Leptogenesis in the Kadanoff-Baym formalism

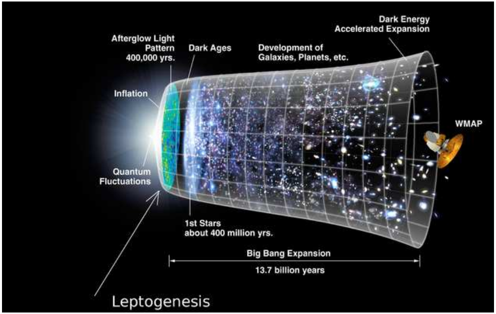
A. Kartavtsev

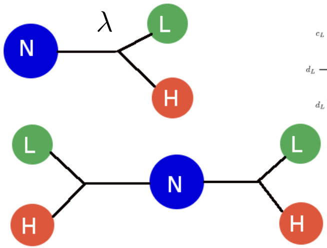
in collaboration with

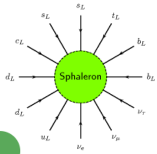
M. Garny, A. Hohenegger and M. Lindner

MPI für Kernphysik, Heidelberg

Baryogenesis via leptogenesis







The universe is baryonically asymmetric:

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = 6.1^{+0.3}_{-0.2} \times 10^{-10}$$

Baryogenesis via leptogenesis:

- The Majorana mass term violates lepton number;
- Complex Yukawa couplings λ induce CP-violation;
- Expansion of the universe provides deviation from equilibrium;
- Sphalerons convert the lepton asymmetry to the baryon asymmetry.

Canonical approach

The Boltzmann equation for the lepton (baryon) distribution function:

$$
\begin{aligned}
p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2} \int d\Pi_\alpha d\Pi_\beta \ldots d\Pi_i d\Pi_j \ldots \\
\times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \ldots - p_i - p_j) \\
\times [|\mathcal{M}|^2_{i+j+\ldots\to L+\alpha+\beta\ldots} f_i f_j \ldots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
- f_L f_\alpha f_\beta \ldots (1 \pm f_i)(1 \pm f_j) \ldots |\mathcal{M}|^2_{L+\alpha+\beta\ldots\to i+j+\ldots}].
\end{aligned}
$$

Pros:

- Easy to derive and solve numerically.

Cons:

- Relies on the quasiparticle picture;
- Matrix elements $\mathcal{M}$ are calculated in *vacuum*.
- Double-counting problem;

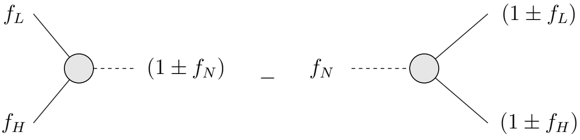
Canonical approach

The Boltzmann equation for the lepton (baryon) distribution function:

$$
\begin{aligned}
p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2} \int & d\Pi_\alpha d\Pi_\beta \ldots d\Pi_i d\Pi_j \ldots \\
& \times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \ldots - p_i - p_j) \\
& \times \big[|\mathcal{M}|^2_{i+j+\ldots \to L+\alpha+\beta\ldots} f_i f_j \ldots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
& - f_L f_\alpha f_\beta \ldots (1 \pm f_i)(1 \pm f_j) \ldots |\mathcal{M}|^2_{L+\alpha+\beta\ldots \to i+j+\ldots} \big].
\end{aligned}
$$

Cons:

- Double-counting problem:



$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 (1 - \epsilon)$ $\qquad$ $|\mathcal{M}|^2 = |\mathcal{M}_0|^2 (1 + \epsilon)$

Canonical approach

The Boltzmann equation for the lepton (baryon) distribution function:

$$
\begin{aligned}
p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2} \int d\Pi_\alpha d\Pi_\beta \ldots d\Pi_i d\Pi_j \ldots \\
\times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \ldots - p_i - p_j) \\
\times [|\mathcal{M}|^2_{i+j+\ldots\to L+\alpha+\beta\ldots} f_i f_j \ldots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
- f_L f_\alpha f_\beta \ldots (1 \pm f_i)(1 \pm f_j) \ldots |\mathcal{M}|^2_{L+\alpha+\beta\ldots\to i+j+\ldots}].
\end{aligned}
$$

Cons:

- Double-counting problem:

