

Dark Matter and Missing Energy at the LHC

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Heidelberg

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Outline

Mass reconstruction (relativistic kinematics)

Underlying parameters (statistics and errors)

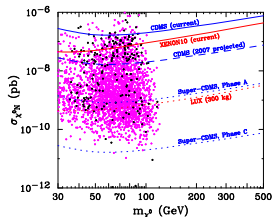
Effective Standard Model in the LHC era

Expectations from the LHC [Uli Baur's rule: 'there is always new physics at higher scales']

- find light Higgs?
- find new physics stabilizing Higgs mass?
- see dark-matter candidate (WIMP)?

Particle theory and new physics

- model-independent analyses likely not helpful
- testing testable hypotheses [theory: e.g. Higgs sector and underlying theory?]
 - discrete hypotheses: spins,....
 - continuous hypotheses: masses,....
- link to other observations [DM+Tevatron: Hooper, TP, Valinotto]
- reconstruction of Lagrangian [theory+experiment]



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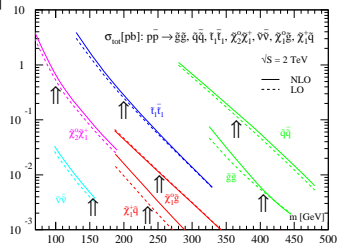
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- beyond inclusive searches [that was Tevatron]
lots of strongly interacting particles
cascade decays to DM candidate
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- ⇒ **aim at underlying theory**



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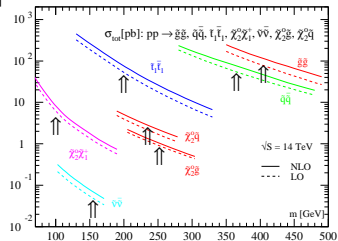
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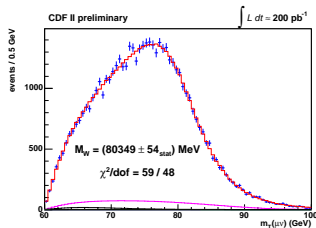
Transverse mass

Learning from neutrinos...

- $W \rightarrow \ell \nu$ decay to invisible particles
- longitudinal boost unobserved at hadron colliders
- transverse instead of invariant W mass $[E_T^2 = \vec{p}_T^2 + m^2, \text{ observed 2D } \vec{p}_T]$

$$\begin{aligned}
 m_{T,W}^2 &= (E_T^{\text{miss}} + E_{T,\ell})^2 - (\vec{p}_T^{\text{miss}} + \vec{p}_{T,\ell})^2 \\
 &= m_\ell^2 + m_{\text{miss}}^2 + 2(E_{T,\ell} E_T^{\text{miss}} - \vec{p}_{T,\ell} \cdot \vec{p}_T^{\text{miss}}) \leq m_W^2
 \end{aligned}$$

⇒ m_W from endpoint and shape



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⇒ m_W from endpoint and shape

- two invisible particles: $H \rightarrow WW \rightarrow \ell \nu \ell \nu$

$$\begin{aligned} m_{T,WW}^2 &= \left(E_T^{\text{miss}} + E_{T,\ell\ell} \right)^2 - \left(\vec{p}_T^{\text{miss}} + \vec{p}_{T,\ell\ell} \right)^2 \\ &= m_{\ell\ell}^2 + m_{\text{miss}}^2 + 2 \left(E_{T,\ell\ell} E_T^{\text{miss}} - \vec{p}_{T,\ell\ell} \cdot \vec{p}_T^{\text{miss}} \right) \end{aligned}$$

- assumption needed:
 $m_{\text{miss}} = m_{\ell\ell}$ sharp peak
 $m_{\text{miss}} = 0$ endpoint [general $m_T < m$]

⇒ $H \rightarrow WW$ at Tevatron and LHC [Kauer, TP, Rainwater, Zeppenfeld; Barr, Gripaios, Lester]

⇒ massive invisible particles and decay chains...?

