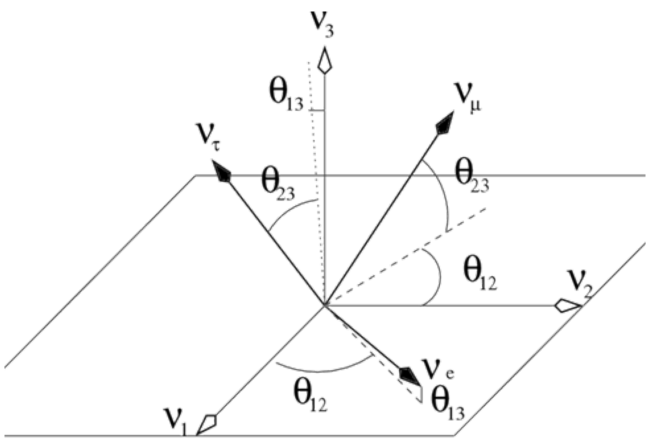
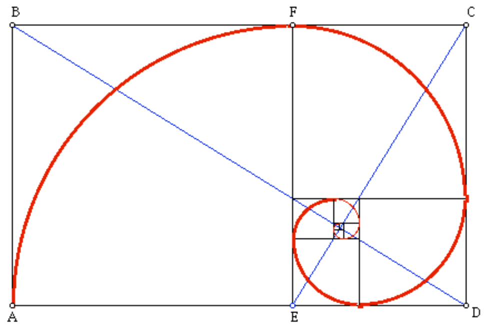
Flavor Symmetries: Models and Implications

Lisa L. Everett
U. Wisconsin, Madison



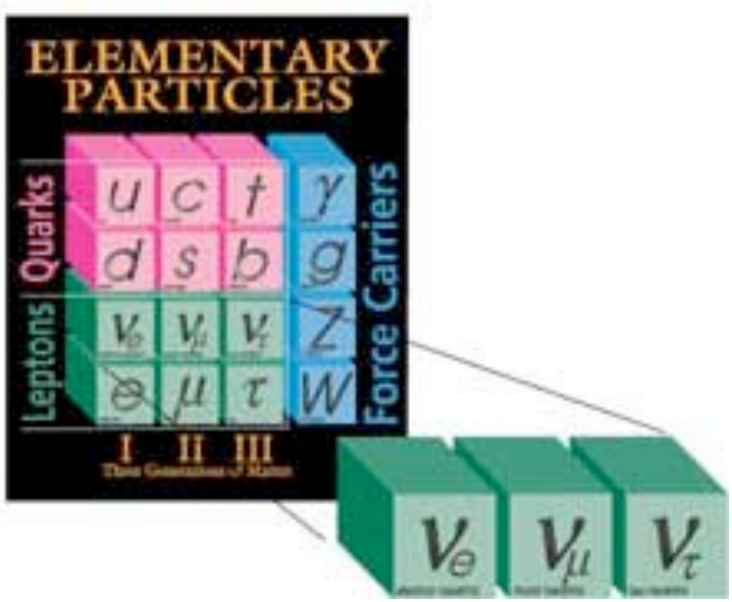


Introduction/Motivation

Neutrino Oscillations:

$$\mathcal{P}_{\nu_\alpha \to \nu_\beta}(L) = \sum_{ij} \mathcal{U}_{i\alpha} \mathcal{U}^*_{i\beta} \mathcal{U}^*_{j\alpha} \mathcal{U}_{j\beta} e^{-\frac{i\Delta m^2_{ij} L}{2E}}$$

massive neutrinos
observable lepton mixing



First particle physics evidence for physics beyond SM!

SM flavor puzzle $\longrightarrow$ ν SM flavor puzzle

Ultimate goal: satisfactory and credible flavor theory

(very difficult!)

fits: Schwetz, Tortola, Valle '08

The Data: Neutrino Masses

Homestake, Kam, SuperK, KamLAND, SNO, SuperK, MINOS, miniBOONE,...

