial KM matrix may arise. The resulting mass matrix must be re–diagonalized and small, acceptable FCNC's may arise.

These model independent features work very generally for top condensation models. Thus these models produce systematically phenomenological desirable features.

3 A Partial List of Models

First models of top condensation assumed simple attractive Nambu–Jona-Lasinio (NJL) type four–fermion interactions. The well known BHL model [2] is a nice example where the above model independent features are realized in a very economic way. Unfortunately the BHL model is theoretically not fully satisfactory and predicts a too high top mass. Nevertheless the predicted top mass is of the correct order of magnitude. The BHL scenario was quickly generalized to extensions of the Standard Model like the two Higgs model [9], models with more generations [10] and supersymmetric extensions [11].

The NJL type models are unsatisfactory for a number of reasons. One problem is that these models are non–renormalizable and do not make much sense from a fundamental point of view. Consequently there were attempts to generate the four–fermion interactions from the exchange of heavy bosons. Among the first models of this type were so called "Topcolor" models [12] where $SU(3)_c$ is a subgroup of a broken $SU(3)_1 \times SU(3)_2$ gauge group. Extensions of these Topcolor models were systematically discussed [13] later. Another direction of extended gauge groups was to embed $U(1)_Y$ into a $U(1)_1 \times U(1)_2$ gauge group [14]. Further examples are embeddings of the Standard Model into a $SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_V$ gauge group [15] or embeddings into even larger groups [16]. More attempts can be found in ref. [17].

All these models have good and weak points. It is probably fair to say that none of these models appears to be a final scenario. In other words the model independent features and the idea are still better than all the existing realizations. But this may just mean that some essential ingredient is still missing. It might easily be that there exist further fields in nature (like e.g. Majorana neutrinos, extra fermions, gauge fields or "unusual" representations) which have to be included before one can arrive at a satisfying theory.

4 A Scale of New Physics $\Lambda = \mathcal{O}(TeV)$

One important aspect is that eq. (4) together with $F_i \simeq 123\ GeV$ and $m_t \simeq 175\ GeV$ seems to require very large values of Λ. On the theoretical side one would however like to keep Λ of the order of several TeV in order to solve the hierarchy problem. In that case eq. (4) would predict a top mass which si to large by about a factor 2. Therefore one may ask if it is possible to have small Λ and an acceptable top mass prediction simultaneously. It is now shown that this is possible.

Assume the formation of a $\bar{t}t$ condensate is driven by some "pairing force" responsible for a dynamical breaking of the electro–weak symmetry. We can study this problem by assuming to know the solution $\Sigma_t(p^2)$ of the relevant Schwinger–Dyson ("gap") equation for the dynamically generated top mass[4]. Thus we pretend to know the electro–weak symmetry breaking top propagator to be

$$S_t(\mathrm{p}^2) = \frac{i}{\not{p} - \Sigma_t(\mathrm{p}^2)} \ , \tag{7}$$

with the (pole) top mass $m_t = \Sigma_t(m_t^2)$. All other quarks and leptons are assumed to be massless at this level. Without specifying the gap equation we assume furthermore that for the theory under consideration[†] $\Sigma_t(p^2) \stackrel{p^2\to\infty}{\longrightarrow} 0$ and that there is only one unique solution for m_t.

The breaking of the electro–weak symmetry (i.e. $\Sigma_t \neq 0$) is assumed to be the result of unspecified new strong forces acting only on the known quarks and leptons and especially on the $t-b$ doublet. The emergence of the top condensate breaks the global symmetries as discussed and the resulting Goldstone Bosons are "eaten" in a dynamical Higgs mechanism such that W and Z become massive. Presumably such a theory does not change significantly if the weak $U(1)_Y$ coupling g_1 is sent to zero[‡]. In the limit $g_1 = 0$ the corrections which

[†]This is e.g. justified for asymptotically free theories where chiral symmetry breaking disappears as $p^2 \to \infty$.

