Nuclear matrix elements for neutrino-less double beta decay

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Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics.

**APS Joint Study on the Future of Neutrino Physics (physics/0411216)**

*We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first in the list of recommendations)*

**ASPERA road map:**

- Requirement for construction and operation of two double-beta decay experiments with a European lead role or shared equally with non-European partners *(GERDA, COBRA, CUORE, SuperNEMO)*
- Different nuclear isotopes are necessary to minimize the impact of uncertainties in matrix elements to the extracted information about neutrino properties.
- We finally reiterate the importance of assessing and reducing the uncertainty in our knowledge of the corresponding nuclear matrix elements, experimentally and theoretically.
### Lepton Family Number Violation

#### New Physics

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\nu_{e,\mu,\tau} \leftrightarrow \nu_{e,\mu,\tau}$</th>
<th>$\nu_{e,\mu,\tau} \leftrightarrow \bar{\nu}_{e,\mu,\tau}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^+ \to e^+ + \gamma$</td>
<td>$R \leq 1.2 \times 10^{-11}$</td>
<td>$K^+ \to \pi^- + e^+ + \mu^+$</td>
</tr>
<tr>
<td>$\mu^+ \to e^+ + e^- + e^+$</td>
<td>$R \leq 1.0 \times 10^{-12}$</td>
<td>$\tau^- \to \pi^- + \pi^+ + e^+$</td>
</tr>
<tr>
<td>$K^+ \to \pi^+ + e^- + \mu^+$</td>
<td>$R \leq 4.7 \times 10^{-12}$</td>
<td>$W^- + W^- \to e^- + e^-$</td>
</tr>
<tr>
<td>$\tau^- \to e^- + \mu^+ + \mu^-$</td>
<td>$R \leq 1.8 \times 10^{-6}$</td>
<td>$(A, Z) \to (A, Z + 2) + e^- + e^-$</td>
</tr>
<tr>
<td>$Z^0 \to e^+ + \mu^+$</td>
<td>$R \leq 1.7 \times 10^{-6}$</td>
<td>$\mu \bar{\nu} + (A, Z) \to (A, Z - 2) + e^+$</td>
</tr>
<tr>
<td>$\mu^- + (A, Z) \to (A, Z) + e^-$</td>
<td>$R \leq 1.2 \times 10^{-11}$</td>
<td>$e^- + e^- \to \pi^- + \pi^-$</td>
</tr>
</tbody>
</table>

### Lepton Universality

#### Total Lepton Number Violation

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Anti – p.</th>
<th>mass [MeV]</th>
<th>$L_e$</th>
<th>$L_\mu$</th>
<th>$L_\tau$</th>
<th>life – time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>$e^-$</td>
<td>$e^+$</td>
<td>0.511</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>el.neutrino</td>
<td>$\nu_e$</td>
<td>$\bar{\nu}_e$</td>
<td>$&lt; 2.2 \times 10^{-6}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>muon</td>
<td>$\mu^-$</td>
<td>$\mu^+$</td>
<td>105.6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$2.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>muon neutr.</td>
<td>$\nu_\mu$</td>
<td>$\bar{\nu}_\mu$</td>
<td>$&lt; 0.19$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>stable</td>
</tr>
<tr>
<td>tau</td>
<td>$\tau^-$</td>
<td>$\tau^+$</td>
<td>1777.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$2.9 \times 10^{-13}$</td>
</tr>
<tr>
<td>tau neutrino</td>
<td>$\nu_\tau$</td>
<td>$\bar{\nu}_\tau$</td>
<td>$&lt; 18.2$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>stable</td>
</tr>
</tbody>
</table>
The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

\[ \frac{1}{T_{1/2}} = G_{\nu}^{0\nu}(E_0, Z) |M_{\nu}\nu| ^2 |m_{\beta\beta}|^2, \quad m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i \]

<table>
<thead>
<tr>
<th>Absolute $\nu$ mass scale</th>
<th>Normal or inverted Hierarchy of $\nu$ masses</th>
<th>CP-violating phases</th>
</tr>
</thead>
</table>

\[ U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} \]

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.
The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei.
Double Beta Decay Nuclei of experimental interest

Emission of two electrons

\[(A,Z) \rightarrow (A,Z+2) + e + e\]

Double electron capture

\[e_b + e_b + (A,Z) \rightarrow (A,Z^*)\]

Preferable nuclear systems with large \(\Delta M_A\) \((E^5)\)

Nuclear systems with small \(\Delta M_A\) might be also important (resonant enhancement)

Signal from \(\gamma\)- and X-rays

11/11/2009

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If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved:

1) What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).

2) How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements.
The $0\nu\beta\beta$-decay mechanisms

Two basic categories are

long-range (exchange of light Majorana $\nu$) and

short-range (exchange of heavy $\nu$, squarks, gluinos ...) contributions to the $0\nu\beta\beta$-decay
**Light neutrino Mass mechanism**

\[ \mathcal{H}_W^\beta = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\alpha(1 + \gamma_5)\nu_e j_\alpha + h.c. \]

\[ \nu_{eL} = \sum_k U^L_{tk}\chi_{kL} \]

\[ \langle \nu_{eL}(x_1)\nu_{eL}^T(x_2) \rangle = -\sum_k (U^L_{ek})^2 \frac{1 + \gamma_5}{2} \xi_k S_k(x_1 - x_2) \frac{1 + \gamma_5}{2} C \]

\[ = \frac{i}{(2\pi)^4} \sum_k (U^L_{ek})^2 \xi_k m_k \int \frac{e^{iq(x_1 - x_2)} dq}{q^2 + m_k^2} \frac{1 + \gamma_5}{2} C \]

