Exercises for: „Introduction to the Standard model II“
Winter term 11/12

Prof. Dr. M. Lindner and Dr. T. Schwetz-Mangold 01.12.11

Sheet 7

Exercise 15: Fate of the false vacuum

Part I

Consider the quantum field theory of a single scalar field in four dimensional space-time with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi).$$

The potential has the following form

$$U(\phi) = U_+ + \frac{\epsilon}{2a}(\phi - a) \quad \text{and} \quad U_+ = \frac{\lambda}{8}\left(\phi^2 - \frac{\mu^2}{\lambda}\right)^2,$$

with a small positive constant ϵ.

a) Sketch the potential. Think how a classical field would behave in such a potential. Is there a difference to a quantum field?

The rate for the wall penetration is given to first order in $\hbar$ by the exponential of the euclidean action (where $\tau = it$).

$$\Gamma = A\,\exp[-S_E] \text{ and } \quad S_E = \int d\tau d^3x \left\{\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}\left(\nabla\cdot\phi\right)^2 + U(\phi)\right\}$$

We can argue that now the solution will have an underlying $O(4)$ symmetry and hence it is sensible to introduce the variable $\rho = \sqrt{\tau^2 + \vec{x}^2}$ on which the solution will depend only.

b) Simplify the action using the symmetry assumption and derive the equation of motion.

Tipp:You should get:

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = \frac{dU}{d\phi}.$$

Part II

Thin wall approximation: Suppose now that around a value of $\rho = R$ the damping (term proportional to one over rho) can be neglected and also the epsilon part of the potential can be dropped.

c) Show now that under those assumptions the equation of motion is solved for arbitrary $U_+(\phi)$ by:

$$x = \int_0^{\phi 1} \frac{d\phi}{\sqrt{2\,U_+(\phi)}}.$$

d) Convince yourself that for the U_+ in this problem the solution is given by ϕ_1 and sketch it.

$$\phi_1 = a\,\tanh[\frac{1}{2}\mu x]$$

Note: This is a so called solitonic wall.

e) Assuming the above form of the solution compute the action and vary it with respect to R.

f) Interpret your result. What does it mean for the fate of our universe?

Exercise sessions:
Albert Überle Str. 3-5 Seminarraum I

Tutoring:
Juri Smirnov, E-Mail: juri.smirnov@mpi-hd.mpg.de