[‡]Indirectly (via vacuum alignment) a small $U(1)_Y$ coupling could be very important such that $g_1 = 0$ should be understood as the result of the limiting procedure $g_1 \to 0$.

give mass to the W_3 and $W_\pm$ propagators must be induced by those fermions which are representations under both $SU(2)_L$ and the new strong force. We should therefore study the contributions of Σ_t to the vacuum polarizations of the W and Z propagators. In an expansion in powers of g_2^2 the leading contribution is given by diagrams which connect the $W_\pm$ or W_3 line to a fermion pair from both sides. There are two ways [4] how the four fermion lines can be connected: By inserting twice the full fermionic propagators or by inserting once the full four–fermion Kernel of the new, strong interaction. Note that in leading order g_2^2 the fermion propagators and the Kernel do not contain any electro–weak gauge boson propagation themself since this would cost at least an extra power of g_2^2. Insertions of fermionic vacuum polarizations into higher order electro–weak loop diagrams, for example, are suppressed by corresponding powers of g_2^2. Thus in leading order g_2^2, but exact in the new strong coupling, the W propagator is corrected by two types of diagrams. The first contribution is a generalization of the leading Standard Model fermion loop with hard fermion masses replaced by the dynamically generated Σ's, i.e. the sum of all one particle irreducible diagrams which generate the fermion masses. The second contribution conects the four fermion lines via the exact four–fermion Kernel K of the strong forces responsible for condensation. This Kernel is connected via a fermion loop (with full propagators) on both sides to the external W–lines. It is useless to expand the four–fermion Kernel perturbatively in powers of the coupling constant of the new strong force. The Goldstone theorem tells us however that the Kernel must contain poles of massless Goldstone Bosons due to the breaking of global symmetries by the fermionic condensates. This can be expressed by writing

$$K = P \cdot \frac{i}{q^2} \cdot P + \tilde{K} \ , \tag{8}$$

where P is a function describing the coupling of Goldstone Bosons with the propagator $\frac{i}{q^2}$ to fermion pairs and where $\tilde{K}$ is the part of the Kernel which does not contain any further massless poles. But $\tilde{K}$ may (and typically will) contain all sort of massive bound states which could e.g. be vectors, Higgs–like scalars etc. in all possible channels.

The Goldstone Boson contributions[§] of eq. (8) were used by Pagels and Stokar [3] to obtain a relation between the Σ's and the Goldstone Boson decay constants. Their derivation uses the fact that only the Goldstone Bosons contribute a term proportional $p_\mu p_\nu/p^2$ to the W polarization at vanishing external momentum, but this method ignores possible contributions from $\tilde{K}$ which enter indirectly via the use of Ward identities. The $p_\mu p_\nu/p^2$ contributions to $\Pi_{\mu\nu}$ are balanced (up to small corrections from $\tilde{K}$) by $g_{\mu\nu}$ terms created by pure fermionic loop mentioned first. Following ref. [4] we derive now relation (2) from these $g_{\mu\nu}$ terms. The result can be compared with the Pagels–Stokar relation and we will see that contributions from $\tilde{K}$ are significantly suppressed.

Let us therefore work with rescaled fields such that gauge couplings appear in the kinetic terms of the gauge boson Lagrangian like $(-1/4g^2)\,(W_{\mu\nu})^2$. Since we do not include any propagating W bosons we need not gauge fix at this stage. The inverse W propagator is written as

$$\frac{1}{g_2^2} D^{-1}_{W,\mu\nu}(p^2) = \frac{1}{g_2^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\, p^2 - \Pi_{\mu\nu}(p^2) \ , \tag{9}$$

[§] Which are essential for a gauge invariant dynamical Higgs mechanism.

with the polarization tensor $\Pi_{\mu\nu}(p^2) = (-g_{\mu\nu}p^2 + p_\mu p_\nu)\,\Pi(p^2)$. At vanishing external momentum the pure fermion loop contributes to $\Pi_{\mu\nu}$

$$\Pi_{\mu\nu} = -iZ^2 N_c \int \frac{d^4k}{(2\pi)^4} \, \frac{Tr\,[\Gamma_\mu(\not{k} + \Sigma_1(k))\Gamma_\nu(\not{k} + \Sigma_2(k))]}{(k^2 - \Sigma_1(k)^2)(k^2 - \Sigma_2(k)^2)} \;, \tag{10}$$

