LYTH BOUND ; SEMINAR, BRANIMIR RADOVČIĆ

- FRW Spacetime

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right)$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \boxed{M_{PL} \equiv (8\pi G)^{-1/2}} = 2.4 \cdot 10^{18}\ \text{GeV}$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$

$$\boxed{\begin{aligned} H^2 \equiv \left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \\ \frac{\ddot a}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned}} \qquad \boxed{\begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{1}{3M_{PL}^2}\rho \\ \frac{\ddot a}{a} &= -\frac{1}{6M_{PL}^2}(\rho + 3p) \end{aligned}}$$

$$\frac{d}{dt}\left(\rho a^3\right) = -p\frac{d}{dt}a^3 \Rightarrow \dot\rho + 3\frac{\dot a}{a}(p+\rho) = 0$$

$$p = w\rho \quad ; \quad w = -1 \Rightarrow \rho(a) = \text{const} \Rightarrow a(t) = e^{\int^t H(t')\,dt'}$$

$$\boxed{\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi)}$$

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V' = 0}$$

$$\boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)}$$

SLOW ROLL

1.) $\dot{\phi}^2 << V(\phi)$

2.) $\ddot{\phi} << H\dot{\phi}$

$$\boxed{H^2 = \frac{1}{3M_{Pl}^2} V(\phi)}$$

$$\boxed{\dot{\phi} = -\frac{V'}{3H}}$$

$$\boxed{\varepsilon(\phi) = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V}\right)^2}$$

$$\boxed{\eta(\phi) = M_{Pl}^2 \frac{V''}{V}}$$

$$\varepsilon, \eta < 1$$

$$\boxed{\varepsilon(\phi_{end}) = 1}$$

$$\boxed{\varepsilon(\phi) = \frac{M_{Pl}^2}{2} \left(\frac{-\cancel{3}\cancel{H}\dot{\phi}}{\cancel{3}H^{\cancel{2}}M_{Pl}^2}\right)^2 = \frac{1}{2M_{Pl}^2} \left(\frac{\dot{\phi}}{H}\right)^2}$$

- number of e-folds

$$N(\phi) \equiv \ln \frac{a_{end}}{a(\phi)} = \int_t^{t_{end}} H\,dt = \int_\phi^{\phi_{end}} \frac{H}{\dot\phi}\,d\phi =$$

$$\dot\phi = -\frac{V'}{3H}$$

$$\int_\phi^{\phi_{end}} \frac{H}{-\frac{V'}{3H}}\,d\phi = \int_{\phi_{end}}^{\phi} \frac{3H^2}{V'}\,d\phi = \int_{\phi_{end}}^{\phi} \frac{1}{M_{PL}^2}\frac{V}{V'}\,d\phi \underset{\text{def. }\varepsilon}{\overset{\uparrow}{=}}$$

$$H^2 = \frac{1}{3M_{PL}^2} V(\phi)$$

$$N(\phi) = \int_{\phi_{end}}^{\phi} \frac{d\left(\frac{\phi}{M_{PL}}\right)}{\sqrt{2\varepsilon}}$$

<u>example 1</u>

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$V'(\phi) = m^2\phi \qquad V''(\phi) = m^2$$

$$\varepsilon(\phi) = 2\cdot\frac{M_{PL}^2}{\phi^2} \qquad \eta(\phi) = \frac{2M_{PL}^2}{\phi^2}$$

$\Rightarrow$ from $\varepsilon(\phi) \leq 1 \Rightarrow \phi > M_{PL}$

$$\varepsilon(\phi_{end}) = 1 \quad \Rightarrow \quad \phi_{end} = \sqrt{2}\,M_{PL}$$

$$N(\phi) = \int_{\phi_{end}}^{\phi} \frac{1}{M_{PL}^2}\frac{\phi'}{2}\,d\phi' = \left.\frac{\phi^2}{4M_{PL}^2}\right|_{\phi_{end}}^{\phi} = \frac{\phi^2}{4M_{PL}^2} - \frac{1}{2}$$

for $N(\phi) = 60 \Rightarrow \quad \phi = 15.6\,M_{PL}$, but $m^2 = ?$ $\quad V(\phi) = ?$

