Sterile Neutrinos and Dark Matter

1. Introduction to particle dark matter (DM)
    - 1.1 Motivation for particle dark matter
    - 1.2 WIMPs as typical CDM candidates
    - 1.3 Thermal Production of WIMPs in standard cosmology

2. Sterile neutrinos as warm dark matter (WDM)
    - 2.1. Motivation
    - 2.2 Constraints on sterile neutrino dark matter
        - 2.2.1 X-rays
        - 2.2.2 Low mass bounds from dwarf spheroidal (dSph)
        - 2.2.3 – " – Lyman-α forest data
    - 2.3 Sterile neutrino production in the early Universe
        - 2.3.1 Active-sterile mixing
        - 2.3.2 Particle decay

3. Numerical simulations of Large-scale structure formation
    - 3.1 Overview of current N-body simulations
    - 3.2 Small-scale issues of ΛCDM + possible solutions
        - 3.2.1 Cusps vs. cores
        - 3.2.2 Satellites and subhalos

4. Summary and Conclusion

1. Introduction to particle dark matter

1.1 Motivation for particle dark matter

Cosmological and astrophysical observations show that large part of the matter density must be in a non-baryonic, weakly interacting form of matter

WMAP 7 year (arXiv: 1001.4538)

baryonic matter: $\Omega_b = 0.0455 \pm 0.0028$

dark matter: $\Omega_{cdm} = 0.228 \pm 0.027$

- One of the first hints:

  1933 Zwicky observed a dominance of "dark" matter in galaxy clusters Helv. Phys. Acta, 6: 110-127 (1933)

- Up to now: hints on different scales
  - galaxy rotation curves (stars in galaxies) → see next page
  - galaxy clusters (rotation curves, bullet cluster)
  - large-scale structure (grav lensing, the cosmic web, CMB)

- Does not have to be particle, may be a new theory of gravitation

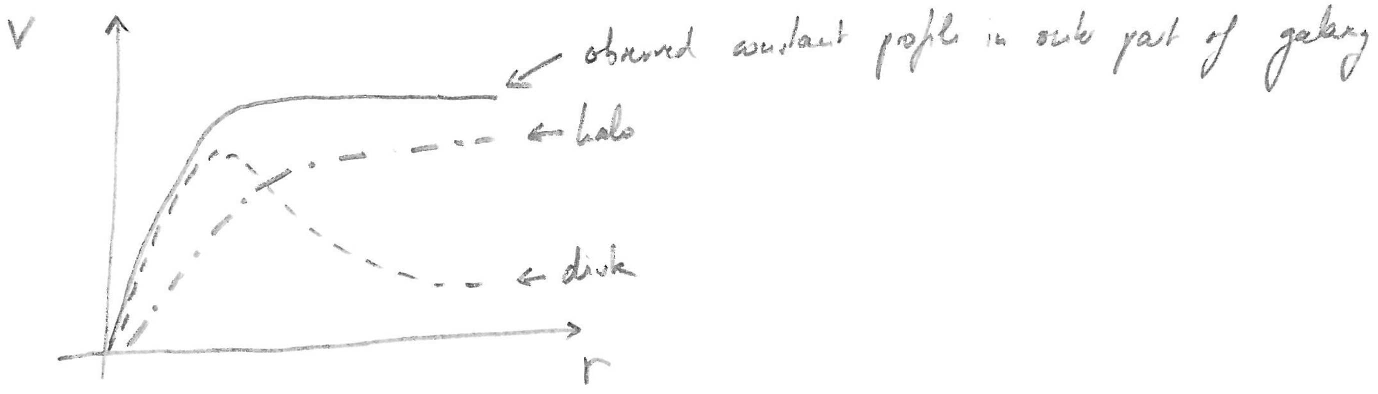
  e.g. MOND (modified Newtonian dynamics)

  however: notoriously difficult to reproduce all hints on DM
  (grav lensing may give a possibility to distinguish MOND and CDM)

↳ strong motivation for particle dark matter

What type of particle could it be?

rotation curves of galaxies



$F_{Grav} = F_{centripetal}$

$$G \frac{m M}{r^2} = m \frac{v^2}{r}$$

$$M = \int \rho(r')\, dV = 4\pi \int_0^r \rho(r')\, r'^2\, dr'$$

$$\Leftrightarrow v = \left(\frac{GM}{r} \right)^{1/2} = \left[4\pi G \frac{1}{r} \int_0^r \rho(r')\, r'^2\, dr' \right]^{1/2}$$

(i) DM in outer part: constant profile: $M \propto r \Leftrightarrow \rho(r) \propto \frac{1}{r^2}$
$v = \text{const.}$

(ii) disk: Keplerian second law

"A line joining a planet and the sun sweeps out equal area during equal interval of time."

$r^2 \dot{\theta} = \text{const.}$

deduction: $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$

total area: $A = \pi a b \longrightarrow$ period $P = \frac{2\pi}{n}$: $\pi a b = P \cdot \frac{1}{2} r^2 \dot{\theta}$

$\longrightarrow v \propto \frac{1}{r}$

1) Can diffuse baryons be DM?

→ 3 methods to determine baryon fraction in the high-redshift universe:

(i) primordial nucleosynthesis of ^4He, ^2H, ^7Li $(z \simeq 10^9)$

$\Omega_b = 0.04 \pm 0.02$ ✓

(ii) relative heights of acoustic peaks in CMB $(z \simeq 1000)$

agreement w/ (i)

(iii) Lyman α forest of intergalactic medium $(z \simeq 3)$

→ present epoch $(z = 0)$: intergalactic medium dominates known baryon fraction

~30% of total baryon fraction

- most reliable measurement: galaxy clusters (assumed to have retained their primordial baryon fraction)

~15%, consistent w/ WMAP

2) light neutrinos?