**Effective mass of Majorana neutrinos**

\[ m_{\beta\beta} = \sum_k (U^L_{ek})^2 \xi_k m_k \]

\[
\begin{pmatrix}
\bar{\nu}_L & (\nu_R)^c
\end{pmatrix}
\begin{pmatrix}
0 & m_D \\
m_D & M_R
\end{pmatrix}
\begin{pmatrix}
(\nu_L)^c \\
\nu_R
\end{pmatrix}
\]

\[ m_1 = m_D^2/M_R \ll m_D \quad m_2 \approx M_R \]

\[ \nu_1 = \nu_L - m_D/M_R (\nu_R)^c \quad \nu_2 = \nu_R + m_D/M_R (\nu_L)^c \]
Neutrino vertex

\[ \mathcal{L}^{LH} = \frac{G_F}{\sqrt{2}} \sum_i U_{ei} \left( \bar{e}_\alpha (1 - \gamma_5) \nu \right) \left( \bar{u}_\alpha (1 - \gamma_5) d \right) + h.c. \quad (V - A) \]

R-parity violating SUSY vertex

\[ \mathcal{L}_{\text{SUSY}}^{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \frac{1}{4} \eta_{(q)LR} \sum_i U_{ei}^* (\bar{\nu}(1 + \gamma_5)e) (\bar{u}(1 + \gamma_5)d) \right) \quad (S, P) \]

\[ + \frac{1}{8} \eta_{(q)LR} \sum_i U_{ei}^* (\bar{\nu}\sigma_{\alpha\beta}(1 + \gamma_5)e) (\bar{u}\sigma^{\alpha\beta}(1 + \gamma_5)d) + h.c. \right) \quad (\text{Tensor}) \]


LN-violating parameter

\[ \eta_{(q)LR} = \sum_k \frac{\lambda'_{11k} \lambda'_{1k1}}{8\sqrt{2}G_F} \sin 2\theta^{d}_{(k)} \left( \frac{1}{m_{\tilde{d}_1}^2(k)} - \frac{1}{m_{\tilde{d}_2}^2(k)} \right) \]
gluino/neutralino exchange R-parity breaking
SUSY mechanism of the $0\nu\beta\beta$–decay

$d+d \rightarrow u + u + e^- + e^-$

exchange of squarks, neutralinos and gluinos

$(\lambda'_{111})^2$ mechanism

1987 R. Mohapatra, J.D. Vergados

R–parity violation
nucleon level

Light neutrino exchange

Heavy neutrino exchange

two-pion exchange (heavy neutrino)
Sterile neutrino in $0\nu\beta\beta$-decay

\[
[T_{1/2}^{0\nu}]^{-1} = G_{01} \left| \frac{(m_{\nu})_{ee} M_{\nu}}{m_e} + U_{eh}^2 \frac{m_h}{m_e} M_{0\nu}(m_h) \right|^2.
\]

Matrix element depends on $\nu$-mass

\[
F_{\nu}(m_h) = \frac{m_h}{m_e} M_{0\nu}(m_h)
\]

\[
|U_{eh}|^2 \leq \frac{1}{|F_{\nu}(m_h)|} \frac{1}{\sqrt{T_{1/2}^{0\nu\beta\beta}} G_{01}}.
\]

Calculation of Nuclear Matrix Elements
Nuclear Matrix Elements

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus.

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them.

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge the quality of the result.
• Exact methods exist up to $A=4$
• Computationally exact methods for $A$ up to 16
• Approximate many-body methods for $A$ up to 60
• Mostly mean-field pictures for $A$ greater than 60 or so

With newer methods and powerful computers, the future of nuclear structure theory is bright!
Many-body Hamiltonian

- Start with the many-body Hamiltonian
  \[ H = \sum_i \frac{\vec{p}_i^2}{2m} + \sum_{i<j} V_{NN}(\vec{r}_i - \vec{r}_j) \]

- Introduce a mean-field \( U \) to yield basis
  \[ H = \sum_i \left( \frac{\vec{p}_i^2}{2m} + U(r_i) \right) + \sum_{i<j} V_{NN}(\vec{r}_i - \vec{r}_j) - \sum_i U(r_i) \]

  Residual interaction

The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- The mean field determines the shell structure
- In effect, nuclear-structure calculations rely on perturbation theory
Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers.

Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field.

Two complementary procedures are commonly used:

• Nuclear shell model (NSM)
• Quasiparticle Random Phase Approximation (QRPA)

In **NSM** a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$-decay calculations.

In **QRPA** a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more $0nbb$-decay calculations.
Nuclear Shell Model

\[ H = \sum_a e_a a_a^+ a_a - \sum_{abcd} \langle j_a j_b ; JT | V | j_c j_d ; JT \rangle_A \left[ a_a^+ \otimes a_b^+ \right]^{JT} \otimes [\tilde{a}_c \otimes \tilde{a}_d]^{JT} \]

- Define a valence space
- Derive an effective interaction \( H \Psi = E \Psi \rightarrow H_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}} \)
- Build and diagonalize Hamiltonian matrix (1010)
- Transition operator \( <\Psi_{\text{eff}} | O_{\text{eff}} | \Psi_{\text{eff}} > \)
- Phenomenological input:
  Energy of states, systematics of B(E2) and GT transitions (quenching f.)

