GUTs and Sterile Neutrinos

Outline:

1) GUTs
2) Fermion Masses in GUTS
3) light sterile neutrinos vs. GUTs

GUTs

SM: particle content, 3 generations

$$\boxed{\begin{aligned} Q &= \begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1/6) \\ u^c &\sim (\bar{3}, 1, -2/3) \\ d^c &\sim (\bar{3}, 1, -1/3) \\ L &\sim (1, 2, -1/2) \\ e^c &\sim (1, 1, 1) \end{aligned}}$$

⌈ $Q = I_3 + Y$ ⌋

- scalar $H \sim (1, 2, 1/2)$ $\qquad \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \longrightarrow G_{SM} = SU(3)_c \times SU(2)_L \times U(1)$

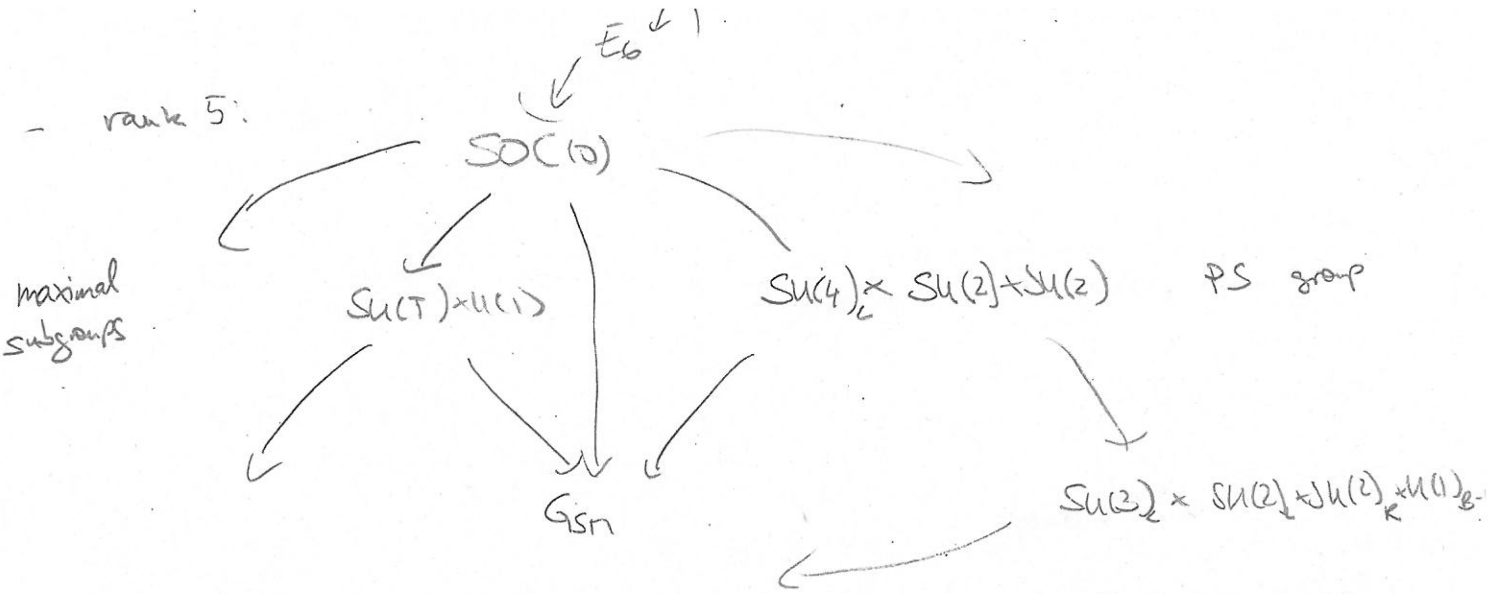
$$\downarrow$$

$$U(1)_{em}$$

- Can there be a larger semi-simple group from which the SM emerges?
  - smallest option $SU(3)_c \times SU(2)_L \times U(1) \subset SU(5)$

    (both) rank $2+1+1=4$ (# of diagonal generators)

- rank 5:



<u>First hint for GUTs</u>: all SM representations fit into small GUT representation w/o exotics.

