Dark Matter (WS 2017/18) - Problem sheet 5

Lectures: Prof. Manfred Lindner and Dr. Giorgio Arcadi

Tutorials: Dominick Cichon

Time: One-time swap with lecture! Next tutorials on 14.12.2017, 09:15 - 11:00 (lecture on 13.12.2017).

Venue: Philosphenweg 12, kHS

Deadline for this sheet: 14.12.2017

1 Sterile neutrinos as dark matter

Scott Dodelson and Lawrence M. Widrow proposed in 1993 (accessible on arXiv:hep-ph/9303287) that dark matter could be explained by sterile neutrinos. They argue: "By far the simplest dark matter candidate, at least from the point of view of particle physics is the neutrino. Massive neutrinos require only the addition of right-handed or sterile neutrino fields to the standard model". At tree level, the most efficient way to produce sterile neutrinos is via oscillations: $\nu_L \to \nu_R$. The main task of this problem sheet is to go through a calculation of oscillation probabilities, assuming only one generation of neutrinos for reasons of simplicity.

1.1 Neutrinos in general *4 Points*

Answer the following questions:

1. What is the chirality of a particle?

2. Which neutrino flavors are known to exist? Which chiralities do they have (particle and anti-particle)?

3. What is the difference between neutrino mass eigenstates and flavor eigenstates?

4. Why are sterile neutrino called "sterile"?

1.2 Oscillation probability *2 Points*

The time evolution of a neutrino state in the mass basis, which at $t = 0$ is in the flavor state $|\nu_a\rangle$ for some flavor a, can be written as a plane wave:

$$|\nu_a(t)\rangle = \sum_j U_{aj}^* e^{-iE_j t} |\nu_j\rangle, \tag{1}$$

with U_{aj} being the mixing matrix, E_j the energy of the mass eigenstate j and t the time. The oscillation probability P is given by the absolute square of the transition amplitude A:

$$P(\nu_a \to \nu_b; t) = |A(\nu_a \to \nu_b; t)|^2 = |\langle \nu_b | \nu_a(t) \rangle|^2, \tag{2}$$

with a and b denoting two different flavors. Compute a general expression for P.

1.3 Oscillation of active to sterile neutrinos *3 Points*

If you consider only a left and right-handed neutrino, then the mixing matrix can be parametrized as:

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{3}$$

Approximate the mass eigenstate energies as $E_i \approx p + \frac{m_i^2}{2p}$ (ultrarelativistic limit, equal momenta) and calculate $P(\nu_L \to \nu_R; t)$. You should arrive, after having used appropriate trigonometric identities, at:

$$P(\nu_L \to \nu_R; t) = \sin^2(2\theta_0) \sin^2 \left(\frac{\Delta m^2}{4p} t \right), \tag{4}$$

with $\Delta m^2 = m_2^2 - m_1^2$.

1.4 Detecting sterile neutrinos *1 Point*

How could sterile neutrinos be detected?