Neutrinos and cosmology

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Relic neutrino background:

- **Temperature:**
  \[ T_{\nu,0} = \left( \frac{4}{11} \right)^{1/3} T_{CMB,0} = 1.95 \text{ K} \]

- **Number density per flavour:**
  \[ n_{\nu,0} = \frac{6}{4 \pi^2} \zeta(3) T_{\nu,0}^3 = 112 \text{ cm}^{-3} \]

- **Energy density per flavour:**
  \[ \Omega_{\nu} h^2 = \frac{m_\nu}{93 \text{ eV}} \]

If \( m_\nu > 1 \text{ meV} \) Neutrino dark matter
Neutrino dark matter...

- Neutrino oscillations:

\[ \Delta m_{\text{atm}}^2 \sim 10^{-3} \text{ eV}^2 \quad \Delta m_{\text{sun}}^2 \sim 10^{-5} \text{ eV}^2 \]

Minimum amount of neutrino dark matter

\[ \min \sum m_\nu \sim 0.05 \text{ eV} \quad \min \Omega_\nu \sim 0.1\% \]
Upper limit on neutrino masses from tritium decay:

\[
m_e \equiv \left( \sum_i |U_{ei}|^2 m_i^2 \right)^{1/2} \lesssim 2.2 \text{ eV}
\]

Large mixing means \( |U_{ei}|^2 \sim O(0.1 \rightarrow 1) \)

\[
\max \sum m_\nu \sim 7 \text{ eV} \Rightarrow \max \Omega_\nu \sim 12 \%
\]

Light neutrinos cannot be the only dark matter component.
Neutrino dark matter is **hot**...

- **Large velocity dispersion:**

  \[
  \langle v_{\text{thermal}} \rangle \approx 81 (1 + z) \left( \frac{\text{eV}}{m_{\nu}} \right) \text{ km s}^{-1}
  \]

  - A **dwarf galaxy** has a velocity dispersion of 10 km s\(^{-1}\) or less, a galaxy about 100 km s\(^{-1}\).

  - Sub-eV neutrinos have **too much thermal energy** to be packed into galaxy-size self-gravitating systems.

- Neutrinos **cannot** be the **dominant** Galactic dark matter.
Why study neutrinos in cosmology...

- Hot dark matter leaves a **distinctive imprint** on the **large-scale structure distribution**.
  - We can learn about neutrino properties from cosmology.

- **Cosmological probes** are getting ever **more precise**:
  - Even a small neutrino mass can **bias** the inference of other cosmological parameters.
The concordance framework...

- We work within the **CDM framework** extended with a **subdominant** component of **massive neutrino dark matter**.
  - Flat geometry.
  - Main **dark matter** is cold.
  - Initial conditions from single-field slow-roll inflation.
The concordance framework...

- We work within the CDM framework extended with a subdominant component of massive neutrino dark matter.
  - Flat geometry.
  - Main dark matter is cold.
  - Initial conditions from single-field slow-roll inflation.
Plan...

- What we can do **now**

- What we can do **in the future**

- The **nonlinear matter power spectrum**
1. What we can do now...
Two effects of massive neutrinos...

- **On the background:**
  - Shift in time of matter radiation equality.

- **On the perturbations:**
  - Suppression of growth.
Sub-eV neutrinos become nonrelativistic at $z<1000$:  

- **Radiation** at early times.
- **Matter** at late times.

Comoving matter density **today ≠** Comoving matter density **before recombination**

- Shift in matter-radiation equality relative to model with zero neutrino mass.
Two effects of massive neutrinos...

- **On the background:**
  - Shift in time of matter radiation equality.

- **On the perturbations:**
  - Suppression of growth.
At low redshifts, neutrinos become nonrelativistic:
- But still have large thermal speed: \( c_v \approx 81 (1+z) \left( \frac{eV}{m_v} \right) \text{ km s}^{-1} \)
  \( \rightarrow \) hinder clustering on small scales.