- are hot dark matter (HDM): relativistic @ time of structure formation
- thermally produced in the early universe, decoupled relativistic @ $T \approx 1$ MeV

$\hookrightarrow \Omega_\nu h^2 = \frac{m_\nu}{90\,eV}$; $m_\nu = \sum_{i=1}^{3} m_{\nu_i}$

- free-streaming length $\lambda_{FS} \sim 20 \left(\frac{30\,eV}{m_\nu}\right) Mpc$

$\hookrightarrow$ spectrum of density perturbations should be suppressed for scales beyond free-streaming length: for $m_\nu \simeq eV$ size of superclusters (Coma, Virgo)

- top-down formation: small structures form by fragmentation of larger ones. Observations show that galaxies are older than superclusters
- assuming adiabatic, scale-inv, Gaussian power spectrum + WMAP data

$\longrightarrow \sum m_\nu < 2.11\,eV$ (95% CL)

including more obs (e.g. large-scale structure): $\sum m_\nu < 0.17\,eV$ (95% CL)

1.2 WIMPs as typical DM candidate

most-studied candidate : WIMPs

→ WIMPs in chemical equilibrium in the early Universe naturally have the correct abundance to be CDM

→ interactions that give right WIMP density allow detection

ballpark argument: density per comoving volume of non-rel. particles in eq. in the early Universe decreases exp. w/ decreasing temperature (Boltzmann factor) until reactions changing particles # become inefficient (annihilation rate ≤ Hubble expansion rate)

↳ WIMP # per comoving volume becomes constant

the larger the ann. cross section σ_{ann}, the later chemical decoupling (= freeze-out) happens → smaller WIMP densities

⇝ present relic density of WIMPs

$$\Omega h^2 \simeq \frac{3\times10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle\sigma_{ann} v\rangle}$$

for weak cross-sections right order of magnitude for DM density ($T_{fo} \simeq m/20$ for a WIMP of mass m)

Important: BBN (200s after BB, $T \simeq 0.8\,\mathrm{MeV}$) : earliest episode w/ traces (D, ^4He, ^7Li)

↓

CMB (3.8×10^5 yrs after BB, $T \simeq \mathrm{eV}$)

↓

LSS

WIMPs freeze out @ $T_{fo} \simeq m/20$

↳ for $m \gtrsim 100\,\mathrm{MeV}$: before BBN → earliest remnants

1.3 Thermal production of WIMPs in standard cosmology

Assumptions about pre-BBN epoch:

- standard calculation:
  - Entropy of matter + radiation conserved
  - WIMPs produced thermally (interactions w/ particles in the plasma)
  - decoupling while Universe radiation-dominated
  - kinetic + chemical equilibrium before decoupling
- conditions do not need to hold in all cosmological models
  - for BBN and subsequent evolution of Universe only necessary: $T_{RH} > 4\,\text{MeV}$

    (earliest = highest temp. in the radiation dominated epoch)
  - possible effects:
    - density may be decreased by reducing the rate of thermal production ($T_{RH} < T_{fo}$)
    - production of radiation after freeze-out (entropy dilution)
    - non-thermal production (decay of particles)
    - increased expansion rate at time of freeze-out
- non-thermal production also possible in standard cosmologies
  - WIMP production through out-of-eq decays of particles, whose density was fixed by thermal processes
  - WIMPZILLA (heavy WIMPs): formed during the reheating phase at the end of inflationary period by gravitational interaction
  - qm oscillation: sterile ν's may be produced by oscillation of active into sterile ν's in the early Universe

Standard WIMP production mechanism

thermal plasma in the early Universe (radiation dominated epoch).

$$\text{WIMP} \longrightarrow \chi\bar{\chi} \longleftrightarrow e^+e^-, \mu^+\mu^-, q\bar{q}, W^+W^-, ZZ, HH, \ldots$$

for $T \gg m_\chi$: equilibrium between creation and annihilation

$$\Gamma_{\text{ann}} = \underset{\substack{\uparrow \\ \text{average over thermal distribution}}}{\langle \sigma_{\text{ann}} v \rangle}\, \overset{\swarrow \#\text{ density}}{n_{eq}}$$

- Universe expands $\longrightarrow T \downarrow$

  $\hookrightarrow$ # WIMPs produced decreases as $\exp(-m_\chi/T)$ (Boltzmann Factor)

  $\hookrightarrow$ expansion of Universe decreases number density and thus Γ_{ann}

- When $\Gamma_{\text{ann}} < H$ (expansion rate of Universe):

  ($\hat{=}$ mean free path for WIMP producing collisions $\geq$ Hubble radius)

  chemical decoupling = freeze-out

  $\hookrightarrow$ # of WIMPs in comoving Volume remain constant

- assume WIMPs to be Majorana $\rightarrow$ WIMP density = anti-WIMP density

//

rate equation for WIMP number density:

$$\frac{dn}{dt} = \underbrace{-3Hn}_{\text{exp. of the Universe}} - \underbrace{\langle \sigma_{\text{ann}} v \rangle \left(n^2 - n_{eq}^2\right)}_{\text{annihilation}}$$

law of entropy conservation:

$$\frac{ds}{dt} \overset{\swarrow \text{ entropy density}}{=} -3Hs$$

$\nwarrow$ time

$\longrightarrow$ for $Y = \frac{n}{s}$ (w/ $x = m/T$, T: photon temp.) : $\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx} \langle \sigma v \rangle \left(Y^2 - Y_{eq}^2\right)$