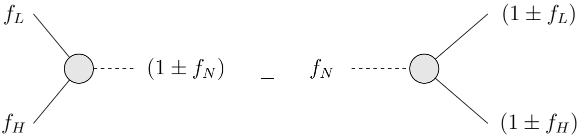
Canonical approach

The Boltzmann equation for the lepton (baryon) distribution function:

$$
\begin{aligned}
p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2} \int & d\Pi_\alpha d\Pi_\beta \ldots d\Pi_i d\Pi_j \ldots \\
& \times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \ldots - p_i - p_j) \\
& \times [|\mathcal{M}|^2_{i+j+\ldots \to L+\alpha+\beta\ldots} f_i f_j \ldots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \\
& - f_L f_\alpha f_\beta \ldots (1 \pm f_i)(1 \pm f_j) \ldots |\mathcal{M}|^2_{L+\alpha+\beta\ldots \to i+j+\ldots}].
\end{aligned}
$$

Cons:

- Double-counting problem:



$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$ $\qquad$ $|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$

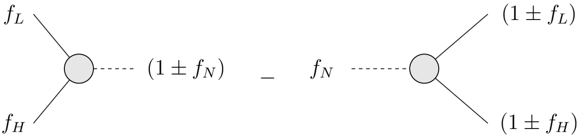
Open questions

Despite the advances a number of questions remains uninvestigated:

- Applicability of the RIS procedure in the resonant regime ?

Canonical approach:

- Solution to the double-counting problem:



$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon) \qquad |\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$$

Resonant regime:

$\Delta M \sim \Gamma$

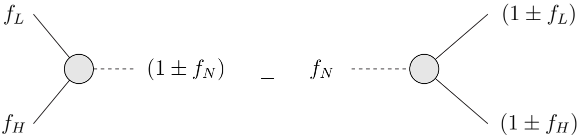
Open questions

Despite the advances a number of questions remains uninvestigated:

- Applicability of the RIS procedure in the resonant regime ?
- Expression for the CP-violating parameter in the resonant case ?

Canonical approach:

- Solution to the double-counting problem:



$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$ $\qquad$ $|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$

Resonant regime:

$\Delta M \sim \Gamma$

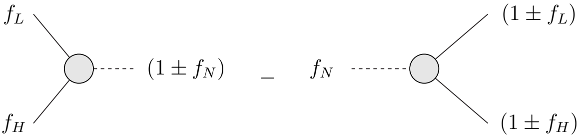
Open questions

Despite the advances a number of questions remains uninvestigated:

- Applicability of the RIS procedure in the resonant regime ?
- Expression for the CP-violating parameter in the resonant case ?
- Medium corrections to the CP-violating parameter ?

Canonical approach:

- Solution to the double-counting problem:



$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$ $\qquad$ $|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$

Resonant regime:

$\Delta M \sim \Gamma$

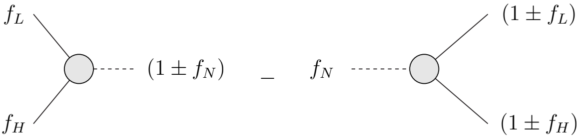
Open questions

Despite the advances a number of questions remains uninvestigated:

- Applicability of the RIS procedure in the resonant regime ?
- Expression for the CP-violating parameter in the resonant case ?
- Medium corrections to the CP-violating parameter ?
- Influence of the expansion of the Universe ?

Canonical approach:

- Solution to the double-counting problem:



$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon) \qquad |\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$$

Resonant regime:

$\Delta M \sim \Gamma$

From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha f_L = \frac{1}{2}\int d\Pi_\alpha d\Pi_\beta \ldots d\Pi_i d\Pi_j \ldots$$

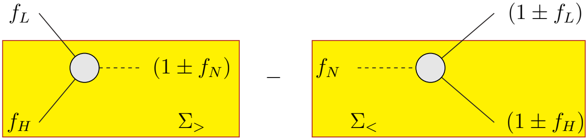
$$\times (2\pi)^4 \delta(p_L + p_\alpha + p_\beta \ldots - p_i - p_j) \qquad \Sigma_< \text{, gain term}$$

$$\times \left[|\mathcal{M}|^2_{i+j+\ldots \to L+\alpha+\beta\ldots} f_i f_j \ldots (1 \pm f_\alpha)(1 \pm f_\beta)(1 \pm f_L) \right.$$

$$\left. - f_L f_\alpha f_\beta \ldots (1 \pm f_i)(1 \pm f_j) \ldots |\mathcal{M}|^2_{L+\alpha+\beta\ldots \to i+j+\ldots} \right].$$

$$\Sigma_> \text{, loss term}$$

Gain and loss terms:



$|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$ $\qquad$ $|\mathcal{M}|^2 = |\mathcal{M}_0|^2(1+\epsilon)$

From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2}\left[\Sigma_<(X,p)(1 \pm f_L) - f_L \Sigma_>(X,p)\right].$$

From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha f_L = \tfrac{1}{2}[\Sigma_<(X,p)(1 \pm f_L) - f_L \Sigma_>(X,p)]\,.$$

Now we change the notation.

