Exercises for: „Introduction to the Standard model II“
Winter term 11/12

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Sheet 4

Exercise 10: Pion decay II

a) Consider again the pion decay: $\pi^+ \to \mu^+ + \nu_\mu$.

The effective Lagrangian describing this decay is the Fermi type interaction:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} \cos\theta_c \left[\bar{u}\gamma^\mu(1-\gamma^5)d\right]\left[\bar{\mu}\gamma_\mu(1-\gamma_5)\nu_\mu\right].$$

Calculate the decay rate for this processes and use the measured lifetime $\tau_\pi = 2.6 \cdot 10^{-8}\, s$ to determine the constant f_π (was introduced in exercise 8).

b) Show that as the V-A (Vector minus Axial-vector) theory, the amplitude for the decay is proportional to m_μ, and to m_e in the decay $\pi^+ \to e^+\nu_e$. Discuss this fact also in a more pictorial way.

Exercise 11: SU(2) gauge theory

Consider a $SU(2)$ gauge theory, where the particle content is given by a left handed fermionic lepton doublet and a right handed singlet:

$$L = \begin{pmatrix} l_L \\ \nu_L \end{pmatrix} \qquad \text{and} \qquad l_R.$$

Furthermore a complex $SU(2)$ doublet ϕ is introduced such that a Higgs mechanism is at work. The Lagrangian of the theory comprises therefore the following parts:

$$\mathcal{L}_{Gauge} = -\frac{1}{2} tr\left\{F_{\mu\nu}F^{\mu\nu}\right\},$$

$$\mathcal{L}_{Fermion} = i\,\bar{L}\,\gamma^\mu D_\mu L + i\,\bar{R}\,\gamma^\mu D_\mu R,$$

$$\mathcal{L}_{Scalar} = (D_\mu\phi)^\dagger(D^\mu\phi) - \left(\frac{\lambda}{2}\left(\phi^\dagger\phi\right) - \frac{v^2}{2}\right)^2,$$

$$\mathcal{L}_{Yukawa} = -g_Y \left(\bar{L}\, \phi\, l_R + h.c\right).$$

Consider the theory, when the scalar field is in the minimum of the potential. Expand around this minimum and call the scalar perturbation Higgs. Now analyze the Lagrangian and find all the possible interactions. Draw the Feynmann graphs for those interactions.

Exercise 12: An other vacuum state

A vacuum state $|\Omega\rangle$ is not always the „empty" no particle state $|0\rangle$. You can verify this with the following Hamiltonian:

$$H = \frac{5}{3}\,\hat{a}^\dagger\,\hat{a} + \frac{2}{3}\,(\hat{a}^\dagger)^2 + \frac{2}{3}\,\hat{a}^2$$

With $\hat{a}^\dagger$ and $\hat{a}$ having the usual properties: $\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$ and $\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$. The particle number operator $\hat{N} = \hat{a}^\dagger\hat{a}$ does not commute with $\hat{H}$, and hence particle number is not conserved.

Find the lowest energy state of $\hat{H}$. Build for this purpose 2 operators $\hat{Q}^\pm$ with the commutation relation:

$$[\hat{H}, \hat{Q}^\pm] = \pm\hat{Q}^\pm\ ,$$

Hence, those operators rise or lower the energy of a given state. Note that the lowering operator applied to the vacuum state annihilates it.

$$\hat{Q}^-|\Omega\rangle = 0 \tag{1}$$

One can now write the vacuum as: $\Omega = \sum\limits_n c_n\,(\hat{a}^\dagger)^n\,|0\rangle$ (This is a usual change of basis). Use equation (1) to find the explicit form of $|\Omega\rangle$.

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