· Friedman equation $\longrightarrow H^2 = \frac{8\pi}{3M_p^2}\rho$ $\quad M_p: 1.22 \times 10^{19}\,\text{GeV}$

$$\rho = \frac{\pi^2}{30} g_{eff}(T)\, T^4 \quad ; \quad s = \frac{2\pi^2}{45} h_{eff}(T)\, T^3$$

g_{eff}/h_{eff}: effective degrees of freedom

$$g_*^{1/2} = \frac{h_{eff}}{g_{eff}^{2}}\left(1 + \frac{1}{3}\frac{T}{h_{eff}}\frac{dh_{eff}}{dT}\right)$$: degrees of freedom parameter

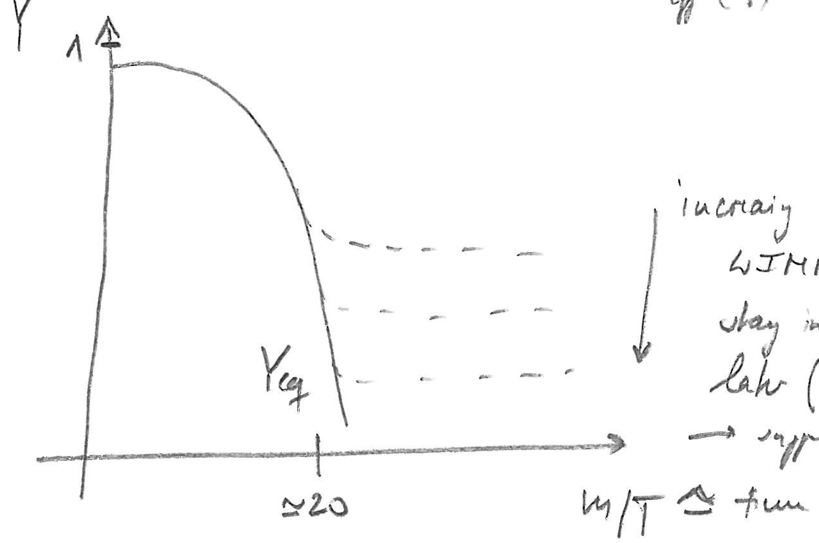
$$\hookrightarrow \quad \frac{dY}{dx} = -\left(\frac{45}{\pi M_p^2}\right)^{-1/2} \frac{g_*^{1/2}\, m}{x^2} \langle \sigma v \rangle \left(Y^2 - Y_{eq}^2\right)$$

solve numerically ($Y = Y_{eq}$ @ $x \simeq 1$) $\longrightarrow$ present WIMP abundance Y_0

$$\hookrightarrow \quad \Omega_\chi h^2 = \frac{\rho_\chi^0 h^2}{\rho_c^0} = \frac{m_\chi s_0 Y_0 h^2}{\rho_c^0} = 2.755 \times 10^8\, Y_0\, m_\chi / \text{GeV}$$

index "0": current values $\quad T_0 = 2.726\,\text{K}$

$h_{eff}(T_0) = 3.91$ (photon + 3 ν's)



increasing $\langle\sigma v\rangle$

WIMPs w/ strong interactions stay in eq. longer and decouple later (colder universe)

→ suppr. by smaller Boltzmann factor

$\longrightarrow$ WIMP speed @ freeze-out: $v_{fo} = (3T_{fo}/2m_\chi)^{1/2} \cong 0.27c$

· freeze-out key plays important role in determining WIMP relic density

but: depends on mass and interaction of WIMP, **and** on content of the Universe (through H)

2. keV sterile neutrinos as warm dark matter

2.1 Motivation

warm DM: intermediate situation between cold (usual explanation) and hot (excluded) dark matter

↳ may provide solutions to the problems of DM simulations
- too many dwarf satellite galaxies
- cusps in DM halos

} → later in this talk

natural WDM candidate: light sterile neutrinos

simple realization: νMSM
- 3 singlet fermions + SM
- Majorana mass + Dirac mixing w/ ordinary active neutrinos
- mass O(keV), no explanation for that → talk of Julian
- small mixing w/ active ν's → lifetime exceeding age of the universe
- problem/virtue: sterile ν w/ small mixing (only interaction via Yukawa couplings) never enter into thermal equilibrium

↳ have to be produced by nonthermal mechanism

If neutrinos would enter into thermal equilibrium at some moment in the early Universe

$$\Omega_\nu h^2 = \frac{m_\nu}{90\,\mathrm{eV}} \rightarrow m_\nu \gtrsim 90\,\mathrm{eV} \text{ would overclose the Universe}$$

(see earlier discussion)

↳ see talk of Klev Sham for details of the νMSM

I will talk about general features of ν's as WDM

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{N}_I \, i\partial_\mu \gamma^\mu N_I - F_{\alpha I} \, \bar{L}_\alpha N_I \tilde{\Phi} - \frac{M_I}{2} \bar{N}_I^c N_I + h.c.$$

$$\tilde{\Phi}_i = \varepsilon_{ij} \Phi_j^*$$

N_I : singlet Majorana fermions (number unfixed, 3 for anomaly cancellation)