where N_c is the number of colors, $Z^{-1} = \sqrt{2}, 2$ in the charged and neutral channel, respectively, $\Gamma_\alpha = \left(\frac{1-\gamma_5}{2}\right)\gamma_\alpha$, and $+\mathbf{i}\epsilon$ is generally implied in the denominator. In the neutral channel we get corrections from $\bar{t}t$ (i.e. $\Sigma_1 = \Sigma_2 = \Sigma_t$), $\bar{b}b$ (i.e. $\Sigma_1 = \Sigma_2 = \Sigma_b \equiv 0$) and in the charged channel contributes only $\bar{t}b$ or $\bar{b}t$ (i.e. $\Sigma_1 = \Sigma_t$, $\Sigma_2 = \Sigma_b \equiv 0$). By naive power counting eq. (10) has quadratic and logarithmic divergences. Since we assume $\Sigma_i(p^2) \overset{p^2\to\infty}{\longrightarrow} 0$ for the top quark and all other fermions we find that the divergences of $\Pi_{\mu\nu}(p^2)$ are identical to those calculated for $\Sigma_i \equiv 0$. It makes therefore sense to split $\Pi_{\mu\nu}(p^2) = \Pi^0_{\mu\nu}(p^2) + \Delta\Pi_{\mu\nu}(p^2)$ where $\Pi^0_{\mu\nu}$ is defined as $\Pi_{\mu\nu}$ for $\Sigma_i \equiv 0$. $\Pi^0_{\mu\nu}$ is then an uninteresting Σ_i independent constant which contains all divergences and needs renormalization. Contrary the interesting Σ_i dependent piece $\Delta\Pi_{\mu\nu} = \Pi_{\mu\nu} - \Pi^0_{\mu\nu}$ is finite, even when the external momentum is sent to zero. Thus

$$\begin{aligned}
\Delta\Pi_{\mu\nu} &= -iZ^2 N_c \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{Tr\,[\Gamma_\mu(\not{k} + \Sigma_1)\Gamma_\nu(\not{k} + \Sigma_2)]}{(k^2 - \Sigma_1^2)(k^2 - \Sigma_2^2)} - \frac{Tr\,[\Gamma_\mu \not{k} \Gamma_\nu \not{k}]}{k^4} \right\} &(11)\\
&= -g_{\mu\nu} \frac{Z^2 N_c}{(4\pi)^2} \int_0^\infty dk^2 \; \frac{k^2(\Sigma_1^2 + \Sigma_2^2) - \Sigma_1^2\Sigma_2^2}{(k^2 - \Sigma_1^2)(k^2 - \Sigma_2^2)} \;, &(12)
\end{aligned}$$

where angular integration was performed in Euclidean space and subsequently continued back to Minkowski space. Under the integral one has as usual $Tr\,[\Gamma_\mu \not{k} \Gamma_\nu \not{k}] = -g_{\mu\nu}k^2$ and $Tr\,[\Gamma_\mu\Gamma_\nu] = 0$. Note that this separation procedure for $\Delta\Pi_{\mu\nu}$ does not spoil gauge invariance.

The Goldstone Boson decay constants F_i^2 are the poles of $\Pi(p^2)$ at vanishing external momentum. For our definition of $\Pi_{\mu\nu}$ we find that F_i^2 is identical to eq. (12) without the factor $-g_{\mu\nu}$. Using the above values of Z for the charged and neutral channel one finds

$$F_\pm^2 = \frac{N_c}{32\pi^2} \int_0^\infty dk^2 \; \frac{\Sigma_t^2}{k^2 - \Sigma_t^2} \;, \quad F_3^2 = \frac{N_c}{32\pi^2} \int_0^\infty dk^2 \; \frac{k^2\Sigma_t^2 - \frac{1}{2}\Sigma_t^4}{(k^2 - \Sigma_t^2)^2} \;, \tag{13}$$

such that

$$F_3^2 - F_\pm^2 = \frac{N_c}{64\pi^2} \int_0^\infty dk^2 \; \frac{\Sigma_t^4}{(k^2 - \Sigma_t^2)^2} \;. \tag{14}$$