48Ca → 48Ti

Small calculations

76Ge → 76Se

76Se in the valence
6 protons and 14 neutrons

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Quasiparticle Random Phase Approximation (QRPA) and its variants

- Large model space (up 23 s.p.l, $^{150}$Nd – 60 active prot. and 90 neut.)
- Spin-orbit partners included
- Possibility to describe all multipolarities of the intermed. nucl. $J^\pi (\pi=\pm1, J=0...9)$

$$H = H_0 + g_{ph} V_{ph} + g_{pp} V_{pp}$$

quasiparticle mean field

Residual interaction

Only Bratislava-Tuebingen group
Realistic NN-interactions used in the QRPA calculations

Modern (phase-shift equivalent) NN potentials

- Nijmegen I - \( P_0 = 5.66\% \) - 41 parameters - \( \chi^2/N_{data} = 1.03 \)
- Nijmegen II - \( P_0 = 5.64\% \) - 47 parameters - \( \chi^2/N_{data} = 1.03 \)
- Argonne \( V_{18} \) - \( P_0 = 5.76\% \) - 40 parameters - \( \chi^2/N_{data} = 1.09 \)
- CD Bonn - \( P_0 = 4.85\% \) - 43 parameters - \( \chi^2/N_{data} = 1.02 \)

Based upon the OBE model

(1999 NN Database: 5990 pp and np scattering data)

Brueckner G-matrices from Tuebingen (H. Muether group)

Bethe-Goldstone equation

\[
G = V + V \frac{Q}{W - H_0 + i\epsilon} G
\]

Renormalization of the NN interaction

Difficulty in the derivation of \( V_{eff} \) from any modern NN potential: existence of a strong repulsive core which prevents its direct use in nuclear structure calculations.

Traditional approach to this problem: Brueckner G-matrix method. The G matrix is model-space dependent as well as energy dependent.
The vectors $X$ and $Y$ are obtained by solving the equations of motion:

\[
\begin{pmatrix}
A & B \\
-B & -A
\end{pmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix} = \omega
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\]

with

\[
A^J_{pn',p'n'} = \langle O | (c^\dagger_p c^\dagger_n)(J^M)^\dagger \hat{H}(c^\dagger_{p'} c^\dagger_{n'})^J | O \rangle \\
= \delta_{pn,p'n'} (E_p + E_n) \\
+ (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) g_{ph} \langle pn^{-1}, J | V | p'n'^{-1}, J \rangle \\
+ (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) g_{pp} \langle pn, J | V | p'n', J \rangle ,
\]

\[
B^J_{pn',p'n'} = \langle O | \hat{H}(c^\dagger_p c^\dagger_n)^{(J-M)} (-1)^M (c^\dagger_{p'} c^\dagger_{n'})^J | O \rangle \\
+ (-1)^J (u_p v_n u_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}) g_{ph} \langle pn^{-1}, J | V | p'n'^{-1}, J \rangle \\
- (-1)^J (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) g_{pp} \langle pn, J | V | p'n', J \rangle .
\]
The $0
\nu\beta\beta$-decay NME: $g_{pp}$ fixed to $2\nu\beta\beta$-decay

Each point: (3 basis sets) x (3 forces) = 9 values

By adjusting of $g_{pp}$ to $2\nu\beta\beta$-decay half-life the dependence of the $0\nu\beta\beta$-decay NME on other things that are not a priori fixed is essentially removed.

The Interacting Boson Model

- The low-lying states of the nucleus, composed by $n$ and $z$ valence nucleons, are modeled in terms of $(n+z)/2$ bosons.
- The bosons have either $L = 0$ (s boson) or $L = 2$ (d boson).
- The bosons can interact through one-body and two-body forces giving rise to bosonic wave functions.
- Any observable can be calculated using these wave functions provided that the relevant operator is employed.

"F. Iachello and A. Arima, The Interacting Boson Model, Cambridge University Press, 1987"
Projected Hartree-Fock-Bogoliubov Model

**PHFB Model**

States of good angular momentum $J$

$$
| \Psi^J_M \rangle = \frac{2J + 1}{8\pi^2 a_J} \int d\Omega \, D^J_{MK} (\Omega) \hat{R} (\Omega) | \Phi^K \rangle
$$

Axially symmetric HFB intrinsic state

$$
| \Phi_0 \rangle = \prod_{im} \left( u_{im} + v_{im} b^+_i b^+_m \right)
$$

where

$$
b^+_i = \sum_m C_{i\alpha_m} a^+_m \quad \quad b^+_m = \sum_i (-1)^{i+j} C_{i\alpha_m} a^+_i a^+_m
$$

Hamiltonian:

$$
H = H_{sp} + V(P) + \zeta_{qq} V(QQ)
$$

Only quadrupole interaction, 
GT interaction is missing
The $0\nu\beta\beta$-decay NME (light $\nu$ exchange mech.)

The $0\nu\beta\beta$-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z) |M^{0\nu}\beta\beta|^2 \langle m_{\beta\beta}\rangle^2,$$

NME = sum of Fermi, Gamow-Teller and tensor contributions

$$M^{0\nu}\beta\beta = \left(\frac{g_A}{1.25}\right)^2 \langle f | - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_{GT} + M^{0\nu}_T | i \rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)q^2}{q^2 + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \quad f_T(qr_{12}) = -j_2(qr_{12})$$

Induced pseudoscalar coupling (pion exchange)

Form-factors:

- Finite nucleon size

$$h_F = g_V^2(q^2)$$

$$h_{GT} = g_A^2 \left[ 1 - \frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} + \frac{1}{3} \left( \frac{q^2}{q^2 + m_\pi^2} \right)^2 \right]$$

$$h_T = g_A^2 \left[ \frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} - \frac{1}{3} \left( \frac{q^2}{q^2 + m_\pi^2} \right)^2 \right]$$

Jastrow f. s.r.c.

