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"Learning from Astroparticle, Underground, Neutrino Physics and Cosmology"

Nuclear matrix elements for neutrino-less double beta decay

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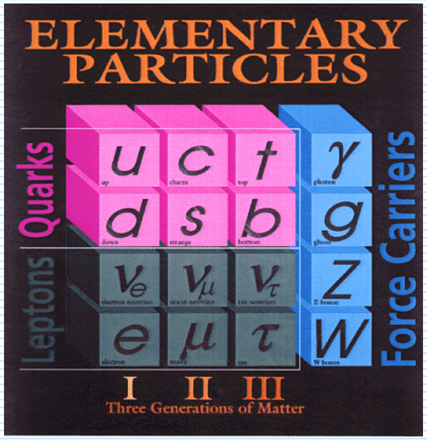
Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics

***APS Joint Study on the Future of Neutrino Physics* (physics/0411216)**
We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first in the list of recommendations)

ASPERA road map:

- ***Requirement for construction and operation of two double-beta decay experiments with a European lead role or shared equally with non-European partners (GERDA, COBRA, CUORE, SuperNEMO)***
- ***Different nuclear isotopes are necessary to minimize the impact of uncertainties in matrix elements to the extracted information about neutrino properties.***
- ***We finally reiterate the importance of assessing and reducing the uncertainty in our knowledge of the corresponding nuclear matrix elements, experimentally and theoretically.***



Standard Model Lepton Universality

Particle	Symbol	Anti − p.	mass [MeV]	L_e	L_μ	L_τ	life − time [s]
electron	e^-	e^+	0.511	1	0	0	stable
el.neutrino	ν_e	$\overline{\nu}_e$	$< 2.2\ 10^{-6}$	1	0	0	stable
muon	μ^-	μ^+	105.6	0	1	0	$2.2\ 10^{-6}$
muon neutr.	ν_μ	$\overline{\nu}_\mu$	< 0.19	0	1	0	stable
tau	τ^-	τ^+	1777.	0	0	1	$2.9\ 10^{-13}$
tau neutrino	ν_τ	$\overline{\nu}_\tau$	< 18.2	0	0	1	stable

Lepton Family Number Violation

NEW PHYSICS
massive neutrinos, SUSY...

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}$, $\overline{\nu}_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	observed
$\mu^+ \to e^+ + \gamma$	$R \le 1.2 \times 10^{-11}$
$\mu^+ \to e^+ + e^- + e^+$	$R \le 1.0 \times 10^{-12}$
$K^+ \to \pi^+ + e^- + \mu^+$	$R \le 4.7 \times 10^{-12}$
$\tau^- \to e^- + \mu^+ + \mu^-$	$R \le 1.8 \times 10^{-6}$
$Z^0 \to e^\pm + \mu^\mp$	$R \le 1.7 \times 10^{-6}$
$\mu_b^- + (A,Z) \to (A,Z) + e^-$	$R \le 1.2 \times 10^{-11}$

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	not observed
$K^+ \to \pi^- + e^+ + \mu^+$	$R \le 5 \times 10^{-10}$
$\tau^- \to \pi^- + \pi^+ + e^+$	$R \le 1.9 \times 10^{-6}$
$W^- + W^- \to e^- + e^-$	
$(A,Z) \to (A,Z+2) + e^- + e^-$	$T^{0\nu} \ge 1.9 \times 10^{-25}$
$\mu_b^- + (A,Z) \to (A,Z-2) + e^+$	$R \le 3.6 \times 10^{-11}$
$e^- + e^- \to \pi^- + \pi^-$	?

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \;,$$

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \; m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

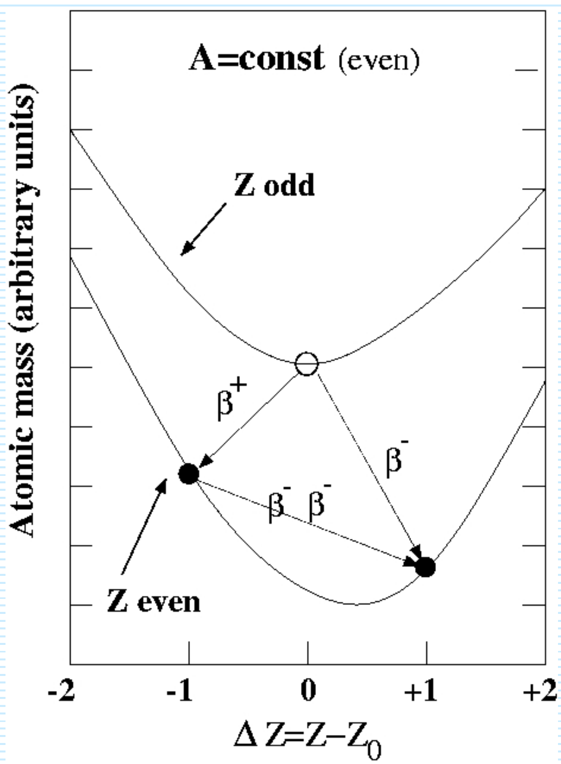
CP-violating phases

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$\beta\beta$ emitters with $Q_{\beta\beta}$ >2 Mev

Transition	$Q_{\beta\beta}$ (keV)	Abundance (%) ($^{232}Th = 100$)
$^{110}Pd \to ^{110}Cd$	2013	12
$^{76}Ge \to ^{76}Se$	2040	8
$^{124}Sn \to ^{124}Te$	2288	6
$^{136}Xe \to ^{136}Ba$	2479	9
$^{130}Te \to ^{130}Xe$	2533	34
$^{116}Cd \to ^{116}Sn$	2802	7
$^{82}Se \to ^{82}Kr$	2995	9
$^{100}Mo \to ^{100}Ru$	3034	10
$^{96}Zr \to ^{96}Mo$	3350	3
$^{150}Nd \to ^{150}Sm$	3667	6
$^{48}Ca \to ^{48}Ti$	4271	0.2

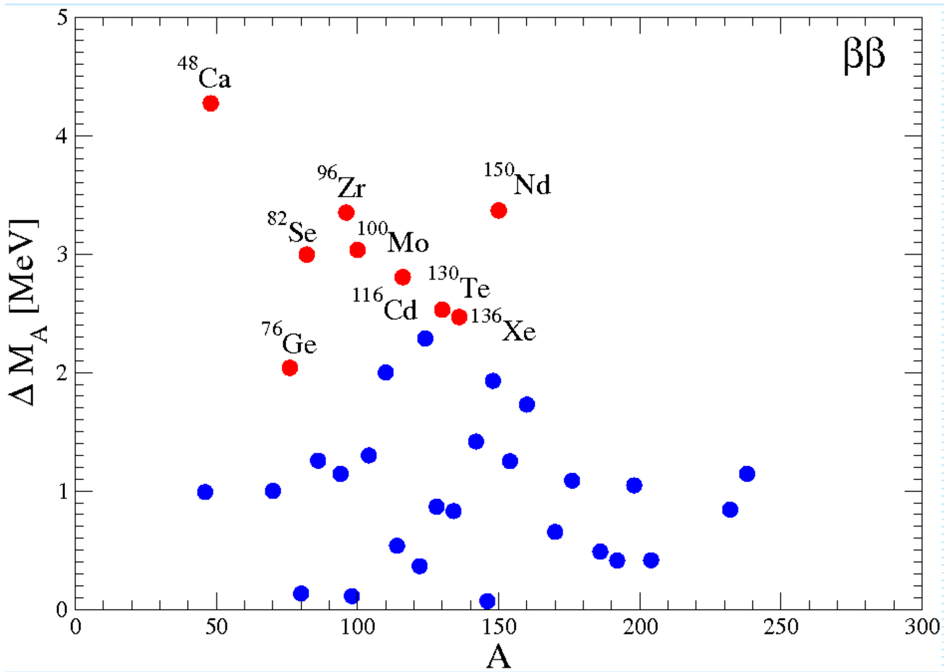
Double Beta Decay Nuclei of experimental interest

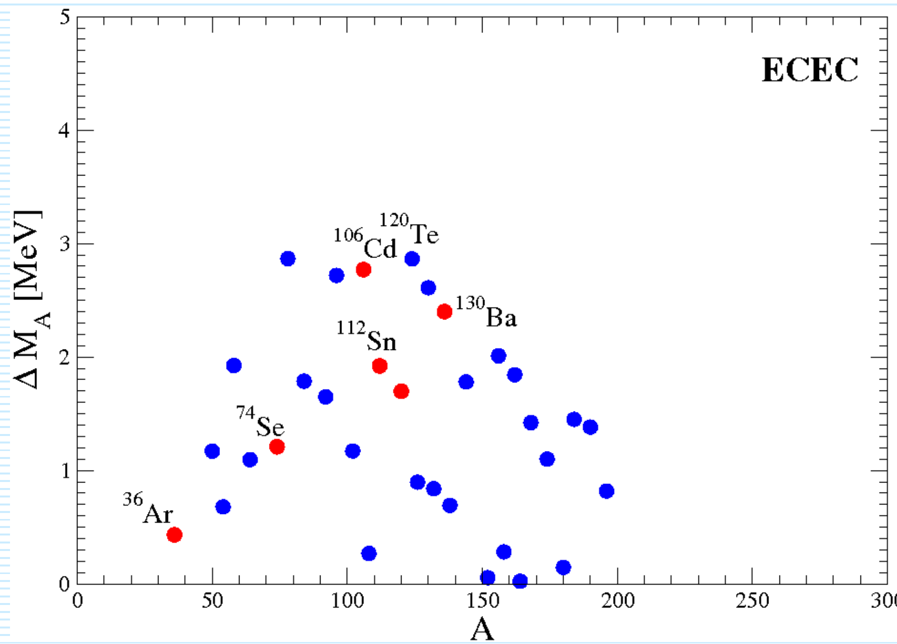
Emission of two electrons

$(A,Z) \rightarrow (A,Z+2) + e + e$

Double electron capture

$e_b + e_b + (A,Z) \rightarrow (A,Z)^*$





Preferable nuclear systems with large ΔM_A (E^5)

Nuclear systems with small ΔM_A might be also important (resonant enhancement)

Signal from γ- and X-rays

If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved

1) *What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).*

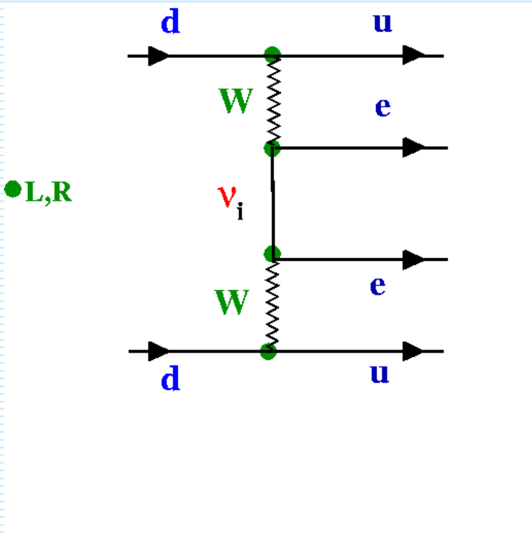
2) *How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements.*

The $0\nu\beta\beta$-decay mechanisms

Two basic categories are
long-range (exchange of light Majorana ν)
and
short-range (exchange of heavy ν, squarks, gluinos …)
contributions to the $0\nu\beta\beta$-decay

Light neutrino Mass mechanism

$$\mathcal{H}^{\beta}_{W} = \frac{G_F}{\sqrt{2}}\, \bar{e}\gamma_\alpha(1+\gamma_5)\nu_e\, j_\alpha + h.c. \qquad \nu_{eL} = \sum_k U^L_{lk}\, \chi_{kL}$$



$$< \nu_{eL}(x_1)\nu_{eL}{}^T(x_2) > = -\sum_k \left(U^L_{ek}\right)^2 \xi_k \frac{1+\gamma_5}{2} S_k(x_1-x_2)\frac{1+\gamma_5}{2}C$$

$$= \frac{i}{(2\pi)^4}\sum_k \left(U^L_{ek}\right)^2 \xi_k m_k \int \frac{e^{iq(x_1-x_2)}dq}{q^2+m_k^2}\frac{1+\gamma_5}{2}C$$

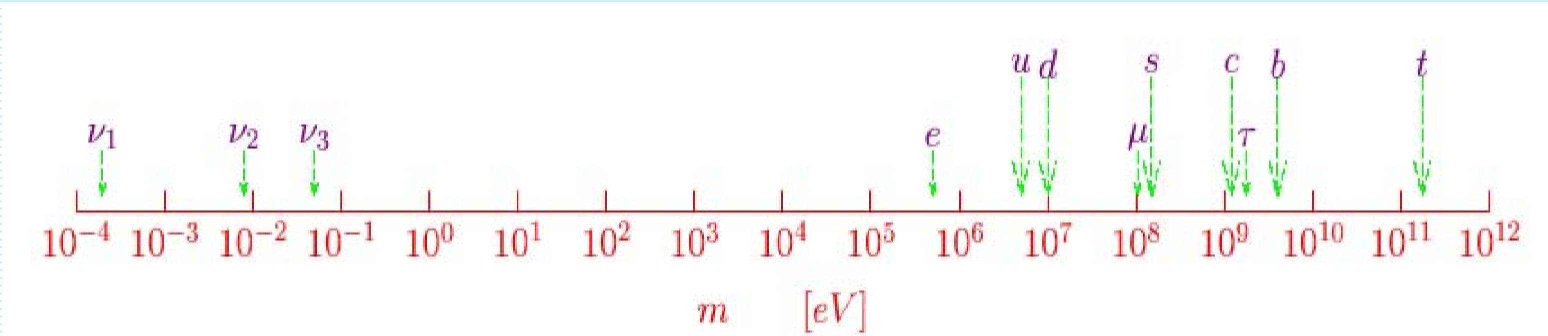
Effective mass of Majorana neutrinos

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\left(\overline{\nu}_L \quad \overline{(\nu_R)^c}\right)\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$

$m_1 = m_D^2/M_R \ll m_D \quad m_2 \approx M_R$

$\nu_1 = \nu_L - m_D/M_R\,(\nu_R)^c \quad \nu_2 = \nu_R + m_D/M_R\,(\nu_L)^c$



Squark mixing RPV SUSY

Neutrino vertex

$$\mathcal{L}^{LH} = \frac{G_F}{\sqrt{2}} \sum_i U_{ei} \, (\bar{e}\gamma_\alpha(1-\gamma_5)\nu) \, (\bar{u}\gamma^\alpha(1-\gamma_5)d) + h.c. \quad (V-A)$$

R-parity violating SUSY vertex

Hirsch, Klapdor-Kleingrothaus, Kovalenko
PLB 372 (1996) 181

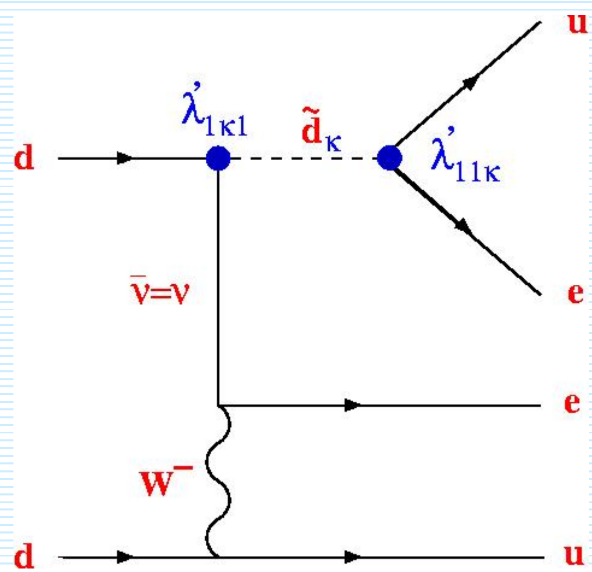
$$\mathcal{L}^{eff}_{SUSY} = \frac{G_F}{\sqrt{2}} \left(\frac{1}{4}\eta_{(q)LR} \sum_i U^*_{ei} \, (\bar{\nu}(1+\gamma_5)e) \, (\bar{u}(1+\gamma_5)d) \quad (S,P) \right.$$

$$\left. + \frac{1}{8}\eta_{(q)LR} \sum_i U^*_{ei} \, (\bar{\nu}\sigma_{\alpha\beta}(1+\gamma_5)e) \, (\bar{u}\sigma^{\alpha\beta}(1+\gamma_5)d) + h.c. \right) \quad (Tensor)$$

Paes, Hirsch, Klapdor-Kleingrothaus,
PLB 459 (1999) 450

LN-violating parameter

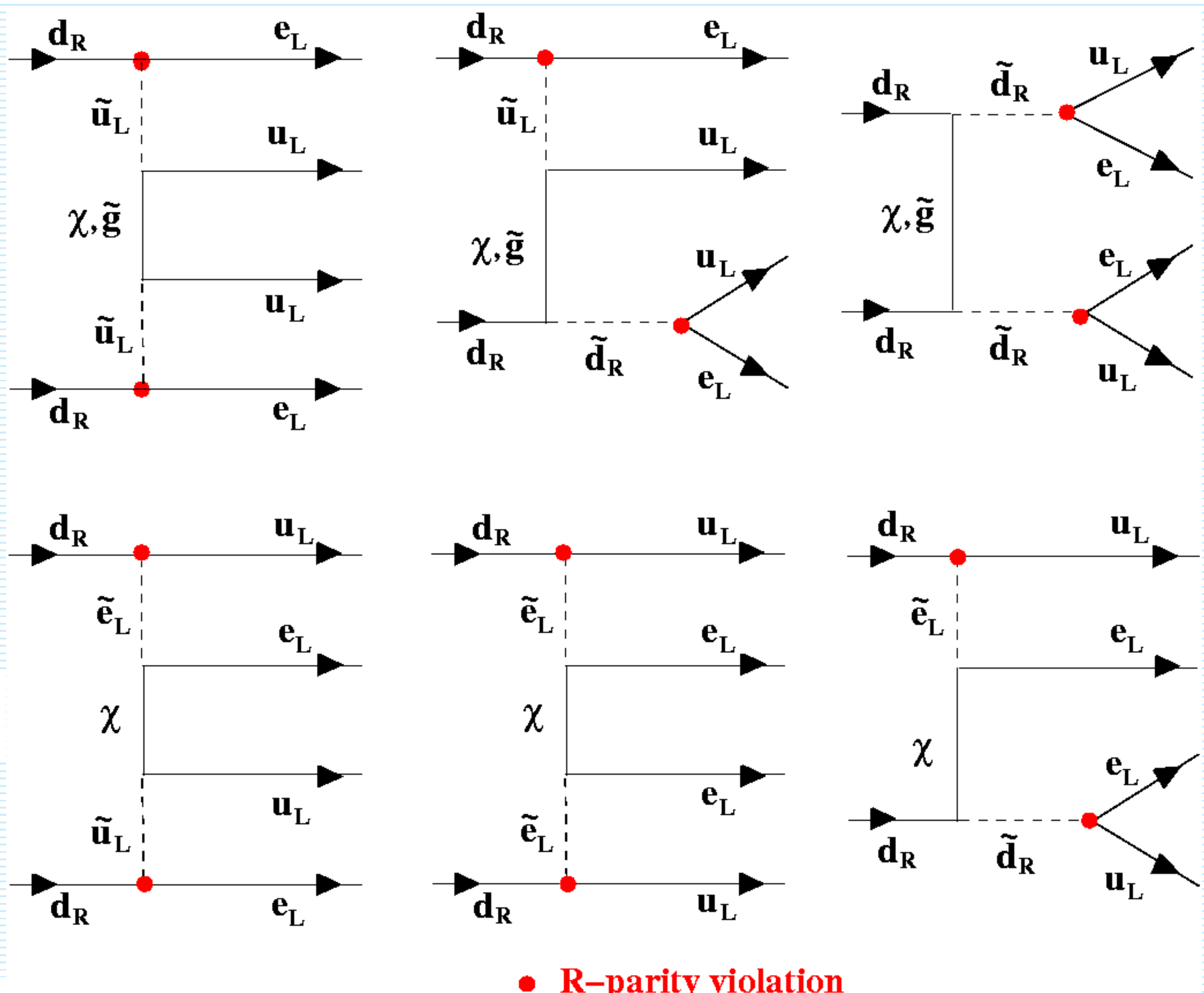
$$\eta_{(q)LR} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{8\sqrt{2}G_F} \sin 2\theta^d_{(k)} \left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}} \right)$$



gluino/neutralino exchange R-parity breaking SUSY mechanism of the $0\nu\beta\beta$–decay