<u>SU(5)</u>: write SM generators as

$$SU(3):\ T_A = \begin{pmatrix} \frac{1}{2}\lambda_A & \\ & 0 \end{pmatrix}_{A=1,\dots,8} ; \quad SU(2): \begin{pmatrix} 0 & \\ & \sigma^{A-20}/2 \end{pmatrix}_{A=21,22,23} ; \quad U(1): \sqrt{\tfrac{3}{5}} \begin{pmatrix} -\frac{1}{3}\mathbb{1}_{3\times 3} & 0 \\ 0 & \frac{1}{2}\mathbb{1} \end{pmatrix}$$

Gell-Mann matrices — Pauli matrices

SM fields fit as

$$\bar{5}_F: \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e \\ \nu \end{pmatrix} \qquad 10_F: \left(\begin{array}{ccc|cc} 0 & u^c_3 & -u^c_2 & u^1 & d^1 \\ & 0 & u^c_1 & u^2 & d^2 \\ & & 0 & u^3 & d^3 \\ \hline & & & 0 & e^c \\ & & & & 0 \end{array}\right)$$

gauge transformation $\bar{5}_F \to U^* \bar{5}_F$, $10_F \to U 10_F U^T$

↳ covariant derivative $\boxed{D_\mu = (\partial_\mu - i g_u A_\mu^A T^A)}$

because of the requirement $tr[T^A T^B] = \frac{1}{2}\delta^{AB}$

$$g_1 = g_2 = g_3 = g_u \qquad (1)$$

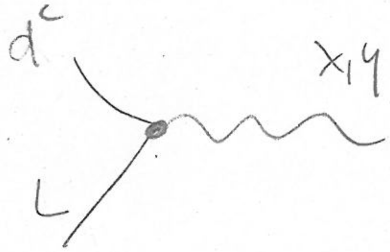
where

$$g_3 = g_s, \quad g_2 = g \qquad g_1 = \sqrt{\frac{5}{3}}\, g'.$$

$T_A \quad A = 9, \ldots, 20 \qquad X, Y$ - Bosons

$$X \quad \left(\begin{array}{ccc|cc} & & & 1 & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{array}\right) \qquad Y \quad \left(\begin{array}{ccc|cc} & & & -i & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ \hline i & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{array}\right)$$

off-diagonal couplings transform quarks into leptons



↳ Nota bene: funny $U(1)_Y$-charges only possible choice if one considers gauge anomalies (i.e. if the theory is supposed to make sense on the quantum level)

↳ Volker

Second Hint for GUTs:

$$\underbrace{\alpha_i^{-1}(M_Z)}_{\text{known}} = \underbrace{\alpha_{GUT}^{-1}}_{\text{unknown}} + \underbrace{\frac{b_i}{2\pi}}_{\substack{\text{known}\\ \text{(model dependent)}}} \ln \underbrace{\frac{M_{GUT}}{M_Z}}_{\text{unknown}} \qquad \left[\alpha_i = \frac{g_i^2}{4\pi}\right]$$

$\Longrightarrow$ eliminate unknown parameters

$$\boxed{\alpha_3^{-1}(M_Z) = (1+B)\,\alpha_2^{-1} - B\,\alpha_1^{-1}}$$

with $\quad B = \dfrac{b_3 - b_2}{b_2 - b_1}$.

$\alpha_3^{-1} = 8.50 \pm 0.14$, $\quad \alpha_2^{-1} = 29.57 \pm 0.02$ $\quad ; \alpha_3^{-1} = 59.00 \pm 0.02$

$$\Longrightarrow \underline{\underline{B = 0.716 \pm 0.005 \pm \underbrace{0.03}_{\text{estimated sys error}}}}$$

Models:

SM:

$b_3 = 11 - \frac{4}{3} n_g$ $\qquad$ n_g = # generations (3)

$b_2 = \frac{22}{3} - \frac{4}{3} n_g - \frac{1}{6} n_H$ $\qquad$ n_H = # Higgs doublets

$b_3 = -4/3\, n_g - \frac{1}{10} n_H$

$$\Longrightarrow \quad B(n_H = 1, n_g = 3) = 0.53$$

$\Longrightarrow$ no unification in SM

MSSM:

$$b_3 = 9 - n_g$$
$$b_2 = 6 - 2n_g - \tfrac{1}{2} n_H$$
$$b_1 = -2n_g - \tfrac{3}{10} n_H$$

$\overset{n_H=2}{\underset{n_g=3}{\Longrightarrow}}$ $B = \frac{5}{7} = 0.714$ ✓

unification @ one-loop in MSSM GUT. 2-loop unification needs large threshold corrections