Free-streaming length scale & wavenumber:

\[
\lambda_{FS} \equiv \sqrt{\frac{8\pi^2 c_v^2}{3 \Omega_m H^2}} \approx 4.2 \sqrt{\frac{1+z}{\Omega_{m,0}}} \left( \frac{eV}{m_v} \right) h^{-1} \text{ Mpc}
\]
\[
k_{FS} \equiv 2\pi \frac{\lambda_{FS}}{\lambda_{FS}}
\]

\( \lambda \gg \lambda_{FS} \) Clustering
\( k \ll k_{FS} \)
\( \lambda \ll \lambda_{FS} \) Non-clustering

Gravitational potential wells
In turn, free-streaming (non-clustering) neutrinos slow down the growth of gravitational potential wells on scales $\lambda << \lambda_{FS}$ or wavenumbers $k >> k_{FS}$.

Clustering → potential wells become deeper

Some time later...

Both CDM and neutrinos cluster

Only CDM clusters
The presence of HDM slows down the growth of CDM perturbations at large wavenumbers $k$.

- The density perturbation spectrum acquires a step-like feature.
Describing perturbations: CDM...

- Cold dark matter = collisionless, pressureless fluid:

\[
\begin{align*}
\dot{\delta}_c + \theta_c &= 0 \\
\dot{\theta}_c + H \theta_c + \nabla^2 \Phi &= 0 \\
\nabla^2 \Phi &= \frac{3}{2} H^2 \Omega_m \left[ f_c \delta_c + f_v \delta_v \right] 
\end{align*}
\]

- Density perturbations
- Continuity eqn
- Euler eqn
- Gravitational source
- Velocity divergence
- Expansion
- Neutrino fraction
- Poisson eqn
Describing perturbations: Neutrinos...

- Free-streaming neutrinos cannot be described by a perfect fluid.
  - Must solve (linearised) collisionless Boltzmann equation:

\[
\frac{\partial (\delta f)}{\partial \tau} + \frac{p}{m_\nu a} \cdot \nabla (\delta f) - a m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial p} = 0
\]

\[
f(x, p, \tau) = f_0 + \delta f
\]

Phase space density
Describing perturbations: Neutrinos...

- **Free-streaming neutrinos cannot** be described by a **perfect fluid**.
  - Must solve (linearised) **collisionless Boltzmann equation**:

\[
\frac{\partial (\delta f)}{\partial \tau} + \frac{p}{m_v a} \cdot \nabla (\delta f) - a m_v \nabla \Phi \cdot \frac{\partial f_0}{\partial p} = 0
\]

- **Momentum moments**:
  - Density perturbation:
    \[
    \delta_v \equiv \frac{1}{\bar{\rho}_v} \int d^3 p \ (\delta f)
    \]
  - Velocity divergence:
    \[
    \theta_v \equiv \frac{1}{\bar{\rho}_v} \int d^3 p \ \frac{p_i}{a m_v} \partial_i (\delta f)
    \]
  - Pressure and anisotropic stress:
    \[
    \sigma_{ij} \equiv \frac{1}{\bar{\rho}_v} \int d^3 p \ \frac{p_i p_j}{a^2 m_v^2} (\delta f)
    \]

Describing perturbations: Neutrinos...

- **Free-streaming neutrinos** cannot be described by a **perfect fluid**.

  - Must solve (linearised) **collisionless Boltzmann equation**:

\[
\frac{\partial (\delta f)}{\partial \tau} + \frac{p}{m_\nu a} \cdot \nabla (\delta f) - a m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial p} = 0
\]

  - **Momentum moments**:

\[
\delta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 p (\delta f) \\
\theta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 p \frac{p_i}{a m_\nu} \partial_i (\delta f) \\
\sigma_{ij} \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 p \frac{p_i p_j}{a^2 m_\nu^2} (\delta f) \quad \vdots
\]

  - **Density perturbation**

\[
f(x, p, \tau) = f_0 + \delta f
\]

  - **Phase space density**

  - **Velocity divergence**

  - **Pressure and anisotropic stress**

Give rise to free-streaming behaviour
Massive neutrinos, $m_\nu=1$ eV

Clustering regime

$$k = 10^{-2} \, h \, \text{Mpc}^{-1} \ll k_{FS}$$

Lesgourgues and Pastor 2006
Massive neutrinos, \( m_\nu = 1 \text{ eV} \)

Non-clustering regime

\[
k = 1 \, h \, \text{Mpc}^{-1} \gg k_{FS}
\]

Lesgourgues and Pastor 2006

Non-clustering regime

\[
\delta_{\nu} = \frac{\Omega_{\nu}}{\Omega_m}
\]