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

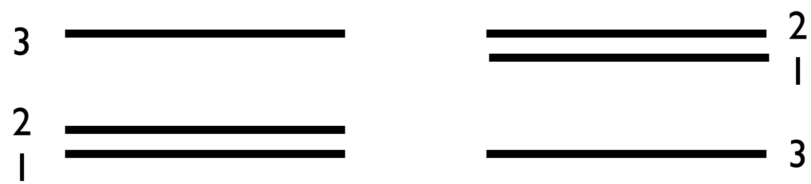
Assume: 3 neutrino mixing

Solar: $$\Delta m_\odot^2 = |\Delta m_{12}^2| = 7.65^{+0.23}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2$$

(best fit $\pm 1\sigma$)

Atmospheric: $$\Delta m_{31}^2 = \pm 2.4^{+0.12}_{-0.11} \times 10^{-3}\,\mathrm{eV}^2$$

Normal Hierarchy Inverted Hierarchy



Cosmology (WMAP): $$\sum_i m_i < 0.7\,\mathrm{eV}$$

fit: Schwetz, Tortola, Valle '08

The Data: Lepton Mixing

Homestake, Kam, SuperK, KamLAND, SNO, SuperK, Palo Verde, CHOOZ, MINOS...

$$\mathcal{U}_{\rm MNSP} = \mathcal{R}_1(\theta_\oplus)\mathcal{R}_2(\theta_{13}, \delta_{\rm MNSP})\mathcal{R}_3(\theta_\odot)\mathcal{P}$$

Maki, Nakagawa, Sakata
Pontecorvo

$$|\mathcal{U}_{\rm MNSP}| \simeq \begin{pmatrix} \cos\theta_\odot & \sin\theta_\odot & \epsilon \\ -\cos\theta_\oplus \sin\theta_\odot & \cos\theta_\oplus \cos\theta_\odot & \sin\theta_\oplus \\ \sin\theta_\oplus \sin\theta_\odot & -\sin\theta_\oplus \cos\theta_\odot & \cos\theta_\oplus \end{pmatrix}$$

Solar: $\theta_\odot = \theta_{12} = 33.4^\circ \pm 1.4^\circ$

Atmospheric: $\theta_\oplus = \theta_{23} = 45.0^\circ\,{}^{+4.0}_{-3.4}$ (best fit $\pm 1\sigma$)

Reactor: $\epsilon = \sin\theta_{13}, \quad \theta_{13} = 5.7^\circ\,{}^{+3.5}_{-5.7}$

2 large angles, 1 small angle (no constraints on CP violation)

Compare: Quark Mixing

Cabibbo; Kobayashi, Maskawa

$$\mathcal{U}_{\mathrm{CKM}} = \mathcal{R}_1(\theta_{23}^{\mathrm{CKM}})\mathcal{R}_2(\theta_{13}^{\mathrm{CKM}}, \delta_{\mathrm{CKM}})\mathcal{R}_3(\theta_{12}^{\mathrm{CKM}})$$

Mixing Angles: $\theta_{12}^{\mathrm{CKM}} = 13.0^\circ \pm 0.1^\circ \longleftrightarrow$ Cabibbo angle θ_c

$$\theta_{23}^{\mathrm{CKM}} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{13}^{\mathrm{CKM}} = 0.2^\circ \pm 0.1^\circ$$

CP violation: $J \equiv \mathrm{Im}(\mathcal{U}_{\alpha i}\mathcal{U}_{\beta j}\mathcal{U}^*_{\beta i}\mathcal{U}^*_{\alpha j})$

Jarlskog
Dunietz, Greenberg, Wu

$$J_{\mathrm{CP}}^{(\mathrm{CKM})} \simeq \sin 2\theta_{12}^{\mathrm{CKM}} \sin 2\theta_{23}^{\mathrm{CKM}} \sin 2\theta_{13}^{\mathrm{CKM}} \sin \delta_{\mathrm{CKM}}$$