- Spectral function:

$$D_\rho(X,p) = (2\pi)\mathrm{sign}(p_0)\delta(p^2 - m^2), \quad p^\alpha \mathcal{D}_\alpha D_\rho(X,p) = 0\,.$$

- Statistical propagator:

$$D_F(X,p) = \left[\tfrac{1}{2} \pm f_L(X,p)\right] D_\rho(X,p)\,.$$

- For further convenience:

$$D_>(X,p) \equiv [1 \pm f_L(X,p)]\, D_\rho(X,p)\,, D_<(X,p) \equiv f_L(X,p) D_\rho(X,p)\,.$$

From Boltzmann to Kadanoff-Baym

The Boltzmann equation for the lepton (baryon) distribution function:

$$p^\alpha \mathcal{D}_\alpha D_F(X,p) = \frac{1}{2}[\Sigma_<(X,p) D_>(X,p) - D_<(X,p)\Sigma_>(X,p)]\,,$$
$$p^\alpha \mathcal{D}_\alpha D_\rho(X,p) = 0\,.$$

What have we gained ?

- So far, only a change of notation;
- However, coincide with the equations that can be derived from the Kadanoff-Baym equations.
- The Kadanoff-Baym gain and loss terms $\Sigma_{\gtrless}$ differ from the Boltzmann gain and loss terms.

Kadanoff-Baym equations

Kadanoff-Baym equations – a coupled system of integro-differential equations for the spectral function and statistical propagator:

$$[\Box_x + m^2(x)] D_F(x,y) = \int_0^{y^0} \mathscr{D}^4 z \, \Sigma_F(x,z) D_\rho(z,y) - \int_0^{x^0} \mathscr{D}^4 z \, \Sigma_\rho(x,z) D_F(z,y) \,,$$

$$[\Box_x + m^2(x)] D_\rho(x,y) = \int_{x^0}^{y^0} \mathscr{D}^4 z \, \Sigma_\rho(x,z) D_\rho(z,y) \,.$$

Pros:

- Do not rely on the concept of quasiparticles;
- Take the medium effects into account;
- Free of the double counting problem.

Cons:

- Complexity.

Schwinger–Dyson equation

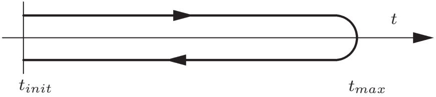
Schwinger–Dyson equation relates full propagator to the corresponding free-field propagator and the self-energy:

$$D^{-1}(x,y) = \mathscr{D}^{-1}(x,y) - \Sigma(x,y)$$

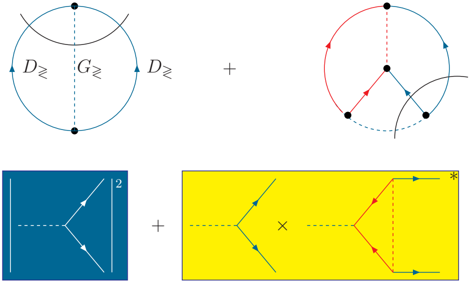
where

$$D(x,y) = D_F(x,y) - \tfrac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) D_\rho(x,y)$$
$$\Sigma(x,y) = \Sigma_F(x,y) - \tfrac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) \Sigma_\rho(x,y)$$

The arguments of the propagators and self-energy are defined on the closed-time-path contour:



Vertex contribution



Prescription:

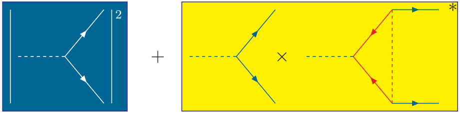
- Draw all 2PI diagrams and read off the effective action;
- Cut one line to obtain the self-energies;
- Cut the diagrams in two pieces to understand which processes it describes.

At one-loop level the decay self-energies read:

$$\Sigma_{\gtrless}(X,p) = -\,|g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \times [1 + \epsilon_i(X,p_1,p_2)] \tilde{G}^{ii}_{\gtrless}(X,p_1) \tilde{D}_{\lessgtr}(X,p_2)\,.$$

- The loop correction is the same for both the gain and loss terms.