\( (f_\nu = \Omega_{\nu}/\Omega_m) \)
CMB

Galaxy clustering surveys

Lyman-α

Large scale matter power spectrum, \( P(k) \)

Wavelength, \( \lambda \) [\( h^{-1} \) Mpc]

Wavenumber, \( k \) [\( h \) Mpc\(^{-1} \)]

\[ \Omega_\nu h^2 = \sum \frac{m_\nu}{93 \text{ eV}} \]

\[ \frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m} \]

\[ \Sigma m_\nu = 0.0 \text{ eV} \]

\[ \Sigma m_\nu = 1 \times 1.2 \text{ eV} \]

\( f_\nu = \text{Neutrino fraction} \)
\[ \text{CMB} \]

Lyman-\( \alpha \)

\[ \sum m_\nu = \frac{\Omega_\nu}{\Omega_m} \approx \frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m} \]

\[ \Omega_\nu h^2 = \sum \frac{m_\nu}{93 \text{eV}} \]

Galaxy clustering surveys

CMB

Lyman-\( \alpha \)

\( f_\nu = \text{Neutrino fraction} \)
\[ CMB \] 
\[ \text{Lyman-} \alpha \] 
\[ \sum m_\nu \approx 0.0 \text{ eV} \] 
\[ \sum m_\nu = 1 \times 1.2 \text{ eV} \] 
\[ \sum m_\nu = 3 \times 0.4 \text{ eV} \] 
\[ \Delta \equiv \frac{k^3 P(k)}{2 \pi^2} \ll 1 \] 
\[ \Delta P \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m} \] 
\[ \Omega_\nu h^2 = \sum \frac{m_\nu}{93 \text{ eV}} \]
Present status...

95% C.L. upper limit

- WMAP5 only
  Dunkley et al. 2008

- + Galaxy clustering
  Reid et al. 2009

- + Galaxy + SN + HST
  Reid et al. 2009
  Break degeneracies

- + Weak lensing
  Tereno et al. 2008
  Ichiki et al. 2008

... and many more.
2. What we can do in the future...
Photometric galaxy surveys with lensing capacity, $z_{\text{max}} \sim 3$

High-z spectroscopic galaxy surveys, $z > 2$

Radio arrays, $5 < z < 15$
Possible new techniques...

- **Weak lensing**
  - of galaxies
  - of the CMB

  Song & Knox 2004
  Hannestad, Tu & Y$^3$W 2006
  Kitching et al. 2008
  Lesgourgues et al. 2006
  Perotto, Lesgourgues, Hannestad, Tu & Y$^3$W, 2006

- **21 cm emission**

  Mao et al. 2008
  Pritchard & Pierpaoli 2008
  Metcalf 2009

- **ISW effect**

  Ichikawa & Takahashi 2005
  Lesgourgues, Valkenburg & Gaztañaga 2007

- **Cluster abundance**

  Wang et al. 2005
Weak lensing of galaxies/Cosmic shear...

- Distortion (magnification or stretching) of distant galaxy images by foreground matter.

Distortions probe both luminous and dark matter (no galaxy bias problem!)
Galaxies are randomly oriented, i.e., no "preferred direction".

Lensing leads to a "preferred direction".

"Average" galaxy shapes over cell

Lensing leads to a "preferred direction".
Shear map

Weak lensing theory predicts:

Cluster

Void
Weak lensing theory predicts:

- Cluster
- Void

Shear map → Convergence map (projected mass)
Tomography = bin galaxy images by redshift

Tomography probes spectrum evolution and the growth function.
Future surveys with lensing capacity

Pan-STARRS

Dark Energy Survey

Large Synoptic Survey Telescope (LSST)

SNAP

Ground-based

Space-based

2008

2010

2015

201X

Time
Projected sensitivities...

Currently disfavoured at 95% C.L.

Planck (1 year)
Lesgourgues et al. 2006
Perotto, Lesgouruges, Hannestad, Tu & Y^3W 2006

+ Weak lensing with LSST (tomography)
Hannestad, Tu & Y^3W 2006
Kitching et al. 2008
Projected sensitivities...

WARNING!
These sensitivities can be achieved only if theoretical predictions of the matter power spectrum are accurate to \( \sim 1\% \).
3. The nonlinear matter power spectrum...
Nonlinearities...