Eq. (13) for $F_\pm^2$ is equivalent to the result obtained by Pagels and Stokar [3] from the $q_\mu q_\nu / q^2$ contributions of Goldstone Bosons to $\Pi_{\mu\nu}$. The result for the neutral channel in eq. (13) looks however somewhat different. But by using the integral identity

$$\int_0^\infty dx \; \frac{x^2 f(x)' - f(x)^2}{\left(x - f(x)\right)^2} = f(\infty) \;, \tag{15}$$

for $x = k^2$ and $f = \Sigma_t^2$ we can rewrite eq. (13) into

$$F_3^2 = \frac{N_c}{32\pi^2} \int_0^\infty dk^2 \; k^2 \; \frac{\Sigma_t^2 - k^2 \Sigma_t \Sigma_t'}{(k^2 - \Sigma_t^2)^2} \;, \tag{16}$$

where $\Sigma_t' = d\Sigma_t/dk^2$. Even though this looks now formally like the Pagels–Stokar result it differs by a factor 2 in front of the derivative term in the nominator of eq. (16). This difference may appear less important, but we will see that in the limit of a hard top mass our method produces the correct ρ–parameter, while the Pagels–Stokar result produces 3/2 times the correct answer. Additionally our expression leads also to a better numerical estimate of f_π if we follow the methods of ref. [3].

The ρ–parameter [18] is defined as $\rho := F_\pm^2/F_3^2$ which can now be written as

$$\rho = 1 + \Delta\rho = \frac{F_\pm^2}{F_3^2} = \left(1 + \frac{(F_3^2 - F_\pm^2)}{F_\pm^2}\right)^{-1} \simeq 1 - 2\,\frac{(F_3^2 - F_\pm^2)}{v^2} \;, \tag{17}$$

and from eq. (14) we find the contribution of the $t-b$ doublet

$$\Delta\rho = \frac{-N_c}{32\pi^2 v^2} \int_0^\infty dk^2 \; \frac{\Sigma_t^4}{(k^2 - \Sigma_t^2)^2} \;, \tag{18}$$

where we used $F_\pm^2 = v^2/2$ with $v \simeq 175\ GeV$ in the denominator. Model independent parametrizations of radiative corrections parametrize the information contained in $\Delta\rho$ essentially in the variables T [19] or ϵ_1 [20].

With the expressions for $\Delta\rho$ in eq. (18) and F_i^2 in eq. (13) we can calculate for given $\Sigma_t(p^2) \stackrel{p^2\to\infty}{\longrightarrow} 0$ three independent observable quantities which are one of the weak gauge boson masses (either $M_W^2 = g_2^2 F_\pm^2$ or $M_Z^2 = (g_1^2 + g_2^2) F_3^2$), $\Delta\rho$ and furthermore the physical top mass m_t. These three quantities are dominated by different momenta and therefore $\Sigma \neq constant$ leads to a different relation than a constant, i.e. hard mass. It is instructive to look at the degree of convergence of the involved integrals. The Goldstone Boson decay constants F_i^2 are formally log. divergent, but are finite with our assumption on $\Sigma_t(p^2)$. In that case renormalization is not needed, but due to the formal log. divergence Σ_t contributes with equal weight at all momentum scales. In other words, the magnitude of F_i^2 depends crucially on the high energy tail of Σ_t. The difference $F_\pm^2 - F_3^2$ has better convergence properties and is always finite, even for $\Sigma_t(p^2) = constant$. This implies that $\Delta\rho$ is finite, as it should be, and it is most sensitive to infrared scales somewhat above m_t. Finally m_t is of course only sensitive to one point, namely $m_t = \Sigma_t(m_t^2)$.

We would like to study now corrections in the relation between m_t, M_W and $\Delta\rho$ when Σ_t is the solution of a hypothetical Schwinger–Dyson equation which deviates from $\Sigma_t = m_t = constant$. First we would like to see if the correct Standard Model result emerges for a $t-b$ doublet when $\Sigma_t \to m_t = constant$. Therefore we take the ansatz (3) and ignore again the b quark mass. From eq. (18) we obtain then

$$\Delta\rho = \frac{N_c m_t^2}{32\pi^2 v^2}\left(\frac{1}{1 - m_t^2/\Lambda^2}\right) \stackrel{\Lambda\to\infty}{\longrightarrow} \Delta\rho^{SM} = \frac{N_c \alpha_{em}}{16\pi \sin^2\theta_W \cos^2\theta_W}\,\frac{m_t^2}{M_Z^2} \;. \tag{19}$$