2.2 Constraints (Cosmology + Astrophysics) on Sterile Neutrino Dark Matter

2.2.1 X-rays:

non-zero Higgs-vev induces mixing between active neutrino states and sterile ν N_1

$$\theta_{\alpha 1} = \frac{F_{\alpha 1} v}{M_1}$$ admixture of sterile ν in every active ν flavor

↳ sterile ν unstable

\# main channel: $$\Gamma_{N_1 \to 3\nu} = \frac{G_F^2}{96\pi^3} M_1^5 \theta_1^2$$ ← Fermi constant $G_F = 1{,}166 \times 10^{-5}\,\mathrm{GeV}^{-2}$

$$\hookrightarrow T_{1/2} = 10^{14}\,\text{years} \left(\frac{10\,\mathrm{keV}}{M_1}\right)^5 \left(\frac{10^{-8}}{\theta_1^2}\right)$$

$$\theta_1 = \frac{m_1}{M_1} \qquad m_1^2 = \sum_{\alpha = e,\mu,\tau} |F_{\alpha 1} v|^2$$

take $m_1 \sim O(1\,\mathrm{eV})$; $M_1 \sim O(1\,\mathrm{keV})$

$\Rightarrow$ Lifetime $\sim 10^{17}$ y $\geq$ age of the Universe $(14 \times 10^9\,\mathrm{y})$

stability favors small values of the mass and mixing:

$$\left(\frac{M_1}{10\,\mathrm{keV}}\right)^5 \left(\frac{\theta_1^2}{10^{-4}}\right) \lesssim 1$$

\# subdominant channel: $N_1 \to \nu\gamma$

$$\Gamma_{N \to \gamma\nu} = \frac{9\,\alpha_{em}\, G_F^2}{256 \cdot 4\pi^2} \sin^2(2\theta_1)\, M_1^5 \simeq 1.1 \times 10^{-21}\, \theta_1^2 \left[\frac{M_1}{\mathrm{keV}}\right]^5 s^{-1}$$ ← fine structure const.

⟶ narrow photon line $E_\gamma = M_1/2$

non-observation puts stringent constraint.

$$\Theta_1^2 < F(M_1) \longrightarrow \text{see fig.} \qquad \Theta_1^2 \lesssim 1.8\times 10^{-5}\left(\frac{1\,\text{keV}}{M_s}\right)^5$$

bounds 6 orders of magnitude stronger than bounds from stability

→ lifetime of DM sterile ν ~~[illegible]~~ ($M_1 \lesssim 1$ MeV) should exceed $10^6 t_u$, so that decay has not been seen by now

↑ age of the universe

- this bound is robust, does not depend on sterile neutrino production mechanism in the early universe

<u>Experimental search</u>: use the same radiative two-body decay

- energy flux from DM decay:

$$F = \frac{\Gamma_{rad}\,\Omega}{8\pi}\int \rho_{DM}(r)\,dr$$

↙ solid angle $\Omega \ll 1$

line of sight ↖ DM density profile

good objects: F big, X-ray background small

	flux	background
→ MW halo signal	1	10^{-2}
: clusters (Coma, Virgo)	~1	strong X-ray emission ~1
→ dSph (Draco, Ursa Minor)	~3	dark

<u>however</u>: Chandra + XMM-Newton maximally improvement of factor 10 (energy resolution largely exceeds width of DM line)

needed:
- $\Delta E/E \sim 10^{-3}$
- field of view ~1° (size of dSph)
- wide energy scan $O(100\,\text{eV}) \to O(100\,\text{keV})$

laboratory constraints much weaker than those from X-ray (β decay of tritium)