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f, J} \sum_{pn'p'n'} (-1)^{j_n + j_p' + J + J} \sqrt{2J + 1} \left\{ j_p \quad j_n \quad J \right\}$$

Jastrow f. s.r.c.

$$\langle p(1), p'(2) : J \mid f(r_{12}) O_K f(r_{12}) \mid n(1), n'(2) : J \rangle \times \langle 0_f^+ \mid [c_p^+ \tilde{c}_{n'}]_J \mid J^\pi k_f \mid J^\pi k_i \rangle \langle J^\pi k_f \mid [c_p^+ \tilde{c}_{n'}]_J \mid 0_i^+ \rangle$$

$$J^\pi = 0^+, 1^+, 2^+, ..., 0^-, 1^-, 2^-,...$$
The $0\nu\beta\beta$-decay NMEs (Status: 2009)

Nobody is perfect:
- LSSM (small m.s., negative parity states)
- PHFB (GT force neglected)
- IBM (Hamiltonian truncated)
- (R)QRPA (g.s. correlations not accurate enough)
A claim of evidence and other experiments (current status)

Faessler, Figli, Lisi, Rodin, Rotunno, Šimkovic,
Anatomy of the $0\nu\beta\beta$-decay NMEs
Decomposition in pp and nn channels

$\langle p(1), p'(2); J \parallel f(r_{12})O_Kf(r_{12}) \parallel n(1), n'(2); J \rangle$

$^{76}\text{Ge}: M^0 = 3.98$

$^{100}\text{Mo}: M^0 = 2.74$

$^{130}\text{Te}: M^0 = 2.67$

$J=0$
Pairing mode

$J\neq0$
Non-pairing mode

The radial dependence of $M^{0n}$ for the three indicated nuclei. The contributions summed over all components shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below. Note that these two parts essentially cancel each other for $r > 2$-3 fm. This is a generic behavior. Hence the treatment of small values of $r$ and large values of $q$ are quite important.
Large Scale Shell Model
Menendez, Poves, Caurier, Nowacki,
Arxive:0901.3760 [nucl-th]

Nuclear physics

Nucleon physics

PHFB
P.Rath, R. Chandra, K. Chaturverdi,
P.Raina, J.G. Hirsch,
to be published in PRC
A consistent approach for the $0\nu\beta\beta$-decay (pairing, s.r.c, g.s.c. calculated with the same NN potential- BonnCD, Argon)

Neutrino potential: $I(r)/r$

$$I(r) = \frac{2}{\pi} \int_0^\infty \frac{\sin(qr)}{(q + E_{\text{aver}})(1 + q^2/E_{\text{cut}}^2)^{1/4}} dq$$

$$|\Psi>|_{\text{corr.}} = f(r_{12}) |\Psi>$$
$$O_{\text{corr.}}(r_{12}) = f(r_{12})O(r_{12})f(r_{12})$$

Two-nucleon short range correlations

Nucleon–Nucleon Potential

Wave function

NN potential
Neutrinoless double beta decay matrix elements

F.Š., Faessler, Muether, Rodin, Stauf, PRC 79, 055501 (2009)
It is of interest to see the contribution of individual orbits to the $0\nu\beta\beta$ matrix element. Within QRPA and its generalization this can be done by using the basic formula:

$$M_K = \sum_{J^\pi, k_z, k_f, \mathcal{J}} \sum_{pnp', n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

$$\sqrt{2\mathcal{J} + 1} \left\{ \begin{array}{ccc}
    j_p & j_n & J \\
    j_{n'} & j_{p'} & \mathcal{J}
\end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} | \tilde{f}(r_{12}) O_K \tilde{f}(r_{12}) | n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0^+_f | \left[ c^+_p c^-_{n'} \right]_J | J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_{n'} \rangle \langle J^\pi k_{n'} \rangle [c^+_p c^-_{n'}]_J | 0^+_i \rangle$$

Summing over all indeces except $n, n'$ (or $p, p'$) will tell give us the required contribution. Note that it can be positive or negative.
Contribution of individual neutron orbits to $M^{0\nu}$ for $^{76}\text{Ge}$ $0\nu\beta\beta$ decay

Note the large positive contributions along the diagonal (pairing) and the negative off-diagonal contributions (higher seniority). The valence orbits dominate, but some also contribute significantly.
How good is the closure approximation?

Comparison between the QRPA $M^{0\nu}$ with the proper energies of the virtual intermediate states (symbols with arrows) and the closure approximation (lines) with different $\langle E_n - E_i \rangle$.

Note the mild dependence on $\langle E_n - E_i \rangle$ and the fact that the exact results are reasonably close to the closure approximation results for $\langle E_n - E_i \rangle < 20$ MeV.
Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons.

Explaining $2\nu\beta\beta$-decay is necessary but not sufficient.
\[2\nu \beta \beta \]

Observed for 10 isotopes: $^{48}\text{Ca}$, $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{96}\text{Zr}$, $^{100}\text{Mo}$, $^{116}\text{Cd}$, $^{128}\text{Te}$, $^{130}\text{Te}$, $^{150}\text{Nd}$, $^{238}\text{U}$, $T_{1/2} \approx 10^{18}-10^{24}$ years

1967: $^{130}\text{Te}$, Kirsten et al, Takaoka et al, (geochemical)
1987: $^{82}\text{Se}$, Moe et al. (direct observation)
2008: $^{100}\text{Mo}$, NEMO 3 coll. $\sim$ 300 000 events

SM forbidden, not observed yet: $T_{1/2}$ ($^{76}\text{Ge}$)$>10^{25}$ years
$2\nu\beta\beta$-decay

Gamow-Teller transitions

Continuum states

$0^+, 1^+$ states

GT resonance

QRPA, RQRPA

SSD hypothesis

$0^+$, $1^+$, $2^+$ states

$M_{2\nu}^{GT} = \sum_{m} \frac{<0_j^+|\tau^+\sigma|1_m^+><1_m^+|\tau^+\sigma|0_i^+>}{E_m - E_i + \Delta}$

Differences in NME: by factor $\sim 10$

Deduced from measured $T_{1/2}^{2\nu}$

$G^{2\nu}|M_{2\nu}^{GT}|^2 = \left( T_{1/2}^{2\nu} \right)^{-1}$
A sum over intermediate nuclear states represents a sum over all meson and gamma exchange correlations of two beta decaying nucleons.