quark-level diagrams

$d + d \rightarrow u + u + e^- + e^-$

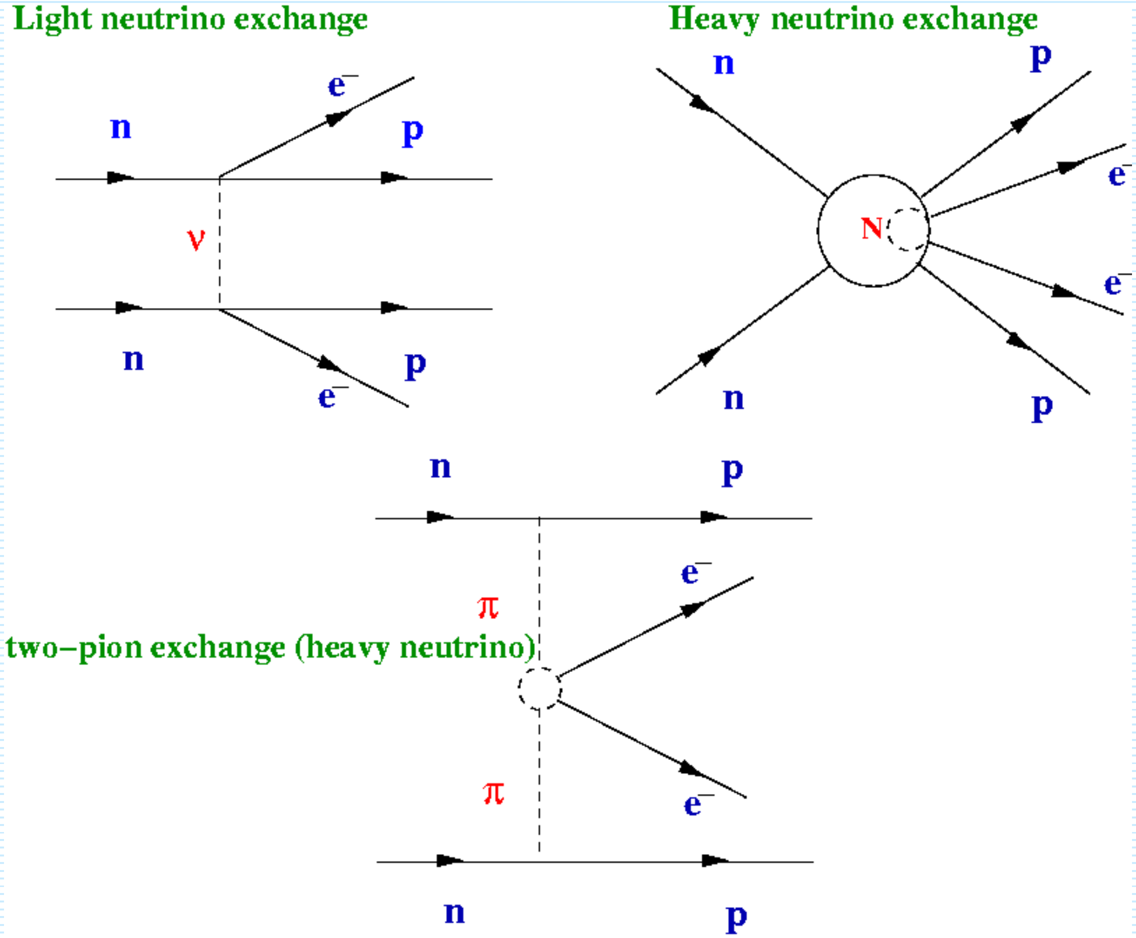


exchange of **squarks, neutralinos** and **gluinos**

$(\lambda'_{111})^2$ mechanism

1987 R. Mohapatra, J.D. Vergados

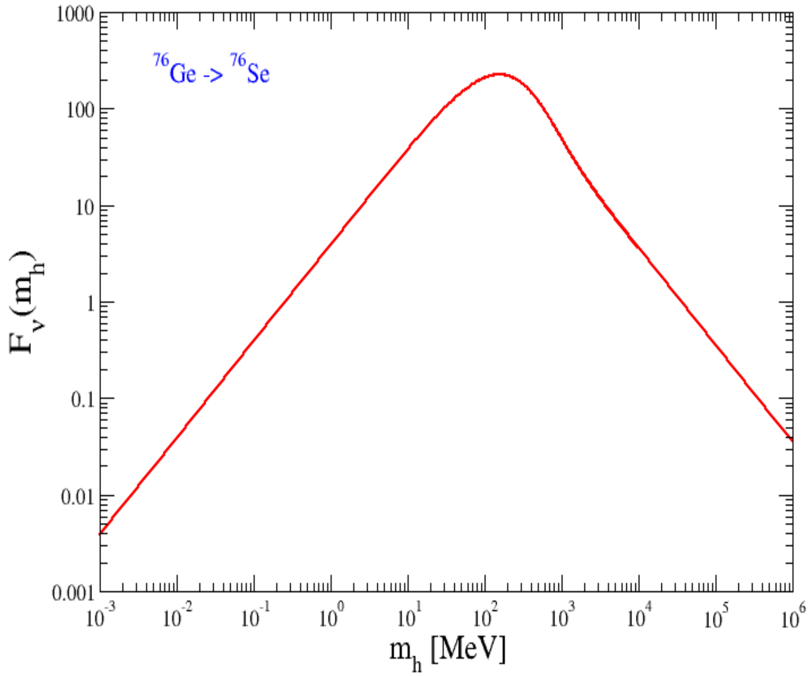
nucleon level

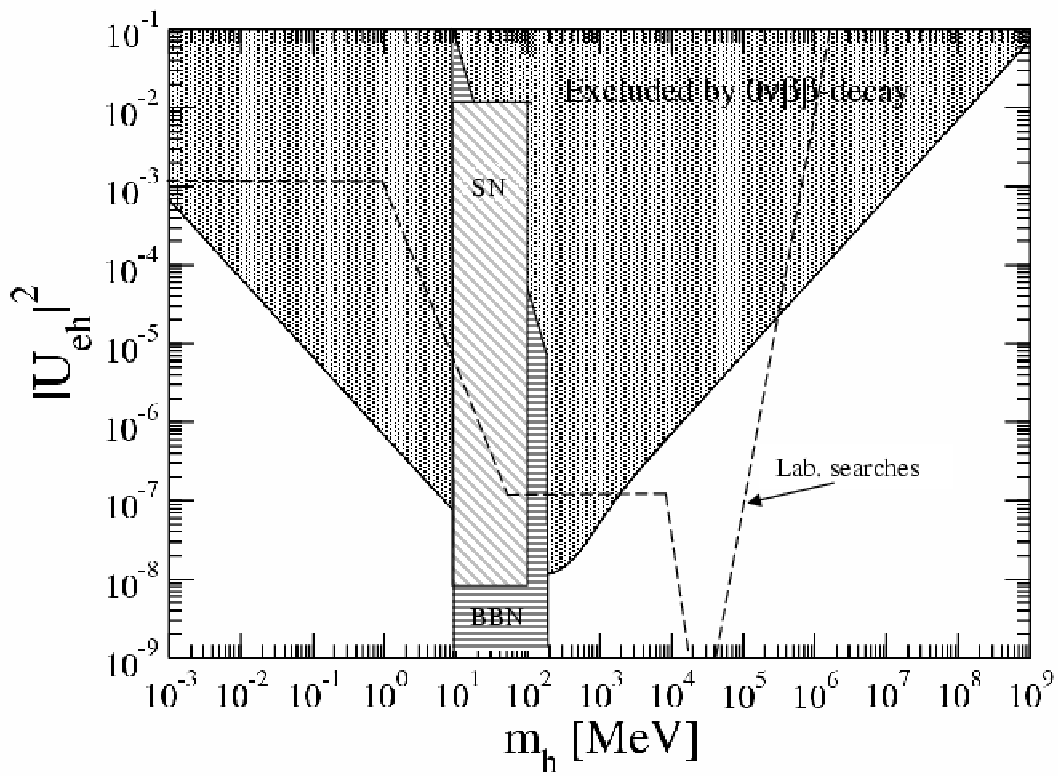


Sterile neutrino in 0νββ-decay

Matrix element depends on ν-mass

$$[T_{1/2}^{0\nu}]^{-1} = G_{01} \left| \frac{\langle m_\nu \rangle_{ee}}{m_e} M_\nu^{light} + U_{eh}^2 \frac{m_h}{m_e} M^{0\nu}(m_h) \right|^2 .$$





$$F_\nu(m_h) = \frac{m_h}{m_e} M^{0\nu}(m_h)$$

$$|U_{eh}|^2 \le \frac{1}{|F_\nu(m_h)|} \frac{1}{\sqrt{T_{1/2}^{0\nu-exp} G_{01}}},$$

Beneš, Faesler, F.Š., Kovalenko, PRD 71 (2005) 077901

Calculation of Nuclear Matrix Elements

Nuclear Matrix Elements

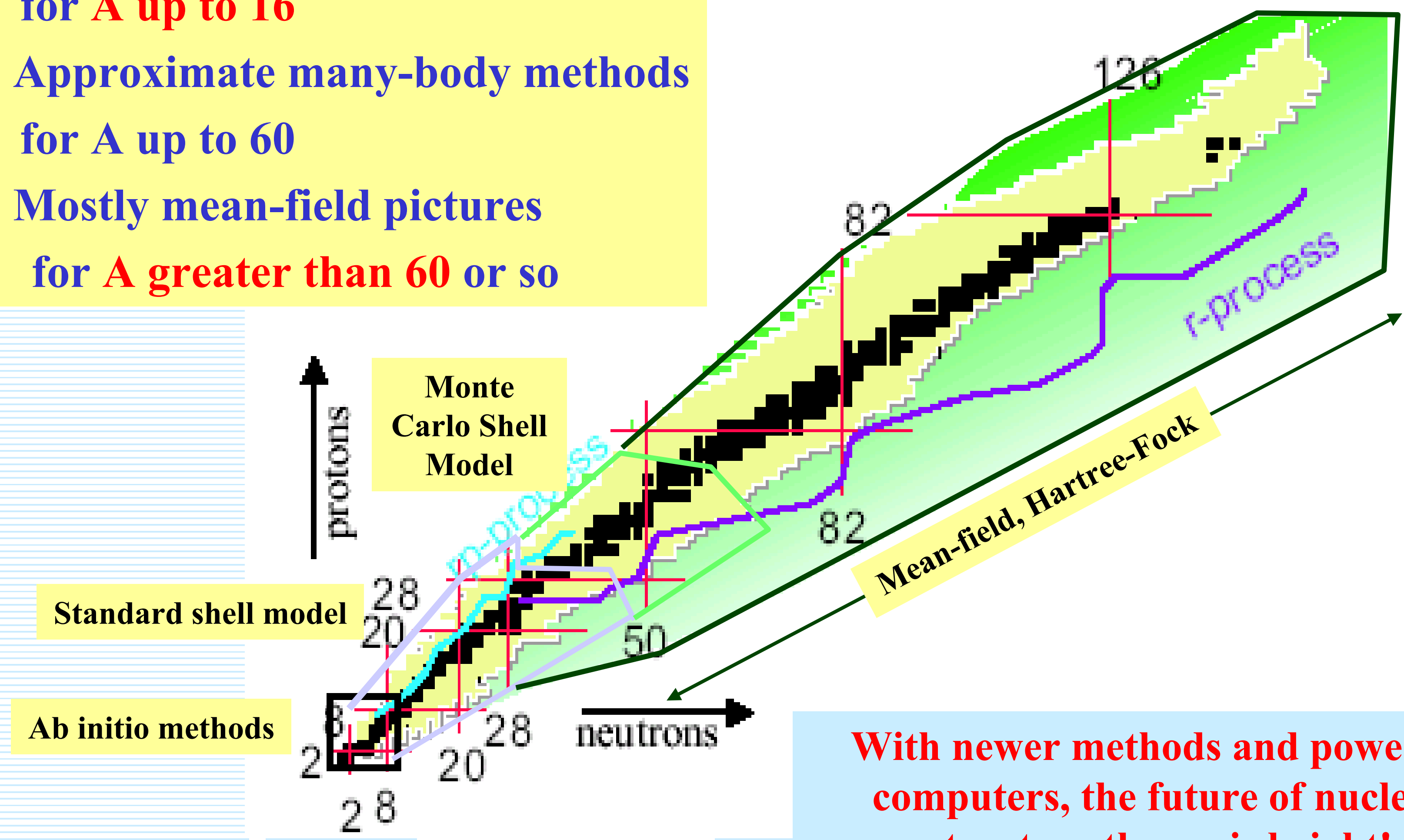
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge the quality of the result.

Nuclear Structure

- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so



With newer methods and powerful computers, the future of nuclear structure theory is bright!

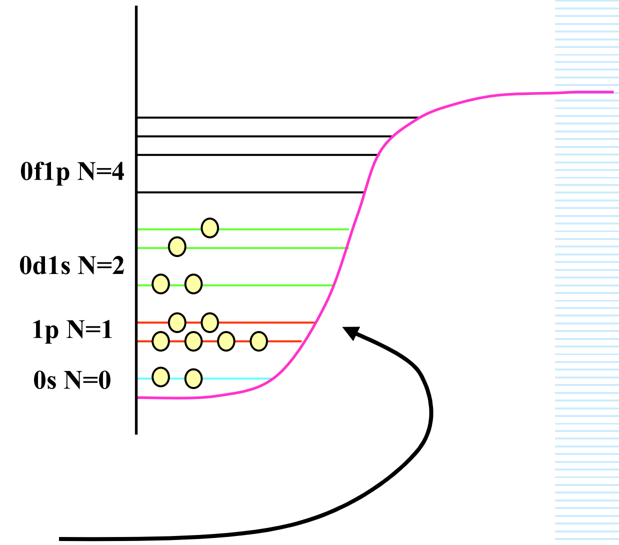
Many-body Hamiltonian

- **Start with the many-body Hamiltonian**

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right)$$

- **Introduce a mean-field *U* to yield basis**

$$H = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + U(r_i)\right) + \underbrace{\sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right) - \sum_i U(r_i)}_{\text{Residual interaction}}$$



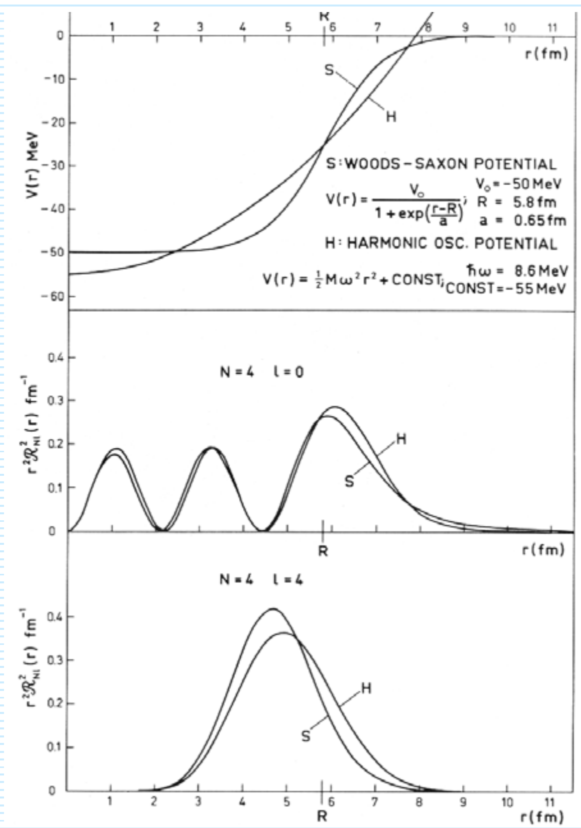
The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- **The mean field determines the shell structure**
- **In effect, nuclear-structure calculations rely on perturbation theory**

Origin of the shell model

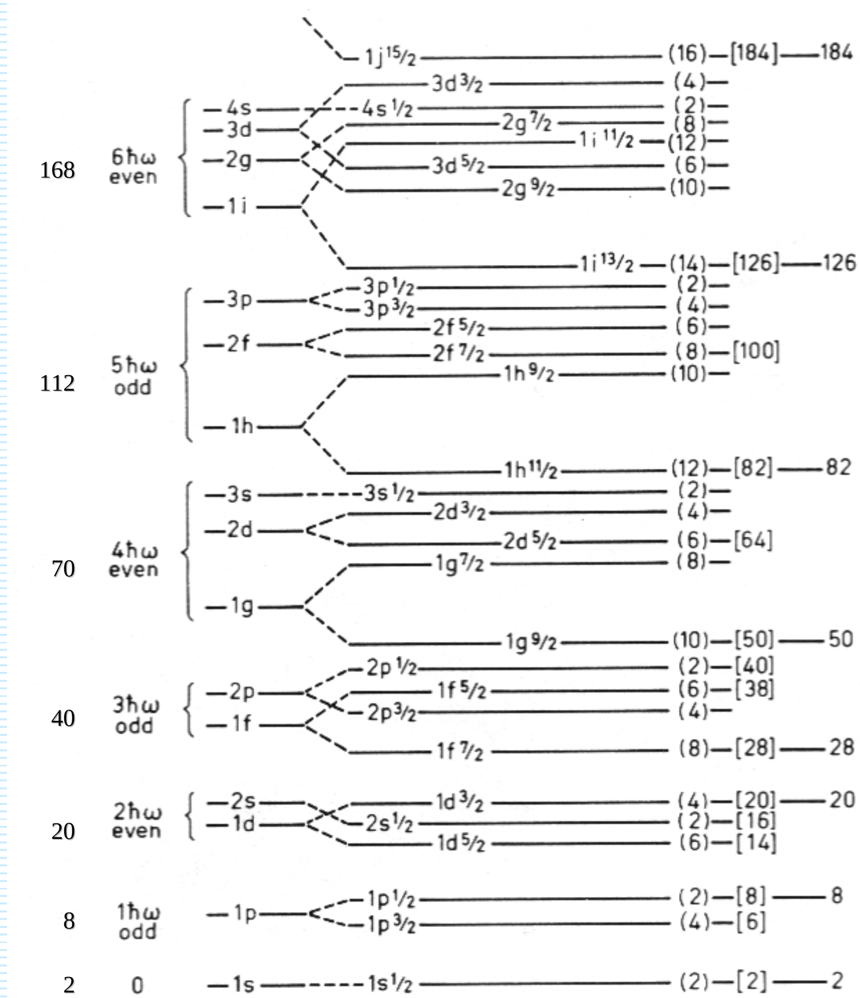
Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers

Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field



$$V(r) = \frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$$V(r) = \tfrac{1}{2}M\omega^2 r^2 + \mathrm{CONST}$$



M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955

Two complementary procedures are commonly used:

- **Nuclear shell model (NSM)**
- **Quasiparticle Random Phase Approximation (QRPA)**

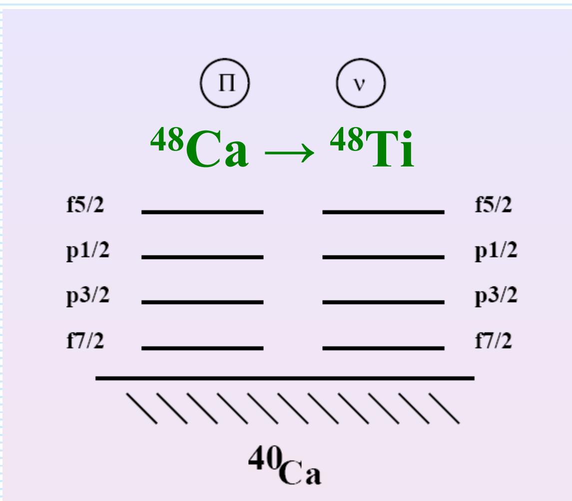
In NSM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$-decay calculations

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations

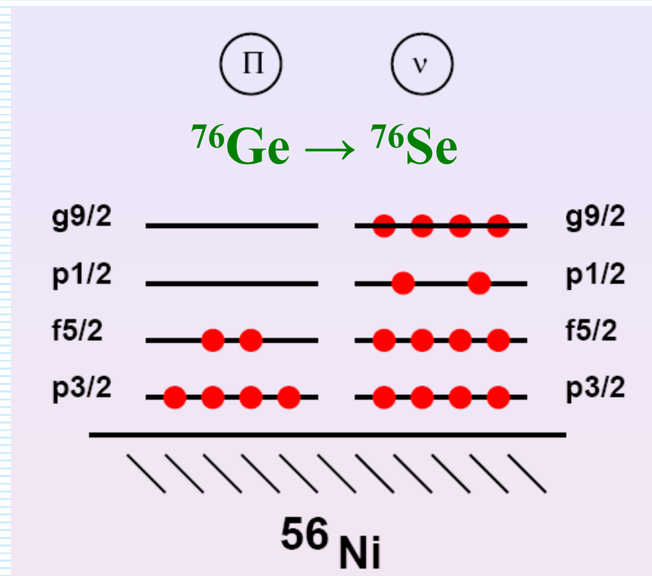
Nuclear Shell Model

$$H = \sum_a \varepsilon_a a_a^+ a_a - \sum_{abcd} \frac{\langle j_a j_b; JT | V | j_c j_d; JT \rangle_A}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \left[\left[a_a^+ \otimes a_b^+ \right]^{JT} \otimes \left[\tilde{a}_c \otimes \tilde{a}_d \right]^{JT} \right]^{00}_{00}$$

- Define a valence space
- Derive an effective interaction $H\,\Psi = E\,\Psi \rightarrow H_{eff}\,\Psi_{eff} = E\,\Psi_{eff}$
- Build and diagonalize Hamiltonian matrix (10^{10})
- Transition operator $<\Psi_{eff}\,|\,O_{eff}\,|\,\Psi_{eff}>$
- Phenomenological input:
  Energy of states, systematics of B(E2) and GT transitions (quenching f.)



Small calculations

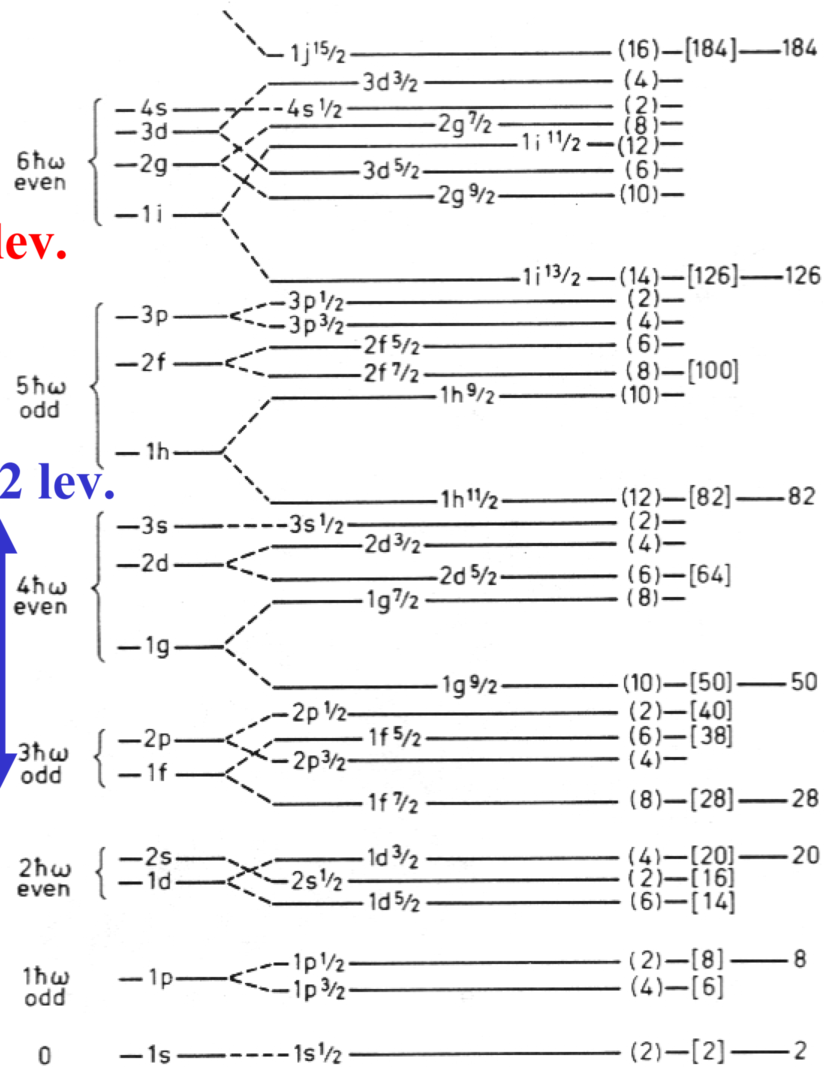


$^{76}Se_{42}$ in the valence
6 protons and 14 neutrons

Quasiparticle Random Phase Approximation (QRPA) and its variants

21 lev.