Note that in the limit $\Lambda \to \infty$ (i.e. a hard, constant top mass) we obtain correctly the leading Standard Model value while the Pagels–Stokar relation would produce 3/2 times the Standard Model result. For finite Λ eq. (19) describes furthermore the modification of the Standard Model result due to a physical high energy momentum cutoff. Such a cutoff makes $\Delta\rho$ a little bit more positive than in the Standard Model which implies for a fixed experimental value of $\Delta\rho$ a lower top mass prediction. From eq. (13) it is in addition possible to determine M_W for the ansatz eq. (3)

$$M_W^2 = g_2^2 F_\pm^2 = \frac{g_2^2 N_c}{32\pi^2} \int\limits_0^{\Lambda^2} dk^2 \, \frac{\Sigma_t^2}{k^2 - \Sigma_t^2} = \frac{g_2^2 N_c}{32\pi^2} \, m_t^2 \, \ln\left(\frac{\Lambda^2 - m_t^2}{m_t^2}\right) . \tag{20}$$

Taking as experimental input $M_W = 80.2\ GeV$, $\Delta\rho = 0.004$, $\alpha_{em}^{-1}(M_Z^2) = 127.9$ and $\sin^2\theta_W^{eff}(M_Z^2) = 0.2318$ we plot in Fig. 1 the two central top mass values resulting from eqs. (19) and (20) as a function of Λ (dashed lines).

The ansatz eq. (3) can be viewed as the result of a Nambu–Jona-Lasinio (NJL) gap equation of top condensation as for example in the BHL model [2]. In fact a NJL gap equation is the simplest conceivable Schwinger–Dyson equation where Σ_t is forced to be a constant. Fig. 1 shows clearly that very high values of Λ and experimental errors of the input data are required to get the two top mass values in agreement. For such high Λ the effective Lagrangian is valid for many orders of magnitude which led in the BHL analysis to the so–called "renormalization group improvement". This means in the current language that $\Sigma_t = constant$ is replaced by $\Sigma_t = g_t(p^2)v$, where $v = 175\ GeV$ and $g_t(p^2)$ is the solution of the one–loop renormalization group equation. In BHL the predicted top mass is then the "effective fixedpoint" of the renormalization group flow. The BHL scenario has however phenomenological problems. First the very high value of Λ is nothing else then the old hierarchy problem which appears now as a fine–tuning of the four–fermion coupling G. Furthermore the infrared fixedpoint prediction is higher than the dashed curve resulting from eq. (20) which is shown in Fig. 1 and has (within newest experimental errors) no intersection with the line resulting from eq. (19). Thus this simplest scenario seems unacceptable even for very high values of Λ.

Remembering that $\Delta\rho$ and M_W are sensitive to details of Σ_t in a different way one may ask if the above problems can be solved by moderate modifications of the solution $\Sigma_t(p^2)$. The answer is yes, and we illustrate therefore now the two most important type of changes: The addition of a slowly falling tail and/or the addition of a "bump" somewhat above m_t.

First we consider a very rough ansatz for a "bump" between Λ_1 and Λ with $m_t < \Lambda_1 < \Lambda$ by modifying eq. (3)

$$\Sigma_t(p^2) = \begin{cases} 0 & \text{for } p^2 > \Lambda^2 \ ; \\ \sqrt{r}\cdot m_t & \text{for } \Lambda_1^2 \le p^2 \le \Lambda^2 \ ; \\ m_t & \text{for } p^2 < \Lambda_1^2 \ , \end{cases} \tag{21}$$

where Σ is changed between $\Lambda_1 < \Lambda$ and Λ. For $r > 1$ there is an extra "bump" between Λ_1 and Λ which affects $\Delta\rho$. For $\Lambda^2, \Lambda_1^2 \gg m_t^2, rm_t^2$ we obtain

$$\Delta\rho \simeq \frac{N_c m_t^2}{32\pi^2 v^2}\left(1 + \frac{m_t^2}{\Lambda^2} - \left[\frac{m_t^2(\Lambda^2 - \Lambda_1^2)}{\Lambda^2\Lambda_1^2}\,(r^2 - 1)\right]\right) , \tag{22}$$

where the leading extra contributions due to $r \neq 1$ and $\Lambda_1 \neq \Lambda$ are isolated in square brackets. We can see that the bump counteracts the effect of the cutoff and makes $\Delta\rho$ less positive. In principle the bump can even be chosen to make $\Delta\rho$ vanish. The relation eq. (20) between m_t and M_W becomes also modified. For $\Lambda^2, \Lambda_1^2 \gg m_t^2, r m_t^2$ we obtain approximately

$$M_W^2 \simeq \frac{g_2^2 N_c}{32\pi^2}\, m_t^2 \, \left(\ln\left(\frac{\Lambda^2 - m_t^2}{m_t^2}\right) + \left[(r-1)\ln\left(\frac{\Lambda^2}{\Lambda_1^2}\right)\right]\right) \, , \tag{23}$$

where extra contributions due to the bump are again isolated in square brackets.