\[ T(\mu(x_1)\nu(x_2)) = J_\mu(x_1)J_\nu(x_2) + \Theta(x_{20} - x_{10})[J_\nu(x_2), J_\mu(x_1)] \]

\[ J_\alpha(0, \bar{x}) = \sum_n \tau_n^+ (\delta_{\alpha 4} + ig_A(\bar{\sigma})_k \delta_{\alpha k}) \delta(\bar{x} - \bar{x}_n) \]

\[ J_{\mu\nu}^{2\beta\beta}(p_1, p_2, k_1, k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k} \times 2\pi \delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \ k = 1, 2, 3, \]
**Integral representation of $M_{GT}$**

$$M_{GT} = \frac{i}{2} \int_{0}^{\infty} \left(e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t}\right)M_{AA}(t)dt$$

*with*

$$M_{AA}(t) = \langle 0^+_f | \frac{1}{2} [A_k(t/2), A_k(-t/2)] | 0^+_i \rangle$$

$$A_k(t) = e^{iHt} A_k(0) e^{-iHt}, \quad A_k = \sum_{i} \tau_{i}^{+} (\vec{\sigma}_{i})_{k}, \quad k = 1, 2, 3.$$  

$$A_k(t) = e^{itH} A_k(0) e^{-itH} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \underbrace{[H[H[...[H, A_k(0)]]]]}_{n \text{ times}}$$

### Completeness:

$$\Sigma_{n} |n><n|=1$$

$$\langle A'| J_{\alpha}(x_1) J_{\beta}(x_2) | A \rangle = \sum_{n} \langle A'| J_{\alpha}(0, \vec{x}_1) | n \rangle \langle n | J_{\beta}(0, \vec{x}_2) | A \rangle \times e^{-i(E' - E_n)x_{10}} e^{-i(E_n - E)x_{20}}$$

$$\int_{0}^{\infty} e^{-iat} dt \Rightarrow \lim_{\epsilon \to 0} \int_{0}^{\infty} e^{-i(a-i\epsilon)t} dt = \lim_{\epsilon \to 0} \frac{-i}{a - i\epsilon}$$

$$M_{GT} = \sum_{n} \frac{\langle 0^+_f | A(0)_k | 1^+_n \rangle \langle 1^+_n | A(0)_k | 0^+_i \rangle}{E_n - E_i + \Delta}$$

11/11/2009
\[ r_{12} \text{-dependence of the } 2\nu\beta\beta\text{-decay NME} \]

\[ M_{GT}^{2\nu} = \sum_{J^\pi, k_i, k_f, J} \sum_{pn p'n'} (-1)^{j_n + j_{p'} + J + J} \times \sqrt{2J + 1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & J \end{array} \right\} \times \langle p(1), p'(2); J || \sigma(1) \cdot \sigma(2) || n(1), n'(2); J \rangle \times \langle 0^+_f || [\tilde{c}_p, \tilde{c}_n], J || J^\pi k_f \rangle \langle J^\pi k_f || J^\pi k_i \rangle \langle J^\pi k_i || [c_p, c_n], J || 0^+_i \rangle \]

\[ C^{2\nu}(r) \]

\[ r \text{ [fm]} \]

\( 76 \text{ Ge} \)

\( 100 \text{ Mo} \)

\( 130 \text{ Te} \)
Decomposition of $C^{2\nu}$
on multipole contributions

\[ M_{GT}^{2\nu} = \langle 0^+_f | \tau^+ \sigma \sum_{J^\pi_m, m} |J^\pi_m\rangle \langle J^\pi_m| \tau^+ \sigma |0^+_i\rangle \]
\[ = \sum_m \frac{\langle 0^+_f | \tau^+ \sigma |1^+_m\rangle \langle 1^+_m| \tau^+ \sigma |0^+_i\rangle}{E_m - (E_i + E_f)/2} \]

\[ \int C_J(r) \, dr = M_{GT}^{2\nu} \text{ for } J^\pi = 1^+ \]
\[ = 0 \text{ for } J^\pi \neq 1^+ \]
Constraining the $0\nu\beta\beta$-decay NMEs

Nucleons that change from neutrons to protons are valence neutrons
Proton, neutron removing transfer reaction

\[ ^{76}\text{Ge} \rightarrow ^{76}\text{Se} \]

John Schiffer, P. Grabmayr et al

\[ n_j^{exp} = \langle 0^+_{\text{init}} | \sum_m c^+_{j,m} c_{j,m} | 0^+_{\text{init}} \rangle \]

QRPA(A) \equiv BCS (WS)

QRPA(B) \equiv BCS (AWS) Suhonen, Civitarese, ovic

PLB 668, 277 (2008)
How can we take into account theoretically the constraint represented by the experimentally determined occupancies?

The experiment fixes

$$n_j^{\text{exp}} = \langle 0^+_\text{init} | \sum c^\dagger_{j,m} c_{j,m} | 0^+_\text{init} \rangle \text{ and the same for the final nucleus}$$

In BCS

$$n_j^{\text{BCS}} = v_j^2 \times (2j+1) \text{ depends only on } v_j \text{ which in turn depends on the mean field eigenenergies}$$

In QRPA the ground state includes correlations and thus

$$n_j^{\text{QRPA}} = (2j+1) \times [v_j^2 + (u_j^2 - v_j^2) \xi_j]$$

$$\xi_j = (2j+1)^{-1/2} \langle 0^+_{\text{qrpa}} | [a^\dagger_j a_j]^0 | 0^+_{\text{qrpa}} \rangle$$

depends on the quasiparticle content of the correlated ground state
Initial and adjusted mean field levels

While $n_j^{\text{exp}}$ and $n_j^{\text{BCS}}$ are constrained by $\sum n_j = N$ (or $Z$) the $n_j^{\text{QRPA}}$ are not constrained by that requirement. The particle number is not conserved, even on average. Thus the QRPA must be modified to remedy this ⇒ Selfconsistent Renormalized QRPA

