Exercises for: „Introduction to the Standard model II"
Winter term 11/12

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Sheet 11

Exercise 24: Debris from last time

For those who were interested, a classic paper on the left-right-symmetric electroweak model with a Higgs triplet is (http://prd.aps.org/abstract/PRD/v44/i3/p837_1).

Exercise 25: Additional Higgs triplet and type II See-Saw

We are focusing on the leptonic sector of the standard model and extend it by an additional Higgs triplet field. The particle spectrum with transformation properties under $SU(2)_L \otimes U(1)_Y$ is now the following:

$$L_a : (2\,,\,-1)\ ;\ l_{aR} : (1\,,\,-2)\ ;\ \phi : (2,\,1)\ ;\ \Delta : (3\,,\,2)\,.$$

With a left handed lepton doublet, a right handed lepton singlet, a scalar Higgs doublet and a scalar Higgs triplet respectively. Remember that the Higgs triplet in the representation as a 2×2 matrix is given in terms of the electric charge eigenstates as:

$$\Delta = \begin{pmatrix} H^{+} & \sqrt{2}H^{++} \\ \sqrt{2}H^{0} & -H^{+} \end{pmatrix}.$$

The lepton mass terms are:

$$\mathcal{L}_Y = \sum_{a,b} \left\{ -c_{ab}\bar{l}_{aR}\phi^{\dagger}L_b + \frac{1}{2} f_{ab}\bar{L}_a^c\, i\,\tau_2 \Delta L_b \right\} + h.c.$$

a) Convince yourself that this Yukawa part of the Lagrangian is a singlet under the $SU(2)_L \otimes U(1)_Y$ gauge group.

The Higgs potential is given by:

$$V(\phi,\Delta) = a\phi^\dagger\phi + \frac{b}{2}Tr\left[\Delta^\dagger\Delta\right] + c\left(\phi^\dagger\phi\right)^2 + \frac{d}{4}\left(Tr\left[\Delta^\dagger\Delta\right]\right)^2 + \frac{e-h}{2}\phi^\dagger\phi Tr\left[\Delta^\dagger\Delta\right] +$$

$$+\frac{f}{4}Tr\left[\Delta^\dagger\Delta^\dagger\right]Tr\left[\Delta\Delta\right] + h\phi^\dagger\Delta^\dagger\Delta\phi + \left(t\phi^\dagger\Delta\left(i\tau_2\phi^*\right) + h.c\right).$$

b) Use now the condition that the vacuum states $\langle\phi\rangle_0$ and $\langle\Delta\rangle_0$ (with the values denoted by v and v_T respectively) must be electrically neutral and find the form of $V(\langle\phi\rangle_0\,,\,\langle\Delta\rangle_0)$.

c) Define the coefficients as: $t = |t|e^{i\omega}$, $v_T = \alpha e^{i\gamma}$ and $\alpha = |v_T|$ and minimize the Potential w.r.t. v, α, γ (Hint: start with γ).

d) Show that with the assumptions $a,\, b \backsim v^2$; $c, d, e, f, h \sim 1$ and $|t| \ll v$ the minimum conditions are equivalent to the approximate statements

$$v \cong -\frac{a}{c} \text{ and } \alpha = |\frac{|t|\,v^2}{b + \frac{e-h}{2}v^2}\,.$$

e) What does this mean for the mass phenomenology of the leptonic sector?

f) Assume now that $\sqrt{b} = m_\Delta$ is very large compared to the electroweak scale v and that $t^2 \backsim b$. Find the relations between the vev's under this condition and analyze their implications for the lepton masses. Why is this scenario called type II See-Saw mechanism?

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