Now we add a slowly falling high energy tail to the last ansatz eq. (21)

$$\Sigma_t(p^2) = \begin{cases} \text{equation (21)} & \text{for } p^2 < \Lambda^2 \, ; \\ \sqrt{r} m_t \, \left(\frac{p^2}{\Lambda^2}\right)^{-\alpha} & \text{for } p^2 > \Lambda^2 \, , \end{cases} \tag{24}$$

where $\alpha > 0$ is assumed. This high energy tail which is parametrized by α leads to

$$\Delta\rho \simeq \frac{N_c m_t^2}{32\pi^2 v^2}\left(1 + \frac{m_t^2}{\Lambda^2} - \left[\frac{m_t^2(\Lambda^2 - \Lambda_1^2)}{\Lambda^2 \Lambda_1^2}\,(r^2-1)\right] - \left\{\frac{r^2}{4\alpha+1}\,\frac{m_t^2}{\Lambda^2}\right\}\right) \, , \tag{25}$$

and

$$M_W^2 = \frac{g_2^2 N_c}{32\pi^2}\, m_t^2 \, \left(\ln\left(\frac{\Lambda^2 - m_t^2}{m_t^2}\right) + \left[(r-1)\ln\left(\frac{\Lambda^2}{\Lambda_1^2}\right)\right] + \left\{\frac{r}{2\alpha}\right\}\right) \, , \tag{26}$$

where the leading extra corrections due to the tail are isolated in curly brackets.

Note that we are looking for a scenario which simultaneously avoids the fine–tuning problem and which is phenomenologically acceptable. Consequently Λ and Λ_1 should be TeV–ish and the top mass values required from the $\Delta\rho$– and M_W–data should agree. This requires consequently some gap equation with a generic condensation scale $\mathcal{O}(TeV)$ capable of producing a bump, and a tail – maybe of the type discussed in ref. [4]. The asymptotic high energy behaviour of Σ_t might be described by a renormalization group equation if the spectrum of the theory does not contain further mass thresholds. This would imply a logarithmic tail and the parameter α should be very small. We could for example fix α in the minimal scenario by expanding the Higgs less one–loop renormalization group equation for g_t in the Standard Model . This would lead to $\alpha \simeq 0.04$. For such small values of α the tail leads to mild effects in the ρ–parameter and drastic changes in the M_W–m_t relation.

We can illustrate the effects of the combined bump and tail by plotting eqs. (25) and (26) in Fig. 1 as solid lines for the parameters $r = 2$, $\Lambda = 2\Lambda_1$, $\Lambda_1 = 2m_t$ and $\alpha = 0.04$. The small value of α (corresponding to a logarithmic high energy tail of Σ_t) influences mostly the M_W–m_t relation while the bump affects essentially only the $\Delta\rho$–m_t relation. Taking into account experimental and theoretical errors the two top mass values can agree for low values of Λ consistent with the above assumptions and avoiding fine–tuning. We have thus illustrated that solutions of Σ_t with moderate structure can solve the fine–tuning problem, i.e. allow for Λ–values within a few TeV. Furthermore the predicted m_t–M_W–$\Delta\rho$ relations are modified to be consistent with the data on M_W and $\Delta\rho$. The predicted top mass differs however typically somewhat from its Standard Model value – something that is be tested by a direct search for the top quark. It will therefore be extremely interesting what the final value of m_t from direct searches will be. A bump and a tail as discussed could for example be relevant in models of top condensation where heavy gauge bosons trigger condensation [14] or in bootstrap scenarios where the t–channel effects of a composite Higgs are non–negligible [21].

There are other electro–weak observables which are sensitive to the top mass value like for example the $Z\bar{b}b$ vertex. If m_t is replaced by Σ_t in the relevant diagrams then one finds however that the top mass dependence is replaced by sensitivity to Σ_t at low momenta. Thus in a first approximation these quantities depend essentially on the pole mass. There are however corrections which should become observable if high enough precision can be reached.

