Dark Matter (WS 2017/18) - Problem sheet 1

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Time: Wd. 09:15 - 11:00

Venue: Philosphenweg 12, kHS

Deadline for this sheet: 02.11.2017

1 Dark matter in galaxies

1.1 The virial theorem *5 Points*

The first indication for dark matter is given in a paper written by F. Zwicky in 1933. He observed the radial velocities of galaxies in the Coma cluster and used the virial theorem to calculate its mass. The state of the system is often quoted in terms of the virial ratio:

$$Q_{vir} = \frac{< T >}{- < \Omega >}. \tag{1}$$

A system which is in virial equilibrium has $Q_{vir} = 0.5$, with Ω being the potential energy and T the kinetic energy. Derive an expression for the total mass of a gravitationally bound system as a function of the dispersion velocity $\langle v^2 \rangle$. Zwicky measured that the size of the Coma cluster is around $R = 1.9 \cdot 10^{22}\, m$ with an average velocity of $\langle v^2 \rangle = 5 \cdot 10^{11}\, \frac{m^2}{s^2}$ and contains approximately 1000 galaxies. If you assume an average luminosity of $8.5 \cdot 10^7$ solar luminosities per galaxy, derive the mass to light ratio for the Coma cluster. What does it tell you if you expect a ratio at the order of $3 \frac{M_\odot}{L_\odot}$?

1.2 Rotation curves: Prediction vs. measurement *5 Points*

Measurements of galactic rotation curves imply an apparent mass deficit in galaxies (see publications by V. Rubin *et al.* from the 1970s/80s). Under the assumption of spherical symmetry of a rotating galaxy one can calculate the mass inside a sphere of a given radius from the circular velocity of the stars at its surface and compare it to an estimate from visible stars.
Give a formula which expresses the circular velocity in terms of the enclosed mass and the distance to the galactic center. Then, assume the simplest case of a constant mass density ρ_0 inside a radius r_0. Draw a sketch of how the rotation curve looks like inside and outside of r_0. Now consider a more realistic distribution in the form of:

$$\rho(r) = \frac{\rho_0}{(1 + r/r_0)^\alpha}. \tag{2}$$

Determine a value for α which gives a flat rotation curve at $r >> r_0$, as indicated by measurements, and derive the rotation curve $v(r)$ for this value.

1.3 Estimation of dark matter *2 Points*

At $r = 10^5$ light years a star velocity measurement yields $v_{meas} = 225\,km/s$, while the expected value is $v_{calc} = 15\,km/s$. Calculate the visible as well as the true galaxy mass. What is the percentage of dark matter in the galaxy? How large is the average dark matter mass density? (Use: $G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$).