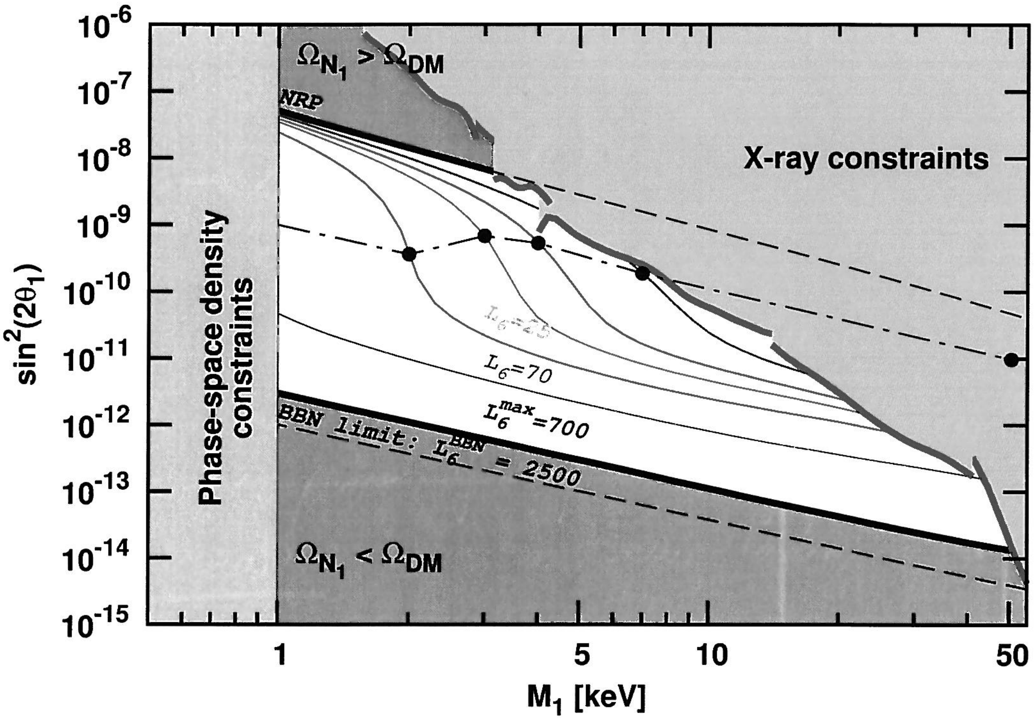


Figure 4: The allowed region of parameters for DM sterile neutrinos produced via mixing with the active neutrinos (*unshaded region*). The two thick black lines bounding this region represent production curves for nonresonant production (NRP) (*upper line*, $L_6 = 0$) and for resonant production (RP) (*lower line*, $L_6^{max} = 700$) with the maximal lepton asymmetry, attainable in the νMSM [53, 48]. The thin colored curves between these lines represent production curves for (*from top to bottom*) $L_6 = 8, 12, 16, 25$, and 70. The red shaded region in the upper right corner represents X-ray constraints [77, 78, 80, 88, 89] (rescaled by a factor of two to account for possible systematic uncertainties in the determination of DM content [86, 80]). The black dashed-dotted line approximately shows the RP models with minimal $\langle q \rangle$ for each mass, i.e., the family of models with the largest cold component. The black filled circles along this line are compatible with the Lyman-α bounds [90], and the points with $M_1 \leq 4$ keV are also compatible with X-ray bounds. The region below 1 keV is ruled out according to the phase-space density arguments [34]. Abbreviation: BBN, big bang nucleosynthesis.

arXiv:0901.0011

$$L_6 \equiv 10^6 \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{s}$$

$$\Delta_L^{max} = 0.2 \mathrel{\hat{=}} L_6^{max} \approx 700$$

Considerably smaller than BBN bound.
$L_6^{BBN} \approx 2500$

Non-res prod. specifies a minimal amount of DM that will be produced for given M_1 and θ_1 → 100% DM by this mech. gives upper bound on θ_1 for a given mass

2.2.2 Lower mass bound from dwarf spheroidals

X-ray bound → no abs. value can be deduced, if mixing angle is not found (may be zero)

- robust and model independent limit:

rotational curves of dSph (dwarf spheroidal satellites of the MW)

if DM are fermions, their phase-space density cannot exceed that of a degenerate Fermi gas

spherically symmetric distribution of a DM-dominated object w/ mass M and radius R   max Fermi velocity does not exceed the escape velocity $v=(2G_N M/R)^{1/2}$

$$\hookrightarrow \quad M_1 > \left(\frac{9\pi}{4\sqrt{2}\, M^{1/2} R^{3/2} \underset{\substack{\uparrow \\ \text{Newton's constant}}}{G_N^{3/2}}}\right)^{1/4} \simeq 0.4 \text{ keV}$$

(current limit)

- conservative limit, can be strengthened in a model dependent manner.

→ possible improvement: if DM are collisionless

⇒ max value of phase-space density f_{max} does not change during evolution

↳ if observed coarse grained PSD does not exceed the initial max. value → limit on mass of DM

(not based on Pauli exclusion principle → also valid for bosons)

- model for final distribution: isothermal sphere, core radius r_c + one-dim. velocity dispersion σ

$$f_{max} = \frac{9\sigma^2}{4\pi G_N (2\pi\sigma^2)^{3/2} r_c^2}$$

Compare this w/ different primordial distributions: at $T_\nu \simeq 1\,\text{MeV}$
(decoupling of active ν)

(i) fermionic DM, which was in thermal eq., and decoupled while relativistic is described by Fermi-Dirac statistics

$$f_{FD} = \frac{\overset{\swarrow \#\ \text{spin states}}{g}}{e^{p/T_{FD}}+1}$$

$T_{FD} < T_\nu$: temperature accounting for entropy prod. so that right DM abundance

↳ Tremaine-Gunn mass bound $M_1 > \left[\frac{9(2\pi)^{1/2}}{g G_N \sigma r_c^2}\right]^{1/4}$

$\rightsquigarrow M_1 > .5\,\text{keV}$

(ii) $f = \frac{g\chi}{e^{p/T_\nu}+1}$ $\chi \ll 1$ determined by observed DM abundance

appears e.g. if DM sterile ν's are produced thermally via active-sterile mixing

$\longrightarrow M_1 > 1.7\,\text{keV}$

2.2.3 Lower mass bounds from Ly-α forest data

if sterile neutrino $\mathcal{O}(\text{keV})$ → WDM

sterile neutrino free-streaming length: $\lambda_{FS} \sim 1\,\text{Mpc} \left(\frac{1\,\text{keV}}{M_1}\right)\left(\frac{\langle p_s/T\rangle}{3.15}\right)$
(at matter-rad. equality)

mass inside λ_{FS}: $M_{FS} \sim 3 \times 10^{8}\, M_\odot \left(\frac{10\,\text{keV}}{M_1}\right)^3 \left(\frac{\langle p_s/T\rangle}{3.15}\right)^3$

$\langle p_s\rangle$: average momentum of sterile neutrinos at T_ν
$M_\odot$: solar mass

↳ primordial perturbations w/ size smaller λ_{FS} are erased
+ structures w/ mass smaller than M_{FS} are unlikely to be formed

more quantitatively: power spectrum P_{WDM} suppressed w/ respect to P_{CDM} for large k

$$T^2(k) = \frac{P_{WDM}(k)}{P_{CDM}(k)} < 1$$

$$\longrightarrow \quad T(k) = \left[1 + (\alpha k)^{2\nu}\right]^{-5/\nu}$$

(fit to numerical simulations)

- too small value of M_1 → hot dark matter

try to extract the matter power spectrum at smallest possible scales

limit depending on details $M_1 \gtrsim 2\,\text{keV}$

note: one neutrino does not take part in seesaw
↳ one extremely light active ν.