Vertex contribution



Prescription:

- Draw all 2PI diagrams and read off the effective action;
- Cut one line to obtain the self-energies;
- Cut the diagrams in two pieces to understand which processes it describes.

At one-

Conclusions:

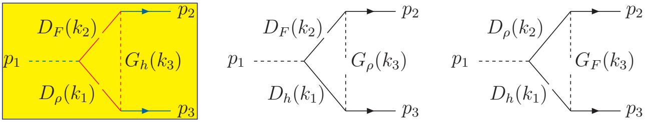
- No need for Real Intermediate State subtraction;

$$\Sigma_{\gtrless}(X,p) = -\,|g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \times \left[1 + \epsilon_i(X,p_1,p_2)\right] \tilde{G}^{ii}_{\gtrless}(X,p_1)\,\tilde{D}_{\lessgtr}(X,p_2)\,.$$

- The loop correction is the same for both the gain and loss terms.

Vertex contribution

Vertex contribution:



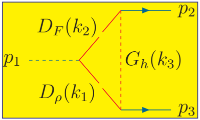
- One off-shell and two on-shell internal lines;
- Only one 'thermal' internal line;
- Only the diagram with two on-shell 'baryons' contributes;

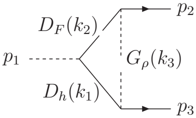
Explicit form

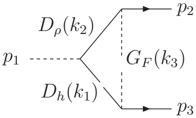
$$\epsilon_i(p_1,p_2) = -\frac{1}{8\pi}\frac{|g_j|^2}{M_i^2}\mathrm{Im}\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\int \frac{d\Omega}{4\pi}\frac{1+\bar{f}(E_{k_1})+\bar{f}(E_{k_2})}{M_j^2/M_i^2+\frac{1}{2}(1+\cos\Theta)}$$

Vertex contribution

Vertex contribution:







- One off-shell and two on-shell internal lines;
- Only one ‘thermal’ internal line;
- On

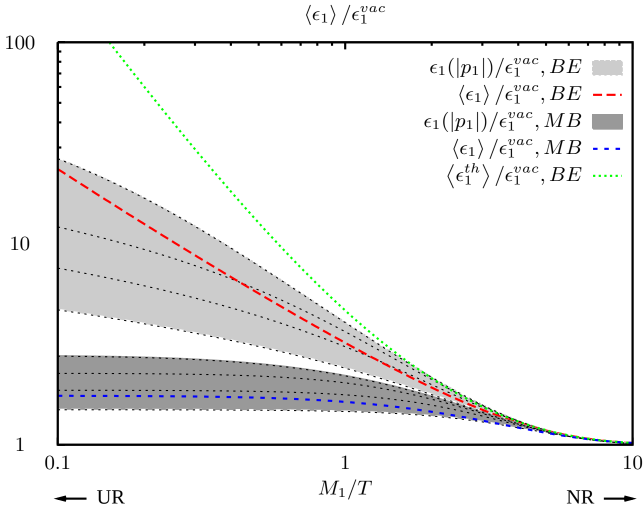
Conclusions:

- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory;

Explicit form:

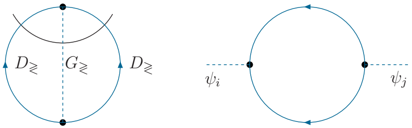
$$\epsilon_i(p_1,p_2) = -\frac{1}{8\pi}\frac{|g_j|^2}{M_i^2}\mathrm{Im}\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\int\frac{d\Omega}{4\pi}\frac{1+\bar f(E_{k_1})+\bar f(E_{k_2})}{M_j^2/M_i^2+\frac{1}{2}(1+\cos\Theta)}$$

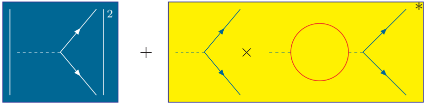
Vertex contribution



Temperature dependence of the CP-violating parameter.

Self-energy contribution





Prescription:

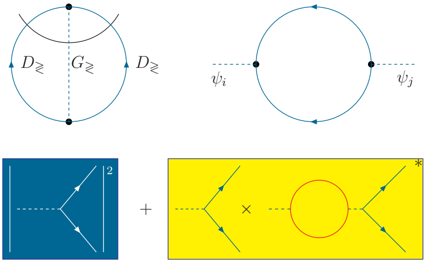
- Draw all 2PI diagrams and read off the effective action;
- Cut one line to obtain the self-energies;
- Cut the diagrams in two pieces to understand which processes it describes.