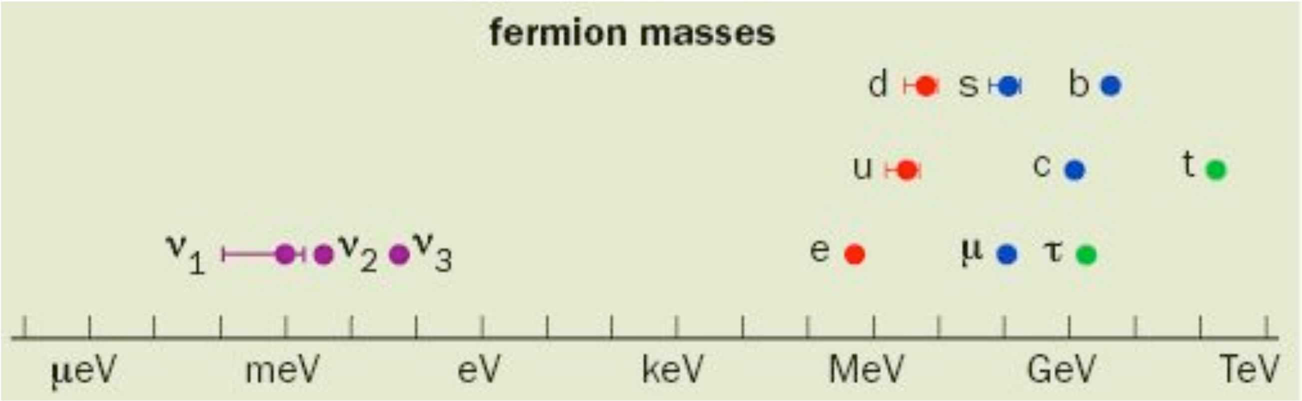
$$J \sim 10^{-5} \qquad \delta_{\mathrm{CKM}} = 60^\circ \pm 14^\circ$$

3 small angles, 1 large phase

A paradigm shift

Strikingly different flavor patterns for quarks and leptons!

- Mass scales, hierarchies of neutral and charged fermions:



Step 1 for theory: suppressing neutrino mass scale

- Mixing Angles: quarks small, leptons 2 large, 1 small

Step 2 for theory: understanding lepton mixing pattern

Step 1: Origin of Neutrino Mass Scale

Charged Fermions: Dirac mass terms

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

parametrized by Yukawa couplings

Neutrinos: beyond physics of Yukawas!

Assuming SM Higgs sector:

$$-\mathcal{L}_\nu = Y_{\nu ij} \bar{L}_{Li} H \nu_{Rj} + \frac{\lambda_{ij}}{\Lambda}(L_{Li}H)(L_{Lj}H) + \frac{1}{2}(M_{ij}\bar{\nu}_{Ri}(\nu_{Rj})^c + h.c.)$$

Majorana masses:

Prototype: Type I seesaw

Minkowski; Yanagida;
Gell-Mann, Ramond, Slansky;...

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$m \sim \mathcal{O}(100\,\mathrm{GeV})$

$M \gg m$

$$m_1 \sim \frac{m^2}{M}$$

$$m_2 \sim M \gg m_1$$

$$\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M}\nu_{R,L}$$

but many other possibilities...

Type II seesaw (Higgs triplets), Type III seesaw (triplet fermions), double seesaw, higher-dimensional operators, supersymmetry +R-parity violation,...

Dirac masses:

issue: Yukawa suppression $Y_\nu \sim 10^{-12}$

options: extra dimensions, flavor symmetries, supersymmetry breaking effects,...

Note: in both cases, many mechanisms exploit SM singlet nature of ν_R

Step 2: Origin of Large Lepton Mixings

Standard paradigm: spontaneously broken flavor symmetry

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\varphi}{M}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

Froggatt, Nielsen

"flavon" fields

Recall for quarks:

- hierarchical masses, small mixings: continuous family symmetries
- CKM matrix: small angles and/or alignment

$$\mathcal{U}_{\mathrm{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda)$$

$$\lambda \sim \frac{\varphi}{M}$$

Wolfenstein parametrization: $\lambda \equiv \sin\theta_c = 0.22$

suggests Cabibbo angle may be a useful flavor expansion parameter

Flavor Model Building in the νSM

- Main issue: what is $\mathcal{U}_{\mathrm{MNSP}}$ in limit of exact symmetry?

for the leptons, large angles suggest

$$\mathcal{U}_{\mathrm{MNSP}} = \mathcal{U}_e \mathcal{U}_\nu^\dagger \sim \mathcal{W} + \mathcal{O}(\lambda')$$

"bare" mixing angles

$(\theta_{12}^0, \theta_{13}^0, \theta_{23}^0)$

flavor expansion parameter

- useful, and motivated in unified/string scenarios, to take

$$\lambda' = \lambda \equiv \sin\theta_c$$

ideas of "Cabibbo haze" and quark-lepton complementarity

Aside: Lepton Mixing Angles are "non-generic"