N-body

Semi-analytic (PT, RG)

Total matter power spectrum, $P(k)$

Galaxies

CMB

<1%

"Linear"

$\Delta \equiv \frac{k^3 P(k)}{2 \pi^2} \ll 1$

Linear theory

+Nonlinear corrections

Wavenumber, $k$ [h Mpc$^{-1}$]
Nonlinearities...

\[ \Delta \equiv \frac{k^3 P(k)}{2 \pi^2} \ll 1 \]

\( \equiv k \)

N-body

Semi-analytic (PT, RG)

Total matter power spectrum, \( P(k) \)

Wavenumber, \( k \) [h Mpc\(^{-1}\)]

Galaxies

CMB

Weak lensing

<1%
N-body simulations with massive neutrinos...

- **Particle representation** for both CDM and neutrinos.
  - Modified GADGET-2.
  - Neutrino particles drawn from Fermi-Dirac distribution.

Brandbyge, Hannestad, Haugbølle & Thomsen 2008
Brandbyge and Hannestad 2008, 2009
Change in the total matter power spectrum relative to the $f_\nu = 0$ case:

$$\frac{\Delta P_m}{P_m} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$

Linear perturbation theory:

$$\frac{\Delta P_m}{P_m} \sim 8 \frac{\Omega_\nu}{\Omega_m}$$

With nonlinear corrections:

$$\frac{\Delta P_m}{P_m} \sim 9.8 \frac{\Omega_\nu}{\Omega_m}$$

Power suppression due to neutrino free-streaming is larger than predicted by linear perturbation theory.
Nonlinearities...

\[ \Delta \equiv \frac{k^3 P(k)}{2 \pi^2} \ll 1 \]

- CMB: \text{<1%}
- Galaxies: \text{"Linear"}
- Weak lensing

\[ \text{Total matter power spectrum, } P(k) \]

\[ \text{Wavenumber, } k \text{ [h Mpc}^{-1}] \]
Perturbation theory and resummation/RG techniques...

- Going beyond linear perturbation theory?

✓ Nonlinear correction

\[
\begin{align*}
\dot{\delta}_c + \theta_c & = 0 \\
\dot{\theta}_c + H \theta_c + \nabla^2 \Phi & = 0
\end{align*}
\]

CDM: Linearised continuity eqn

Linearised Euler eqn

× No nonlinear correction

Neutrinos

\[
\frac{\partial (\delta f)}{\partial \tau} + \frac{p}{m_\nu a} \cdot \nabla (\delta f) - a m_\nu \nabla \Phi \frac{\partial f_0}{\partial p} = 0
\]

Linearised Vlasov eqn

\[
\nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m \left[ f_c \delta_c + f_\nu \delta_\nu \right]
\]

Poisson eqn

But see Shoji & Komatsu 2009

Y^3W in prep
Corrections to the CDM component...

- Fluid description (linear):

  Continuity eqn
  \[ \dot{\delta}_c(k, \tau) + \theta_c(k, \tau) = 0 \]

  Euler eqn
  \[ \dot{\theta}_c(k, \tau) + H \dot{\theta}_c(k, \tau) - k^2 \Phi(k, \tau) = 0 \]

  Poisson eqn
  \[ k^2 \Phi = -\frac{3}{2} H^2 \Omega_m \left[ f_c \delta_c + f_\nu \delta_\nu \right] \]

  \( \delta_c = \) CDM density perturbations
  \( \delta_\nu = \nu \) density perturbations
  \( \theta_c = \) Divergence of velocity field
Corrections to the CDM component...

- Fluid description (incl. **some** nonlinear terms):
  
  **Continuity eqn**
  \[
  \dot{\delta}_c(k, \tau) + \theta_c(k, \tau) = -\int d^3 q_1 d^3 q_2 \gamma_{121}(k, q_1, q_2) \theta_c(q_1, \tau) \delta_c(q_2, \tau)
  \]

  **Euler eqn**
  \[
  \dot{\theta}_c(k, \tau) + H \theta_c(k, \tau) - k^2 \Phi(k, \tau) = -\int d^3 q_1 d^3 q_2 \gamma_{222}(k, q_1, q_2) \theta_c(q_1, \tau) \theta_c(q_2, \tau)
  \]

