# Exercises for: „Introduction to the Standard model II“
## Winter term 11/12

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Sheet 6

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**Exercise 14:** Optical Theorem

a) Derive the optical theorem, which connects the forward scattering amplitude to the total x-section.

$$\sigma_{tot} = \frac{4\pi}{k} Im[A(\theta = 0)]$$

Consider therefore a plane wave propagating in $z$ direction, hitting on an obstacle and being scattered by it. The amplitude viewed from large distance $r$, is given then by:

$$\Psi(r) \cong e^{i\,z\cdot k} + A(\theta)\,\frac{e^{i\,k\cdot r}}{r}$$

Suppose now there is a screen at a large distance behind the obstacle. The intensity of the wave is given by the square of its amplitude. Expand the expression above for large $z$ and small $\theta$. Then integrate the intensity over an area $A_0$ of the distant screen.

From your result you can compute the „effective area“ of the obstacle.

b) The optical theorem also holds in QFT (where $k$ in the denominator is replaced by the Mandelstam variable $s = (p_1 + p_2)^2$).

Use the partial wave decomposition of $A(\theta) = 16\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos(\theta))\,a_l$ (as introduced in the lecture) and the expression for the differential x-section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi s}|A|^2,$$

to obtain an expression for $\sigma$. Now use the optical theorem to get constraints on the $a_l$'s.

**Exercise 15:** Compton scattering in QED (revision)

Consider the reaction:

$$\gamma(k_1)e^-(p_1) \to \gamma(k_2)e^-(p_2)$$

a) Write down the Feynman diagrams contributing to this process.

Tipp: There are two of them.

b)Write down the matrix element for this process, by first writing:

$$\langle 0|a(p_2)a(k_2)\, S\, a^\dagger(p_1)a^\dagger(k_2)|0\rangle.$$

Now expand the scattering operator to second order:

$$S = 1 + i\int d^4x\, T\,\mathcal{L}_I(x) + \frac{(i)^2}{2!}\int d^4x\, d^4y\, T\,\mathcal{L}_I(x)\, T\,\mathcal{L}_I(y) + ...$$

With:

$$\mathcal{L}_I(x) = \bar{\Psi}\gamma_\mu\Psi A^\mu(x).$$

Use Wick's theorem to contract all the fields appearing.

**Exercise sessions:**
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