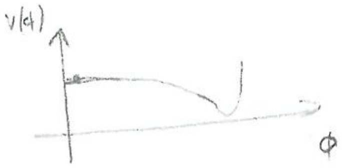
$$\varepsilon(\phi(N)) = \frac{1}{2N} \qquad \eta(\phi(N)) = \frac{1}{2N}$$

<u>example 2</u>

- small field

$$N(\phi) = \int_{\phi_{end}}^{\phi} \frac{d\left(\frac{\phi}{M_{PL}}\right)}{\sqrt{2\varepsilon}} \qquad \text{if } \varepsilon \ll 1$$

$$\varepsilon \sim \frac{V'}{V} \ll 1$$



have to be close to the local maximum $\Rightarrow \eta \gg \varepsilon$

$$\frac{\delta\rho}{\rho} \sim \delta t \cdot H \sim \frac{\delta\phi\, H}{\dot\phi} \sim \frac{H^3}{V'} \sim \frac{H^3 M_{PL}}{\sqrt{\varepsilon}\, V} \sim \frac{H}{M_{PL}} \frac{1}{\sqrt{\varepsilon}}$$

$$\frac{\delta h}{h} \sim \frac{H}{m_p}$$ B modes $\Rightarrow H \Rightarrow V$

scalar perturbations $$\Delta_s^2(k) = \frac{H^2}{(2\pi)^2} \frac{H^2}{\dot\phi^2}\Bigg|_{k=aH} \qquad n_s - 1 \equiv \frac{d\ln \Delta_s^2}{d\ln k} = 2\eta - 6\varepsilon$$

tensor perturbations $$\Delta_T^2(k) = \frac{2}{\pi^2} \frac{H^2}{M_{PL}^2}\Bigg|_{k=aH}$$

$$r \equiv \frac{\Delta_t^2(k)}{\Delta_s^2(k)} = 8 \frac{1}{M_{PL}^2} \frac{\dot\phi^2}{H^2} = \frac{8}{M_{PL}^2}\left(\frac{d\phi}{dt\,H}\right)^2 = \frac{8}{M_{PL}^2}\left(\frac{d\phi}{dN}\right)^2$$

$r = 16\varepsilon$; $\Delta_T^2 \sim H^2 \sim V \Rightarrow$ BICEP $\Rightarrow V^{1/4} = 2 \cdot 10^{16}$ GeV

$$\frac{d\phi}{M_{PL}} = \sqrt{\frac{r}{8}}\, dN \qquad \text{if } r(N) \approx r \quad \text{za} \quad 2 \leq \ell \leq 100 \Leftrightarrow \Delta N = 4$$

LYTH PRL(1997)

$$\frac{\Delta\phi}{M_{PL}} = 0.45 \left(\frac{r}{0.1}\right)^{1/2}$$

ali inflacija se nastavlja dalje ovo je minimum

$$\frac{\Delta\phi}{M_{PL}} = \mathcal{O}(1) \times \left(\frac{r}{0.05}\right)^{1/2} \qquad \text{if } r(N) \approx r \quad \text{za} \quad \Delta N = 60,$$

ali $r(N) \nearrow$ for $N \to 0$

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example

$$V = \frac{1}{2} m^2 \phi^2 \qquad \varepsilon(\phi(N)) = \frac{1}{2N} = \eta(\phi(N))$$

$$n_s = 1 + 2\eta - 6\varepsilon = 1 - \frac{2}{N} \approx 0.966$$

$$r = 16\,\varepsilon = \frac{8}{N} = 0.13$$

za $2 \leq \ell \leq 100 \Leftrightarrow \Delta N = 4$

pa su tu n_s i $r \approx$ const

– Conservative bound, because inflation goes for another $\Delta N = 56$ and $r \sim \varepsilon$ just goes up

if $r = 0.13$; $\frac{\Delta\phi}{M_{PL}} = \mathcal{O}(1) \times \left(\frac{r}{0.05}\right)^{1/2}$ recimo dobit $\Delta\phi = 1.5\, M_{PL}$

jer nismo uključili $r \uparrow$ as $N \to 0$!