<table>
<thead>
<tr>
<th>76 Ge $\rightarrow$ 76 Se</th>
<th>prev.</th>
<th>new</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jastrow s.r.c.</td>
<td>4.24(0.44)</td>
<td>3.49(0.23)</td>
</tr>
<tr>
<td>UCOM s.r.c.</td>
<td>5.19(0.54)</td>
<td>4.60(0.39)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>76 Ge</th>
<th>76 Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$f_{5/2}$</td>
<td>$f_{7/2}$</td>
</tr>
<tr>
<td>neut.</td>
<td>5.65</td>
<td>5.27</td>
</tr>
<tr>
<td>$f_{5/2}$</td>
<td>5.54</td>
<td>5.12</td>
</tr>
<tr>
<td>$f_{7/2}$</td>
<td>7.91</td>
<td>7.67</td>
</tr>
<tr>
<td>$g_{7/2}$</td>
<td>0.01</td>
<td>0.05</td>
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<tr>
<td>$g_{9/2}$</td>
<td>0.03</td>
<td>0.14</td>
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<td></td>
<td>0.09</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
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<td>0.53</td>
</tr>
<tr>
<td></td>
<td>4.63</td>
<td>4.78</td>
</tr>
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</table>

F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)
The cross sections give $B(GT)$ for $\beta^+$ and $\beta^-$, product of the amplitudes ($B(GT)^{1/2}$) gives the numerator of the $M^{2\nu}$ matrix element.

**Constraining the $0\nu\beta\beta$-decay NME**

**charge-exchange reactions**

(t, $^3$He)  
(d, $^2$He)

From D. Frekers, RIKEN 2008 lecture

The cross sections give $B(GT)$ for $\beta^+$ and $\beta^-$, product of the amplitudes ($B(GT)^{1/2}$) gives the numerator of the $M^{2\nu}$ matrix element.
Staircase plot (running sum) of the contributions to the $2\nu\beta\beta$ decay ($^{76}{\text{Ge}} \rightarrow ^{76}{\text{Se}}$)

- Adjusted mean field
- Old Woods-Saxon potential
- Experiment (Frekers)
Shell structure of the mean field changed

Deformation

Anisotropic harmonic oscillator
Nuclear deformation

\[ \beta = \sqrt{\frac{\pi}{5}} \frac{Q_P}{Zr_c^2} \]

**Exp. I** (nuclear reorientation method)
**Exp. II** (based on measured E2 trans.)
**Theor. I** (Rel. mean field theory)
**Theor. II** (Microsc.-Macrosc. Model of Moeller and Nix)

Till now, in the QRPA-like calculations of the $0\nu\beta\beta$-decay NME spherical symmetry was assumed.

The effect of deformation on NME has to be considered.

<table>
<thead>
<tr>
<th>Nucl.</th>
<th>Exp. I</th>
<th>Exp. II</th>
<th>Theor. I</th>
<th>Theor. II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>0.00</td>
<td>0.101</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{48}\text{Ti}$</td>
<td>+0.17</td>
<td>0.269</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>+0.09</td>
<td>0.26</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>$^{76}\text{Se}$</td>
<td>+0.16</td>
<td>0.31</td>
<td>-0.24</td>
<td>-0.24</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>+0.10</td>
<td>0.19</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>$^{82}\text{Kr}$</td>
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<td>0.20</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td></td>
<td>0.081</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
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<td>+0.07</td>
<td>0.17</td>
<td>0.17</td>
<td>0.08</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>+0.14</td>
<td>0.23</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>$^{100}\text{Ru}$</td>
<td>+0.14</td>
<td>0.22</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>+0.11</td>
<td>0.19</td>
<td>-0.26</td>
<td>-0.24</td>
</tr>
<tr>
<td>$^{116}\text{Sn}$</td>
<td>+0.04</td>
<td>0.11</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>+0.01</td>
<td>0.14</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{128}\text{Xe}$</td>
<td></td>
<td>0.18</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>+0.03</td>
<td>0.12</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{130}\text{Xe}$</td>
<td></td>
<td>0.17</td>
<td>0.13</td>
<td>-0.11</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td></td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{136}\text{Ba}$</td>
<td></td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{150}\text{Nd}$</td>
<td>+0.37</td>
<td>0.28</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>$^{150}\text{Sm}$</td>
<td>+0.23</td>
<td>0.19</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>
New Suppression Mechanism of the DBD NME

The suppression of the NME depends on relative deformation of initial and final nuclei
F. Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the $2\nu\beta\beta$-decay NME within deformed QRPA
Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F. Š.,
QRPA with realistic forces in deformed nuclei


\[
\langle pp_n \bar{p} \bar{n} \phi_n \mid G \mid p' p' \bar{n}' \bar{n}' \phi_n' \rangle = \sum_J \sum_{(N_o I_j)_p} \sum_{(N_o I_j)_n} \sum_{(N_b I_j)_p} \sum_{(N_b I_j)_n} B^{(p)}_{(N_b I_j)_p} B^{(n)}_{(N_b I_j)_n} B^{(p')}_{(N_b I_j)_p'} B^{(n')}_{(N_b I_j)_n'}
\]

\[
\times (-1)^{J_n - \Omega_n} (-1)^{J_n' - \Omega_n'} C^{JK}_{\Omega_p \Omega_n} C^{JK}_{\Omega_p' \Omega_n'}
\]

\[
\times \langle (N_o I_j)_p (N_o I_j)_n, J \mid G \mid (N_o I_j)_p (N_o I_j)_n', J \rangle
\]

G-matrix elements in spherical single particle basis

Bonn CD potential
$2\nu\beta\beta$-decay and statistical properties of $\nu$
Mixed statistics for neutrinos

Defninnition of mixed state

\[ |\nu > = \hat{a}^\dagger |0 > = \cos \delta \hat{f}^\dagger |0 > + \sin \delta \hat{b}^\dagger |0 > = \cos \delta |f > + \sin \delta |b > \]

with commutation Relations

\[ \hat{f}\hat{b} = e^{i\phi} \hat{b}\hat{f} \]
\[ \hat{f}^\dagger \hat{b}^\dagger = e^{-i\phi} \hat{b}^\dagger \hat{f} \]