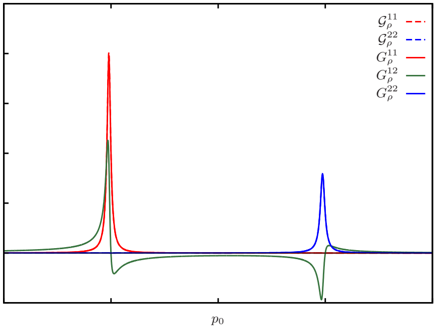
At one-loop level the decay self-energies read:

$$\Sigma_{\gtrless}(X,p) = -\,|g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \times [1 + \epsilon_i(X,p_1,p_2)] \tilde{G}^{ii}_{\gtrless}(X,p_1) \tilde{D}_{\lessgtr}(X,p_2)\,.$$

- The loop correction is the same for both the gain and loss terms.

Self-energy contribution





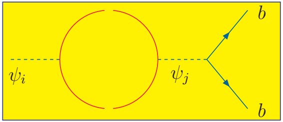
At one-loop level the decay self-energies read:

$$\Sigma_{\gtrless}(X,p) = -\,|g_i|^2 \int d\Pi_{p_1} d\Pi_{p_2} (2\pi)^4 \delta^g(p_1 - p_2 - p) \times [1 + \epsilon_i(X,p_1,p_2)]\tilde{G}^{ii}_{\gtrless}(X,p_1)\tilde{D}_{\lessgtr}(X,p_2)\,.$$

- The loop correction is the same for both the gain and loss terms.

Self-energy contribution

Self-energy contribution in the *hierarchical* case:



- One off-shell and two on-shell internal lines;
- Only one 'thermal' internal line;
- Only the diagram with two on-shell 'baryons' contributes;

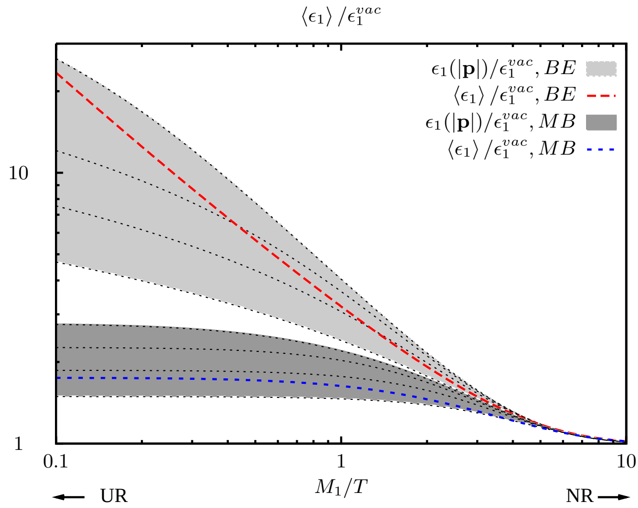
Explicit form:

$M_1^2 \ll M_2^2$
$M_j^2 \gg M_j\Gamma_j$

$$\epsilon_i = -\frac{|g_j|^2}{16\pi}\mathrm{Im}\left(\frac{g_i^* g_j}{g_i g_j^*}\right)\frac{M_i^2 - M_j^2}{(M_i^2 - M_j^2)^2 + (M_j\Gamma_j)^2 L_\rho^2}\cdot L_\rho$$

where $L_\rho \equiv \int \frac{d\Omega}{4\pi}[1 + f_b(E_p) + f_{\bar{b}}(E_p)]$ describes the medium effects.

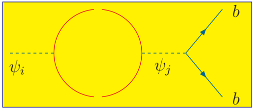
Self-energy contribution

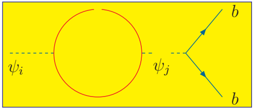


Temperature dependence of the CP-violating parameter.