Classify scenarios by the form of $\mathcal{U}_{\mathrm{MNSP}}$ in symmetry limit

note: lepton mixing angle pattern has the most challenges (w/3 families)

- 3 small angles $\longrightarrow$ $\sim$ diagonal $\mathcal{M}_\nu$
- 1 large, 2 small $\longrightarrow$ $\sim \mathrm{Rank}\mathcal{M}_\nu < 3$
- 3 large angles $\longrightarrow$ "anarchical" $\mathcal{M}_\nu$
- 2 large, 1 small $\longrightarrow$ fine-tuning, non-Abelian

Issues: size of θ_{13}, origin of non-maximal θ_{12}

large angles may suggest discrete non-Abelian family symmetries!

$$\mathcal{U}_{\mathrm{MNSP}} = \mathcal{U}_e \mathcal{U}_\nu^\dagger \sim \mathcal{W} + \mathcal{O}(\overset{\nu}{\lambda'})$$

Classify models by form of $\mathcal{W}(\theta^0_{12}, \theta^0_{13}, \theta^0_{23})$:

- In general: $\theta^0_{23} = 45^\circ$ $\theta^0_{13} = 0^\circ$ (reasonable)
- More variety in choice of bare solar angle θ^0_{12}:
  - "bi-maximal" mixing (quark-lepton complementarity)
  - "tri-bimaximal" mixing Harrison, Perkins, Scott (HPS)
  - "golden ratio" mixing $\phi = (1+\sqrt{5})/2$

or other options...

Scenario 1. Bi-maximal Mixing

"bare" solar angle $\theta_{12}^0 = 45^\circ$ $\tan\theta_{12}^0 = 1$

$$\mathcal{U}_{\rm MNSP}^{\rm (BM)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Requires large perturbations:

$$\theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda) \sim \frac{\pi}{4} - \theta_c$$

"quark-lepton complementarity"

Raidal; Minakata, Smirnov; Frampton, Mohapatra; Xing; Ferrandis, Pakvasa; King; L.E., Ramond; Plentinger, Lindner; Dighe, Rodejohann, many, many others...

Bimaximal mixing scenarios:

useful framework for exploring Cabibbo effects in quark+lepton sectors

$$\frac{m_u}{m_t} \sim \lambda^8 \qquad \frac{m_d}{m_b} \sim \lambda^4 \qquad \frac{m_e}{m_\tau} \sim \lambda^5 \qquad \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_\oplus}} \sim \lambda$$

$$\frac{m_c}{m_t} \sim \lambda^4 \qquad \frac{m_s}{m_b} \sim \lambda^2 \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2$$

$$\theta_{13} \leq \mathcal{O}(\lambda)$$

$$\frac{m_b}{m_\tau} \sim 1 \qquad \frac{m_b}{m_t} \sim \lambda^3$$

(GUT scale)

$$\Delta\theta_{23} < \mathcal{O}(\lambda)$$

$$\theta_{12}^{\rm CKM} \sim \lambda \qquad \theta_{23}^{\rm CKM} \sim \lambda^2 \qquad \theta_{13}^{\rm CKM} \sim \lambda^3$$

but implementation in full grand unified theories: very challenging

recent work in context of discrete non-Abelian family symmetries

Altarelli, Feruglio, and Merlo, '09,...

Scenario II. Tri-bimaximal (HPS) Mixing

"bare" solar angle $\tan\theta_{12}^0 = \frac{1}{\sqrt{2}}$ $\theta_{12}^0 = 35.26^\circ$

Harrison, Perkins, Scott '02

$$\mathcal{U}_{\mathrm{MNSP}}^{(\mathrm{HPS})} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Does not require large perturbations! $\theta_{12} = \theta_{12}^0 + \mathcal{O}(\lambda^2)$

amusing note: MNSP looks like Clebsch-Gordan coeffs Meshkov; Zee,...

Naturally obtained from discrete non-Abelian symmetries (subgroups of $SO(3)$, $SU(3)$)

A Few Examples: $\mathcal{A}_4$ (tetrahedron) — Ma and collaborators (earliest in '01), Altarelli, Feruglio, Babu and He, Valle, Hirsch et al., King et al., many, many others...

$\mathcal{S}_4$ (cube) — Ma; Hagedorn, Lindner, Mohapatra; Cai, Yu; Zhang,...