12 lev.



- **Large model space** **(up 23 s.p.l,** **^{150}Nd – 60 active prot. and 90 neut.)**
- **Spin-orbit partners included**
- **Possibility to describe all multipolarities of the intermed. nucl.** **J^π ($\pi=\pm1$, J=0…9)**

$$H = H_0 + g_{ph}\, V_{ph} + g_{pp}\, V_{pp}$$

quasiparticle mean field

Residual interaction

Only Bratislava-Tuebingen group

Realistic NN-interactions used in the QRPA calculations

Brueckner G-matrices from Tuebingen (H. Muether group)

Bethe-Goldstone equation

$$G = V + V\frac{Q}{W - H_0 + i\varepsilon}G$$

Modern *(phase-shift equivalent) NN* potentials

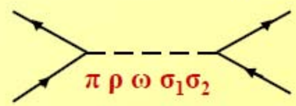
Nijmegen I - (P_D = 5.66%) - 41 parameters - χ^2/N_{data} = 1.03

Nijmegen II - (P_D = 5.64%) - 47 parameters - χ^2/N_{data} = 1.03

Argonne V_{18} - (P_D = 5.76%) - 40 parameters - χ^2/N_{data} = 1.09

CD Bonn - (P_D = 4.85%) - 43 parameters - χ^2/N_{data} = 1.02

based upon the OBE model



(1999 *NN* Database: 5990 *pp* and *np* scattering data)

Renormalization of the *NN* interaction

Difficulty in the derivation of V_{eff} from any modern *NN* potential: existence of a strong repulsive core which prevents its direct use in nuclear structure calculations.

Traditional approach to this problem: Brueckner *G*-matrix method. The *G* matrix is model-space dependent as well as energy dependent.

The vectors X and Y are obtained by solving the equations of motion:

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

Eigenvalue equation for ω^2, unphysical solutions with $\omega^2 < 0$ possible

with

$$\begin{aligned} A^J_{pn,p'n'} &= \langle O|(c^\dagger_p c^\dagger_n)^{(JM)^\dagger} \hat{H} (c^\dagger_{p'} c^\dagger_{n'})^{(JM)}|O\rangle \\ &= \delta_{pn,p'n'}(E_p + E_n) \\ &\quad + (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) g_{ph} \langle pn^{-1}, J|V|p'n'^{-1}, J\rangle \\ &\quad + (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) g_{pp} \langle pn, J|V|p'n', J\rangle \ , \end{aligned}$$

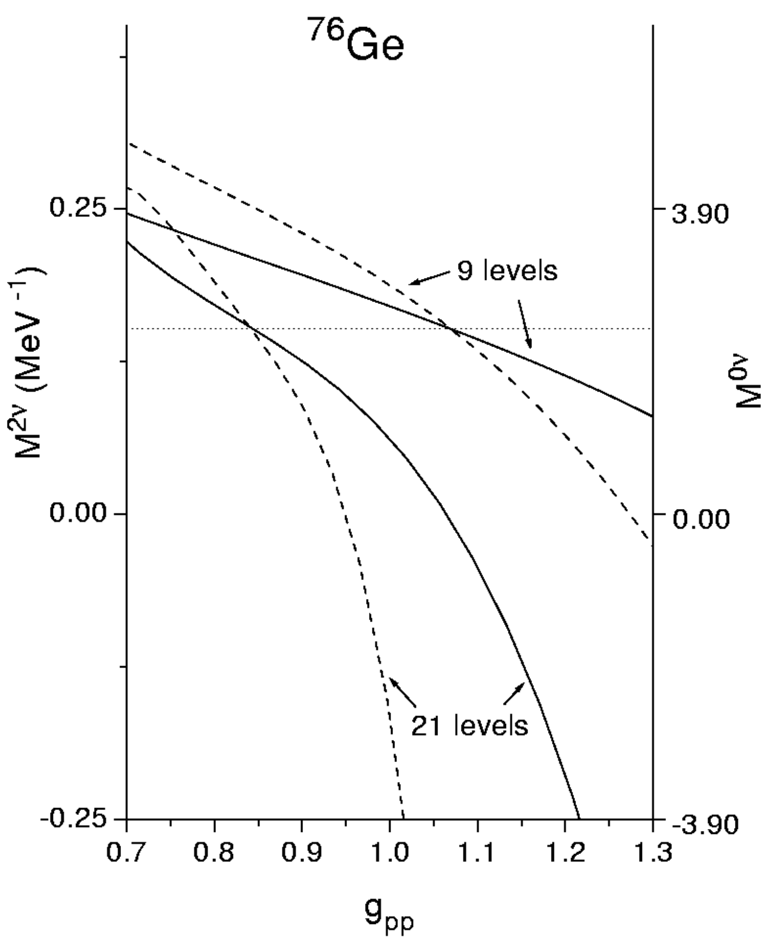
$$\begin{aligned} B^J_{pn,p'n'} &= \langle O|\hat{H}(c^\dagger_p c^\dagger_n)^{(J-M)}(-1)^M (c^\dagger_{p'} c^\dagger_{n'})^{(JM)}|O\rangle \\ &\quad + (-1)^J (u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) g_{ph} \langle pn^{-1}, J|V|p'n'^{-1}, J\rangle \\ &\quad - (-1)^J (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) g_{pp} \langle pn, J|V|p'n', J\rangle \ . \end{aligned}$$

particle-hole

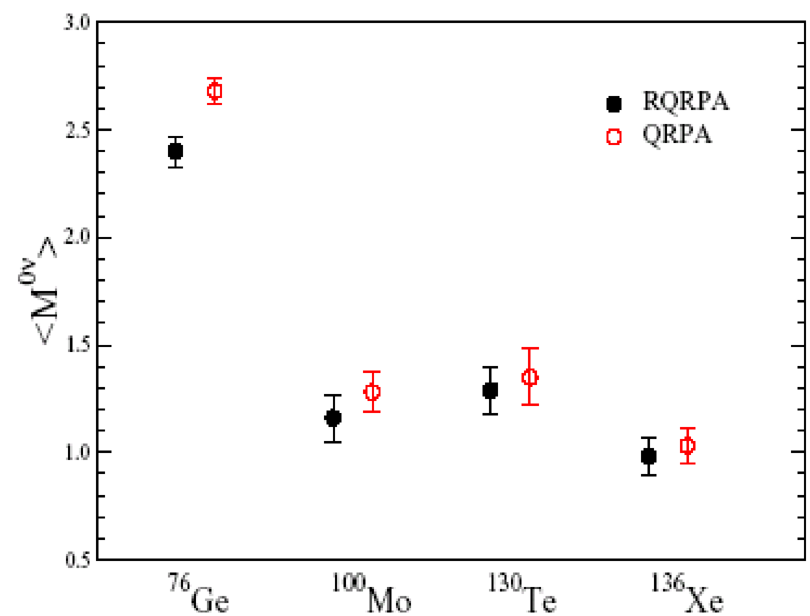
particle-particle

The 0νββ-decay NME: g_{pp} fixed to 2νββ-decay

Each point: (3 basis sets) x (3 forces) = 9 values



By adjusting of g_{pp} to 2νββ-decay half-life the dependence of the 0νββ-decay NME on other things that are not a priori fixed is essentially removed



Rodin, Faessler, F.Š,Vogel, Phys. Rev. C 68, 044302 (2003)

The Interacting Boson Model[¶]

- The low-lying states of the nucleus, composed by n and z valence nucleons, are modeled in terms of (n+z)/2 bosons.
- The bosons have either L = 0 (*s* boson) or L = 2 (*d* boson).
- The bosons can interact through one-body and two-body forces giving rise to bosonic wave functions.
- Any observable can be calculated using these wave functions provided that the relevant operator is employed.

[¶] F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987

Projected Hartree-Fock-Bogoliubov Model

PHFB Model

States of good angular momentum J

$$\left|\Psi_M^J\right\rangle = \frac{2J+1}{8\pi^2 a_J}\int d\Omega\, D_{MK}^J(\Omega)\hat{R}(\Omega)\left|\Phi_K\right\rangle$$

Axially symmetric HFB intrinsic state

$$\left|\Phi_0\right\rangle = \prod_{im}\left(u_{im} + v_{im} b_{im}^{+} b_{i\bar{m}}^{+}\right)$$

where

$$b_{im}^{+} = \sum_m C_{i\alpha m}\, a_{im}^{+} \qquad\qquad b_{i\bar{m}}^{+} = \sum_m (-1)^{l+j-m} C_{i\alpha m}\, a_{i-m}^{+}$$

Hamiltonian:

$$H = H_{sp} + V(P) + \zeta_{qq} V(QQ)$$

Only quadrupole interaction, GT interaction is missing

The 0νββ-decay NME (light ν exchange mech.)

The 0νββ-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)qdq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors:
finite nucleon size

$$h_F = g_V^2(q^2)$$
$$h_{GT} = g_A^2\left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right]$$
$$h_T = g_A^2\left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right]$$

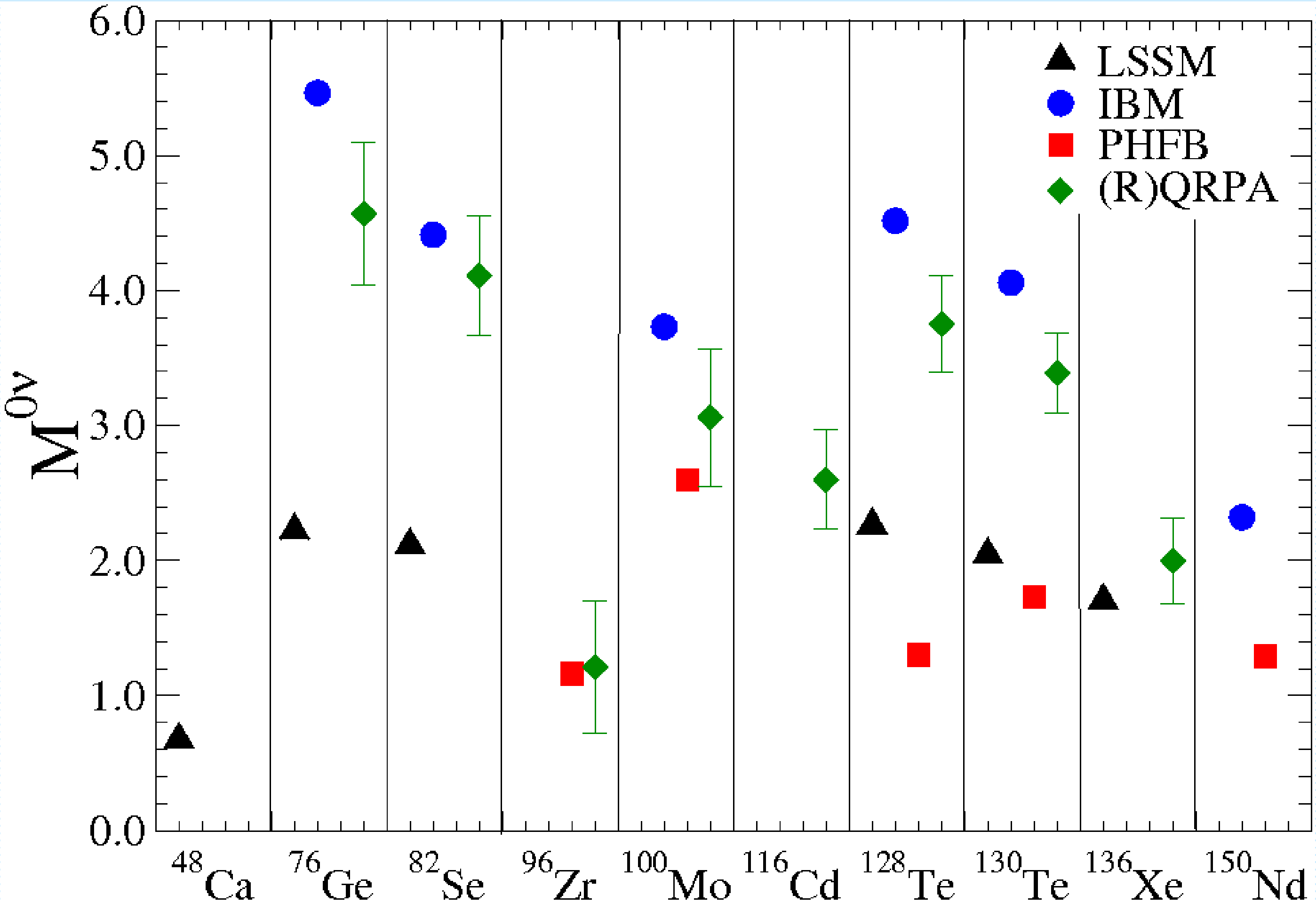
Induced pseudoscalar coupling (pion exchange)

$$M_{K=F,GT,T} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$
$$\times \langle 0_f^+ ||[c_{p'}^+ \tilde{c}_{n'}]_J|| J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f ||[c_p^+ \tilde{c}_n]_J|| 0_i^+\rangle$$

Jastrow f. s.r.c.

J^π = $0^+, 1^+, 2^+$... $0^-, 1^-, 2^-$...

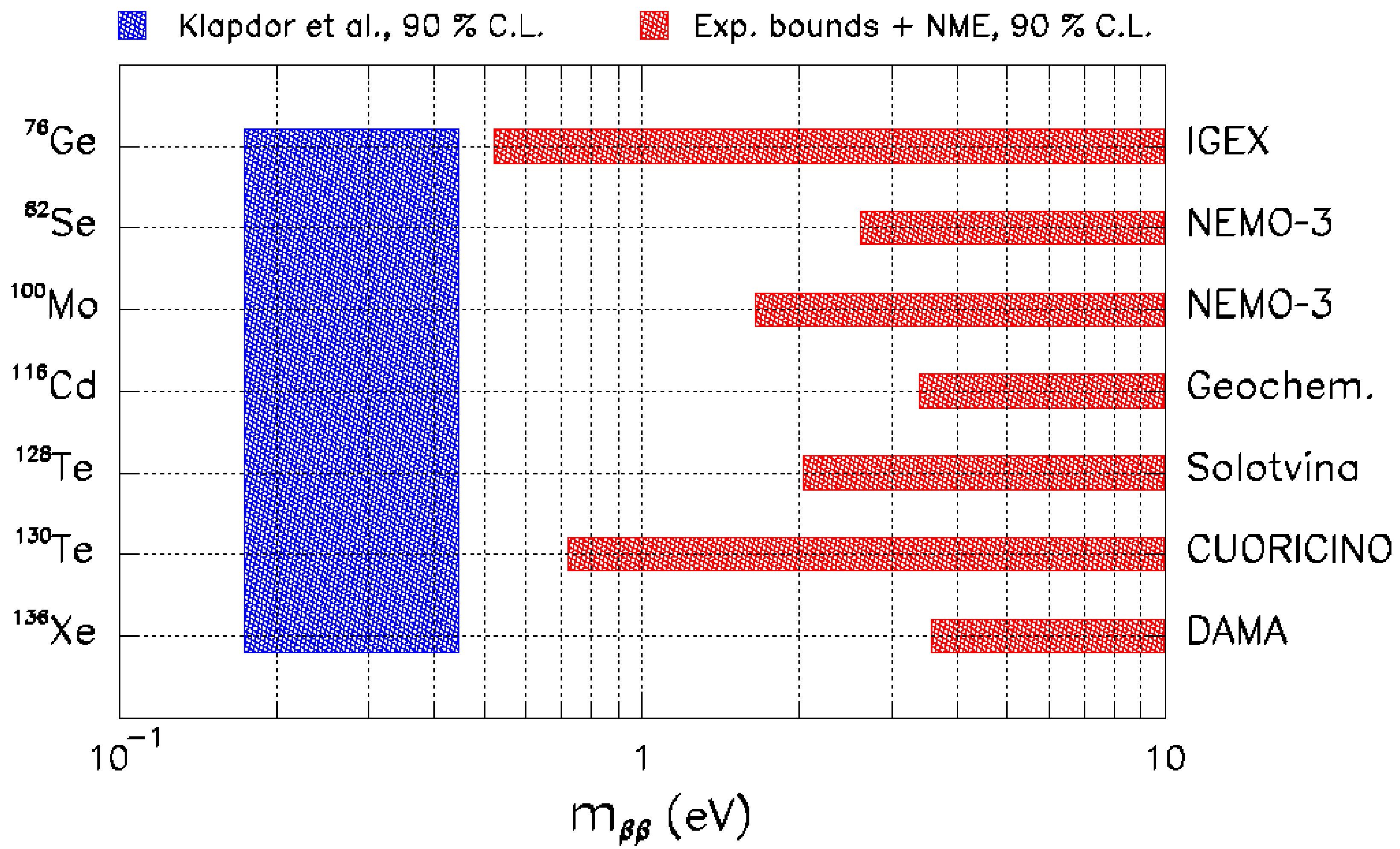
The 0νββ-decay NMEs (Status:2009)



Nobody is perfect:

LSSM (small m.s., negative parity states)
PHFB (GT force neglected)
IBM (Hamiltonian truncated)
(R)QRPA (g.s. correlations not accurate enough)

A claim of evidence and other experiments (current status)

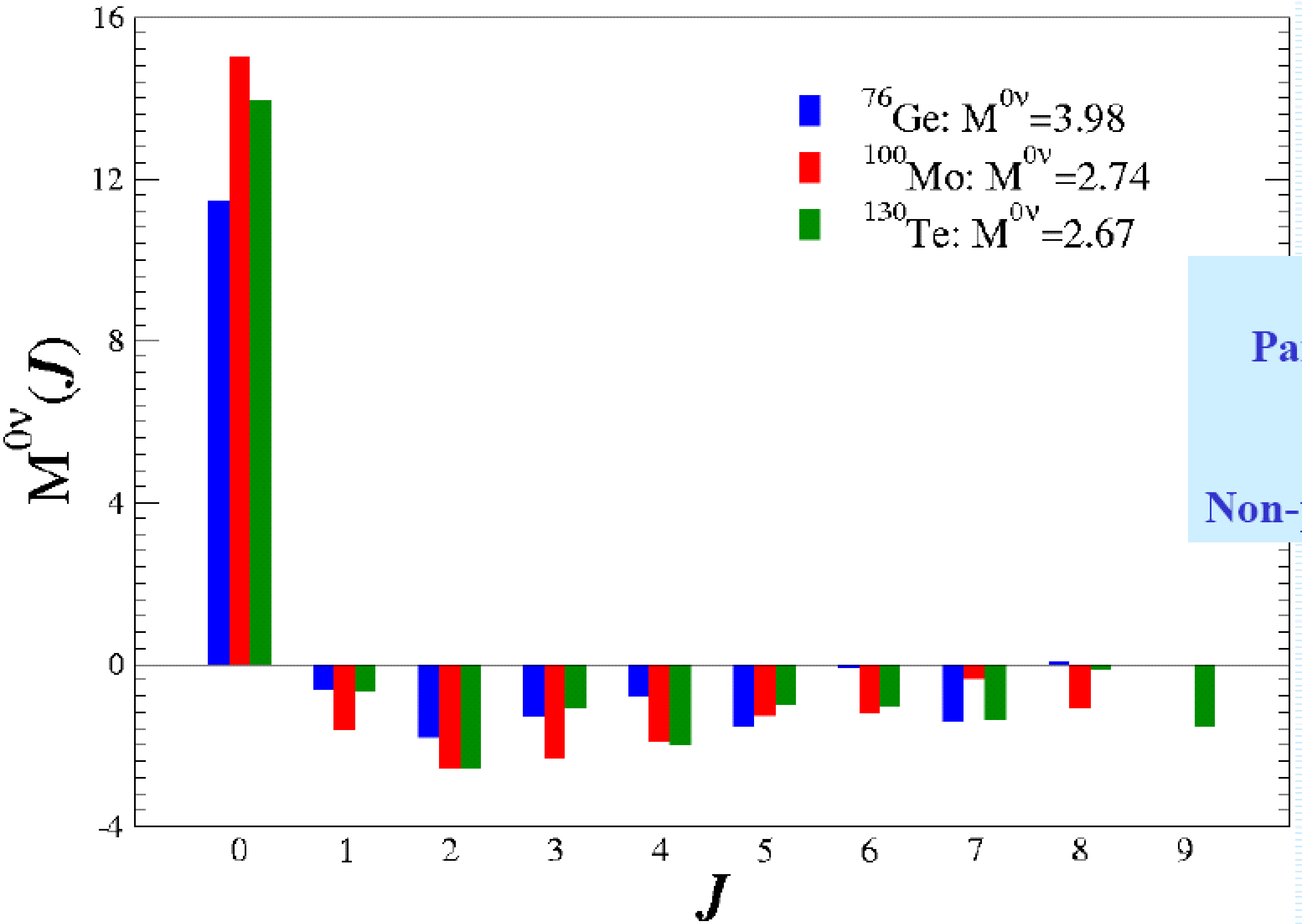


Faessler, Figli, Lisi, Rodin, Rotunno, Šimkovic,

Anatomy of the $0\nu\beta\beta$-decay NMEs

Decomposition in in pp and nn channels

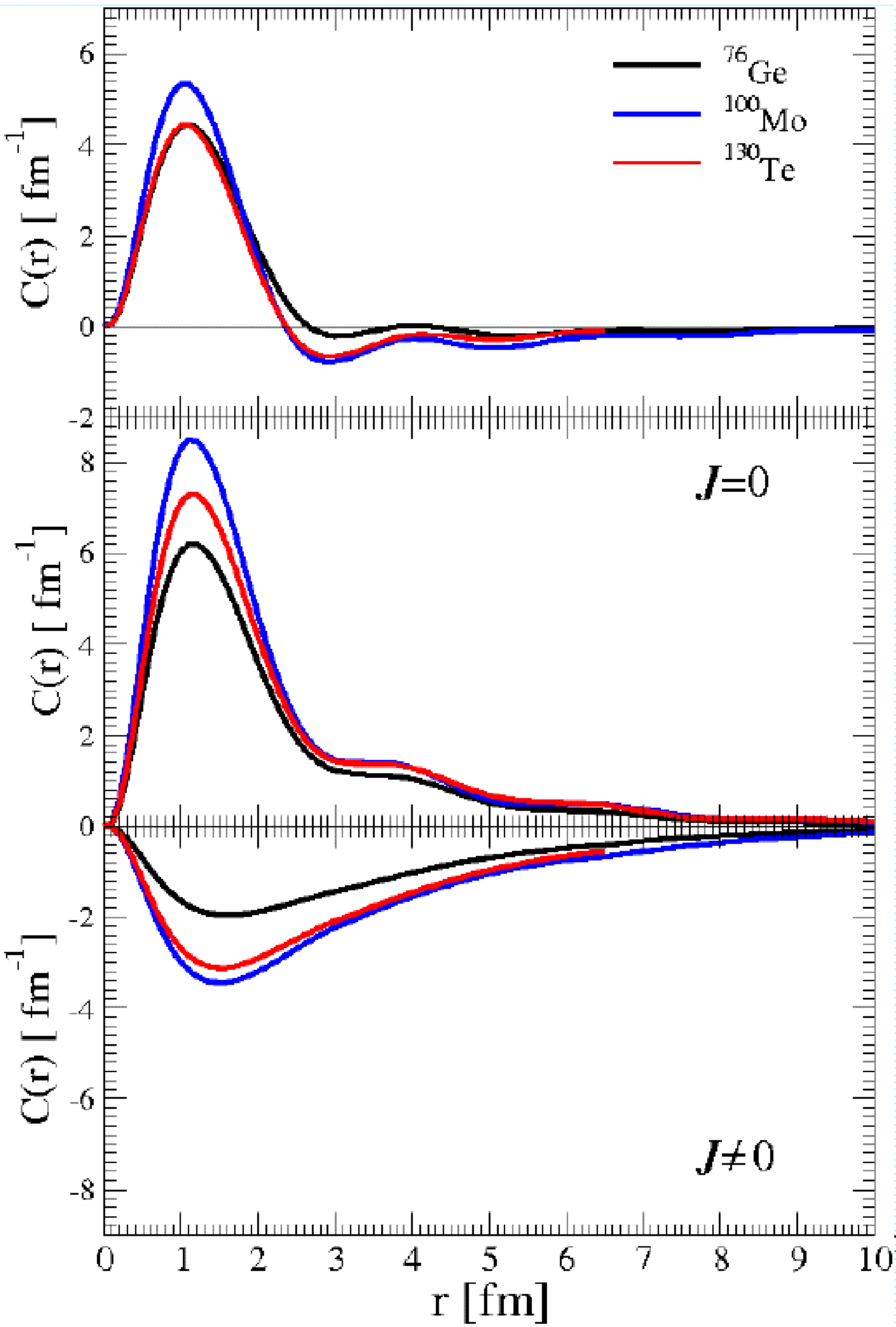
$$\langle p(1), p'(2) ; \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2) ; \mathcal{J} \rangle$$



