Dark Matter (WS 2017/18) - Problem sheet 2

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Tutorials: Dominick Cichon

Time: Wd. 09:15 - 11:00

Venue: Philosphenweg 12, kHS

Deadline for this sheet: 09.11.2017

1 Gravitational lensing

General relativity predicts light to be deflected by the presence of massive objects which can act as lenses. Estimating the frequency of how often such a lens passes the line of sight between a light source and an observer allows to extract information about the distribution of massive compact halo objects (MACHOs) in our galaxy.

1.1 Einstein rings *4 Points*

In the case where a massive object is directly between a light source and an observer, light from the source is deflected such, that the observer will see a luminous ring, called *Einstein ring*, around the massive object. Given figure 1 and the equation for the deflection angle θ:

$$\theta = \frac{4GM}{bc^2},$$

derive an expression for the opening angle α_E of the Einstein ring which only depends on the object's mass M and the distances D_i as given in figure 1. Assume α_E and β to be small in order to approximate trigonometric functions (*Hint:* $\alpha_E \sim M^{1/2}$ *in the final result*).

1.2 Microlensing *2 Points*

While Einstein rings caused by MACHOs are usually too small to be resolved in telescope observations, one can still measure an apparent amplification of a light source if such an object passes the line of sight between source and observer, acting as a lens. If the object moves at a velocity v perpendicular to the line of sight, the typical time within which lensing happens is given by:

$$T = \frac{D_M \alpha_E}{v}$$

Calculate T for an object with $M = 0.2\, M_{\odot}$, $v = 200\,\mathrm{km/s}$ which lies halfway between an observer and a light source at a distance of $D_S = 60\,\mathrm{kpc}$ to the observer.

2 The cosmic microwave background

Temperatures and densities in the early universe were so high, that photons were not able to propagate freely. Instead, they were "coupled" to matter via Compton scattering and other interactions. After the universe had expanded and cooled down sufficiently, photons were able to

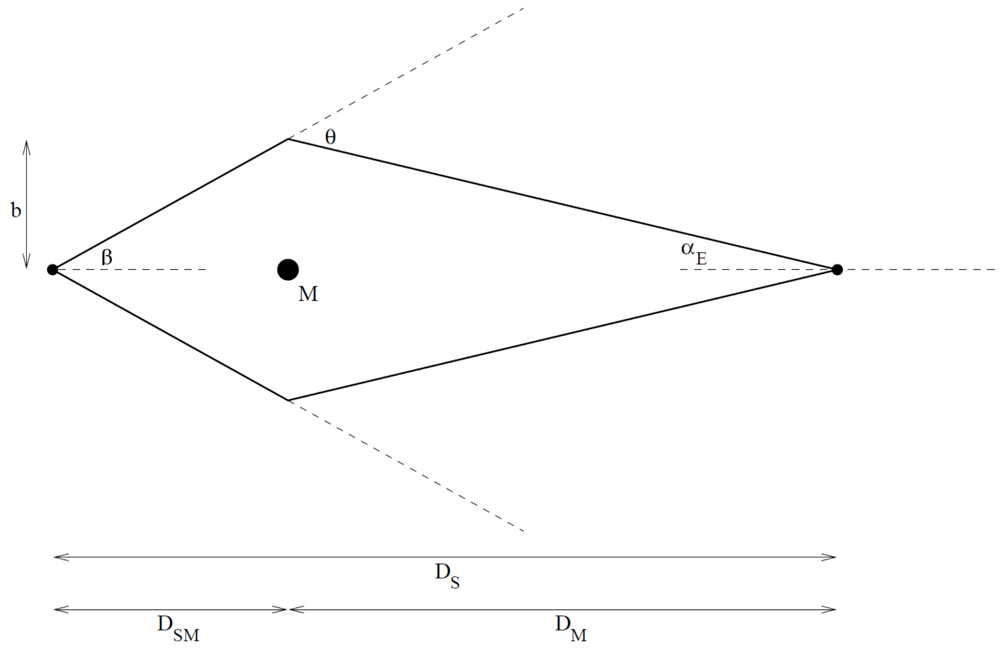

Figure 1: Sketch for task 1.2

propagate freely. These photons from the early universe now make up the so-called cosmic microwave background (CMB). Fluctuations of the CMB spectrum across the sky provide a way to probe physics during the early stages of the universe.

2.1 General properties *3 Points*

a) The temperature of the universe at which photons decoupled from matter is estimated to be $T_{\text{dec}} \simeq 3000\,\text{K}$. Use Wien's law to estimate the peak wavelength of the photon spectrum at the time of decoupling. Would the CMB back then have been visible to the human eye? If so: What color would it have had?

b) The average temperature of the CMB measured today is $T \simeq 2.73\,\text{K}$. What is the corresponding peak wavelength?

c) Assuming the past evolution of the universe to be matter-dominated yields for the time evolution of the scale factor $a(t) \sim t^{2/3}$. Use the fact, that the energy density of a photon gas ρ_{r} is proportional to T^4 while it also depends on the scale factor as $\rho_{\text{r}} \sim a^{-4}$ to give an estimate for the time of decoupling (in years after the Big Bang).

2.2 Parameters *2 Points*

Go to `https://lambda.gsfc.nasa.gov/education/cmb_plotter/` and play around with the CMB plotter. Observe, how the power spectrum changes when modifying the sliders, understand the meaning of the parameters and try to find the best fit values without clicking on the answer button. Why does the flatness of the universe inferred from the fit, which is also shown in the plotter, only depend on the first three parameters, but not on today's value of the Hubble constant, the re-ionization redshift or the spectral index?