Self-energy contribution

Self-energy contribution in the *resonant* case:





- One off-shell and two on-shell internal lines;
- Only one ‘thermal’ internal line;

$$M_i^2 \gg |\Delta M_{ij}^2| \gg M_j \Gamma_j$$

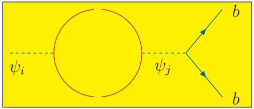
Explicit form:

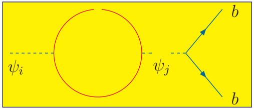
$$\epsilon_i = -\frac{|g_j|^2}{16\pi} \mathrm{Im}\left(\frac{g_i^* g_j}{g_i g_j^*}\right) \frac{\Delta_{ij}}{\cos^2(\delta_{cp})\Delta_{ij}^2 + \sin^2(\delta_{cp})[\tilde{\Delta}_{ij}^2 + (M_j \Gamma_j L_\rho)^2]} \cdot L_\rho \,,$$

where Δ_{ij} is the difference of the *in-medium* masses and we have introduced $\tilde{\Delta}_{ij} = \Delta_{ij} - 2M_j\Gamma_j L_h^{ij}$.

Self-energy contribution

Self-energy contribution in the *resonant* case:





- One off-shell and two on-shell internal lines;
- Only one ‘thermal’ internal line;

$$M_i^2 \gg |\Delta M_{ij}^2| \gg M_j\Gamma_j$$

Explicit f

$\epsilon_i = -$ … L_ρ ,

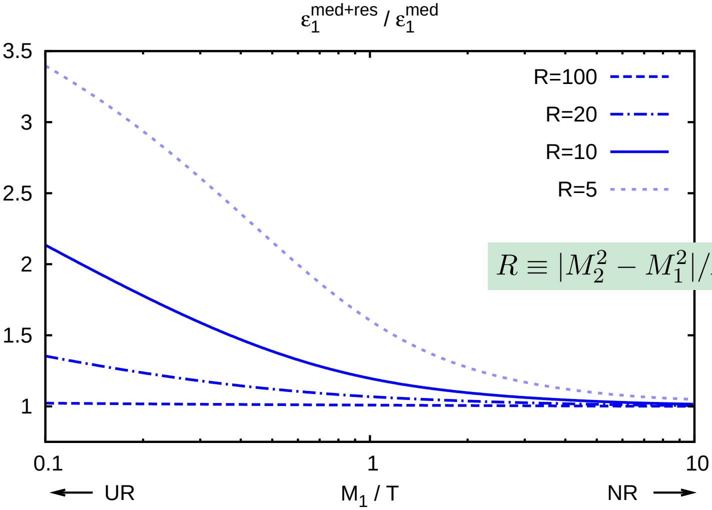
where

introduced $\Delta_{ij} = \Delta_{ij} - 2M_j\Gamma_j L_h^{ij}$.

Conclusions:

- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory;
- The canonical expression for the CP-violating parameter is only applicalble in the hierarchical case;

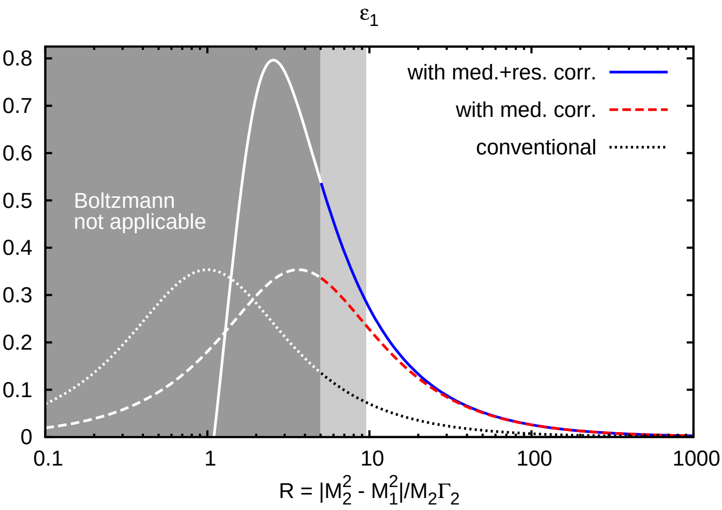
Self-energy contribution



$$R \equiv |M_2^2 - M_1^2|/M_2\Gamma_2$$

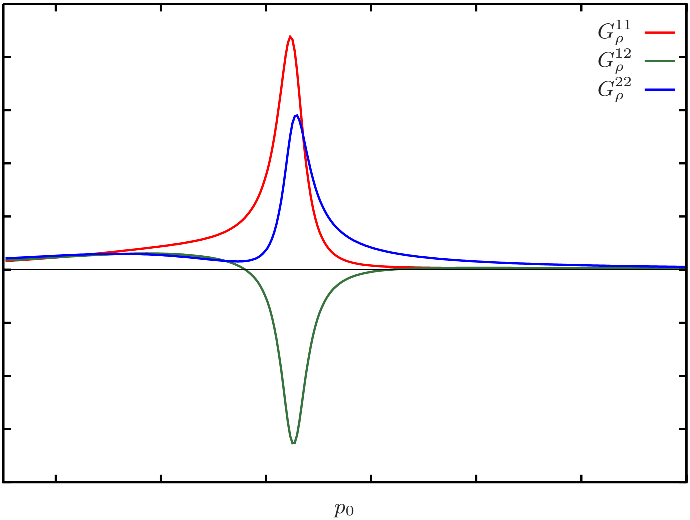
CP-violating parameter in the resonant regime.

