Exercises for: „Introduction to the Standard model II“
Winter term 11/12

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Sheet 5

Exercise 13: Diagonalization of the Fermion mass matrix

The Fermion masses are generated through the coupling of the Fermions to the Higgs, the Yukawa part of the Lagrangian is given by:

$$\mathcal{L}_{Yukawa} = \overline{\psi_{i,L}}\, M_{ij}\, \psi_{j,R} + h.c.$$

M is the Yukawa–Coupling matrix Y times the Higgs vev. A complex $n \times n$ can be diagonalized by a bi-unitary transformation.

$$V^\dagger\, M\, U = M^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, \dots, m_n)$$

Where V and U are unitary.

a) In principle the diagonal elements can be negative. Show that it is always possible to find such unitary transformations that all the diagonal entries are positive.

b) Perform a basis change among the fermions with $f_m = U^{(f)}_{mn} f'_n$ assuming that then M is diagonal. Which part of the Standard Model Lagrangian will this transformation affect non trivially? Write down the altered expression.

Tipp: Introduce the matrix:

$$V_{mn} = \left(\left(U^{(u)} \right)^\dagger U^{(d)} \right)_{mn}$$

c) Count the degrees of freedom of a $n \times n$ unitary matrix. Consider now the rewritten Lagrangian from part (b), not all of the new introduced parameters are physical degrees of freedom. How many degrees of freedom remain after global quark redefinitions?

d) What are the physical consequences of your considerations?

e) In a simplified model, the charged current is given by:

$$J_\mu = \bar{U}\gamma_\mu (1+\gamma_5)\, C\, D$$

with:

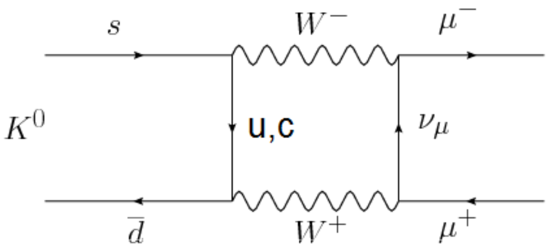
$$U = \begin{pmatrix} u \\ c \end{pmatrix},$$

$$D = \begin{pmatrix} d \\ s \end{pmatrix}$$

and:

$$C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

Explain using this model the long life-time of the neutral Kaon considering the decay precess below.



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