J=0
Pairing mode

J≠0
Non-pairing mode

F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)



nkovic

r-dependence of the 0νββ-decay NME

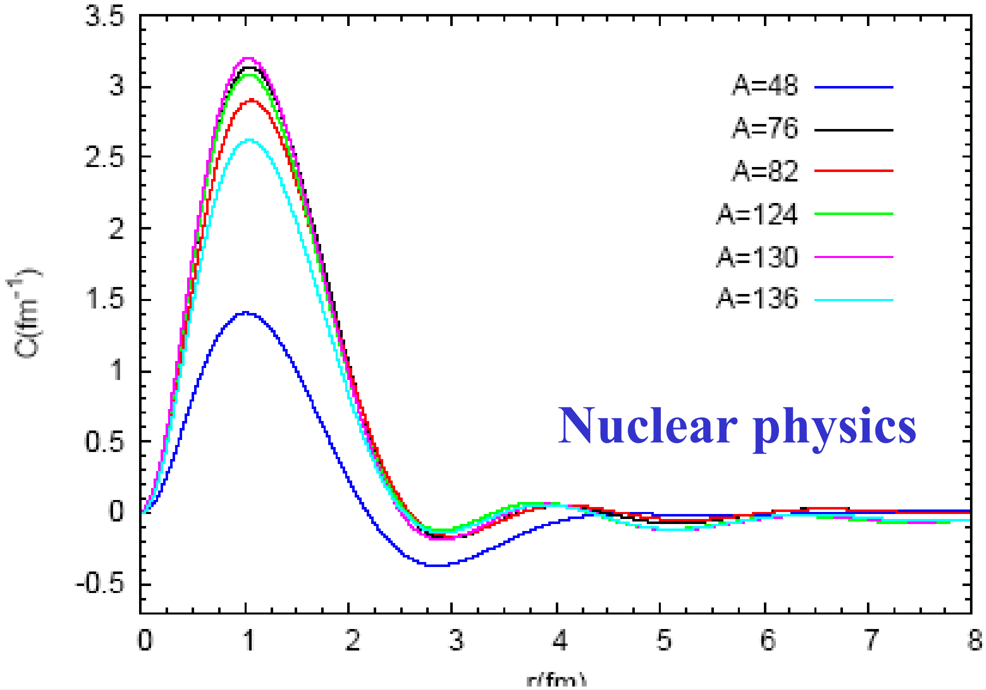
The radial dependence of M^{0n} for the three indicated nuclei. The contributions summed over all components shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below.

Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* and large values of *q* are quite important.

QRPA
F.Š, Faessler, Rodin, Vogel, Engel
PRC 77, 045503 (2008)

Large Scale Shell Model

Menendez, Poves, Caurier, Nowacki,
Arxive:0901.3760 [nucl-th]

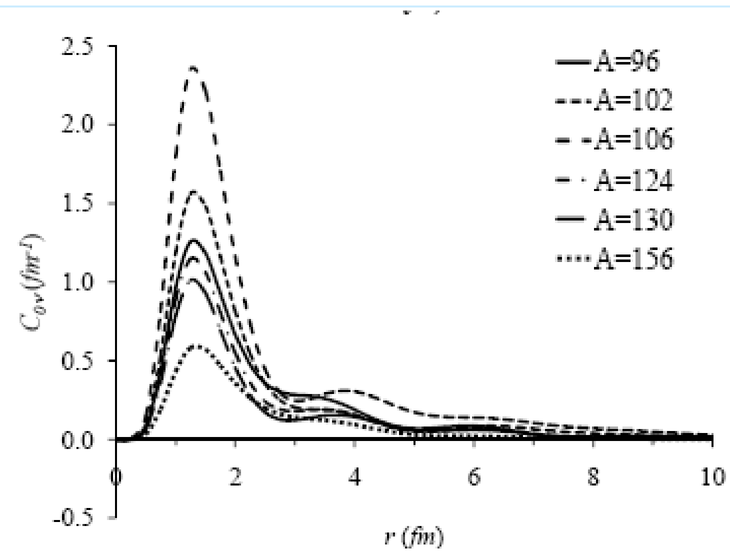


Nuclear physics

Nucleon physics

PHFB

P.Rath, R. Chandra, K. Chaturverdi,
P.Raina, J.G. Hirsch,
to be published in PRC



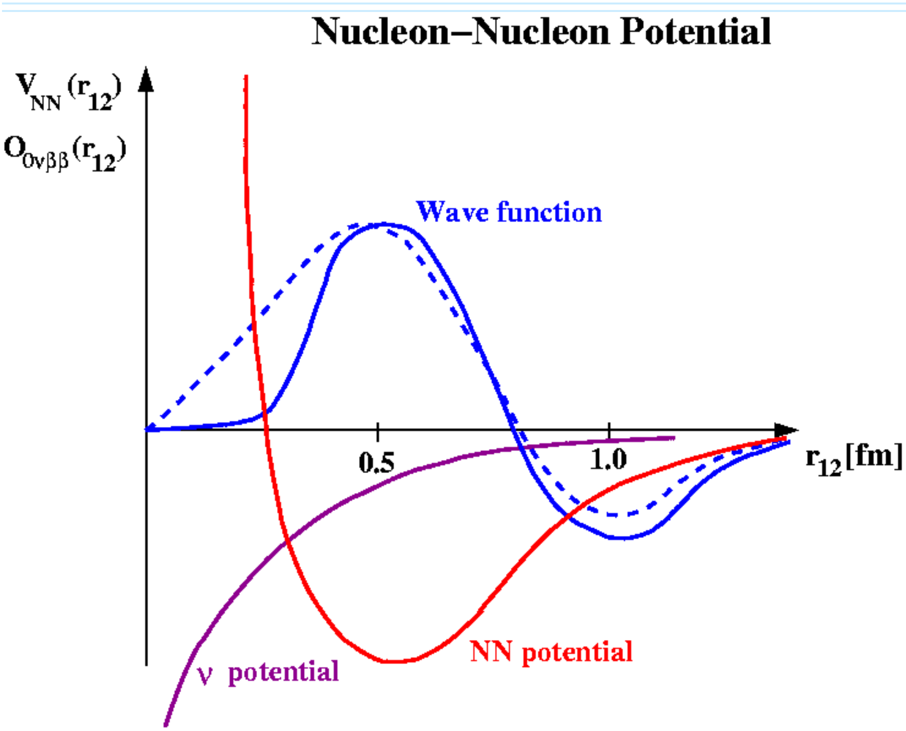
A consistent approach for the 0νββ-decay (pairing, s.r.c, g.s.c. calculated with the same NN potential- BonnCD, Argon)

Neutrino potential: I(r)/r

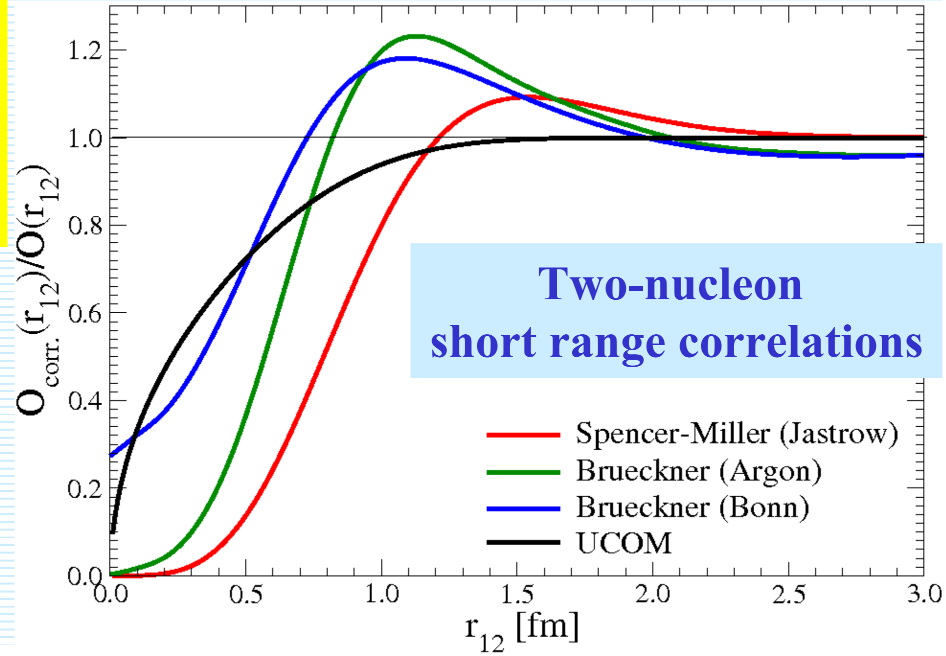
$$I(r) = \frac{2}{\pi}\int_0^\infty \frac{\sin(qr)}{(q+E_{aver})}\frac{dq}{(1+q^2/E_{cut}^2)^4}$$

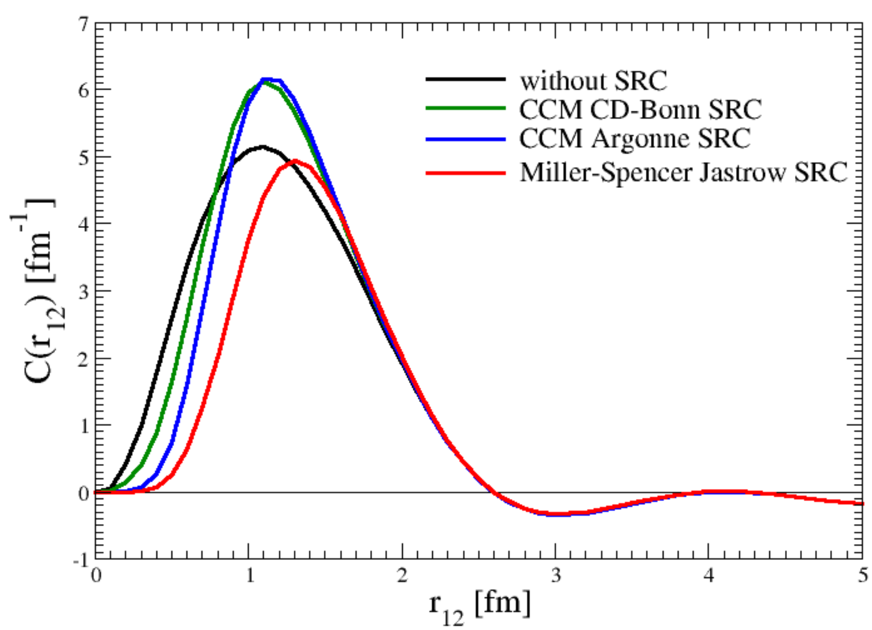
$|\Psi>_{corr.} = f(r_{12})\,|\Psi>$

$O_{corr.}(r_{12}) = f(r_{12})O(r_{12})f(r_{12})$



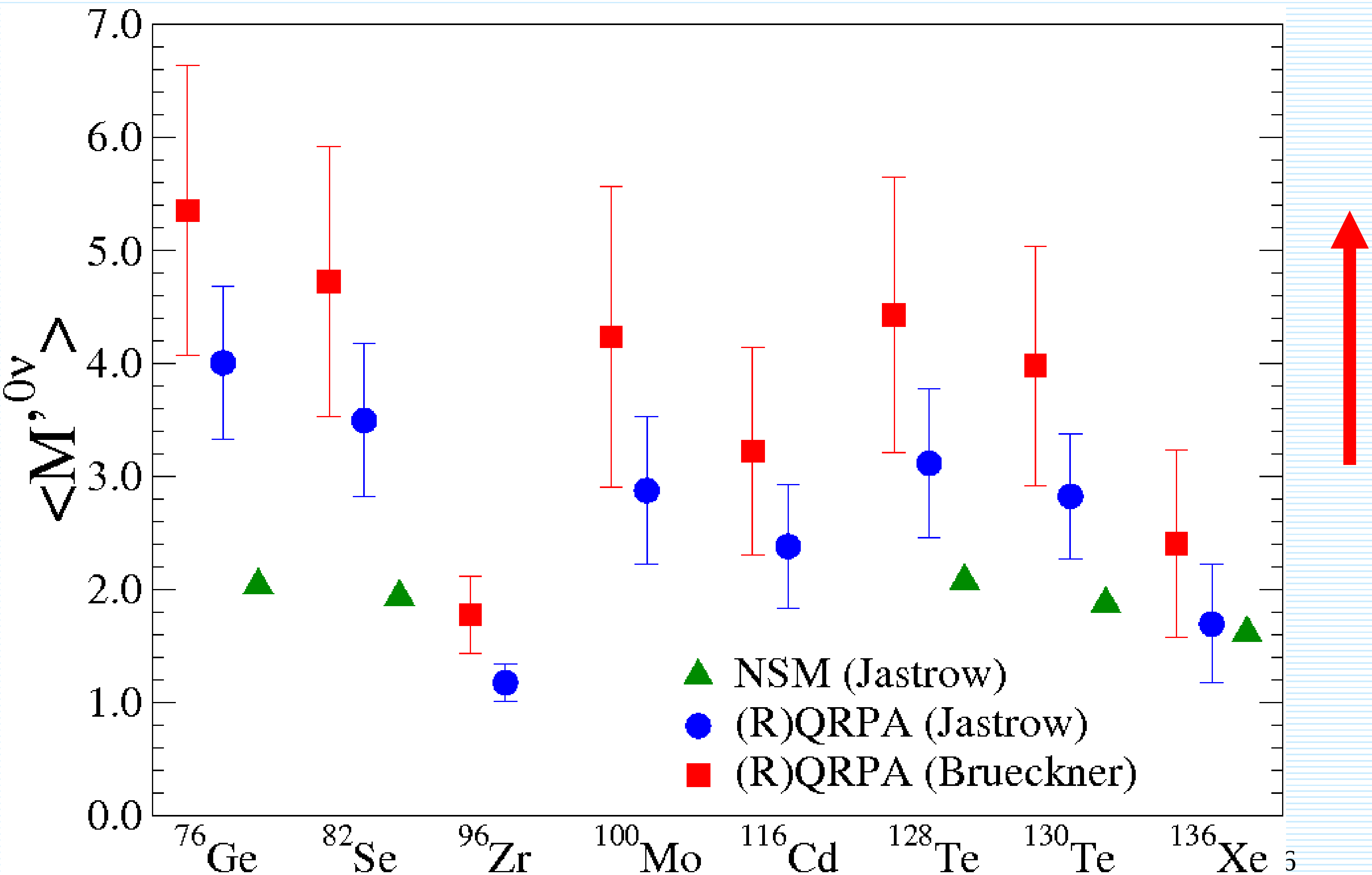
Two-nucleon short range correlations





Neutrinoless double beta decay matrix elements

F.Š., Faessler, Muether, Rodin, Stauf, PRC 79, 055501 (2009)



It is of interest to see the contribution of individual orbits to the 0νββ matrix element. Within QRPA and its generalization this can be done by using the basic formula:

sum over virtual intermediate states

sum over proton and neutron orbits

$$M_K = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

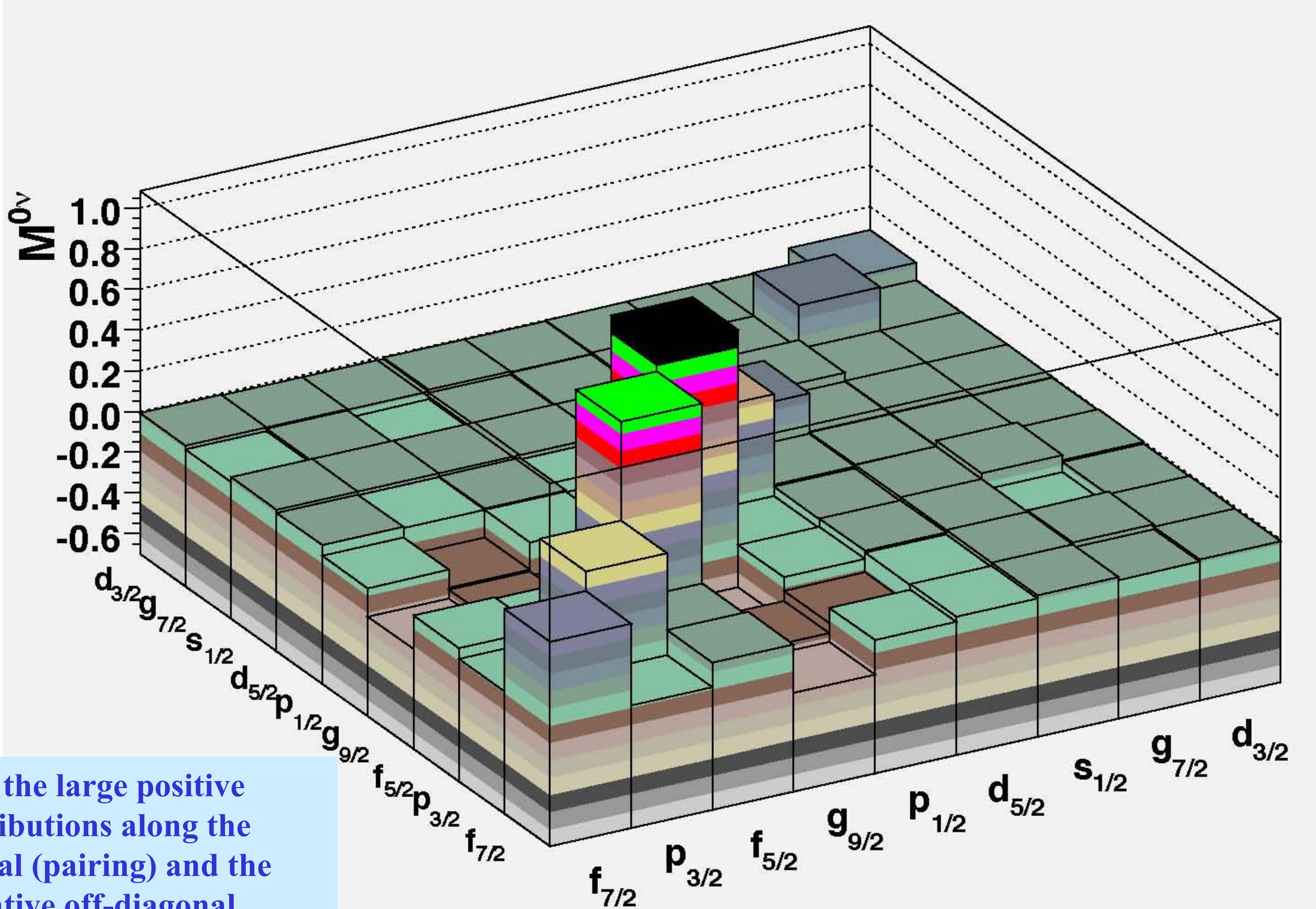
$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel \bar{f}(r_{12}) O_K \bar{f}(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