Self-energy contribution



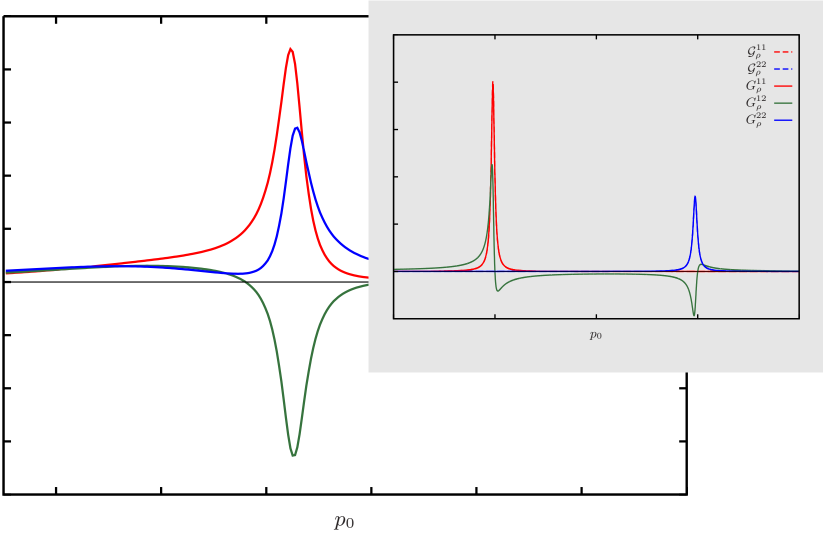
Comparison of the approximations for the CP-violating parameter.

Self-energy contribution



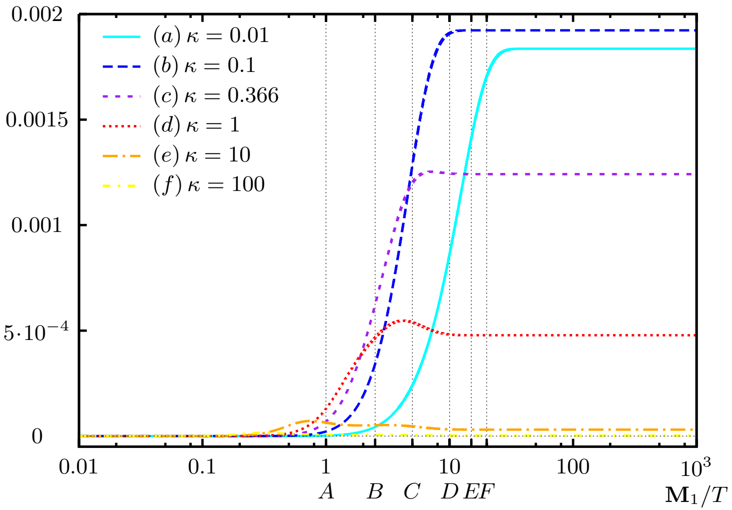
Spectral function in the maximal resonant regime.

Self-energy contribution



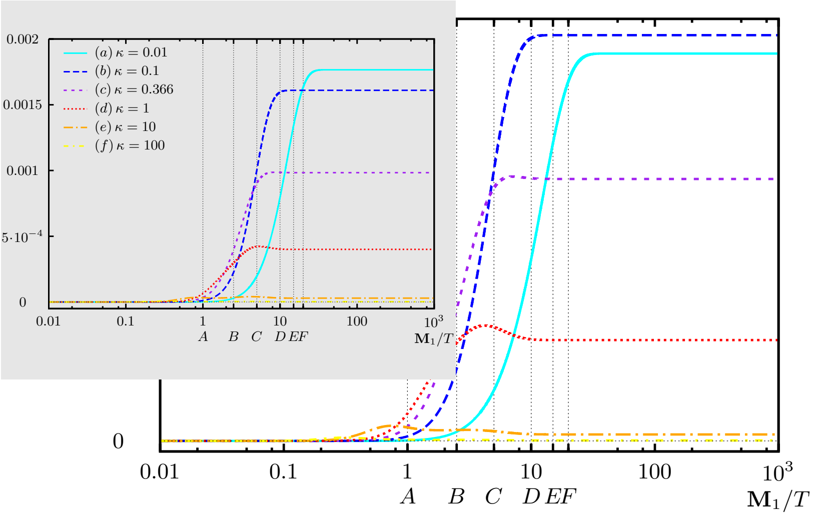
Spectral function in the maximal resonant regime.