ANTUSCH NOLDE 2014

- $r > 0.1$ impossible to reconcile with $\Delta\phi < M_{PL}$ for single field slow roll independently of the form of the potential

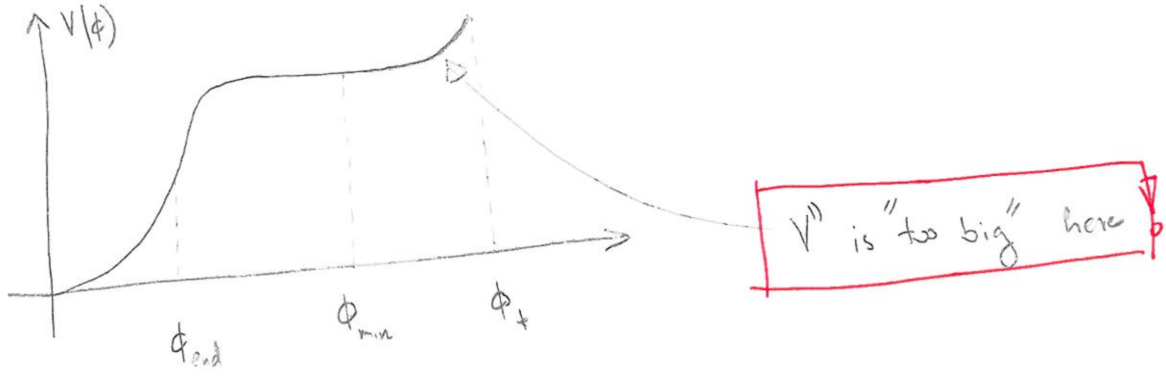
$$N_{tot} = \int_{\phi_{end}}^{\phi_*} d\phi \frac{V(\phi)}{V'(\phi)} < |\phi_* - \phi_e| \left|\frac{V}{V'}\right|_{max} = |\phi_* - \phi_e| \frac{1}{\sqrt{2\epsilon_{min}}}$$

ϕ_* → when CMB scales cross the horizon

- if $\epsilon_{min} \sim \epsilon_*$ ⇒ Lyth bound, i.e. ϵ is smallest at the "start of inflation"
" when the first mode exit Hubble radius

- what if $\epsilon_{min} << \epsilon_*$; large ϵ_* to get large r

- $$N_{tot} = \int_{\phi_{end}}^{\phi_*} \frac{d\left(\frac{\phi}{M_{PL}}\right)}{\sqrt{2\epsilon(\phi)}}$$ confirms that small ϵ leads to smaller $\Delta\phi$



- this works up to a point and Lyth bound extendet during all $\Delta N = 60$ can be weakend.

$\boxed{\text{- in slow roll } \varepsilon \text{ can not vary arbitrarily quickly}}$!

$$\frac{d}{d\phi}\sqrt{2\varepsilon} = \frac{d}{d\phi}\frac{V'}{V} = \frac{V''}{V} - \left(\frac{V'}{V}\right)^2 = \eta - 2\varepsilon$$

droping out M_{PL}

$\eta, \varepsilon \ll 1 \Rightarrow \varepsilon' \ll 1$

cannot change too quickly during slow roll

- integrate from ϕ_{min} to ϕ_*

$$\left(\frac{V'}{V}\right)_* - \left(\frac{V'}{V}\right)_{min} = \int_{\phi_{min}}^{\phi_*} d\phi \frac{d}{d\phi}\sqrt{2\varepsilon} = \int_{\phi_{min}}^{\phi_*} d\phi\,(\eta - 2\varepsilon) = (\phi_* - \phi_{min})\langle\eta - 2\varepsilon\rangle$$