Transverse masses

M_{T2} algorithm [Lester, Summers; Barr, Lester, Stephens]

– SUSY process $pp \rightarrow \tilde{\ell}\tilde{\ell}^* \rightarrow \ell\tilde{\chi}_1^0 \ell\tilde{\chi}_1^0$ [massive particles, balancing \vec{p}_T^{miss}]

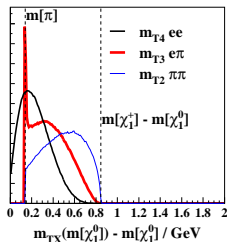
– (in)famous m_{T2} [for m_{miss} hypothesis]

$$m_{T2}(m_{\text{miss}}) = \min_{\vec{p}_T^{\text{miss}} = \vec{q}_1 + \vec{q}_2} \left[\max_j m_{T,j}(\vec{q}_j; m_{\text{miss}}) \right]$$

– $m_{\text{daughter}}^{(1)} + m_{\text{daughter}}^{(2)} < m_{T2} < m_{\text{mother}}$ sharp endpoint [for correct m_{miss}]

– constraint in $m_{\tilde{\ell}} - m_{\tilde{\chi}_1^0}$ plane

m_{TX} with more invisible particles not as promising [$\nu\nu\tilde{\chi}_1^0\tilde{\chi}_1^0$ etc]



Transverse masses

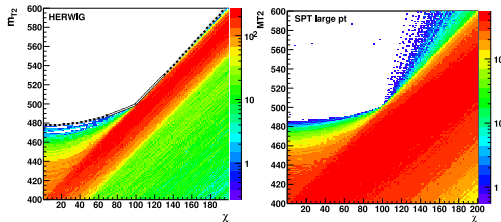
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- constraint in $m_{\tilde{\ell}} - m_{\tilde{\chi}_1^0}$ plane
- $m_{T\chi}$ with more invisible particles not as promising [$\nu\nu\tilde{\chi}_1^0\tilde{\chi}_1^0$ etc]
- m_{T2} boost invariant only for correct m_{miss}
- scan over m_{miss}
- accumulation points for many events [Cho, Choi, Kim, Park; Barr, Gripanos, Lester]
- kink for pairwise three-particle decays $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \rightarrow \tilde{f}\tilde{\chi}_1^0 \tilde{f}\tilde{\chi}_1^0$

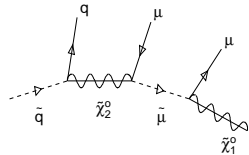
→ determine missing mass



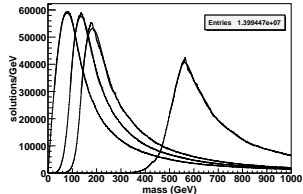
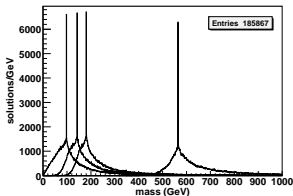
Mass relations

Same masses in all events [Nojiri, Tovey; McElrath et al; Webber]

- on-shell decay chain $\tilde{q}_L \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\ell} \rightarrow \tilde{\chi}_1^0$
- $$(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \vec{\mathbf{p}}^{\text{miss}})^2 = m_{\tilde{q}_L}^2$$
- $$(\mathbf{p}_2 + \mathbf{p}_3 + \vec{\mathbf{p}}^{\text{miss}})^2 = m_{\tilde{\chi}_2^0}^2$$
- $$(\mathbf{p}_3 + \vec{\mathbf{p}}^{\text{miss}})^2 = m_{\tilde{\ell}}^2$$
- $$(\vec{\mathbf{p}}^{\text{miss}})^2 = m_{\tilde{\chi}_1^0}^2$$



- pair production, $\vec{\mathbf{p}}_T^{\text{miss}}$ measured: 4+2 constraints for 8 unknowns [McElrath et al]
- two events: 12+4 constraints for 16 unknowns
solve for invisible momenta, extract masses [quantum effects, detector effects]



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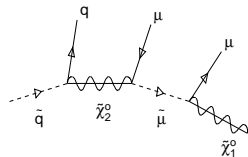
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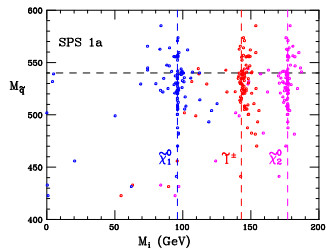
$$(\rho_2 + \rho_3 + \vec{p}^{\text{miss}})^2 = m_{\tilde{\chi}_2^0}^2$$

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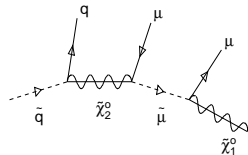
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- **working scheme for LHC** [backgrounds etc to be checked]