2.3 Sterile neutrino production in the early Universe

2.3.1 Active-sterile mixing

only source of DM sterile ν's is active-sterile mixing in the Lagrangian

· rate of DM sterile ν production (temperature below the EW scale)

$$\Gamma_N \sim \Gamma_\nu \, \Theta_M(T)^2$$

← Tibor's talk

· $\Gamma_\nu \sim G_F^2 T^5$: rate of active ν production

· $\Theta_M(T)$: temp.-dependent mixing angle

$$\theta_1 \longrightarrow \Theta_M \simeq \frac{\theta_1}{1 + 2.4 (T/200\,\mathrm{MeV})^6 \, (\mathrm{keV}/M_1)^2}$$

Γ_N roughly peak at: $T_{peak} \sim 130 \left(\frac{M_1}{1\,\mathrm{keV}}\right)^{1/3}$ MeV

⇝ T(QCD cross-over) for keV scale sterile ν's

⇝ strongly suppressed for T >> 100 MeV, $\Gamma_N \propto T^{-7}$

· T > EW scale: rate determined by decay and inv. decay of Higgs:

$$\Gamma_N \sim \frac{\theta_1^2 M_1^2 T}{v^2}$$

In the region (θ_1, M_1) admitted by X-ray, Ly-α, dsph

DM sterile ν's were never in thermal equilibrium in the early Universe

⇒ calc. of abundance: Integrate the rate of production over the whole history of the Universe

but: max. rate near QCD-crossover (quark-gluon plasma strongly coupled)
→ difficult

not: perform calculation, just give results here.

- sterile ν's are produced most intensively @ T_{peak}

  → QCD interactions are strong and cannot be treated perturbatively

  $\Downarrow$ no exact calc. possible (QCD eq for $T \in [10\,\text{MeV}, 1\,\text{GeV}]$ ok)

  ↳ semi-phenomenological model!

turns out that lepton asymmetry ΔL (can be as large as $\Delta_L = 0.2$ in νMSM)

plays important role for DM prod

$\# \nu\text{-species} = f(M_1, \Theta_1, \Delta L)$

→ see fig.: (resonant and non-resonant) reactions $l\bar{l} \to \nu N_1$
$q\bar{q} \to \nu N_1$

produce sterile ν's
abundance must be correct

depending on $\overset{\text{lept}}{\text{asymmetry}}$: between black lines

feature of fig: allowed regime surrounded by constraints

↳ single out $\mathcal{O}(10\,\text{keV})$ region

2.3.2: Particle decays.

before: only neutrino interaction: Yukawa coupling w/ neutrinos

now: couple sterile neutrinos to a singlet scalar field

$$\mathcal{L}_\chi = \frac{f_1}{2} \overline{N_1^c} N_1 \chi$$

for $M_\chi > 2M_1$ → scalar decay to neutrinos

if χ in thermal equilibrium at time of decay

↳ DM sterile neutrino abundance $f(f_1, M_\chi \gg M_1,$ entropy prod. after gen. of neutrinos$)$

$T \simeq M_\chi$: main production

momentum of sterile ν.

distribution function $n(p, t)$

$$\frac{\partial n}{\partial t} - Hp \frac{\partial n}{\partial p} = \frac{2 M_\chi \Gamma}{p^2} \int_{p + M_\chi^2/4p}^{\infty} n_\chi(E)\, dE$$

χ momentum distribution function

inv. decays $N_1 N_1 \to \chi$ are neglected

$\Gamma = f_1^2 M_\chi / 16\pi$

partial width $\chi \to NN$

if effective # of degrees of freedom is time-independent:

asymptotic solution $(t \to \infty)$: $n(x) = \dfrac{16 \Gamma M_0}{3 M_\chi^2} x^2 \displaystyle\int_1^\infty \frac{(y-1)^{3/2}\, dy}{e^{xy} - 1}$

$x = p/T$, $M_0 \simeq M_{Pl} / 1.66 \sqrt{g^*}$

↳ number density $N_0 = \displaystyle\int \frac{d^3 p}{(2\pi)^3} n(p) = \frac{3 \Gamma M_0 \zeta(5)}{2\pi M_\chi^2} T^3$

average momentum $\langle p \rangle \sim 2.45T \approx 0.8\, p_T = 3.15T$

eq. thermal distribution

- Formally $n(x) \to \infty$ for $x \to 0$

  $\Rightarrow$ not valid for small momenta: $x \lesssim (10^5 M_0/M_\chi^2)^2$

  (approx: · negl. Fermi blocking factors
  · negl. inverse decay
  · $M_1 \ll M_\chi$)

  # low momenta rather small → good approx.

- sterile ν produced by this mechanism perfectly consistent w/ bounds: independent of mixing angle
  ↳ X-ray no problem

- Ly-α: $S=1$, $m_\nu > 11.5$ keV, and smaller for more entropy production
- Nature of scalar field not crucial (has to be in thermal equilibrium)
  e.g. inflaton or scalar singlet

