Exercises for: „Introduction to the Standard model II“
Winter term 11/12

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Sheet 8

Exercise 16: The Conjugation operator

The action of the particle-antiparticle conjugation operator on a fermion field ψ is defined as:

$$\hat{C} : \psi \to \psi^C = C\,\bar{\psi}^T \quad , \quad C = i\gamma_2\gamma_0$$

Where the matrix C has following properties:

$$C^\dagger = C^T = C^{-1} = -C, \quad C\gamma_\mu C = -\gamma_\mu^T$$

Show that:

$$(\psi^c)^c = \psi, \quad \bar{\psi^c} = \psi^T C, \quad \bar{\psi}_1\psi_2^c = \bar{\psi}_2^c\psi_1, \quad \bar{\psi}_1 A\psi_2 = \bar{\psi}_2^c(CA^TC^{-1})\psi_1^c$$

with A an arbitrary 4×4 matrix.

Exercise 17: The See–Saw–mechanism

If the Standard model (SM) is extended to contain three right handed neutrinos, the standard Higgs mechanism generates Dirac masses when one introduces the below term in the Lagrange density

$$\mathscr{L}_{\text{Dirac}} = -\overline{\nu_{\text{R}}}\, m_{\text{D}}\, \nu_{\text{L}} + h.c.$$

$\nu_{\text{L}} = (\nu_{\text{L}}^1, \nu_{\text{L}}^2, \nu_{\text{L}}^3)^T$ is a column vector of left handed neutrinos of the (SM), ν_{R} is an analogous vector with the new right handed neutrinos and m_D is a (in general complex) 3×3–matrix.

Since the right handed neutrinos are per definitionem singlets under the (SM) gauge group (i.e. they do not participate in gauge interactions) they can have Majorana masses. The corresponding Lagrange density is

$$\mathscr{L}_{\text{Majorana}} = -\frac{1}{2}\overline{\nu_{\text{R}}}\, m_{\text{R}}\, \nu_{\text{R}}^c + h.c.\ .$$

ν_{R}^{c} is now the charge conjugated field (it holds $(\nu_{\mathrm{R}}^{c})^{c} = \nu_{\mathrm{R}}$) and m_R is a symmetric 3×3–matrix. We assume that the elements of m_R are much larger than the elements of m_D, $m_{\mathrm{D}} \ll m_{\mathrm{R}}$.

a) Show that it is possible to write the total mass term in the Lagrange density

$$\mathscr{L}_{\text{Mass}} = \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Majorana}} \ ,$$

as

$$\mathscr{L}_{\text{Mass}} = -\frac{1}{2}\overline{\Psi^c}\, M\, \Psi + h.c.$$

with

$$\Psi \equiv \begin{pmatrix} \nu_{\mathrm{L}} \\ \nu_{\mathrm{R}}^{c} \end{pmatrix} \quad \text{und} \quad M \equiv \begin{pmatrix} 0 & m_{\mathrm{D}}^{T} \\ m_{\mathrm{D}} & m_{\mathrm{R}} \end{pmatrix} \ .$$

Hint: Use the relation: $\overline{\nu_{\mathrm{L}}^{c}}\, m_{\mathrm{D}}^{T}\, \nu_{\mathrm{R}}^{c} = \overline{\nu_{\mathrm{R}}}\, m_{\mathrm{D}}\, \nu_{\mathrm{L}}$.

b) It is possible to transform the 6×6–matrix M using the transformation $\Psi = U\chi \equiv U\left(\begin{smallmatrix}\chi_1 \\ \chi_2\end{smallmatrix}\right)$ with $U = \left(\begin{smallmatrix} 1 & \rho \\ -\rho^{\dagger} & 1 \end{smallmatrix}\right)$ (change of basis) in such a way that it is block diagonal. i.e, it has the form

$$U^{T} M\, U \simeq \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{1}$$

with (symmetric) 3×3–matrices m_1, m_2. ρ is here a small parameter i.e. terms containing higher than linear powers of rho can be dropped. Compute ρ and m_1, m_2 from (1). What is the relation between the fields χ_1, χ_2 with the initial fields $\nu_{\mathrm{L}}, \nu_{\mathrm{R}}$?

c) Why do you think the mechanism has the name „See-Saw“?

Exercise sessions:

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