Exercises for: „Introduction to the Standard model II“
Winter term 11/12

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Sheet 10

<u>Exercise 22:</u> Can the quantum state be interpreted statistically

a) Review the paper mentioned on Sheet 9 (http://arxiv.org/abs/1111.3328).

Given two copies of a one dimensional Hilbert space with an orthonormal basis $\{\,|0\rangle, |1\rangle\}$ and following definitions of $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

b) Show that the four states ϕ_1, ϕ_2, ϕ_3, ϕ_4 are mutually orthogonal.

$$\phi_1 = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$$

$$\phi_2 = \frac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle)$$

$$\phi_3 = \frac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle)$$

$$\phi_4 = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle)$$

c) Show the relations

$$\phi_1 \perp |0\rangle \otimes |0\rangle\,,$$

$$\phi_2 \perp |0\rangle \otimes |+\rangle\,,$$

$$\phi_3 \perp |+\rangle \otimes |0\rangle\,,$$

$$\phi_4 \perp |+\rangle \otimes |+\rangle.$$

Exercise 23: Left-right symmetric electroweak model

Recall the Left-right symmetric model introduced in the lecture. The Lepton and quark spectrum is analogously given by:

$$Q^i_{L,R} = \begin{pmatrix} U^i_{L,R} \\ D^i_{L,R} \end{pmatrix} \; ; \; L^i_{L,R} = \begin{pmatrix} \nu^i_{L,R} \\ e^i_{L,R} \end{pmatrix} .$$

With transformation properties under $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$ respectively

$$Q_L : \left(2_L,\, 1_R\,,\, \frac{1}{3}\right) ; Q_R : \left(1_L,\, 2_R\,,\, \frac{1}{3}\right) ; L_L : (2_L,\, 1_R\,,\, -1) \,; L_R : (1_L,\, 2_R\,,\, -1) \,.$$

The Higgs sector contains again a bidoublet ϕ, but now instead of the triplets there are scalar doublets $A_{L,R}$ and a fermion singlet χ with the transformation properties under $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively

$$\phi : (2_L,\, 2_R\,,\, 0) \; ; \; A_L : (2_L,\, 1_R\,,\, 1) \; ; \; A_R : (1_L,\, 2_R\,,\, 1) \; ; \; \chi : (1_L,\, 1_R\,,\, 0) \,.$$

The Left-right symmetry implies invariance of the Lagrangian under the transformation (with Ψ symbolizing all fermion fields) :

$$\Psi_L \leftrightarrow \Psi_R \,, \quad A_L \leftrightarrow A_R \,, \quad \phi \leftrightarrow \phi^\dagger .$$

Write down the Lagrangian for the fermion masses of this model (you have to construct singlets under the full gauge group).

Tip: Recall that the bidoublet can be represented by a 2×2 matrix

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} .$$

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