Introduction to sterile neutrinos

From oscillation measurements:
active neutrinos have masses m_1, m_2, m_3

$$\Delta m_\odot^2 = 7.6 \cdot 10^{-5}\,eV^2$$
$$|\Delta m_{atm}^2| = 2.4\ 10^{-3}\,eV^2$$

convention: $m_1 < m_2$

$\longrightarrow$ 2 possibilities

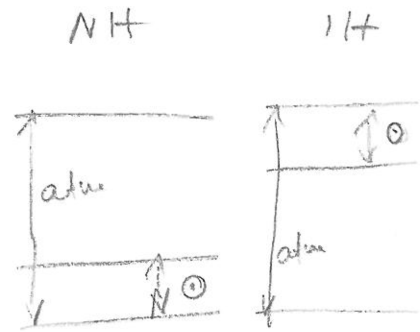
NH $\quad m_1 = m_{min} \longrightarrow m_1 < m_2 < m_3$

$$\Delta m_\odot^2 = \Delta m_{21}^2 > 0$$
$$\Delta m_{atm}^2 = \Delta m_{31}^2 > 0$$

IH $\quad m_3 = m_{min} \longrightarrow m_3 < m_1 < m_2$

$$\Delta m_\odot^2 = \Delta m_{21}^2 > 0$$
$$\Delta m_{atm}^2 = \Delta m_{32}^2 < 0$$

depending on m_{min} $\Rightarrow$ neutrino mass spectra



1) $m_{min} \simeq 0 \qquad m_1 \ll m_2 < m_3$ in NH

$$m_2 \approx (\Delta m_\odot^2)^{1/2} \simeq 0.009\ eV$$
$$m_3 \approx |\Delta m_{atm}^2|^{1/2} \simeq 0.05\ eV$$

$m_3 \ll m_1 < m_2$ in IH

$$m_{1,2} \approx |\Delta m_{atm}^2|^{1/2} \simeq 0.05\ eV$$

2) QD $\quad m_1 \simeq m_2 \approx m_3 \approx m_0 \quad , \quad m_j^2 \gg |\Delta m_{atm}^2|$

$\longrightarrow$ from limits $m_0 \gtrsim 0.1\,eV$

Limits : (Katrin ~ 0.2 eV)

β-decay spectrum $m_{\bar{\nu}_e} \lesssim 2$ eV

Cosmology $\sum_j m_j \leq (0.3 - 1.3)$ eV (WMAP)

Generation of neutrino masses.

SM → neutrinos massless, only left-handed ν_L

for Dirac mass term → introduce ν_R

sterile → not el. charged
not strongly int.
doesn't couple to W, Z
(but can have int. in BSM physics)
↳ eg L-R-Symm.

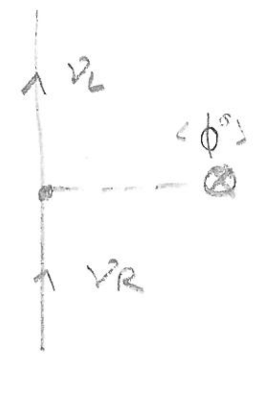
Dirac mass term.

$$-\mathcal{L}_D = m_D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$

weak Isospin conservation → m_D needs to be generated by VEV of neutral comp of Higgs doublet

$$m_D = y \langle \phi_0 \rangle = y \frac{v}{\sqrt{2}}$$

$$\rightarrow -\mathcal{L}_D = y \cdot \bar{\nu}_R \tilde{\Phi}^+ L + h.c.$$



as we have seen: $m_\nu \leq 0.1$ eV

$\longrightarrow y \lesssim 10^{-12}$ (e^- : 10^{-6})

Suppression of y

- symm → y has to vanish → contributions from higher dim operators
- ν_L & ν_R in (large or warped) extra dim

→ overlap of wave fcts

Majorana mass term

no symm forbids Majorana mass term
(no U(1) charge)

Maj mass requires only one Weyl spinor (2-comp.)

eg ν_L →

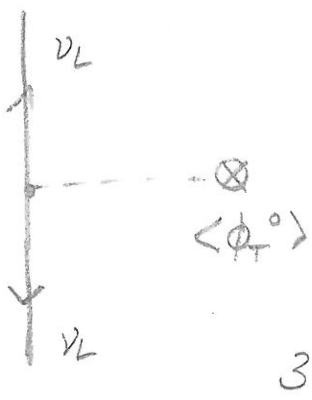
$$-\mathcal{L}_T = \frac{m_L}{2}\left(\overline{\nu_L}\,\nu^c{}_R + \overline{\nu^c}_R\,\nu_L\right)$$

$$= \frac{m_L}{2}\left(\overline{\nu_L}\,C\,\overline{\nu_L}^T + \nu_L^T\,C^{-1}\nu_L\right) = \frac{m_L}{2}\,\overline{\nu_M}\,\nu_M$$

$$\nu_M = \nu_L + \nu^c{}_R = \nu_M{}^c$$

but weak isospin violated by $\Delta T_3 = \pm 1$
(ν_L is part of doublet) → Higgs triplet Δ w/
VEV of neutral comp.