Amplitude for 2νββ

\[ A^{2\nu} = [\cos \delta^4 + \cos \delta^2 \sin \delta^2 (1 - \cos \phi)] A^f + [\cos \delta^4 + \cos \delta^2 \sin \delta^2 (1 + \cos \phi)] A^b \]
\[ = \cos \chi^2 A^f + \sin \chi^2 A^b \]

Decay rate

\[ W^{2\nu} = \cos \chi^4 W^f + \sin \chi^4 W^b \]
\[ = (1 - b^2) W^f + b^2 W^b \]

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

( calculations coming up soon )

11/11/2009 Fedor Simkovic 58
Looking for a signature of bosonic $\nu$

$2\nu\beta\beta$–decay half-lives ($0^+ \rightarrow 0^+_{\text{g.s.}}, 0^+ \rightarrow 0^+_{1}, 0^+ \rightarrow 2^+_{1}$)

- HSD – NME needed
- SSD – log $ft_{\text{EC}}, \log ft_\beta$ needed

\[ \frac{T_{1/2}^{2\nu-SSD}(2^+_f)}{T_{1/2}^{2\nu-SSD}(0^+_f)} = 2.41 \times 10^4 \quad \text{fermionic } \nu \]
\[ = 403 \quad \text{bosonic } \nu \]

\[ T_{1/2}^{2\nu}(2^+) = 1.73 \times 10^{23} \text{years} \]

\[ T_{1/2}^{2\nu-ex}(2^+) > 1.6 \times 10^{21} \text{years} \]

Normalized differential characteristics

- The single electron energy distribution
- The distribution of the total energy of two electrons
- Angular correlations of two electrons
  (free of NME and log $ft$)
Mixed $\nu$ excluded for $\sin^2\chi < 0.6$

$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ (SSD)
Neutrinoless Double Electron Capture
Modes of the $0\nu$ECEC-decay:
$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$
$$+ 2\gamma$$
$$+ e^+ e^-$$
$$+ M$$

Neutrinoless double electron capture

Theoretically, not well understood yet:
- which mechanism is important?
- which transition is important?

\[ \Gamma^{0\nu\gamma} = \frac{\Gamma^r(2p \rightarrow 1s)}{[E_\gamma - Q_{res}]^2 + [\Gamma^r/2]^2} |\langle R^{CC}_{0\nu}\rangle|^2 \]

Fed \[ Q_{res} = E_{s_{1/2}} - E_{p_{1/2}} \]
Mixing of neutral atoms and total lepton number oscillation

\[ n + n \leftrightarrow p + p + e^- + e^- \]

\[ (A, Z) \leftrightarrow (A, Z + 2)^{**} \]

\[ (A, Z) \leftrightarrow (A, Z - 2)^{**} \]

LNV Potential

\[ V_{LNV} \approx m_{\beta\beta} \frac{G_F^2}{4\pi r_a} < \Psi_1(0)\Psi_2(0) \]

\[ V_{LNV} \sim 10^{-24} \text{ eV} \]

\[ m_\nu = 0.5 \text{ eV}, \quad Z = 30, \quad n_i = 1, \quad l_i = 0 \]
Different types of Oscillations (Effective Hamiltonian)

Oscillations of $\nu_l$-$\bar{\nu}_l$ (lepton flavor)

Oscillation of $K_0$-anti{$K_0$} (strangeness)

Oscillation of $n$-anti{$n$} (baryon number)

Oscillation of atoms (total lepton number)

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2} \Gamma_1,$$

$$\lambda_- = M_f - \frac{i}{2} \Gamma - \Delta M + \frac{i}{2} \Gamma_1,$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4} \Gamma^2},$$

$$\Gamma_1 = \frac{V^2}{(M_i - M_f)^2 + \frac{1}{4} \Gamma^2}.$$
### Calculated double electron capture half-lives ($m_{\beta\beta} = 1$ eV)

<table>
<thead>
<tr>
<th>Transition</th>
<th>$M^*<em>{A,Z-2} - M</em>{A,Z-2}$</th>
<th>$M^{**}<em>{A,Z-2} - M</em>{A,Z}$</th>
<th>Holes</th>
<th>$T_{1/2}^{min}$</th>
<th>$T_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{112}<em>{50}\text{Sn} \rightarrow ^{112}</em>{48}\text{Cd}^*$</td>
<td>1871 ± 0.2</td>
<td>−5.9 ± 4.2 ± 2.7</td>
<td>$1s_{1/2} \ 1s_{1/2}$</td>
<td>$2 \times 10^{24}$</td>
<td>$8 \times 10^{30}$</td>
</tr>
<tr>
<td>$^{152}<em>{64}\text{Gd} \rightarrow ^{152}</em>{62}\text{Sm}$</td>
<td>0</td>
<td>−0.3 ± 2.5 ± 2.5</td>
<td>$1s_{1/2} \ 2s_{1/2}$</td>
<td>$5 \times 10^{24}$</td>
<td>$9 \times 10^{29}$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.9 ± 2.5 ± 2.5</td>
<td>$1s_{1/2} \ 3s_{1/2}$</td>
<td>$4 \times 10^{25}$</td>
<td>$8 \times 10^{29}$</td>
</tr>
<tr>
<td>$^{148}<em>{64}\text{Gd} \rightarrow ^{148}</em>{62}\text{Sm}^*$</td>
<td>3045 ± 2</td>
<td>5.7 ± 2.5 ± 2.5</td>
<td>$2s_{1/2} \ 2s_{1/2}$</td>
<td>$8 \times 10^{25}$</td>
<td>$3 \times 10^{32}$</td>
</tr>
<tr>
<td></td>
<td>3045 ± 2</td>
<td>11.8 ± 2.5 ± 2.5</td>
<td>$2s_{1/2} \ 3s_{1/2}$</td>
<td>$3 \times 10^{26}$</td>
<td>$8 \times 10^{33}$</td>
</tr>
<tr>
<td></td>
<td>3045 ± 2</td>
<td>13.3 ± 2.5 ± 2.5</td>
<td>$2s_{1/2} \ 4s_{1/2}$</td>
<td>$4 \times 10^{27}$</td>
<td>$2 \times 10^{35}$</td>
</tr>
<tr>
<td>$^{156}<em>{66}\text{Dy} \rightarrow ^{156}</em>{64}\text{Gd}^*$</td>
<td>3045 ± 2</td>
<td>6.6 ± 2.5 ± 2.5</td>
<td>$2p_{1/2} \ 2p_{1/2}$</td>
<td>$2 \times 10^{29}$</td>
<td>$2 \times 10^{36}$</td>
</tr>
<tr>
<td></td>
<td>1988.5 ± 0.2</td>
<td>7.0 ± 6.6 ± 2.5</td>
<td>$2s_{1/2} \ 2s_{1/2}$</td>
<td>$2 \times 10^{27}$</td>
<td>$8 \times 10^{31}$</td>
</tr>
<tr>
<td></td>
<td>1988.5 ± 0.2</td>
<td>7.9 ± 6.6 ± 2.5</td>
<td>$2p_{1/2} \ 2p_{1/2}$</td>
<td>$8 \times 10^{29}$</td>
<td>$4 \times 10^{35}$</td>
</tr>
</tbody>
</table>