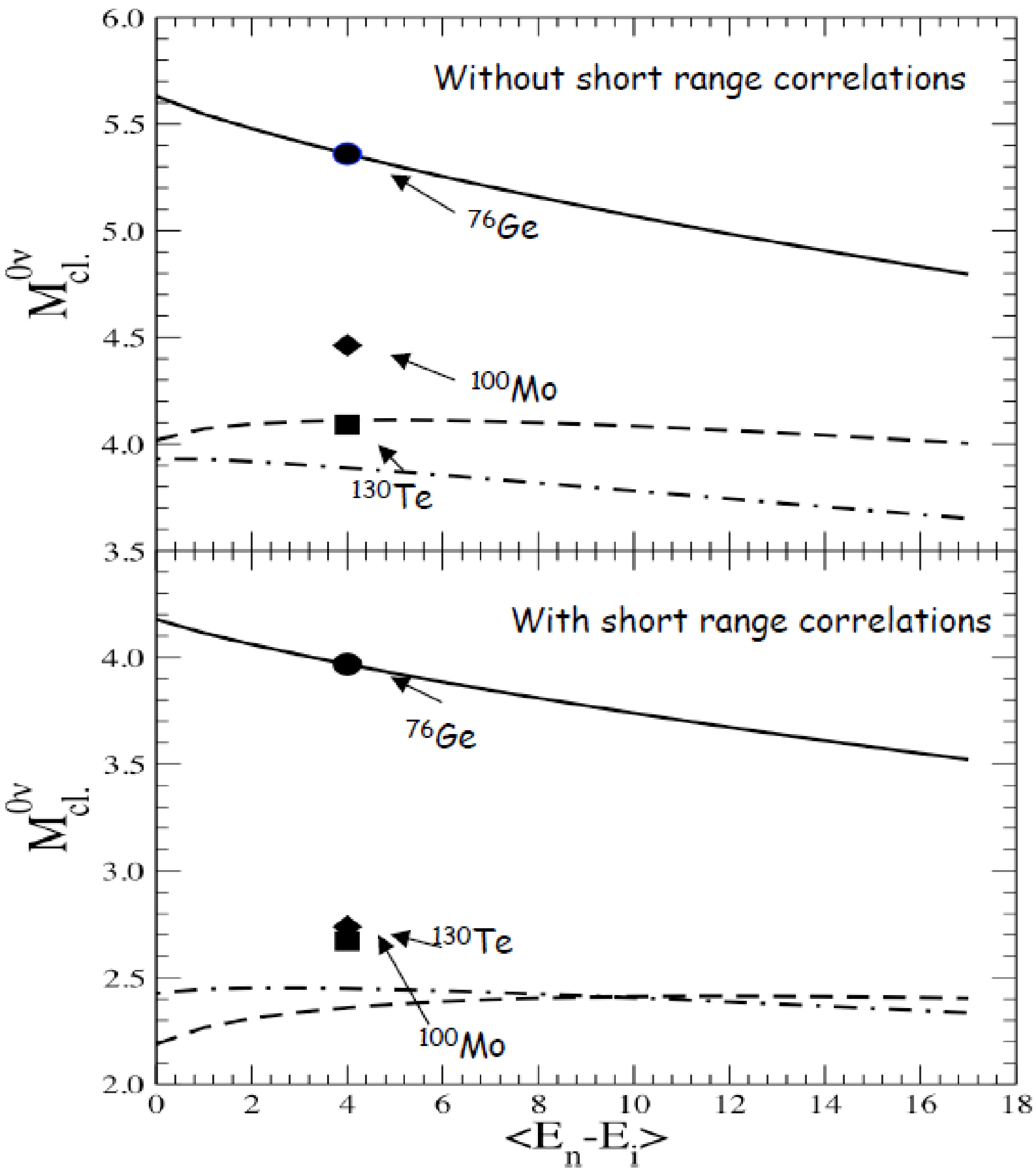
$$\langle 0_f^+ || [\widetilde{c_{p'}^+ \tilde{c}_{n'}}]_J || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i || [c_p^+ \tilde{c}_n]_J || 0_i^+ \rangle$$

Summing over all indeces except *n,n'* (or *p,p'*) will tell give us the required contribution. Note that it can be positive or negative.

Contribution of individual neutron orbits to $M^{0\nu}$ for ^{76}Ge $0\nu\beta\beta$ decay



Note the large positive contributions along the diagonal (pairing) and the negative off-diagonal contributions (higher seniority). The valence orbits dominate, but



How good is the closure approximation?

Comparison between the QRPA $M^{0\nu}$ with the proper energies of the virtual intermediate states (symbols with arrows) and the closure approximation (lines) with different $\langle E_n - E_i \rangle$.

Note the mild dependence on $\langle E_n - E_i \rangle$ and the fact that the exact results are reasonably close to the closure approximation results for $\langle E_n - E_i \rangle < 20$ MeV.

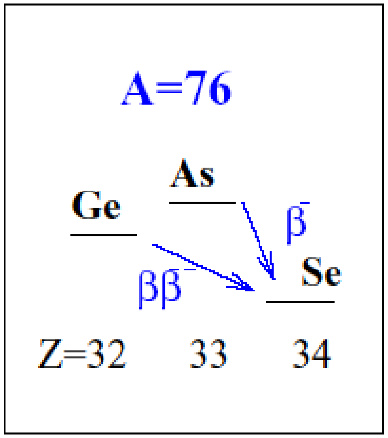
Graph by F. Simkovic

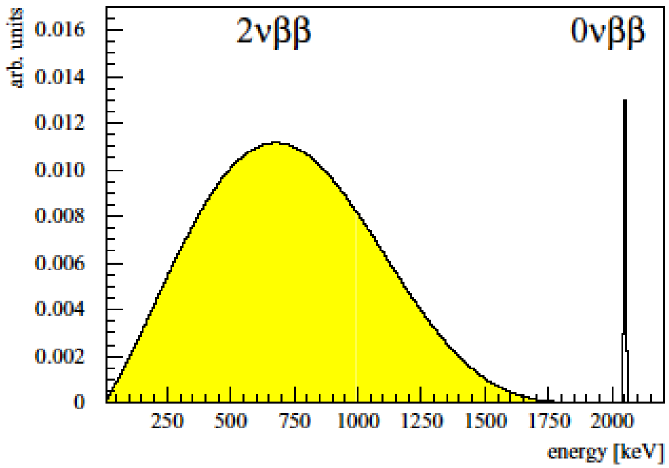
$2\nu\beta\beta$-decay

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.

Explaining $2\nu\beta\beta$-decay is necessary but not sufficient

ββ-decay





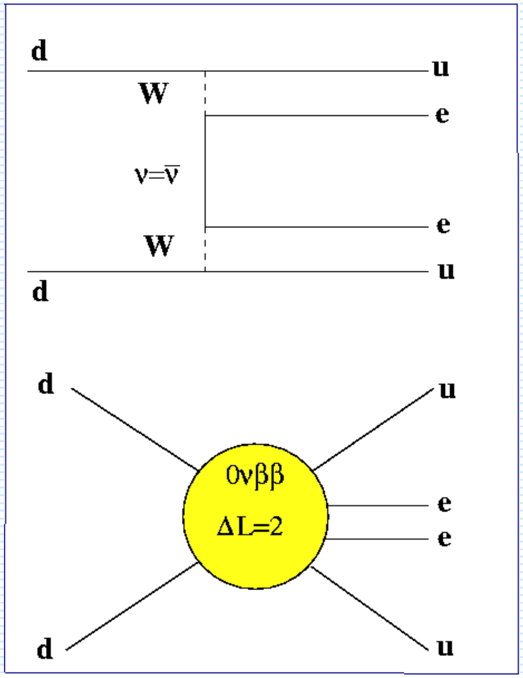
$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

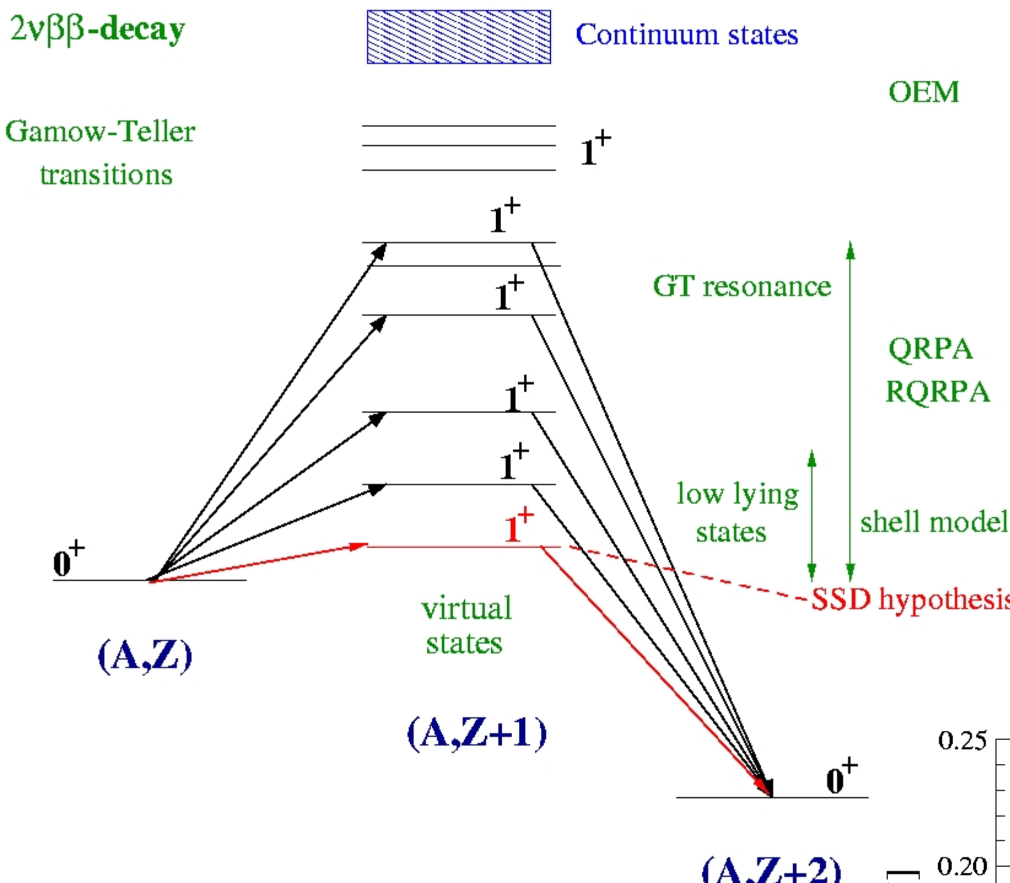
Observed for 10 isotopes: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo. ^{116}Cd. ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, $T_{1/2} \approx 10^{18}$-10^{24} years

1967: ^{130}Te, Kirsten et al, Takaoka et al, (geochemical)
1987: ^{82}Se, Moe et al. (direct observation)
2008: ^{100}Mo, NEMO 3 coll. ~ 300 00 events

$$(A,Z) \rightarrow (A,Z+2) + 2e^-$$

SM forbidden ,not observed yet: $T_{1/2}$ (^{76}Ge)>10^{25} years



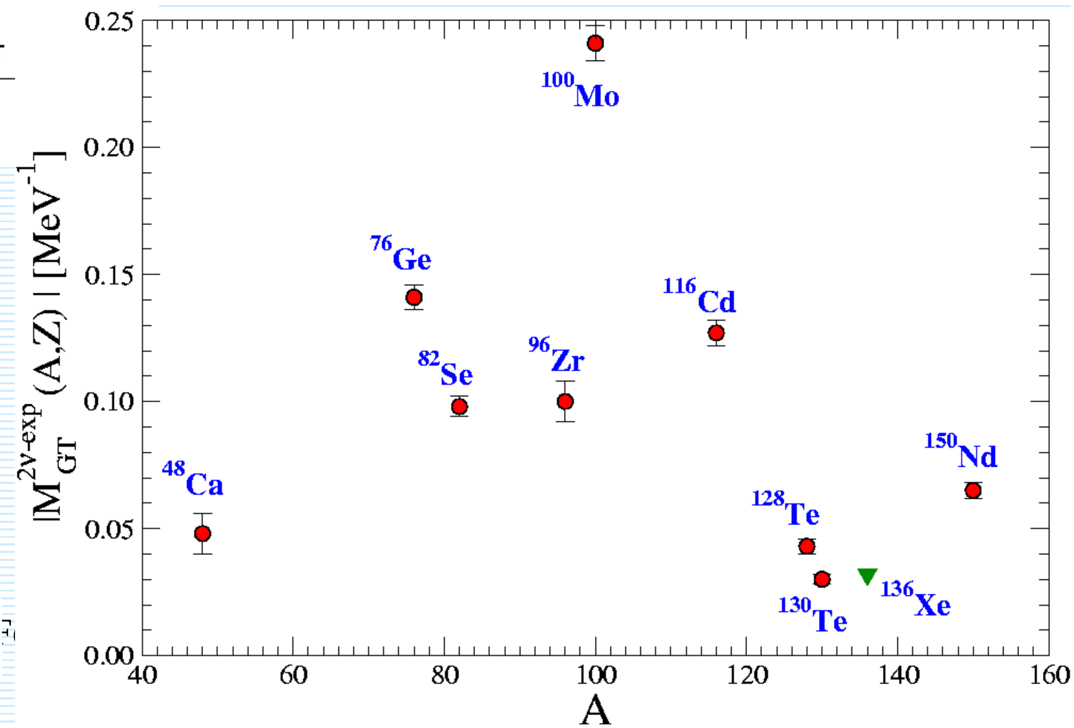


2νββ-decay nuclear matrix elements

$$(T^{2\nu}_{1/2})^{-1} = G^{2\nu}|M^{2\nu}_{GT}|^2$$

Deduced from measured $T_{1/2}^{2\nu}$

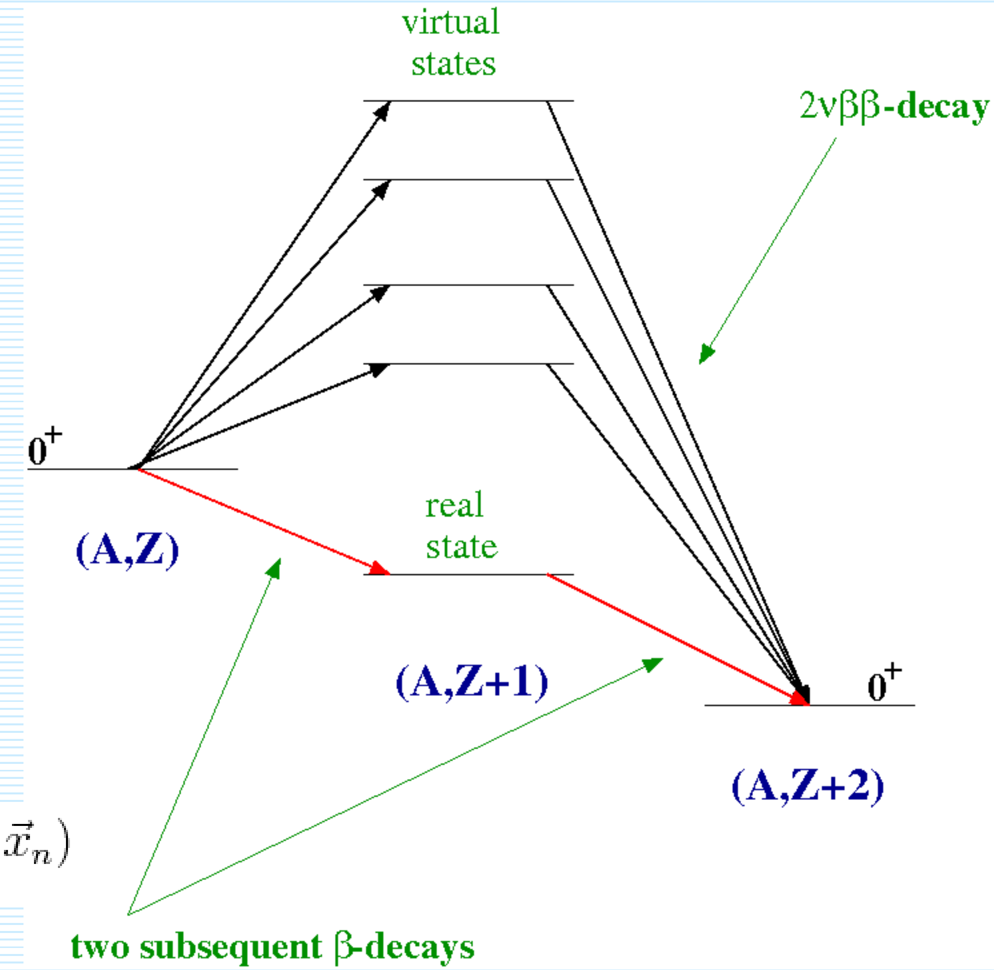
$$M^{2\nu}_{GT} = \sum_m \frac{< 0^+_f||\tau^+\sigma||1^+_m >< 1^+_m||\tau^+\sigma||0^+_i >}{E_m - E_i + \Delta}$$



Differencies in NME: by factor ~ 10

$$T(J_\mu(x_1)J_\nu(x_2)) = J_\mu(x_1)J_\nu(x_2) \qquad (two\ \beta - decays)$$
$$+ \ \Theta(x_{20}-x_{10})[J_\nu(x_2), J_\mu(x_1)] \qquad (2\nu\beta\beta - decay)$$

A sum over intermediate nuclear states represents a sum over all meson and gamma exchange correlations of two beta decaying nucleons



$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+(\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k})\delta(\vec{x}-\vec{x}_n)$$

$$J^{2\beta 2\nu}_{\mu\nu}(p_1,p_2,k_1,k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k}$$
$$\times 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \ k = 1,2,3,$$

Integral representation of M_{GT}

$$M_{GT} = \frac{i}{2}\int_0^\infty (e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}) M_{AA}(t) dt$$

with

$$M_{AA}(t) = < 0_f^+ | \frac{1}{2}[A_k(t/2), A_k(-t/2)] | 0_i^+ >$$

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \qquad A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k, \ \ k = 1,2,3.$$

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

Completeness:
Σ_n |n><n|=1

$$< A'|J_\alpha(x_1)J_\beta(x_2)|A > \ = \ \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times \\ e^{-i(E'-E_n)x_{10}} e^{-i(E_n-E)x_{20}}$$

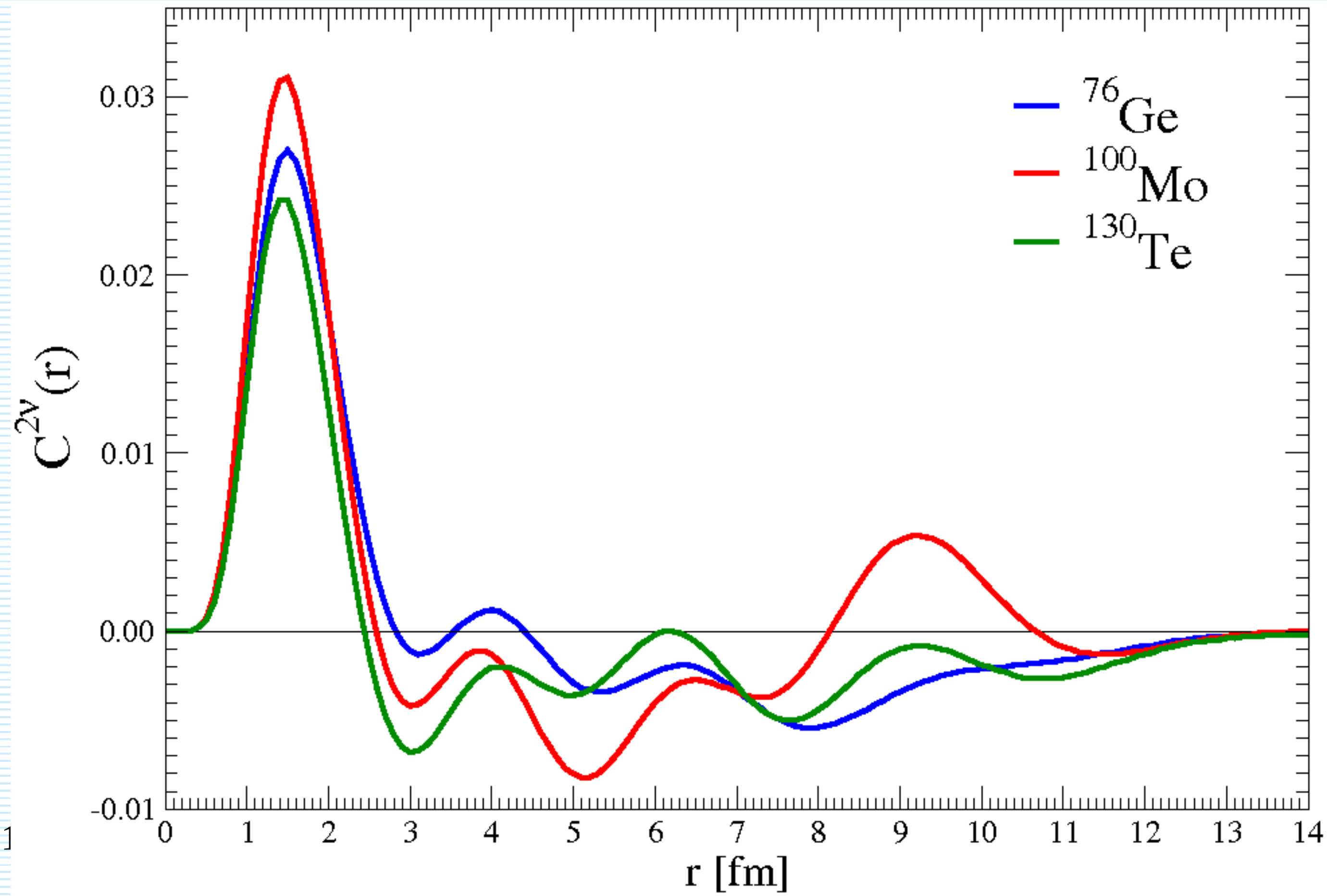
$$\int_0^\infty e^{-iat} dt \Rightarrow \lim_{\epsilon\to 0}\int_0^\infty \ e^{-i(a-i\epsilon)t} dt = \lim_{\epsilon\to 0} \frac{-i}{a - i\epsilon}$$

$$M_{GT} = \sum_n \frac{< 0_f^+ | A(0)_k | 1_n^+ >< 1_n^+ | A(0)_k | 0_i^+ >}{E_n - E_i + \Delta}$$

r_{12} -dependence of the 2νββ-decay NME

$$M^{2\nu}_{GT} = \int C^{2\nu}(r)dr$$

$$M^{2\nu}_{GT} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \times$$
$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$
$$\langle p(1),p'(2);\mathcal{J} \parallel \sigma(1)\cdot\sigma(2) \parallel n(1),n'(2);\mathcal{J}\rangle \times$$
$$\langle 0^+_f || \widetilde{[c^+_{p'}\tilde{c}_{n'}]_J} || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f i || [c^+_p \tilde{c}_n]_J || 0^+_i\rangle$$