$M_{GUT} \sim 10^{16}$ GeV

non-susy options:

many possibilities if one adds stuff to the SM

see e.g. 1204.5465 → slides

- $SU(2)_L$ triplet @ TeV scale → type II see-saw

Proton Decay:

In SM, $U(1)_B$ and $U(1)_L$ are accidental symmetries → have to be broken as GUT gauge interaction change leptons into quarks

→ proton decay → constraint on GUT scale

naive: $\Gamma_p \sim \alpha_{GUT}^2 \frac{m_p^5}{M_V^4}$

$$\Longrightarrow M_V > (2.57 - 3.23) \cdot 10^{15}\ \text{GeV}$$

↳ unification scale better be high ($\sim 10^{16}$ GeV)

(for details ask Pavel)

Fermion masses in GUTS

minimal SU(5)

introduce Higgs $5_H = \Phi = \begin{pmatrix} h^r \\ h^g \\ h^b \\ \phi^+ \\ \phi^0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} h \\ h \\ h \end{matrix}}\right\} SU(3)_c \\ \left.\vphantom{\begin{matrix} h \\ h \end{matrix}}\right\} SU(2)_L \end{matrix}$

(need additional $\Sigma \sim 24$ to break SU(5))

$\longrightarrow$

$$\mathcal{L}_Y = f_d \, \bar{5}_F \, 10_F \, \Phi^* + \frac{1}{2} f_u \, 10_F \, 10_F \, \Phi + h.c.$$

$$\Big\downarrow \langle \phi \rangle = (0,0,0,0,v_W)^T$$

$$\mathcal{L}_M = - f_d v_W (\bar{d}d + \bar{e}e) - f_u v_W \bar{u}u$$

$$\hookrightarrow \boxed{m_d = m_e}$$

- fewer unknowns than in SM
- unfortunately, it does not quite work @ M_{GUT} $m_b = 0.6\, m_\tau$
  - $\rightarrow$ higher dimensional operators
  - $\rightarrow$ add additional Higgs representation (45_H)
    - $\rightarrow$ more couplings / more freedom

Fermion masses in GUTs

$SO(10) = SO(4 \cdot 2 + 2)$ $\qquad U = \exp(i\, \omega^{ab} \sigma_{ab})$

- $\underline{45}$ gauge bosons

$45 = 24 + 1 + 10 + \overline{10}$ under $SU(5)$

- spinorial representation $2^4 = 16$ dimensional $\quad \sigma_{ab} = \frac{1}{2i} [\gamma^a, \gamma^b]$

$\underline{SO(2n+2)}$:

$$\gamma_i^{(n+1)} = \begin{pmatrix} \gamma_i^{(n)} & \\ & -\gamma_i^{(n)} \end{pmatrix} \qquad i = 1, \dots 2n$$

$$\gamma_{2n+1}^{(n+1)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{generalized Pauli matrices}$$

$$\gamma_{2n+2}^{(n+1)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$\hookrightarrow$ define representations as product of 5 $SU(2)$ spinors

$$\chi^{\pm} = \psi_1 \times \psi_2 \times \psi_3 \times \psi_4 \times \psi_5 \qquad \text{with} \quad \prod_i \sigma_3^{\,i} = 1$$

$$|\underbrace{+++}_{\substack{SO(6) \\ \cong \\ SU(4) \\ \supset SU(3)_c}} \underbrace{++}_{\substack{SO(4) \cong SU(2) \times SU(2) \\ \cap \\ S}}\rangle \quad \text{etc.}$$

$$\underline{16} \overset{422}{=} (4,2,1) \oplus (\bar{4},1,2)$$ SU(4)×SU(2)×SU(2)

under G_{321}

$$(4,2,1) \overset{SM}{=} (3,2)_{1/6} \oplus (1,2)_{-1/2}$$

$$(\bar{4},1,2) \overset{SM}{=} (\bar{3},1)_{1/3} \oplus (\bar{3},1)_{-2/3} \oplus (1,1)_1 \oplus (1,1)_0$$ ν^c

$$(4,2,1) = \begin{pmatrix} u_r & u_y & u_b & \nu \\ d_r & d_y & d_b & e \end{pmatrix} \quad ; \quad (\bar{4},1,2) : \begin{pmatrix} d_r^c & d_y^c & d_b^c & e^c \\ u_r^c & u_y^c & u_b^c & \nu^c \end{pmatrix}$$