$$-\mathcal{L}_T = \frac{1}{2} f \cdot L^T C^{-1} i\tau_2 \Delta\, L + h.c.$$



Majorana mass for sterile neutrino ν_R:

$$-\mathcal{L}_S = \frac{m_R}{2}\left(\bar{\nu}^c_L \nu_R + \bar{\nu}_R \nu^c_L\right) =$$
$$= \frac{m_R}{2}\left(\nu_R^T C^{-1} \nu_R + \bar{\nu}_R C \bar{\nu}_R^T\right)$$
$$= \frac{m_R}{2}\left(\bar{N} N\right)$$

$$N_M = \nu^c_L + \nu_R = N_M^c$$

not forbidden by any symmetry
(but eg in L-R-Symm model m_R needs to be generated by VEV)

Maj & Dirac

When both mass terms present $\longrightarrow$ need to diagonalise mass matrix to obtain mass eigenstates, which are a lin comb of ν_L & ν_R

define $\nu^M = \frac{1}{\sqrt{2}}\left(\nu_L + \nu_L{}^c\right)$

$N^M = \frac{1}{\sqrt{2}}\left(\nu_R + \nu_R{}^c\right)$

$$\rightarrow \; -\mathcal{L}_{mass} = \left(\bar{\nu}^M, \bar{N}^M\right) \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu^M \\ N^M \end{pmatrix}$$

Diagonalised by unitary matrix U

Seesaw

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix}$$

m_L : 3×3

M_R : $n_R \times n_R$ $\rightarrow$ $m_D^T = 3\times n_R$

can assume M_R diagonal (sterile!)

if $\frac{m_D}{M_R} \ll 1$ we can block-diag $\mathcal{M}_\nu$, expanding in $\frac{m_D}{M_R}$

$$W^T \begin{pmatrix} m_L & m_D^T \\ m_D & \mathcal{M}_R \end{pmatrix} W = \begin{pmatrix} M_{light} & 0 \\ 0 & \mathcal{M}_{heavy} \end{pmatrix} \quad (*)$$

$$\begin{pmatrix} \nu^M \\ N^M \end{pmatrix} = W \begin{pmatrix} \nu_{light} \\ \nu_{heavy} \end{pmatrix}$$

almost only active (not mass ES!)

almost only sterile

Solve $(*)$ with ansatz

$$W = \begin{pmatrix} \sqrt{1-BB^\dagger} & B \\ -B^\dagger & \sqrt{1-B^\dagger B} \end{pmatrix}$$

sqrts $\hat{=}$ power series

$B = B_1 + B_2 + B_3 \ldots$

$B_j \propto \left(\frac{1}{M_R}\right)^j$

make $\nu^M = \sqrt{1-BB^\dagger}\,\nu_{light} + B\,\nu_{heavy}$

$\Rightarrow$ active sterile mixing $\propto B$

→ $B_1^\dagger \approx M_R^{-1} m_D \quad \left(\frac{m_D}{M_R}\right)$

& $M_{light} \approx m_L - m_D^T M_R^{-1} m_D$

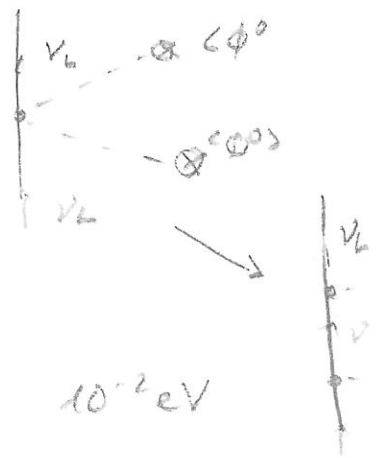
$M_{heavy} \approx M_R$

no triplet term ($m_L = 0$) → seesaw type I

$M_{light} \approx -m_D^T M_R^{-1} m_D$

→ M_R suppresses the mainly active ν

M_{light} ≙ masses of



eg m_D w/ Yukawa ~ $O(1)$

⇒ $m_D \approx 100$ GeV

$M_{light} \approx 10^{-2}$ eV ⇒ $\frac{m_D^2}{M_R} \doteq 10^{-2}$ eV

→ $M_R \sim 10^{15}$ GeV

[of course y can be smaller]

seesaw II → $m_L \neq 0$ & new scale

cancellations
also only $m_L \neq 0$