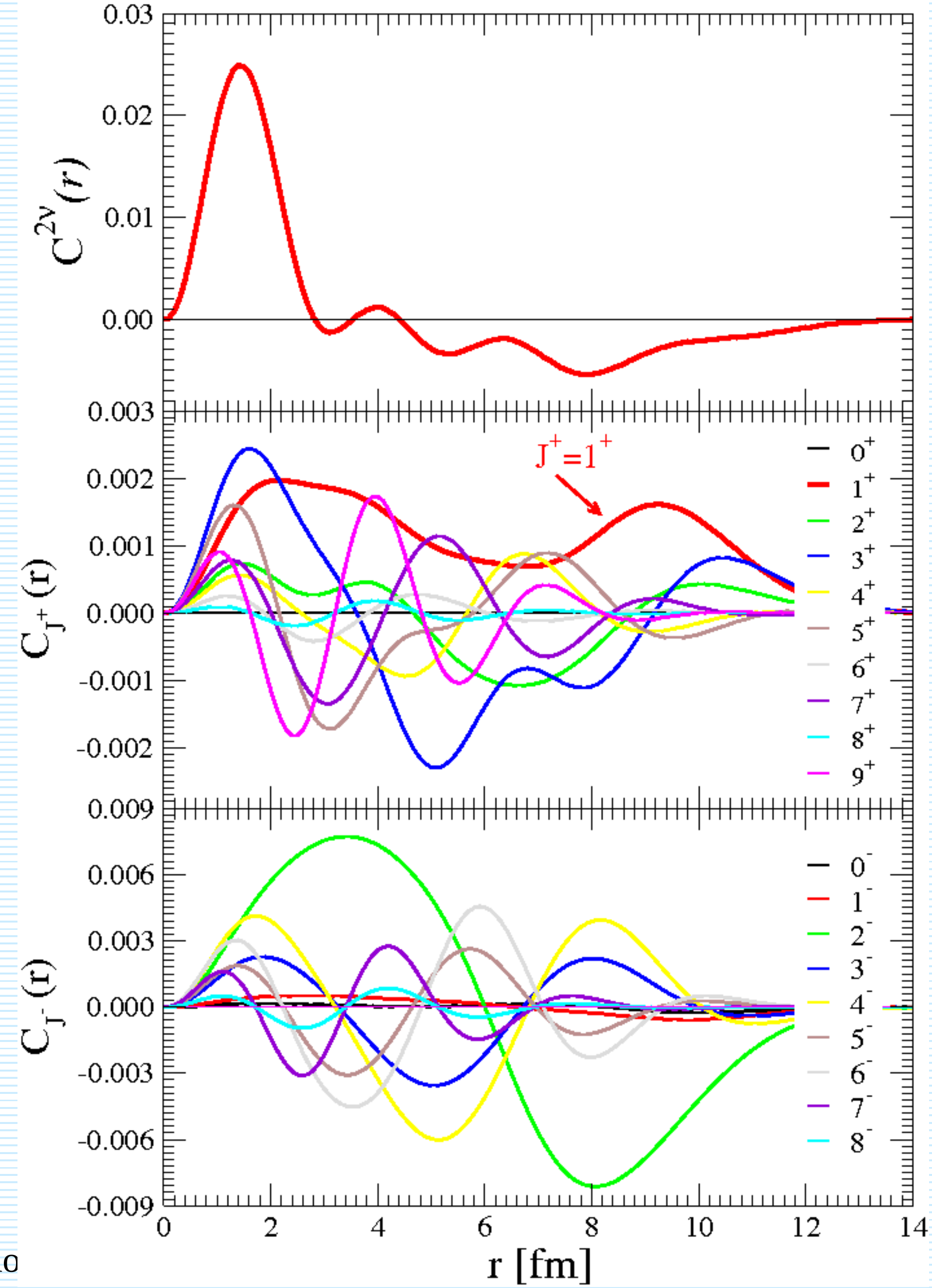
Decomposition of $C^{2\nu}$ on multipole contributions

$$M_{GT}^{2\nu} = <0_f^+|\tau^+\boldsymbol{\sigma} \sum_{J^\pi,m} \frac{|J_m^\pi><J_m^\pi|}{E_m-(E_i+E_f)/2} \tau^+\boldsymbol{\sigma}|0_i^+>$$

$$= \sum_m \frac{<0_f^+||\tau^+\boldsymbol{\sigma}||1_{m'}^+><1_m^+||\tau^+\boldsymbol{\sigma}||0_i^+>}{E_m-(E_i+E_f)/2}$$

$\int C_J(r)\, dr = M_{GT}^{2\nu}$ for $J^\pi = 1^+$

$= 0$ for $J^\pi \neq 1^+$



Constraining the $0\nu\beta\beta$-decay NMEs

Nucleons that change from neutrons to protons are valence neutrons

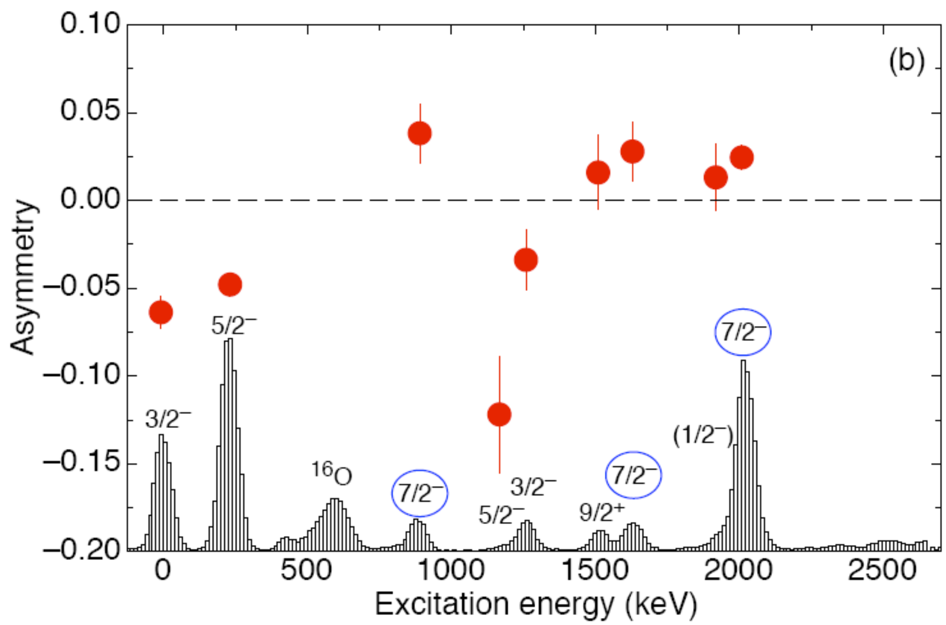
J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008)

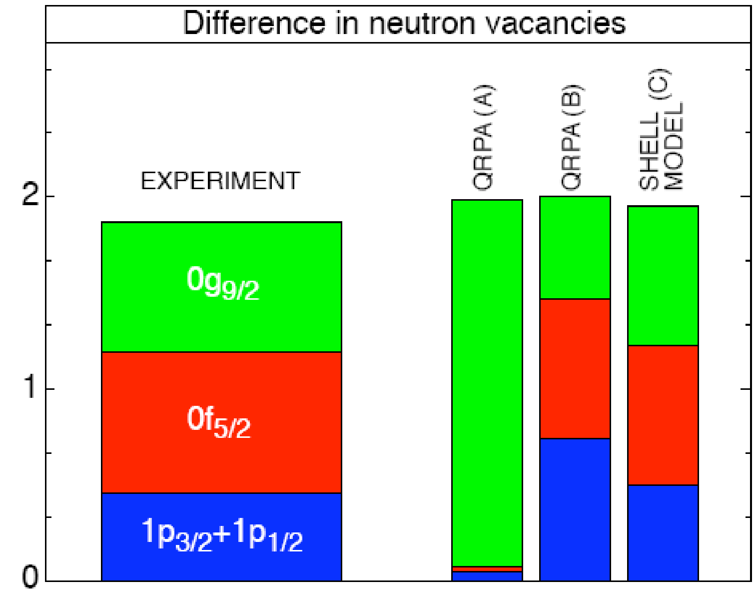
Proton, neutron removing transfer reaction

$^{76}Ge \rightarrow {}^{76}Se$

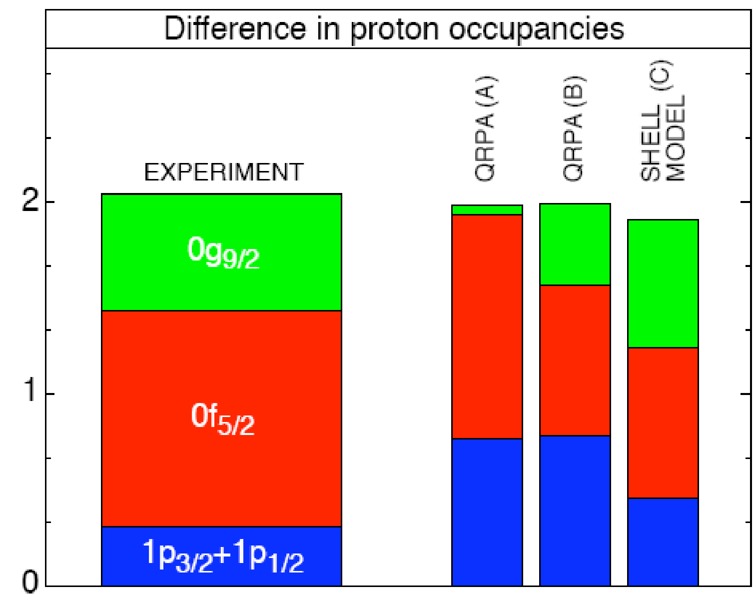
John Schiffer, P.Grabmayr et al

$$n_j^{exp} = \langle 0^+_{init} | \Sigma_m c^+_{j,m} c_{j,m} | 0^+_{init} \rangle$$





Kay et. Al, PRC 79, 021301 (2009)



QRPA(A) ≡ BCS (WS)

QRPA(B) ≡BCS (AWS) **Suhonen, Civitarese,** ovic

PLB 668, 277 (2008)

How can we take into account theoretically the constraint represented by the experimentally determined occupancies?

The experiment fixes $n_j^{exp} = \langle 0^+_{init} | \Sigma\, c_{j,m}^+ \, c_{j,m} \, | 0^+_{init} \rangle$ **and the same for the final nucleus**

particle creation and annihilation operators

In BCS $n_j^{BCS} = v_j^2 \times (2j+1)$ **depends only on v_j which in turn depends on the mean field eigenenergies**

In QRPA the ground state includes correlations and thus

$$n_j^{QRPA} = (2j+1)\times[v_j^2 + (u_j^2 - v_j^2)\xi_j]$$

$$\xi_j = (2j+1)^{-1/2} \langle 0^+_{qrpa} | \, [a^+_j a_j]^0 \, | \, 0^+_{qrpa} \rangle$$

depends on the quasiparticle content of the correlated ground state

quasiparticle creation and annihilation operators

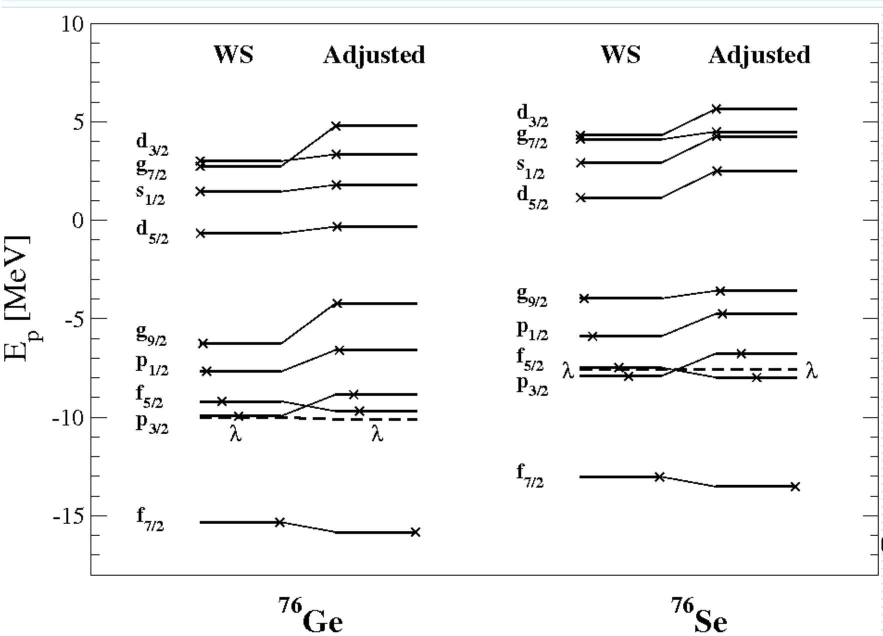
Initial and adjusted mean field levels

While n_j^{exp} and n_j^{BCS} are constrained by $\Sigma n_j = N$ (or Z) the n_j^{QRPA} are not constrained by that requirement. The particle number is not conserved, even on average. Thus the QRPA must be modified to remedy this ⇒
Selfconsistent Renormalized QRPA

$^{76}Ge \rightarrow ^{76}Se$	prev.	new
Jastrow s.r.c.	4.24(0.44)	3.49(0.23)
UCOM s.r.c.	5.19(0.54)	4.60(0.39)

	^{76}Ge				^{76}Se			
neut.	BCS	Q	S	exp	BCS	Q	S	exp
p	5.65	5.27	4.64	4.9±0.2	5.57	5.05	4.12	4.4±0.2
$f_{5/2}$	5.54	5.12	4.34	4.6±0.4	5.53	5.00	3.63	3.8±0.4
$f_{7/2}$	7.91	7.67	7.62	-	7.90	7.54	7.37	-
$s_{1/2}$	0.01	0.05	0.07	-	0.01	0.04	0.08	-
$d_{3/2}$	0.03	0.14	0.15	-	0.02	0.14	0.16	-
$d_{5/2}$	0.09	0.30	0.36	-	0.07	0.27	0.39	-
$g_{7/2}$	0.14	0.53	0.48	-	0.12	0.56	0.58	-
$g_{9/2}$	4.63	4.78	6.35	6.5±0.3	2.78	3.55	5.66	5.8±0.3
prot.								
p	2.23	2.34	1.75	1.77±0.15	2.77	2.76	2.28	2.08±0.15
$f_{5/2}$	1.61	2.27	2.08	2.04±0.25	2.95	2.97	3.03	3.16±0.25
$f_{7/2}$	7.83	7.19	7.13	-	7.76	7.12	7.06	-
$s_{1/2}$	0.00	0.02	0.03	-	0.00	0.03	0.04	-
$d_{3/2}$	0.01	0.07	0.07	-	0.01	0.09	0.09	-
$d_{5/2}$	0.01	0.12	0.15	-	0.02	0.17	0.18	-
$g_{7/2}$	0.02	0.19	0.16	-	0.03	0.31	0.27	-
$g_{9/2}$	0.29	0.85	0.62	0.23±0.25	0.46	1.15	1.04	0.84±0.25

F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)



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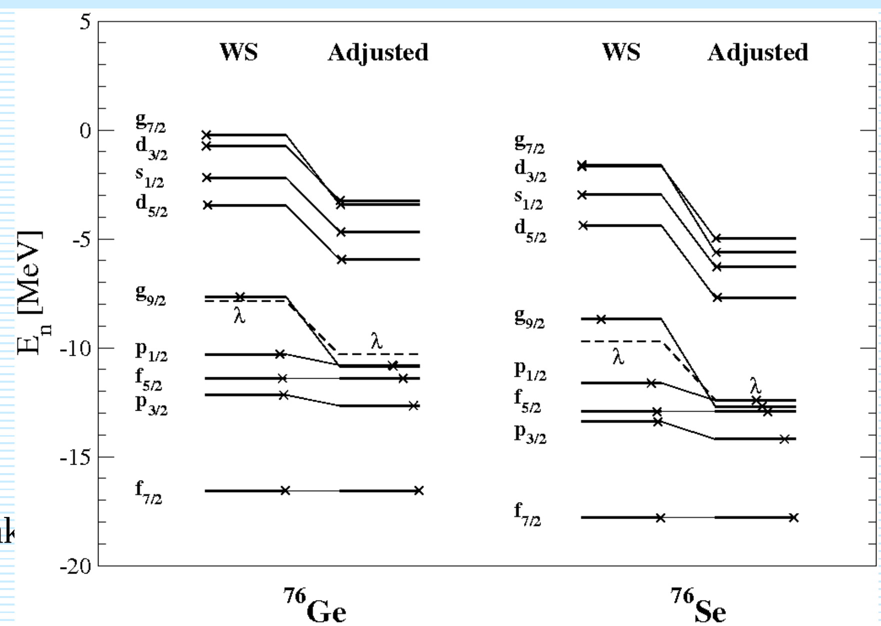
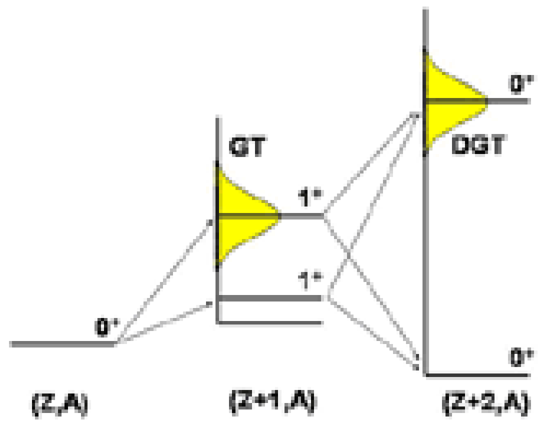


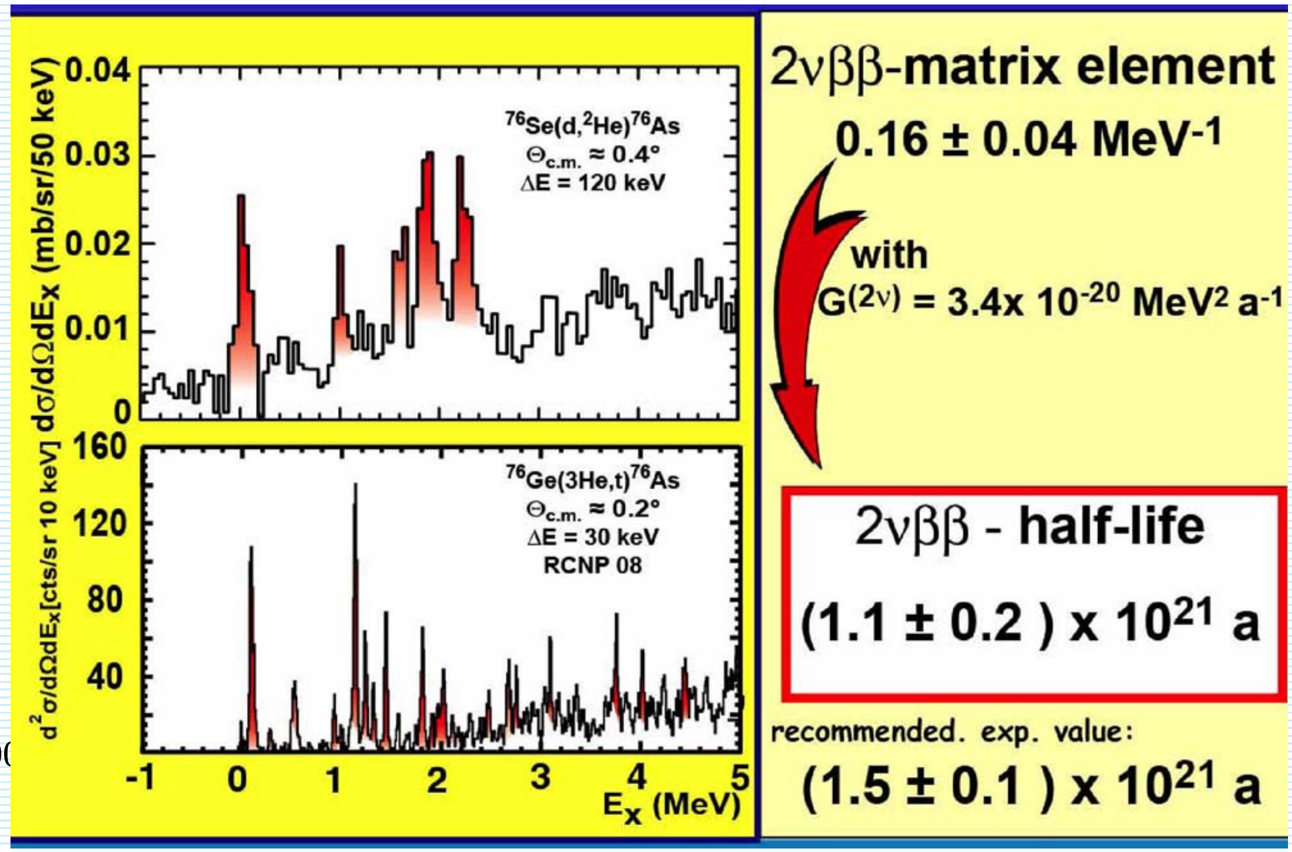
Constraining the 0νββ-decay NME



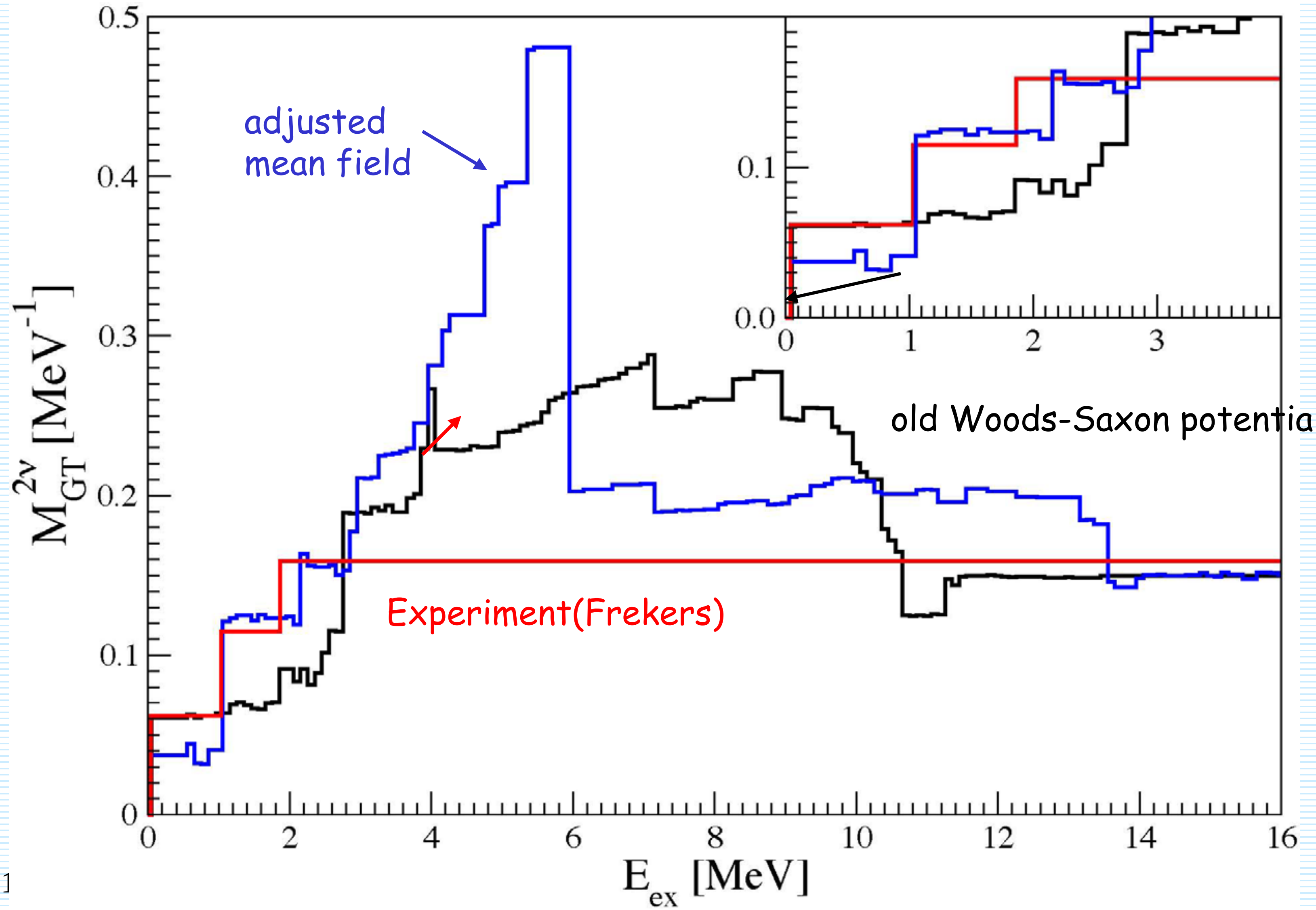
charge-exchange reactions	(t, ^{3}He) (d, ^{2}He)

