Exercises for: „Introduction to the Standard model II“
Winter term 11/12

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Sheet 1

<u>Exercise 1:</u> Massless Vector field

The Lagrange density of a massless Vector (Spin 1) field is given by:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j_\mu A^\mu \,, \tag{1}$$

with the Field-strength tensor being:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{2}$$

a) Vary the Action for the field and derive the equations of motion for A^μ.

b) In Electrodynamics the relation between the four-potential A^μ and the E and B fields is:

$$\begin{aligned} \vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{E} &= -\partial_t \vec{A} - \vec{\nabla} A_0 \end{aligned} \tag{3}$$

Rewrite $F^{\mu\nu}$ as a function of the components E^i and B^i.

c) Show that the e.o.m. for A^μ satisfies the inhomogeneous Maxwell eqns.

$$\begin{aligned} \vec{\nabla}\vec{E} &= \rho \\ \vec{\nabla} \times \vec{B} - \partial_t \vec{E} &= \vec{\jmath} \end{aligned} \tag{4}$$

with ($j^\mu = (\rho, \vec{\jmath})$), while the homogeneous Maxwell eqns are satisfied trivially by (2).

$$\begin{aligned} \vec{\nabla}\vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \partial_t \vec{B} &= \vec{0} \end{aligned} \tag{5}$$

(*Hint for Nr. 1. :* Write the e.o.m with the Field-strength tensor.)

Exercise 2: Symmetry of the QED Lagrangian

Prove the invariance of the QED Lagrange density

$$\begin{aligned} \mathcal{L}_{QED} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad D_\mu = \partial_\mu - ieA_\mu \end{aligned} \tag{6}$$

under local gauge transformations, defined below:

$$\psi \to \psi' = e^{-i\theta(x)}\psi \quad , \quad \bar{\psi} \to \bar{\psi}' = e^{i\theta(x)}\bar{\psi} \quad , \quad A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\theta(x) \tag{7}$$

Exercise 3: Noether Theorem

a) Derive the Noether theorem for a global symmetry transformation of the Lagrange density:

$$\mathcal{L} = \mathcal{L}(\phi_A, \partial_\mu\phi_A)$$

What is the conserved current?

b) Apply your results to the Lagrange density of a massive, complex, scalar field:

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi* - m^2\phi\phi*$$

What is the conserved current here?

Exercise sessions:
Room and time are going to be announced.

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