Some numerical results



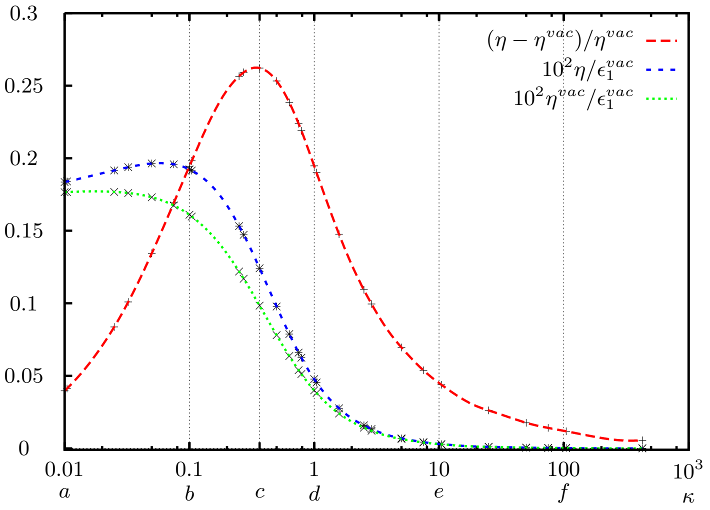
Asymmetry as a function of the inverse temperature.

Some numerical results



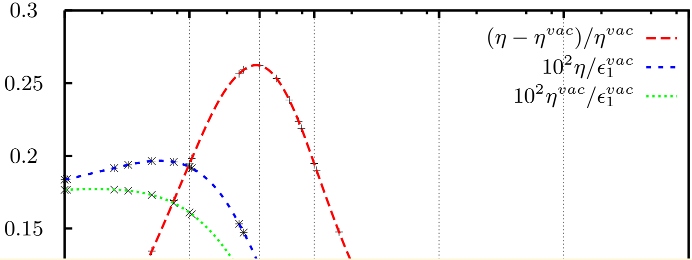
Asymmetry as a function of the inverse temperature.

Some numerical results



Asymmetry as a function of $\kappa \equiv \Gamma/\mathcal{H}$

Some numerical results



Conclusions:

- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory;
- The canonical expression for the CP-violating parameter is only applicalble in the hierarchical case;
- The new contributions are of order of 25 %.

Asymmetry as a function of $\kappa \equiv \Gamma/\mathcal{H}$

Conclusions

Kadanoff-Baym formalism:

Pros.

- Do not rely on the concept of quasiparticles;
- Take the medium effects into account;
- Free of the double counting problem.

Cons.

- Complexity.

Toy model of leptogenesis

Lagrangian of the model reads:

$$\mathcal{L} = \frac{1}{2}\partial^\mu\psi_i\partial_\mu\psi_i - \frac{1}{2}M_i^2\psi_i\psi_i$$
$$+ \partial^\mu\bar{b}\partial_\mu b - m^2\bar{b}b - \frac{\lambda}{2!2!}(\bar{b}b)^2$$
$$- \frac{g_i}{2!}\psi_i bb - \frac{g_i^*}{2!}\psi_i\bar{b}\bar{b}\,, \quad i = 1, 2\,.$$

- Real scalar fields ψ_i imitate the heavy right–handed neutrinos;
- Complex scalar field b models the baryons;
- The baryon number is broken by the last two terms of $\mathcal{L}$;
- Complex couplings g_i induce CP-violation;
- Deviation from equilibrium is due to the universe's expansion;
- The $\lambda(\bar{b}b)^2$ term models fast SM interactions.

Conclusions

Results:

- No need for Real Intermediate State subtraction;
- Our results differ from those of thermal field theory;
- The canonical expression for the CP-violating parameter is only applicable in the hierarchical case;
- The analysis of the maximal resonant regime requires the use of the full Kadanoff-Baym equations;
- The medium contributions are of order of 25 %.

Thank you for your attention