From D. Frekers, RIKEN 2008 lecture

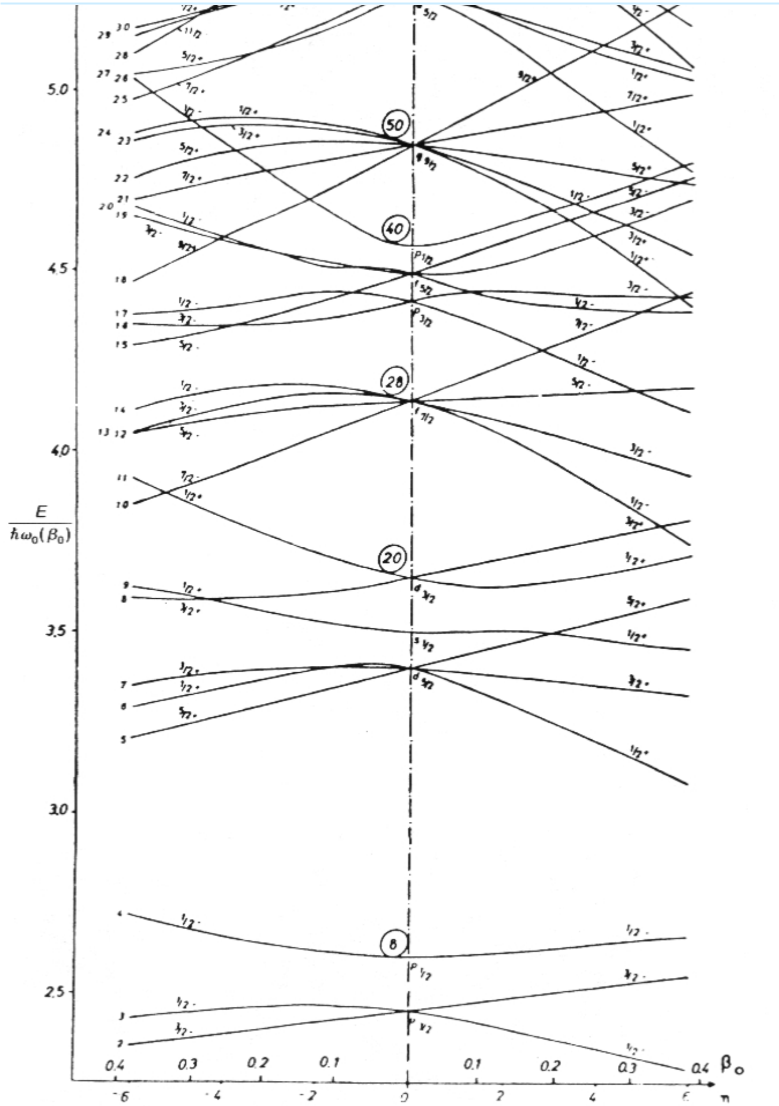
The cross sections give B(GT) for β^{+} and β^{-}, product of the amplitudes ($B(GT)^{1/2}$) gives the numerator of the $M^{2\nu}$ matrix element.



Staircase plot (running sum) of the contributions to the $2\nu\beta\beta$ decay ($^{76}Ge \rightarrow ^{76}Se$)



Shell structure of the mean field changed



Deformation

Anisotropic harmonic oscillator

Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

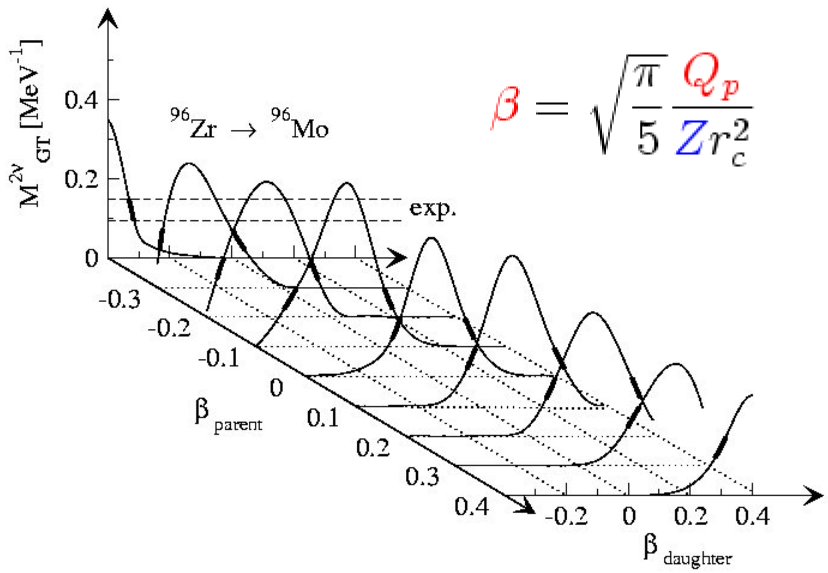
Exp. I (nuclear reorientation method)
Exp.II (based on measured E2 trans.)
Theor. I (Rel. mean field theory)
Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

Till now, in the QRPA-like calculations of the 0νββ-decay NME spherical symetry was assumed

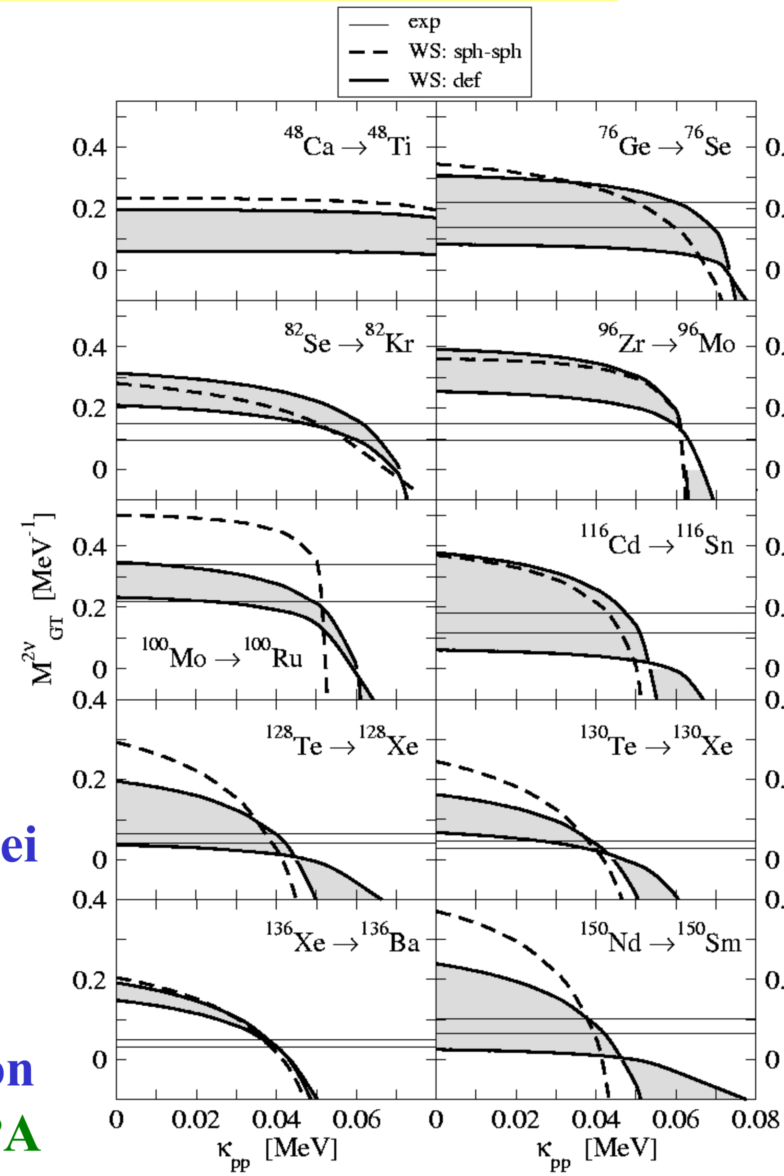
The effect of deformation on NME has to be considered

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
^{48}Ca	0.00	0.101	0.00	0.00
^{48}Ti	+0.17	0.269	-0.01	0.00
^{76}Ge	+0.09	0.26	0.16	0.14
^{76}Se	+0.16	0.31	-0.24	-0.24
^{82}Se	+0.10	0.19	0.13	0.15
^{82}Kr		0.20	0.12	0.07
^{96}Zr		0.081	0.22	0.22
^{96}Mo	+0.07	0.17	0.17	0.08
^{100}Mo	+0.14	0.23	0.25	0.24
^{100}Ru	+0.14	0.22	0.19	0.16
^{116}Cd	+0.11	0.19	-0.26	-0.24
^{116}Sn	+0.04	0.11	0.00	0.00
^{128}Te	+0.01	0.14	-0.00	0.00
^{128}Xe		0.18	0.16	0.14
^{130}Te	+0.03	0.12	0.03	0.00
^{130}Xe		0.17	0.13	-0.11
^{136}Xe		0.09	0.00	0.00
^{136}Ba		0.12	0.00	0.00
^{150}Nd	+0.37	0.28	0.22	0.24
^{150}Sm	+0.23	0.19	0.18	0.21

New Suppression Mechanism of the DBD NME



$$\beta = \sqrt{\frac{\pi}{5}}\frac{Q_p}{Zr_c^2}$$



The suppression of the NME depends on relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez,Sarriguren, Moya,Pacearescu, Faessler, F.Š.,
Phys. Rev. C 70 (2004) 321

QRPA with realistic forces in deformed nuclei

M. Saleh Yousef, V. Rodin, A. Faessler, F.Š, PRC 79 (2009) 014314

$$\langle p\rho_p \bar{n}\rho_n | G | p'\rho_{p'} \overline{n'}\rho_{n'} \rangle = \sum_J \sum_{(N_0lj)_p} \sum_{(N_0lj)_n} \sum_{(N_0lj)_{p'}} \sum_{(N_0lj)_{n'}} B^{(p)}_{(N_0lj)_p} B^{(n)}_{(N_0lj)_n} B^{(p')}_{(N_0lj)_{p'}} B^{(n')}_{(N_0lj)_{n'}}$$

$$\times (-1)^{j_n-\Omega_n} (-1)^{j_{n'}-\Omega_{n'}} C^{JK}_{j_p\Omega_p j_n\Omega_n} C^{JK}_{j_{p'}\Omega_{p'} j_{n'}\Omega_{n'}}$$

$$\times \langle (N_0lj)_p (N_0lj)_n, J | G | (N_0lj)_{p'} (N_0lj)_{n'}, J \rangle$$

G-matrix elements in
spherical single particle basis
Bonn CD potential

$2\nu\beta\beta$-decay
and
statistical properties of ν

Mixed statistics for neutrinos

Definnition of mixed state

$$|\nu> = \hat{a}^\dagger|0> \equiv \cos\delta \; \hat{f}^\dagger|0> + \sin\delta \; \hat{b}^\dagger|0> = \cos\delta \; |f> + \sin\delta \; |b>$$

with commutation Relations

$$\hat{f}\hat{b} = e^{i\phi}\hat{b}\hat{f} \qquad \hat{f}^\dagger\hat{b}^\dagger = e^{i\phi}\hat{b}^\dagger\hat{f}^\dagger$$
$$\hat{f}\hat{b}^\dagger = e^{-i\phi}\hat{b}^\dagger\hat{f} \qquad \hat{f}^\dagger\hat{b} = e^{-i\phi}\hat{b}\hat{f}^\dagger$$

Amplitude for 2νββ

$$A^{2\nu} = [\cos\delta^4 + \cos\delta^2\sin\delta^2(1-\cos\phi)]A^f + [\cos\delta^4 + \cos\delta^2\sin\delta^2(1+\cos\phi)]A^b = \cos\chi^2 A^f + \sin\chi^2 A^b$$

Decay rate

$$W^{2\nu} = \cos\chi^4 \; W^f + \sin\chi^4 \; W^b = (1-b^2)\; W^f + b^2 \; W^b$$

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

(calculations coming up soon)

Looking for a signature of bosonic ν

2νββ–decay half-lives ($0^+ \to 0^+_{g.s.}$, $0^+ \to 0^+_1$, $0^+ \to 2^+_1$)

- HSD – NME needed
- SSD – log ft_{EC}, log ft_β needed

$$\frac{T_{1/2}^{2\nu-SSD}(2_f^+)}{T_{1/2}^{2\nu-SSD}(0_f^+)} = 2.41 \times 10^4 \quad \text{fermionic } \nu \qquad T_{1/2}^{2\nu}(2^+) = 1.73 \times 10^{23} years$$

$$= 403 \quad \text{bosonic } \nu \qquad = 2.74 \times 10^{21} years$$

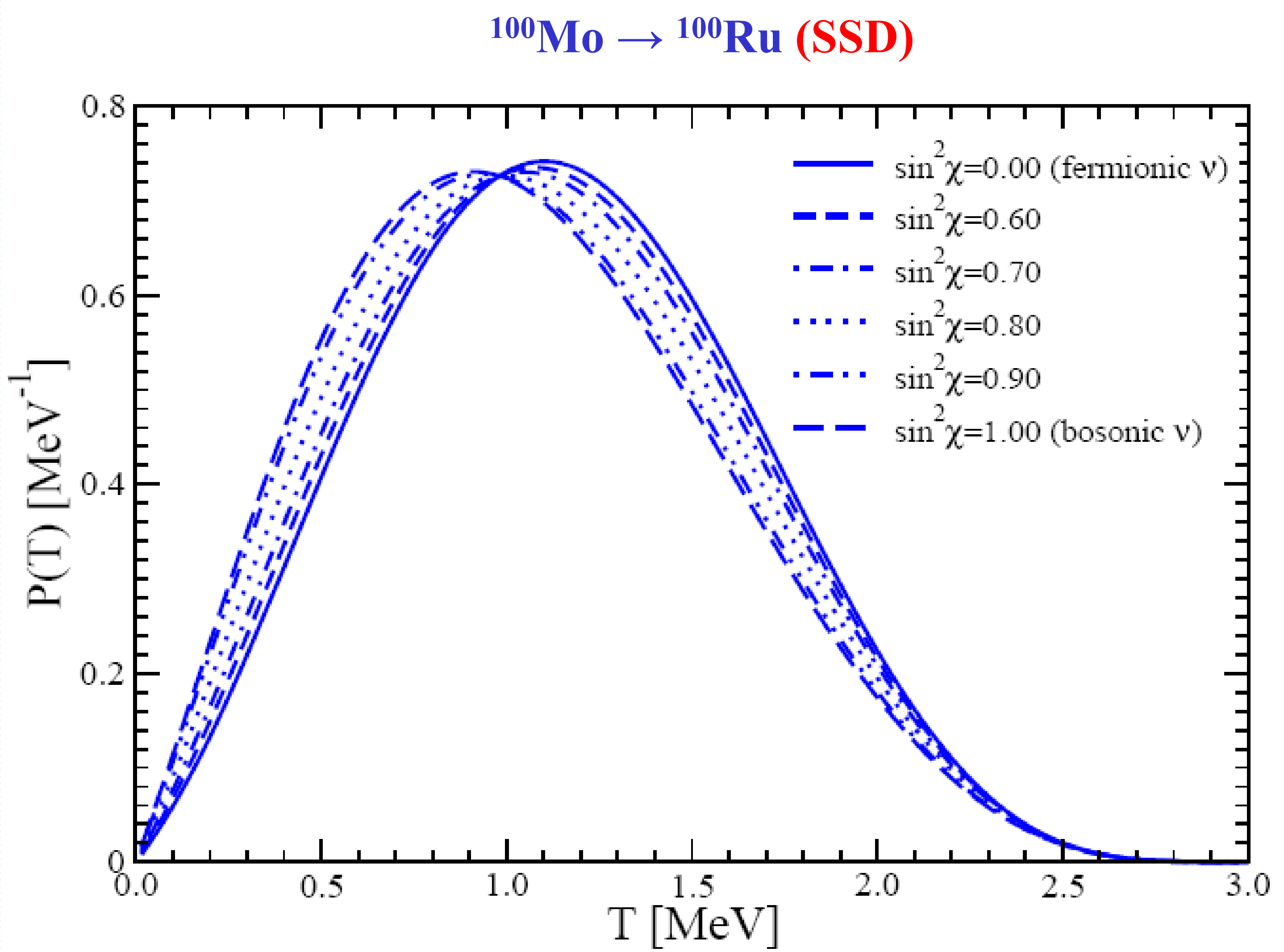
$$T_{1/2}^{2\nu-exp}(2^+) > 1.6 \times 10^{21} years$$

Normalized differential characteristics

- **The single electron energy distribution**
- **The distribution of the total energy of two electrons**
- **Angular correlations of two electrons**

(free of NME and log ft)

Mixed ν excluded for $\sin^2\chi < 0.6$



Neutrinoless Double Electron Capture

Neutrinoless double electron capture

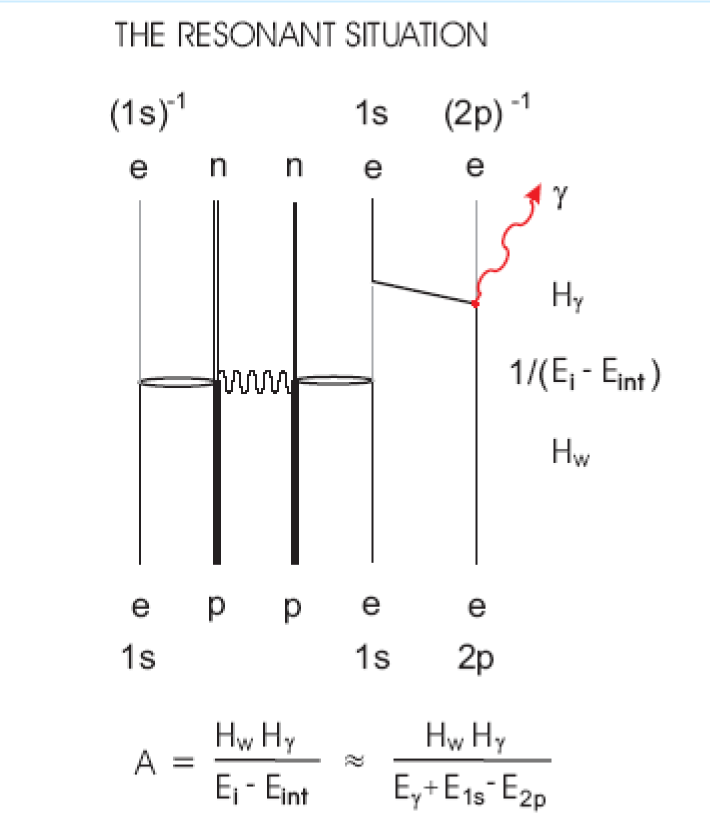
Modes of the 0νECEC-decay:

$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$
$$+ \; 2\gamma$$
$$+ \; e^+e^-$$
$$+ \; M$$