5 Summary

In summary there should exist top condensation models which are phenomenological viable. If Σ_t has suitable structure then it is even possible to choose a low lying scale of new physics Λ. The calculation of Σ_t for low Λ is for a given theory in general very difficult due to the non–perturbative nature of the relevant Schwinger–Dyson equation. An important test of the discussed effects arises from the comparison of the direct top mass with its indirect window from radiative corrections. The better these values agree in the Standard Model the less room is left for structure in Σ_t. If the presented ideas are relevant then the top quark should typically be somewhat above the Standard Model window. We listed some of the models which have been discussed in the literature and expressed the view that none of the models constructed so far appears to be a final scenario. The model independent features in favour of top condensation appear therefore still better than any specific realization. This may simply reflect the fact that some essential ingredient is missing in model building so far. One should also not conclude that top condensation or any Dynamical Symmetry Breaking scenario is incompatible with precision data since Technicolor leads to deviations at the percent level while only effects at the permille level are allowed. Such large deviations have nothing to do with Dynamical Symmetry Breaking but are a consequence of rather low lying (composite or fundamental) vector states. For systematic reasons top condensation models do not have such vector states. The Higgses which do arise should lead to small deviations from the Standard Model independently of their composite nature.

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Figures

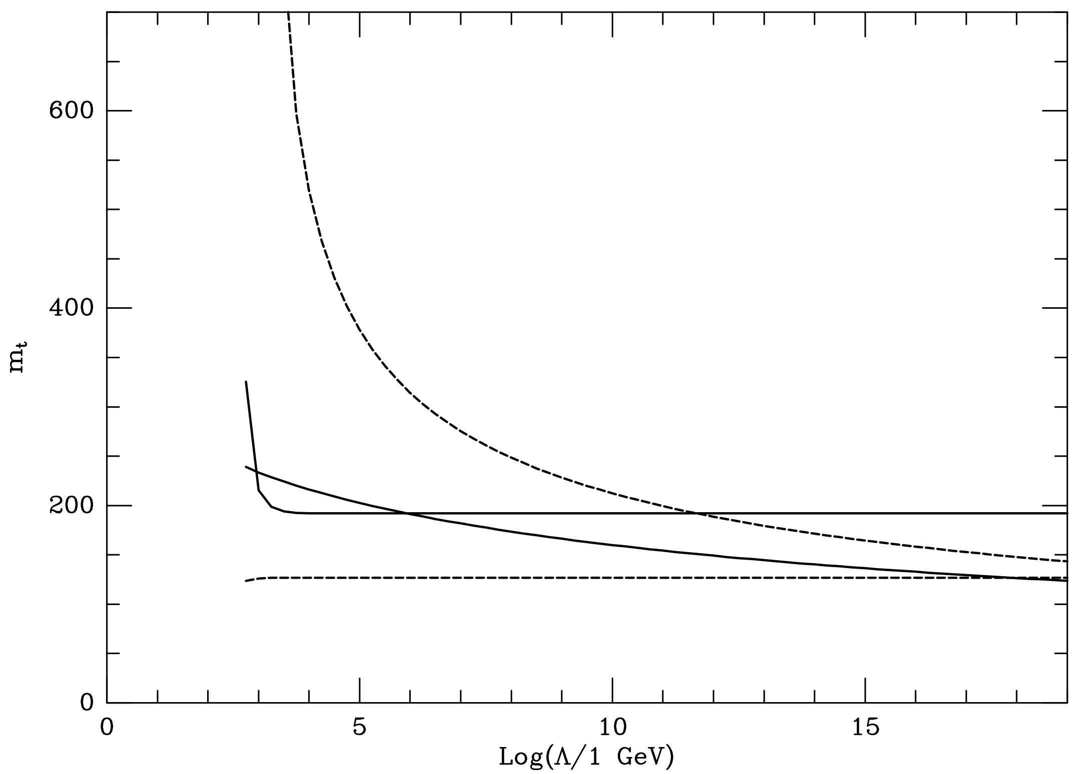

Figure 1: *The predicted (pole) top mass m_t versus the scale of new physics Λ using $\Delta\rho$ and M_W as experimental input. The upper dashed line follows from eq. (20) and the lower dashed line from eq. (19). The solid lines follow from the combined bump and tail ansatz for Σ_t eq. (24) showing that low values of Λ are then possible.*