  **Poisson eqn**
  \[
  k^2 \Phi = -\frac{3}{2} H^2 \Omega_m \left[ f_c \delta_c + f_\nu \delta_\nu \right]
  \]

- Vertex
  \[
  \gamma_{121}(k, q_1, q_2) \equiv \delta D(k - q_{12}) \frac{q_{12} \cdot q_1}{q_1^2}
  \]

- Vertex
  \[
  \gamma_{222}(k, q_1, q_2) \equiv \delta D(k - q_{12}) \frac{q_{12}^2 (q_1 \cdot q_2)}{2 q_1^2 q_2^2}
  \]

- \( \delta_c = \) CDM density perturbations
- \( \delta_\nu = \nu \) density perturbations
- \( \theta_c = \) Divergence of velocity field

Starting point of **most** semi-analytic calculations in the literature.
Standard perturbation theory...

- Solve by perturbative expansion:

\[
\varphi(k, \tau) \equiv \begin{pmatrix}
\delta_c(k, \tau) \\
-\theta_c(k, \tau)/H
\end{pmatrix},
\quad \varphi(k, \tau) = \sum_{m=1}^{\infty} \varphi^{(n)}(k, \tau)
\]

- nth order solution:

\[
\varphi^{(n)}(k, \tau) = g_{ab}(\tau, 0) \varphi^{(n)}_b(k, 0) \\
+ \int d^3 q_1 \int d^3 q_2 \int_0^{\tau} d \tau' g_{ab}(\tau, \tau') \gamma_{bcd}(k, q_1, q_2) \sum_{m=1}^{n-1} \varphi^{(n-m)}_c(q_1, \tau') \varphi^{(m)}_d(q_2, \tau')
\]
\[ \varphi(k) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \ldots \]

Density/Velocity

\[ \varphi = \frac{\varphi^{(1)}(\tau)}{g_{ab}(\tau, 0)} + k \left[ \frac{\varphi^{(1)}(0)}{y} + \tau' \left( k q_1 - q_2 \right) \right] + \ldots \]

Cocce & Scoccimarro 2006
Matarrese & Pietroni 2007
\[ \varphi(k) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \ldots \]

**Density/Velocitiy**

\[ \varphi_{a}^{(1)}(\tau) \frac{k}{g_{ab}(\tau, 0)} \varphi_{b}^{(1)}(0) + \frac{k}{\tau} q_{1} + k q_{2} + \ldots \]

**Power spectrum**

\[ P(k) \delta_{D}(k + k') \equiv \langle \varphi(k) \varphi(k') \rangle = \langle \varphi^{(1)} \varphi^{(1)} \rangle + \langle \varphi^{(2)} \varphi^{(2)} \rangle + 2 \langle \varphi^{(1)} \varphi^{(3)} \rangle + \ldots \]

**Linear**

**“One-loop” correction**
Free-streaming suppression: One-loop corrected...

Thin lines = linear
Thick lines = 1-loop corrected

Change in power spectrum relative to the $f_{\nu} = 0$ case:

$$\frac{\Delta P}{P} \equiv \frac{P_{f_{\nu} \neq 0}(k) - P_{f_{\nu} = 0}(k)}{P_{f_{\nu} = 0}(k)}$$
Free-streaming suppression: One-loop corrected...

Thin lines = linear
Thick lines = 1-loop corrected

$\nu \sim 0.1$

$\nu \sim 0.05$

$\nu \sim 0.01$

N-body simulations, Brandbyge et al. 2008

$Y^3W$ 2008
also Saito et al. 2008, 2009
Many schemes have been proposed that go beyond standard perturbation theory:

- Crocce & Scoccimarro 2006, 2008
- Taruya & Hiramatsu 2007
- McDonald 2007
- Matsubara 2008
- Valageas 2007
- Pietroni 2008
- Hiramatsu & Taruya 2009
- etc.
- Applied to massive neutrino cosmologies:

\[ \sum m_\nu = 0.3\text{ eV} \]

\[ \sum m_\nu = 0.6\text{ eV} \]

Lesgourgues, Matarrese, Pietroni & Riotto 2009
Summary...

- Using the **large-scale structure distribution** to probe **neutrino physics** is still fun.
  - We can do **even better** in the **future** with forthcoming probes/new techniques.

- But we must make sure our **theoretical predictions** are **reliable** (1% accurate) at the (nonlinear) scales of interest.
  - **N-body simulations** are the definitive way to go.
  - **Semi-analytic PT & RG techniques** are also of some (limited) use.