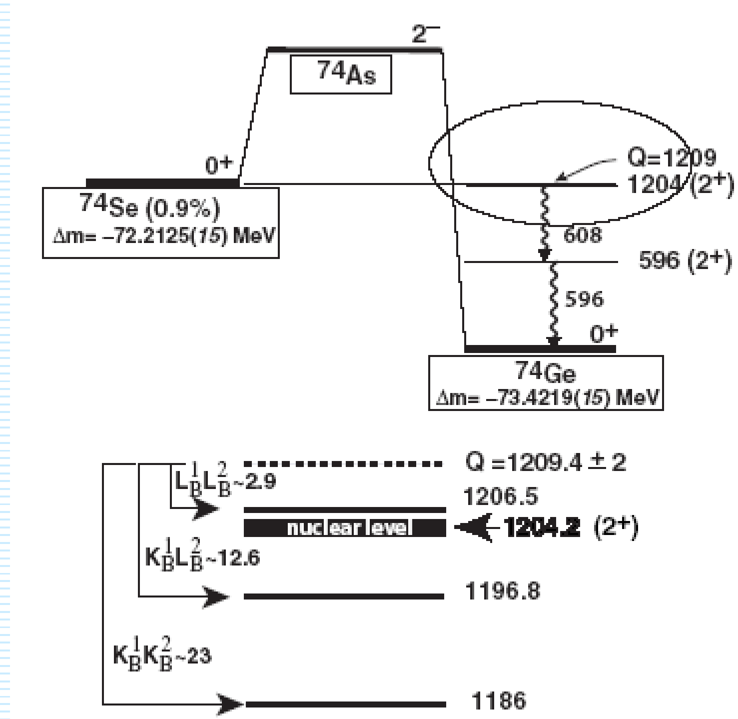
Theoretically, not well understood yet:

- **which mechanism is important?**
- **which transition is important?**

$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$



⇒



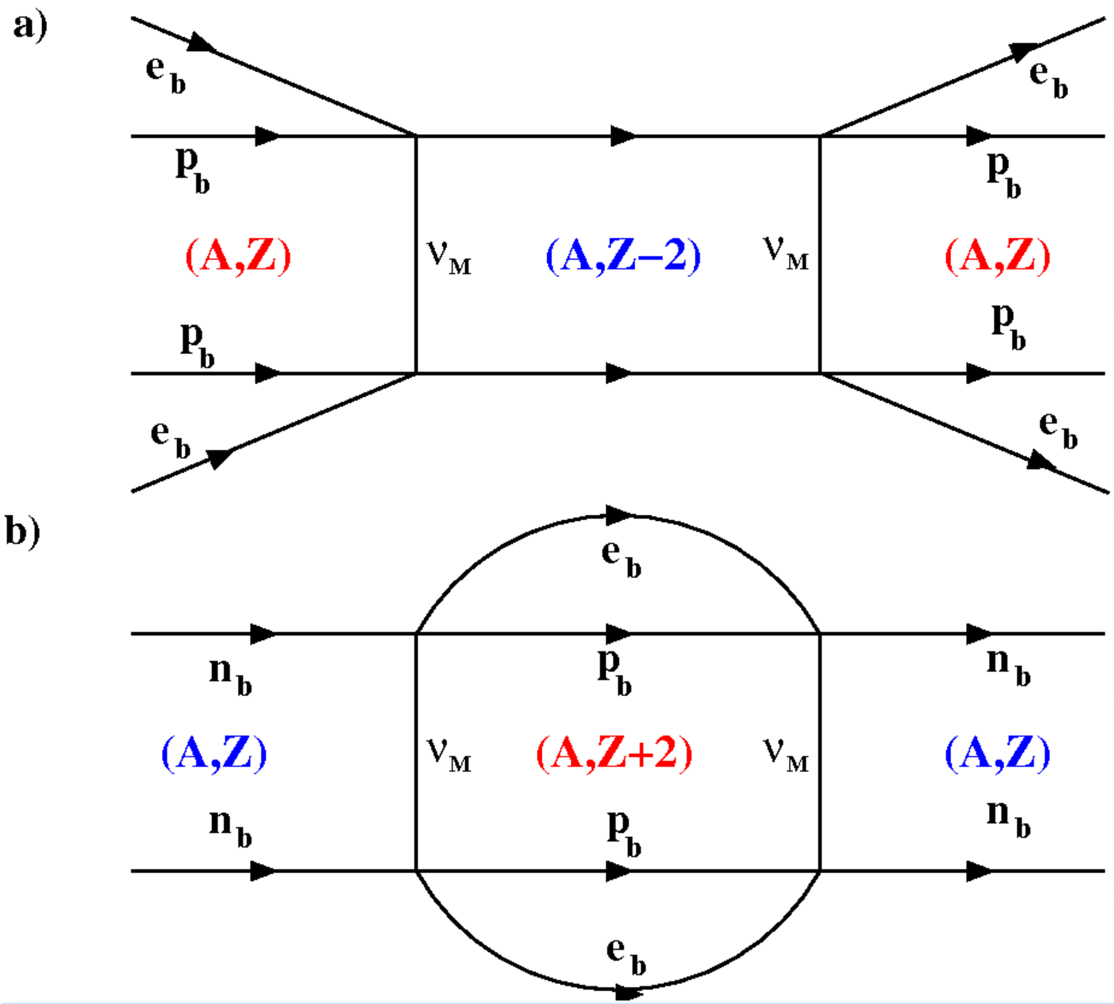
$$\Gamma^{0\nu\gamma} = \frac{\Gamma^r(2p \rightarrow 1s)}{[E_\gamma - Q_{res}]^2 + [\Gamma^r/2]^2} |R^{cc}_{0\nu}|^2$$

Fed $Q_{res} = E_{s_{1/2}} - E_{p_{1/2}}$

Sujkowski, Wycech, PRC 70, 052501 (2004)

D.Frekers, hep-ex/0506002

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.



Mixing of neutral atoms and total lepton number oscillation

$$n + n \leftrightarrow p + p + e_b^- + e_b^-$$

$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}+2)^{**}$$
$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}-2)^{**}$$

LNV Potential

$$V^{LNV} \simeq m_{\beta\beta}\, G_F^2 \;<\frac{1}{4\pi r_a}>\; \Psi_1(0)\Psi_2(0)$$

$$V^{LNV} \sim 10^{-24}\ \mathrm{eV}$$
$$m_\nu = 0.5\ eV,\quad Z=30,\quad n_i=1,\quad l_i=0$$

Different types of Oscillations (Effective Hamiltonian)

Oscillations of ν_l-$\nu_{l'}$
(lepton flavor)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of K_0-anti{K_0}
(strangeness)

$$H_{eff}^{n\overline{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n}
(baryon number)

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of atoms
(total lepton number)

Full width of unstable atom/nucleus

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1,$$
$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$
$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

$J^{\pi}=0^{+}$

Calculated double electron capture half-lives ($m_{\beta\beta}$ = 1 eV)

Transition	$M^*_{A,Z-2} - M_{A,Z-2}$	$M^{**}_{A,Z-2} - M_{A,Z}$	Holes	$T^{min}_{1/2}$	$T_{1/2}$
${}^{112}_{50}Sn \to {}^{112}_{48}Cd^*$	1871 ± 0.2	$-5.9 \pm 4.2 \pm 2.7$	$1s_{1/2}\ 1s_{1/2}$	2×10^{24}	8×10^{30}
${}^{152}_{64}Gd \to {}^{152}_{62}Sm$	0	$-0.3 \pm 2.5 \pm 2.5$	$1s_{1/2}\ 2s_{1/2}$	5×10^{24}	9×10^{29}
	0	$5.9 \pm 2.5 \pm 2.5$	$1s_{1/2}\ 3s_{1/2}$	4×10^{25}	8×10^{29}
	0	$7.4 \pm 2.5 \pm 2.5$	$1s_{1/2}\ 4s_{1/2}$	8×10^{26}	10^{33}
${}^{148}_{64}Gd \to {}^{148}_{62}Sm^*$	3045 ± 2	$5.7 \pm 2.5 \pm 2.5$	$2s_{1/2}\ 2s_{1/2}$	8×10^{25}	3×10^{32}
	3045 ± 2	$11.8 \pm 2.5 \pm 2.5$	$2s_{1/2}\ 3s_{1/2}$	3×10^{26}	8×10^{33}
	3045 ± 2	$13.3 \pm 2.5 \pm 2.5$	$2s_{1/2}\ 4s_{1/2}$	4×10^{27}	2×10^{35}
	3045 ± 2	$6.6 \pm 2.5 \pm 2.5$	$2p_{1/2}\ 2p_{1/2}$	2×10^{29}	2×10^{36}
${}^{156}_{66}Dy \to {}^{156}_{64}Gd^*$	1988.5 ± 0.2	$7.0 \pm 6.6 \pm 2.5$	$2s_{1/2}\ 2s_{1/2}$	2×10^{27}	8×10^{31}
	1988.5 ± 0.2	$7.9 \pm 6.6 \pm 2.5$	$2p_{1/2}\ 2p_{1/2}$	8×10^{29}	4×10^{35}

Transition	J^P	$M^*_{A,Z-2} - M_{A,Z-2}$	$M^{**}_{A,Z-2} - M_{A,Z}$	Holes	$\tilde{T}^{min}_{1/2}$	$\tilde{T}_{1/2}$
${}^{162}_{68}Er \to {}^{162}_{66}Dy^*$	1^+	1745.716 ± 0.007	$-10.1 \pm 3.5 \pm 2.5$	$1s_{1/2}\ 1s_{1/2}$	8×10^{23}	2×10^{29}
${}^{156}_{66}Dy \to {}^{156}_{64}Gd^*$	1^+	1965.950 ± 0.004	$-12.5 \pm 6.6 \pm 2.5$	$1s_{1/2}\ 2s_{1/2}$	10^{25}	3×10^{30}
	1^+	1965.950 ± 0.004	$-5.8 \pm 6.6 \pm 2.5$	$1s_{1/2}\ 3s_{1/2}$	2×10^{26}	2×10^{31}
	1^-	1946.375 ± 0.006	$8.4 \pm 6.6 \pm 2.5$	$1s_{1/2}\ 2s_{1/2}$	8×10^{26}	4×10^{31}
${}^{74}_{34}Se \to {}^{74}_{32}Ge^*$	2^+	1204.204 ± 0.007	$3.0 \pm 1.7 \pm 1.6$	$2p_{1/2}\ 2p_{3/2}$	10^{36}	10^{45}

Lepton number and parity oscillations

$$\Gamma_1 = \frac{4V^2}{4(M_i - M_f) + \Gamma^2}\ \Gamma$$

Double electron capture

$e_{1s1/2} + e_{1s1/2} + {}^{112}Sn \longrightarrow {}^{112}Cd(0^+_3)$

Reletivistic electron w.f. (j=1/2, l=0, l'=1)

$$\Psi^{(\alpha)}_{jm}(\vec{x}) = \begin{pmatrix} f_\alpha(r)\ \Omega_{jlm} \\ (-1)^{\frac{1+l+l'}{2}} g_\alpha(r)\ \Omega_{jl'm} \end{pmatrix} \quad l = j \pm 1/2,\ l' = 2j - l$$

Potential

0.022

$$V^{1s_{1/2}1s_{1/2}}(0^+_3) = \frac{1}{4\pi}\ m_e\ \left(G^2_\beta m^4_e\right)\ \frac{m_{\beta\beta}}{m_e}\ \frac{1}{R\ m_e} \frac{\left(\bar{f}_{1s_{1/2}}\right)^2}{4\pi\ m^3_e} g^2_A\ M^{0\nu}(0^+_3).$$

Width

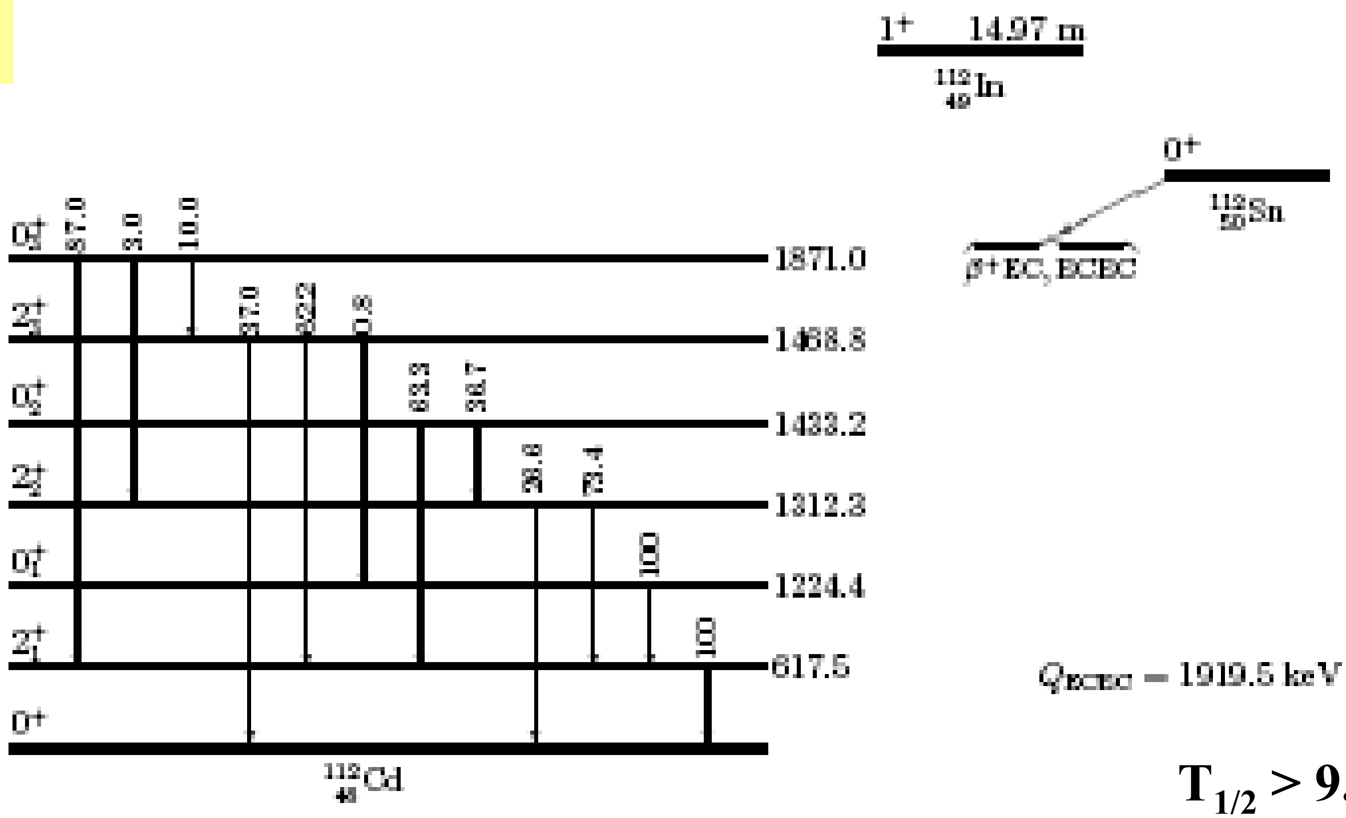
$$\Gamma^{ECEC} = \frac{\left|V^{1s_{1/2}1s_{1/2}}(0^+_3)\right|^2}{(M_i - M_f)^2 + \frac{\Gamma^2_X}{4}}\ \Gamma_X$$

Matrix element

Exc. state	E_{ex} (MeV)	$M^{0\nu}$
$0^+_{g.s.}$	0	2.69
0^+_1 (1 ph.)	1.224	3.02
0^+_2 (2 ph.)	1.433	0.90
0^+_3 (1 ph.)	1.224	2.78

Experimental activities (^{112}Sn)

^{112}Sn



$T_{1/2} > 9.2 \cdot 10^{19}$ years

In comparison with the 0νββ-decay disfavoured due:
- **process in the 3-rd (4th) order in electroweak theory**
- **bound electron wave functions**

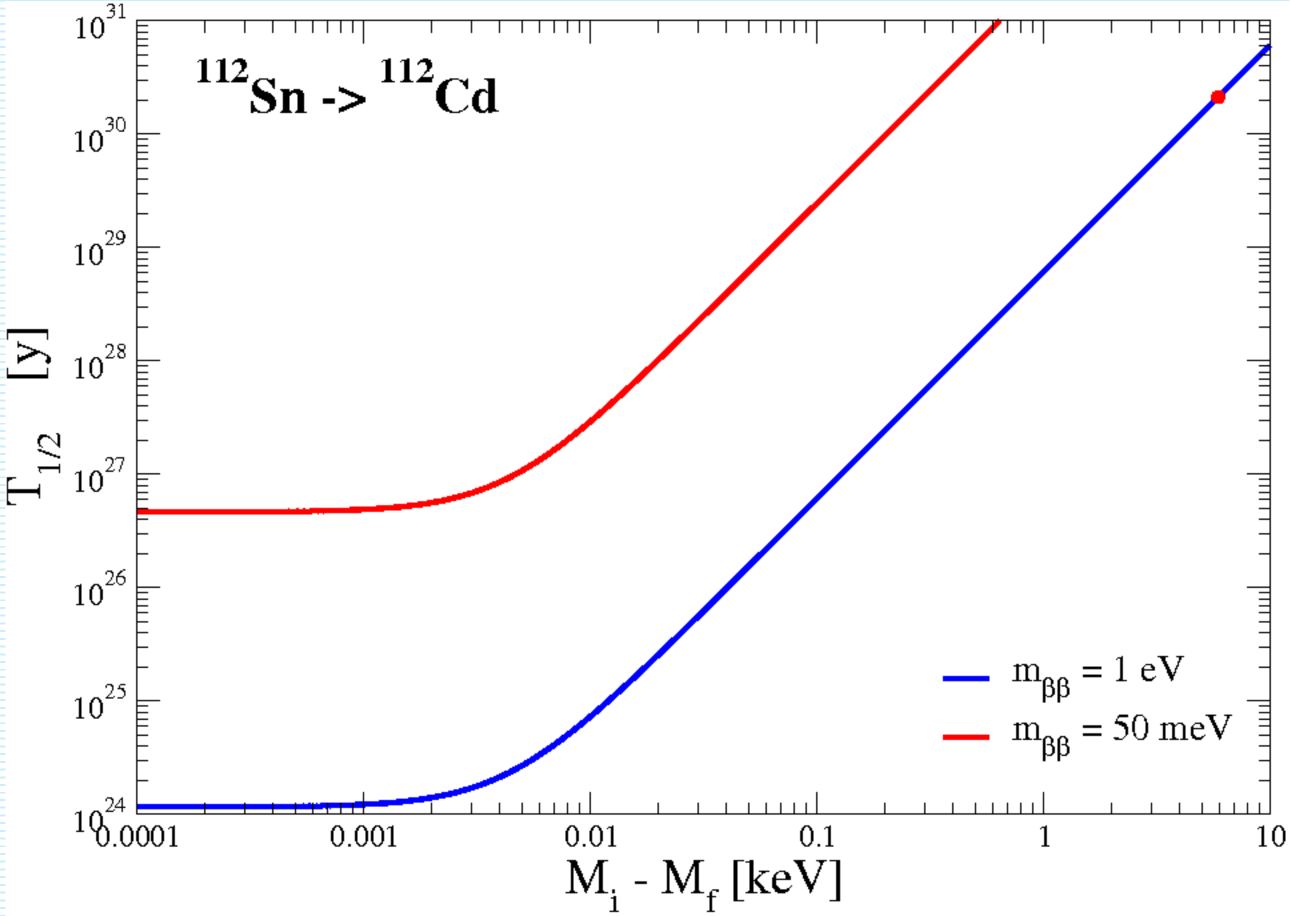
favoured: resonant enhancement ?

A.S. Barabash et al., NPA 807 (2008) 269

Double electron capture of ^{112}Sn (perspectives of search)

F. Šimkovic, M. Krivoruchenko, A. Faessler, to be submitted

M_i-M_f	$T_{1/2}^{ECEC}$ ($m_{\beta\beta}$= 50 meV)
1 keV	$2.44\ 10^{31}$ years
100 eV	$2.45\ 10^{29}$ years
10 eV	$2.91\ 10^{27}$ years
0 eV	$4.67\ 10^{26}$ years



$T_{1/2}^{0\nu}$ (^{76}Ge)= (2.95 – 5.74) 10^{26} years for $m_{\beta\beta}$= 50 meV

Program: Matrix Elements for Fundamental Processes

$0\nu\beta\beta$-decay:

- ***Systematic study of NMEs for all mechanisms (general parameterization), coexistence of different LNV mechanisms, angular distributions, transitions to excited states***
- ***Improvement of the nuclear structure model (QRPA-deformation, non-linear phonon operator, calculation from single vacuum state)***
- ***A detailed comparison with other approaches (NSM, IBM, PHFB) and with available nuclear structure data (occupation numbers, β-strengths)***

$2\nu\beta\beta$-decay:

- ***Better understand the role of residual interaction***
- ***Predictions for transitions to 2^+ states in the context of bosonic neutrinos***

***0νECEC:** resonant transition to excited states of final atoms, Analysis of capture probabilities, calculation of corresponding nuclear and atomic matrix elements*

***Single β-decay:** in the context of neutrino mass (^{3}H, ^{187}Re) and relic neutrino detection*

***dark matter detection:** spin-dependent interaction, scattering of neutralinos on odd-odd nuclei, role of exchange currents*

Frank Avignone:
Nuclear Matrix Elements are as important as DATA

What is the nature of neutrinos?

$\nu \Rightarrow$

theory

Only the 0νββ-decay can answer this fundamental question

^{76}Ge

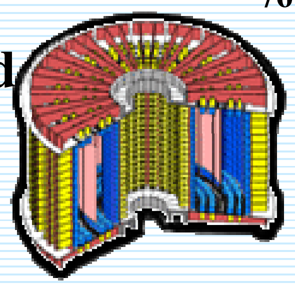
GERDA

^{130}Te

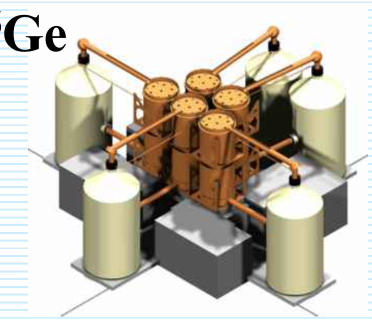
CUORE

^{82}Se

^{150}Nd

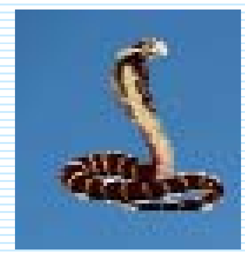


^{76}Ge



^{106}Cd

^{116}Cd



^{100}Mo

^{150}Nd

^{136}Xe

ic

+ (?)

Double electron capture

(Muenster, Dresden, Jyvaskula, Bratislava…col.)