$\mathcal{T}'$ — Aranda, Carone, Lebed; Chen, Mahanthappa,...

$\Delta(3n^2)$ — Luhn, Nasri, Ramond; Ma; King, Ross,...

$\mathcal{A}_5$ (icosahedron) — L.E., Stuart

Most popular scenario! many models, elegant results

issues: incorporating quarks, "vacuum alignment" of flavon fields

Scenario III. Golden Ratio Mixing

Idea: solar angle related to "golden ratio"

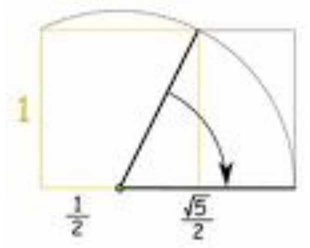
$$\phi = (1+\sqrt{5})/2$$

- Two proposed scenarios:
  - $\tan\theta_{12} = \frac{1}{\phi}$ $\quad\theta_{12} = 31.72^\circ$ L.E., Stuart '08, + work in progress

    icosahedral flavor symmetry $\mathcal{I}\ (\mathcal{A}_5)$
  - $\cos\theta_{12} = \frac{\phi}{2}$ $\quad\theta_{12} = 36^\circ$ Adulpravitchai, Blum, Rodejohann '09

    dihedral flavor symmetry $\mathcal{D}_{10}$

Scenario III: $\tan\theta_{12} = \frac{1}{\phi}$



Ramond et al., '03 (footnote), Kajiyama, Raidal, Strumia '07 $\mathcal{Z}_2 \times \mathcal{Z}_2$

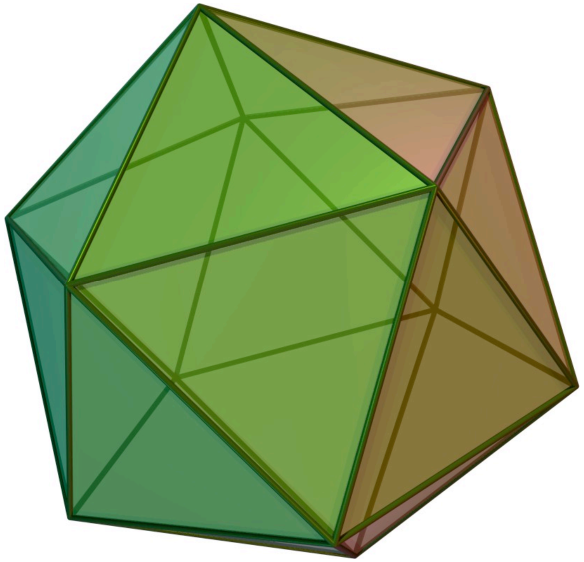
L.E. and Stuart, '08 and continuing... $\mathcal{A}_5$

$$\mathcal{U}_{\mathrm{MNSP}}^{(\mathrm{GR1})} = \begin{pmatrix} \sqrt{\frac{\phi}{\sqrt{5}}} & -\sqrt{\frac{1}{\sqrt{5}\phi}} & 0 \\ \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathcal{A}_5$ isomorphic to icosahedral group, $\mathcal{I}$ $\mathcal{A}_5 \simeq \mathcal{I}$

The (Rotational) Icosahedral Group, *I* ~ A5

Properties of the icosahedron:



20 faces (equilateral triangles)

30 edges (3 sides/triangle, 2 triangles/edge)

12 vertices (3 vertices/triangle, 5 vertices/edge)

Group elements:

Rotations which take vertices to vertices, i.e., by $0,\ \frac{2\pi}{5},\ \frac{4\pi}{5},\ \frac{2\pi}{3},\ \pi$

Rotation by each angle forms a conjugacy class:

$$e,\ 12C_5,\ 12C_5^2,\ 20C_3,\ 15C_2$$

(Schoenflies: C_n^k = $\frac{2\pi k}{n}$ rotation)

order=number of elements: $1 + 12 + 12 + 15 + 20 = 60$

The (Rotational) Icosahedral Group, *I* ~ A5

Theorem: group order = sum of squares of irred. reps

$$1 + 12 + 12 + 15 + 20 = 60 = 1^2 + 3^2 + 3^2 + 4^2 + 5^2.$$

(two triplets!)