Yukawa couplings :

$$16 \otimes 16 = 10 \oplus 120 \oplus 126 \rightarrow \text{most general Higgses}$$ 10_H, → EW sym breaking; 120_H, $\overline{126}_H$ → see-saw

$$\psi_L^T \sigma^2 \psi_L \phi_{10} = \phi_{10}(5)\,(\bar{u}_R u_L + \bar{\nu}_R \nu_L) + \phi_{10}(\bar{5})\,(\bar{d}_R d_L + \bar{e}_R e_L)$$

$$\boxed{m_d = m_e \; ; \; m_u = m_\nu}$$ ⚡

neutrino masses from $\overline{126} \overset{SU(5)}{=} 1 + 5 + 15 + \overline{45} + 50$

$$\psi_L^T \sigma^2 \psi_L \phi_{126} = \phi_{126}(1)\, \nu_R^T \sigma^2 \nu_R + \ldots$$

- $\overline{126}_H$ & 10_H → 2 symmetric Yukawa matrices

→ $\underline{\underline{16}}$ of SO(10) includes SM singlets (sterile neutrinos)

but their masses are usually ~ GUT scale → see-saw model.

$\overline{126}$ can be effective or fundamental

- effective

→ idea: use minimal # Higgs fields

SUSY SO(10) with realistic fermion masses

45_H, $16_H + \overline{16}_H$, 10_H

↳ fermion masses from LO & NLO operators

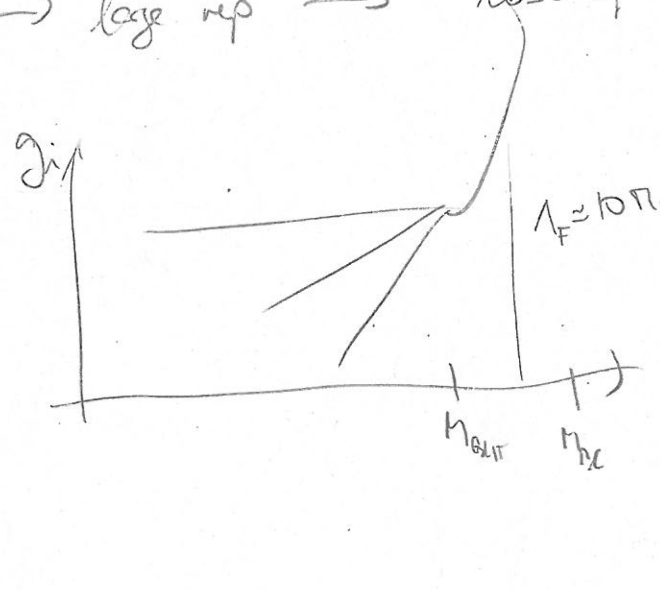
→ not very predictive

- fundamental

minimal SUSY SO(10)

- 10_H, 126_H, $\overline{126}_H$, 210_H

→ large rep → loss of asymptotic freedom



Can you make the sterile neutrinos contained in the 16 of SO(10) light?

<u>NO</u>, not really

- easiest way, usually Froggatt-Nielsen mechanism needs different charge for ν_R, not possible
- can always add extra singlets in GUT models

- double see-saw in SO(10)

<u>Rules of the Game</u> ~~t'Hooft~~ technical ~~naturalness~~

- 'no fine-tuning'
  → otherwise we do not need BSM → UTRSM (Kleiner Shaposhnikov)
  → caveat $y_e \sim 10^{-6}$, $\frac{m_{EW}}{m_{PL}} \sim 10^{-17}$

- in see-saw $m_\nu \rightarrow \frac{(LH)^2}{\Lambda}$

sterile $m_\nu \rightarrow m_S \, \epsilon^2$

130 GeV γ-line and sterile neutrinos

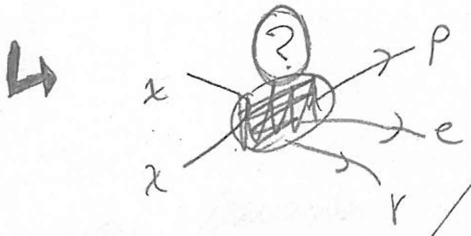
I Introduction

Dark Matter

$$\Omega_{DM} h^2 = 0.11$$

$$\Omega h^2 \simeq \frac{3 \cdot 10^{-27}\ \frac{cm^3}{s}}{\langle \sigma v \rangle_{th}}$$

$$\langle \sigma v \rangle_{th} \sim 3 \cdot 10^{-26}\ \frac{cm^3}{s}$$



II. Diffuse galactic γ-ray emission