<u>Summary:</u>

DM: lightest singlet fermion
- depending on mass and creation mech → warm or cold DM
- warm → can render small-scale problems of CDM
- search: · X-ray spectrometer in space $\delta E/E \sim 10^{-3}-10^{-4}$
  our Galaxy + dwarf satellites
  · lab: detailed analysis of kinematics of β-decay in different isotopes (extremely challenging)

3. Numerical Simulations of Large-scale Structure Formation

3.1 Overview over current N-body simulations

2005: Millennium Simulation (V. Springel et al) arXiv: astro-ph/0504097
Virgo consortium

- N-body simulation to investigate how cosmological structure forms in an almost structureless universe (GADGET code)
- based on dark matter only
- cube of 500 Mpc/h : at $z=0 \hat{=} 685 \times 10^6$ Pc $= 2.2 \times 10^9$ ly

$10 \times 10^{18}\ M_\odot$, 10×10^9 $\overset{= 2160^3}{\text{virtual}}$ particles : 1 particle $\hat{=}\ 10^9 M_\odot$

- small density perturbations (see CMB) in dark matter distribution
- gravitational interaction, 1 step for 1×10^6 y
- expanding universe
- 11.000 steps $\hat{=}$ 14 billion years ($\hat{=}$ age of the universe); starting at CMB epoch ~ 400,000 y aft BB
- modelling normal matter into DM distribution to show galaxies / stars

↳ density perturbations grow and form a structure very similar to today's universe (compare statistically w/ SDSS)

2009: Millennium II simulation arXiv: 0903.3041

- cube of $(400 \times 10^6 \text{ ly})^3$
- 10×10^9 particles, $6.9 \times 10^6\ M_\odot$/particle

2010: Millennium XXL arXiv: 1203.3216

- cube of $(10 \times 10^9 \text{ ly})^3$
- 6720^3 particles, $7 \times 10^9\ M_\odot$/particle
- Volume 216 × MS and 27,000 × MS-II

2008: Aquarius Project : arXiv:0809.0898 (Springel et al.)

subhalos of galactic halos : Milky Way sized DM halo

cube of 137 Mpc

$m_p : 10^3 - 10^5\ M_\odot$, 4 billion particles

- Bolshoi Simulation (arXiv: 1002.3660) hipacc.ucsc.edu/Bolshoi
  - ART code
  - 250 Mpc/h box, ΛCDM
  - 8G particles
  - 1kpc/h force resolution
  - 1e8 Msun/h mass resolution
  - → force and mass resolution 1 order of magnitude better than M-I
  - → force res. ≈ M-I, volume 16 times large

- Big Bolshoi / MultiDark

3.2 Small-scale issues of ΛCDM and possible solutions

- On large scales: ΛCDM perfectly valid
  - CMB power spectrum perfectly agrees w/ ΛCDM
  - Simulations of large-scale structure agree w/ large-scale structure observed in surveys
  - outer parts of galactic haloes: constant rotation profiles
- but: there exist issues on sub-galactic scales
  - (i) ΛCDM predicts DM cusps ($\rho_{DM} \propto r^{-\alpha}$ with $1 < \alpha < 1.5$) which are not found in low surface brightness dwarfs (→ cores)
  - (ii) ΛCDM predicts more subhalos than satellite galaxies (of the Milky Way) are observed
- Viewpoints: (i) Astrophysical processes may reconcile ΛCDM predictions w/ observations (many possibilities: gas bulk flow, SN-driven winds, faint dwarf galaxies are found in future surveys etc.)

  (ii) Modification of DM theory needed.

our viewpoint!

→ We need a form of DM which does not destroy the success of ΛCDM on large scales, but may resolve small-scale issues.

Idea: Size of structures which are influenced by a particular form of DM is given by the free-streaming length. (hot/warm/cold DM)

- hot (= relativistic) DM has high velocities and large free-streaming length → prevents formation of the observed structures in the universe
- warm dark matter: intermediate scenario, smaller free-streaming length, ex.: O(keV) sterile neutrinos

sterile neutrino free-streaming length : $\lambda_{FS} \sim 100\,\mathrm{kpc}\left(\frac{10\,\mathrm{keV}}{M}\right)\left(\frac{\langle p_\nu/T\rangle}{3.15}\right)$
(at matter-rad. equality)

mass inside λ_{FS} : $M_{FS} \sim 3\times 10^7 M_\odot \left(\frac{10\,\mathrm{keV}}{M}\right)^3 \left(\frac{\langle p_\nu/T\rangle}{3.15}\right)^3$

$\langle p_\nu \rangle$: average momentum of sterile neutrino of T_ν ↑ [0.8 - 1.0]

$M_\odot$: solar mass

↳ . primordial perturbation w/ size smaller than λ_{FS} are erased and structures w/ mass smaller than M_{FS} are unlikely to form

. for distances $> \lambda_{FS}$: WDM scenario = CDM scenario

- Milky Way : diameter $\sim O(100\,\mathrm{kpc})$
- dwarf spheroidals : $M \sim 10^7 M_\odot$

↳ study of dSph should allow to decide which scenario

3.2.1 Cusps vs Cores

the Core-Cusp problem: 0910.3538

CDM: particles moving slowly → no primordial phase space constraints that could impose a cosmologically significant scale

⟹ CDM consistent w/ cusp (and not w/ core) dominated halos since the core radius will be negligible.