Conjugacy classes: characterized by trace (character)

Character Table

$\mathcal{I}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	$\mathbf{4}$	$\mathbf{5}$
e	1	3	3	4	5
$12C_5$	1	ϕ	$1-\phi$	-1	0
$12C_5^2$	1	$1-\phi$	ϕ	-1	0
$20C_3$	1	0	0	1	-1
$15C_2$	1	-1	-1	0	1

The (Rotational) Icosahedral Group, *I* ~ A5

From character table, deduce tensor product decomposition:

$$
\boxed{\begin{gathered}
3 \otimes 3 = 1 \oplus 3 \oplus 5 \\
3' \otimes 3' = 1 \oplus 3' \oplus 5 \\
3 \otimes 3' = 4 \oplus 5 \\
3 \otimes 4 = 3' \oplus 4 \oplus 5 \\
3' \otimes 4 = 3 \oplus 4 \oplus 5 \\
3 \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5 \\
3' \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5 \\
4 \otimes 4 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 5 \\
4 \otimes 5 = 3 \oplus 3' \oplus 4 \oplus 5 \oplus 5 \\
5 \otimes 5 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 4 \oplus 5 \oplus 5
\end{gathered}}
$$

Not enough for flavor model building. Need explicit representations!

I not a crystallographic point group, so there was work to be done...

Lepton Flavor Model Building with A5

Mass terms: $$-\mathcal{L}_m = \frac{a_{ij}}{M} L_i H L_j H + Y_{ij}^{(e)} L_i \bar{e}_j H$$

Charge assignments: $$L \rightarrow 3, \;\; \bar{e} \rightarrow 3'$$

$$LL : 3 \otimes 3 = 1 \oplus \cancel{3} \oplus 5, \qquad L\bar{e} : 3 \otimes 3' = 4 \oplus 5$$

Leading order: charged leptons massless, neutrinos degenerate...

Fix it at higher order with flavon sector:

$$\underset{LL}{\xi \rightarrow 5} \qquad \underset{L\bar{e}}{\psi \rightarrow 5, \;\; \chi \rightarrow 4}$$

$$-\mathcal{L}_{mass} = \frac{\alpha_{ijk}}{M^2} L_i H L_j H \xi_k + \frac{\beta_{ijk}}{M} L_i \bar{e}_j H \psi_k + \frac{\gamma_{ijl}}{M} L_i \bar{e}_j H \chi_l$$

Lepton Flavor Model Building with A5 (continued)

Explicit toy example with assumed flavon vevs:

specific "golden prediction" for solar mixing angle, plus

neutrinoless double beta decay:

$$m_{\beta\beta} = \frac{m_1 \phi}{\sqrt{5}} + \frac{m_2}{\phi\sqrt{5}}.$$

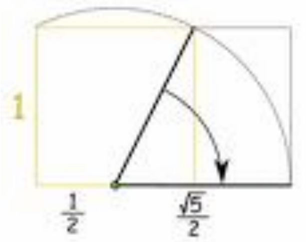
neutrino masses (normal hierarchy)

hierarchical charged lepton masses

In progress: dynamics of flavon sector, quark flavor mixing,...

Rich and virtually unexplored model building territory!

Scenario III: $\cos\theta_{12} = \frac{\phi}{2}$



Rodejohann '08, Adulpravitchai, Blum, and Rodejohann, '09 $\mathcal{D}_{10}$

$$\mathcal{U}_{\rm MNSP}^{\rm (GR2)} = \begin{pmatrix} \frac{\phi}{2} & -\frac{1}{2}\sqrt{\frac{\sqrt{5}}{\phi}} & 0 \\ \frac{1}{2}\sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2}\sqrt{\frac{5}{2\phi}} & \frac{\phi}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

complete flavor theory based on dihedral symmetry

(solar angle prediction based on exterior angle of decagon)

Conclusions/Outlook

- The ν flavor puzzle is intriguing and very rich:
  - Many options for neutrino mass scale suppression, each with implications for particle/astroparticle physics
  - Many theoretically motivated mixing patterns:
    Bi-maximal, tri-bimaximal, mixing, "golden ratio,"...
- Themes: Dirac v. Majorana? role of family symmetries? quark-lepton unification?
- Data will of course continue to be crucial!
- May provide our best window to ultrahigh scale physics!