$\delta\rho/\rho$	ξ_{max}^2	f_{ξ}	f_{cor}	$M_{\tilde{q}} (540)$	$M_{\tilde{\chi}_2^0} (177)$	$M_{\tilde{\ell}} (143)$	$M_{\tilde{\chi}_1^0} (96)$
0	∞	100%	72%	538 ± 20	176 ± 12	143 ± 7	95 ± 10
0	100	80%	76%	539 ± 7	177 ± 1	144 ± 1	96 ± 2
5%	∞	100%	52%	534 ± 28	176 ± 11	143 ± 10	95 ± 13
5%	100	57%	55%	539 ± 9	178 ± 3	144 ± 2	96 ± 4
10%	∞	100%	40%	522 ± 37	171 ± 18	140 ± 17	88 ± 26
10%	200	42%	43%	530 ± 22	173 ± 12	140 ± 12	89 ± 20

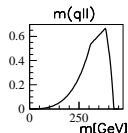
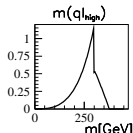
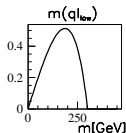
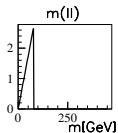
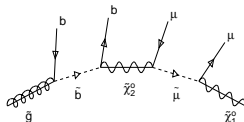
Kinematic endpoints

Cascade decays [Atlas-TDR, Cambridge people]

- new particles strongly interacting and LSP weakly interacting
- long chain $\tilde{g} \rightarrow \tilde{b}\tilde{b} \rightarrow \tilde{\chi}_2^0 b\tilde{b} \rightarrow \mu^+ \mu^- b\tilde{b}\tilde{\chi}_1^0$
- tough: $(\sigma\text{BR})_1/(\sigma\text{BR})_2$ [model dependence, QCD uncertainty]
easier: kinematics
- thresholds & edges

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- new-physics spins from shapes [Barr, Lester, Smillie, Webber; Alves, Eboli, TP;...]



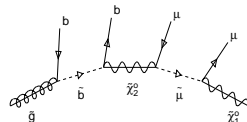
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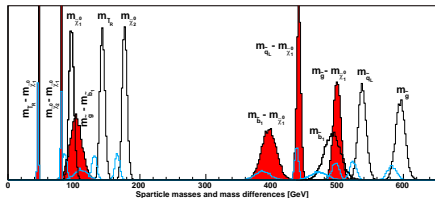
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- no problem: off-shell [Catpiss]
- no problem: jet radiation?
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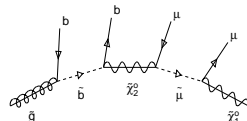
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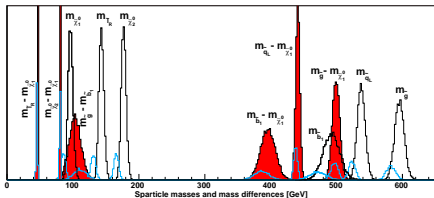
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- \Rightarrow but why physical masses?



Underlying parameters

From kinematics to weak-scale parameters [Fittino; SFitter: Lafaye, TP, Rauch, Zerwas]

- parameters: weak-scale Lagrangian
- measurements: edges or masses,
branching fractions, rates,... [NLO, of course]
ew precision, dark matter,...
- errors: general correlation, statistics & systematics & theory [flat theory errors!]
- problem in grid: no local maximum
problem in fit: no global maximum
problem in interpretation: bad observables, secondary maxima?

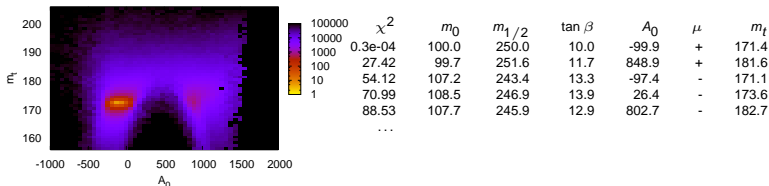
Probability maps of new physics [Baltz,...; Roszkowski,...; Allanach,...; SFitter]

- want probability of model being true $p(m|d)$
- can do exclusive likelihood map $p(d|m)$ over m
- LHC challenge: poor data [e.g. endpoints vs rates]
- Bayesian: $p(m|d) \sim p(d|m) p(m)$ [cosmology, BSM]
frequentist: best-fitting point $\max_m p(d|m)$ [flavor, Higgs@LHC]
- LHC era: (1) compute high-dimensional map $p(d|m)$
(2) find and rank local best-fitting points
(3) predict additional observables