### Lepton number and parity oscillations

Fedor Simkovic

\[
\Gamma_1 = \frac{4V^2}{4(M_i - M_f) + \Gamma^2} \Gamma
\]
Double electron capture 
\[ e_{1s1/2} + e_{1s1/2} + ^{112}\text{Sn} \rightarrow ^{112}\text{Cd}(0^+\_3) \]

Reletivistic electron w.f. \((j=1/2, l=0, l'=1)\)

\[
\Psi^{(\alpha)}_{jm}(\vec{x}) = \left( \begin{array}{c} f_{\alpha}(r) \ \Omega_{jlm} \\ (-1)^{\frac{1+\ell+\ell'}{2}} g_{\alpha}(r) \ \Omega_{j\ell'm} \end{array} \right) \quad l = j \pm 1/2, \ l' = 2j - l
\]

Potential

\[
V^{1s1/21s1/2}(0^+_3) = \frac{1}{4\pi} m_e \left( G^2_{\beta} m_e^4 \right) \frac{m_\beta}{m_e} \frac{1}{R m_e} \left( \frac{\bar{f}_{1s1/2}}{4\pi m_e^3} \right)^2 g_A M^{0\nu}(0^+_3).
\]

Width

\[
\Gamma \text{ECEC} = \frac{\left| V^{1s1/21s1/2}(0^+_3) \right|^2}{(M_i - M_f)^2 + \frac{\Gamma_X^2}{4}} \Gamma_X
\]

Matrix element

<table>
<thead>
<tr>
<th>Exc. state</th>
<th>(E_{ex}) (MeV)</th>
<th>(M^{0\nu})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^+ _gs.</td>
<td>0</td>
<td>2.69</td>
</tr>
<tr>
<td>0^+_1 (1 ph.)</td>
<td>1.224</td>
<td>3.02</td>
</tr>
<tr>
<td>0^+_2 (2 ph.)</td>
<td>1.433</td>
<td>0.90</td>
</tr>
<tr>
<td>0^+_3 (1 ph.)</td>
<td>1.224</td>
<td>2.78</td>
</tr>
</tbody>
</table>

11/11/2009
Fedor Simkovic
In comparison with the $0\nu\beta\beta$-decay disfavoured due:

- process in the 3-rd (4th) order in electroweak theory
- bound electron wave functions favoured: resonant enhancement?

\[ T_{1/2} > 9.2 \times 10^{19} \text{ years} \]

A.S. Barabash et al., NPA 807 (2008) 269
Double electron capture of $^{112}\text{Sn}$
(perspectives of search)

F. Šimkovic, M. Krivoruchenko, A. Faessler, to be submitted

$M_i - M_f$  $T_{1/2}^{\text{ECEC}}$
________________________
$\beta\beta = 50$ meV


|$1 \text{ keV}$ | $2.44 \times 10^{31}$ years
|$100 \text{ eV}$ | $2.45 \times 10^{29}$ years
|$10 \text{ eV}$  | $2.91 \times 10^{27}$ years
|$0 \text{ eV}$  | $4.67 \times 10^{26}$ years

$T_{1/2}^{0\nu}(^{76}\text{Ge}) = (2.95 - 5.74) \times 10^{26}$ years for $\beta\beta = 50$ meV
Program: Matrix Elements for Fundamental Processes

0νββ-decay:
- Systematic study of NMEs for all mechanisms (general parameterization), coexistence of different LNV mechanisms, angular distributions, transitions to excited states
- Improvement of the nuclear structure model (QRPA-deformation, non-linear phonon operator, calculation from single vacuum state)
- A detailed comparison with other approaches (NSM, IBM, PHFB) and with available nuclear structure data (occupation numbers, β-strengths)

2νββ-decay:
- Better understand the role of residual interaction
- Predictions for transitions to 2+ states in the context of bosonic neutrinos

0νECEC: resonant transition to excited states of final atoms, Analysis of capture probabilities, calculation of corresponding nuclear and atomic matrix elements

Single β-decay: in the context of neutrino mass (³H, ¹⁸⁷Re) and relic neutrino detection

Dark matter detection: spin-dependent interaction, scattering of neutralinos on odd-odd nuclei, role of exchange currents

Frank Avignone:
Nuclear Matrix Elements are as important as DATA
What is the nature of neutrinos?

Only the $0\nu\beta\beta$-decay can answer this fundamental question

$\nu \Rightarrow \text{theory}$

Double electron capture

(Muenster, Dresden, Jyvaskula, Bratislava…col.)