WDM: larger particle velocities → lower primordial phase space densities which impose a limit on the central phase space density of collapsed WDM halos

in WDM: core densities decrease, core radii increase (see fig.)

arXiv: astroph/0010388

but arXiv: 0912.3518: tension

needed primordial phase space densities smaller than lower bound

↯ WDM does not resolve cusp issue

new developments

- new observations undermine some previous evidence for dark matter cores in dwarf galaxies → ApJ 745, 92 (*)
  undecided
- the properties of density cores of dwarf spiral galaxies are inconsistent w/ expectations from WDM → ApJ, 710L, 161 (**)
- New simulations show that gas blowout during evolution of dwarf spiral galaxies can remove cusps (***) → Nature 463, 203
- but biggest subhalos in MW size dm simulation may be too dense to host the observed satellites.
  → "too big to fail"

(*) gas kinematics: core; stellar kinematics: NFW (cuspy)
→ stellar more robust (collisionless); gas subject to radial motions, warped disks, pressure support

but also studies which favor core

(**) assuming that core size is set by WDM particle properties: even the smallest core would require primordial phase space density values that are orders of magnitude smaller than lower limits obtained from Ly-α forest
→ no motivation to favor WDM over other models

(***) strong outflows from SN remove low angular momentum gas, which inhibits the formation of bulges and decreases the dark matter density to less than half of what it would be otherwise within the central kiloparsec
dwarf galaxies arise naturally in the simulation

3.2.2 Satellites and subhalos

ΛCDM predicts more DM halos than galaxies at small velocities ($V_{max} \lesssim 50\,km/s$)
(no disagreement for velocities $\geq 60\,km/s$)

e.g.: $\#(\Lambda CDM) = 2\times$ observation @ $V = 40\,km/s$

note: well-established linear relation between galaxy luminosity and circular velocity ($\hat{=}$ mass)

Warm dark matter may resolve this issue: see figures

new developments:

- "too big to fail" problem appears to be the most serious current challenge for ΛCDM
  → need for a more complex theory of DM
- high-resolution ΛCDM simulation substructure is consistent w/ quad-lens radio quasar flux and galaxy-galaxy lensing anomalies and indication of substructure by stellar stream gaps.
- ΛCDM predicts that there is a population of low-luminosity stealth galaxies around the Milky Way. Will new surveys w/ bigger telescopes find them? recently faint dwarfs have been detected that allow

SDSS only reached small part of sky
search for faint MW satellites has just begun
→ Dark Energy Survey: larger region of southern sky
→ LSST will go deeper

too big to fail : ΛCDM subhalos vs Milky way satellites

Aquarius simulation: $> 10^5$ identified subhalos

Milky Way : 12 bright satellites ($L_V > 10^5 L_\odot$)

(astro-ph/9901240: where are the missing Galactic satellites)

. all observed MW dSph are consistent w/ $V_{max} \le 25$ km/s

→ no indication that more massive halos host more luminous galaxies ∀ (linear relation between galaxy luminosity and circular velocity/mass)

→ massive failures: LCDM sims predict ~10 subhalos in the range $25 \le V \le 50$ in MW type galaxies but we don't see any (except Sagittarius)

possible solutions: . The Milky way is anomalous
. -"- has a low mass DM halo
. galaxy formation is stochastic @ low mass
. DM not just CDM – WDM or self interacting

or: high-res. CDM simulations misinterpreted
baryons strongly modify structure of sub-halos?

→ why baryons matter   12.07. 2400

Why baryons matter: the kinematics of dwarf spheroidal satellites

(arXiv: 1207.2468, Brooks, Zolotov)

Simulations of Milky Way-mass galaxies (baryons + dark matter):

baryonic physics (energetic feedback from supernovae and subsequent tidal stripping)

reduces DM mass in the central region of luminous satellite galaxies

↳ naturally explain the observed low dark matter densities in the Milky Way's dSph population

⟹ would resolve tension between CDM theory and observations of galaxies w/o alternative forms of DM

→ see fig

Literatur:

1) Gianfranco Bertone (ed.): Particle Dark Matter, Cambridge University Press 2010

relevant: Ch. 1: Particle Dark Matter
Ch. 2: Simulations of Cold Dark Matter Haloes
Ch. 3: Milky way satellites
Ch. 5: Dark Matter at the Centers of Galaxies
Ch. 7: DM production mechanisms
Ch. 12: Sterile Neutrinos

2) Sterile neutrinos as DM

- νMSM: arXiv: 0901.0011
- keV sterile neutrinos in gauge extension of the SM: arXiv: 0912.4415
  (Fedor, Hans, Manfred)
- Dodelson, Widrow: "Sterile neutrinos as DM" arXiv: hep-ph/9303287
- Dolgov, Hansen: "Massive sterile neutrinos as WDM" arXiv: hep-ph/0009083

3) DM allgemein

- Taoso, Bertone, Masiero: "DM Candidates: A Ten-Point Test", arXiv: 0711.4996

4) Problems of ΛCDM and possible Solutions

- Moore, Nature 370 (1994) 629
- Moore et al.: "DM substructure within galactic halos", arXiv astro-ph/9907411
- Kuzio de Naray et al.: "Case against WDM...", arXiv: 0912.3518
- Boylan-Kolchin et al.: "Too big to fail? ...", arXiv: 1103.0007
- -"- "The MW's bright satellites as an apparent failure of ΛCDM"
  arXiv: 1111.2048
- Lovell, Eke, Frenk et al: "The haloes of bright satellite galaxies in a WDM universe", arXiv: 1104.2923
- Brooks, Zolotov: "Why baryons matter...", arXiv: 1207.2468
- de Blok: "The Core-Cusp Problem", arXiv: 0910.3538