Correlations and errors

Toy model: MSUGRA map from LHC [LHC endpoints with free y_j]

- model unrealistic but useful testing ground
 - SFitter output #1: fully exclusive likelihood map
SFitter output #2: ranked list of local maxima
- ⇒ correlations and secondary maxima significant



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A word on errors

- statistical errors Gaussian
systematic errors Gaussian, correlated
theory errors flat
- RFit scheme

[CKMFitter, profile likelihood inspired]

$$\chi^2 = -2 \log \mathcal{L} = \vec{\chi}_d^T \mathbf{C}^{-1} \vec{\chi}_d$$

$$\chi_{d,i} = \begin{cases} 0 & |d_i - \bar{d}_i| < \sigma_i^{(\text{theo})} \\ \frac{\mathcal{D}|d_i - \bar{d}_i| - \sigma_i^{(\text{theo})}}{\mathcal{D}\sigma_i^{(\text{exp})}} & |d_i - \bar{d}_i| > \sigma_i^{(\text{theo})} \end{cases},$$

$$C_{i,i} = 1 \quad C_{i,j} = C_{j,i} = \frac{0.99 \sigma_i^{(\ell)} \sigma_j^{(\ell)} + 0.99 \sigma_i^{(j)} \sigma_j^{(j)}}{\sigma_i^{(\text{exp})} \sigma_j^{(\text{exp})}}$$

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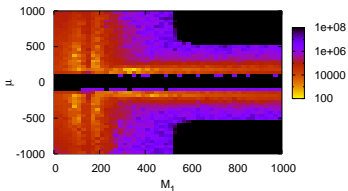
- statistical errors Gaussian
systematic errors Gaussian, correlated
theory errors flat
 - theory error sizeable
- ⇒ endpoints instead of masses

	SPS1a	$\Delta_{\text{zero}}^{\text{theo-exp}}$	$\Delta_{\text{zero}}^{\text{expNoCorr}}$	$\Delta_{\text{zero}}^{\text{theo-exp}}$	$\Delta_{\text{gauss}}^{\text{theo-exp}}$	$\Delta_{\text{flat}}^{\text{theo-exp}}$
		masses		endpoints		
m_0	100	4.11	1.08	0.50	2.97	2.17
$m_{1/2}$	250	1.81	0.98	0.73	2.99	2.64
$\tan \beta$	10	1.69	0.87	0.65	3.36	2.45
A_0	-100	36.2	23.3	21.2	51.5	49.6
m_t	171.4	0.94	0.79	0.26	0.89	0.97

TeV-scale MSSM: SFitter

MSSM map from LHC mass measurements

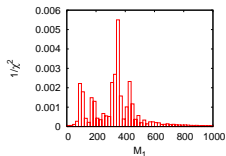
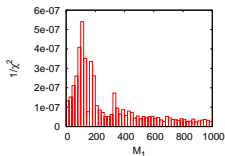
- 19D parameter space [Markov chain globally + hill climber locally]
- SFitter outputs #1 and #2 still the same
- three neutralinos observed [left: Bayesian — right: likelihood]



TeV-scale MSSM: SFitter

MSSM map from LHC mass measurements

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- quality of fit all the same...

	$\mu < 0$				$\mu > 0$			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan \beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
$M_{\tilde{\mu}_L}$	192.7	192.7	192.7	192.9	192.6	192.6	192.7	192.8
$M_{\tilde{\mu}_R}$	131.1	131.1	131.1	131.3	131.0	131.0	131.1	131.2
$A_t (-)$	-252.3	-348.4	-477.1	-259.0	-470.0	-484.3	-243.4	-465.7
$A_t (+)$	384.9	481.8	641.5	432.5	739.2	774.7	440.5	656.9
m_A	350.3	725.8	263.1	1020.0	171.6	156.5	897.6	256.1
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

⇒ combination with astro-particle measurements...

Outlook

Once we actually see LHC data...

- from neutrinos we know invisible particles
 - WIMP the same, but massive
 - transverse masses for short decays
mass relations for pure samples
cascade endpoints including spin info
 - missing: co-transverse mass [Tovey]
 - missing: dark matter sectors [Dama, Pamela, Atic, Fermi]
- ⇒ **LHC more than a discovery machine!**



**Missing Energy at
the LHC**

Tilman Plehn

Mass reconstruction

Parameters