- $$\left(\frac{V'}{V}\right)_* = \sqrt{\frac{r}{8}} \simeq 0.11 \times \sqrt{\frac{r}{0.1}}$$

- $$\phi_* - \phi_{end} = \int_{N_e}^{N_*} dN \frac{d\phi}{dN} = \int_{N_e}^{N_*} dN \frac{V'(\phi)}{V(\phi)} > N_{total}\left(\frac{V'}{V}\right)_{min}$$

$$\left(\frac{V'}{V}\right)_{min} < \frac{\phi_* - \phi_{end}}{N_{total}} = \frac{\Delta\phi}{N_{total}} \lesssim 0.02\,\Delta\phi$$

$$\left(\frac{V'}{V}\right)_* = \left(\frac{V'}{V}\right)_{min} + \left(\phi_* - \phi_{min}\right)\langle\eta - 2\varepsilon\rangle$$

$0.11 \times \sqrt{\frac{r}{0.1}}$ $\quad < \frac{\Delta\phi}{M_{total}}$ $\quad < \Delta\phi \, \langle\eta - 2\varepsilon\rangle$

$$0.11 \times \sqrt{\frac{r}{0.1}} < \Delta\phi\left(\frac{1}{N} + \langle\eta - 2\varepsilon\rangle\right)$$

$$\boxed{\frac{\Delta\phi}{M_{PL}} > \frac{0.11}{\frac{1}{N} + \langle\eta - 2\varepsilon\rangle}\sqrt{\frac{r}{0.1}}} > \frac{0.07}{\langle\eta - 2\varepsilon\rangle}\sqrt{\frac{r}{0.05}}$$

$\sim \varepsilon'$ a to ne može bit preveliko!

- slow roll
- single field
- no specific potential

$\Rightarrow$ if B-modes detected $\Rightarrow \Delta\phi > M_{PL}$

- What about the value for $V = ?$ or for $V = \frac{1}{2}m^2\phi^2$ $m^2 = ?$

$$\Delta_T^2 \sim H^2 \sim V \Rightarrow \text{for } r = 0.2 \Rightarrow V^{1/4} = 2 \cdot 10^{16}\ \text{GeV} \cdot \left(\frac{r}{0.2}\right)^{1/4}$$

- field values over M_{PL} and energy densities under M_{PL}

- higher dimensional operators $\frac{O_D}{M^{D-4}} = \frac{O_D}{M_{PL}^{D-4}}$

- the Eta problem

$$\frac{O_6}{M_{PL}^2} = \frac{O_4}{M_{PL}^2}\phi^2$$

if $\langle O_4 \rangle \sim V$ then correction to inflaton mass $\sim \frac{V}{M_{PL}^2} \sim H$

and this is an order 1 correction to η.

example $V = \frac{1}{2} m^2 \phi^2$ $V = M_{PL}^2 H^2 \Rightarrow m^2 \sim \frac{M_{PL}^2}{\phi^2} H^2 < H^2$

$\Rightarrow$ small inflaton mass is radiatively unstable

$$\mathcal{L}_{eff}(\phi) = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - \sum_{p=1}^{\infty}\left[\lambda_p\phi^4 + \nu_p(\partial\phi)^2\right]\left(\frac{\phi}{M_{PL}}\right)^{2p} + \dots$$

- infinite series of higher dimensional operators

- SHIFT SYMMETRY

$$\phi \to \phi + \text{const}$$

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 - \lambda_p \phi^?$$ technically natural

- requiring symmetry protecting from coupling from Planck scale

=> Planck scale theory should respect this symmetry

ARKANI HAMED

=> - there is no manifest problem, curvature and energy is Planckian

- Something goes wrong in low energy theory where there is a problem with EFT

- no difficulty what so ever

RIOTTO

- abandon the theoretical prejudice that scalar perturbations come from inflation
- or all states coupled to inflaton $\phi > M_{PL}$ are classical black holes

↳ exponential suppression of these states Hawking entropy

- effective operator are given by V and V'', physical quantities and every thing is ok

if only ~~...~~ inflaton + graviton

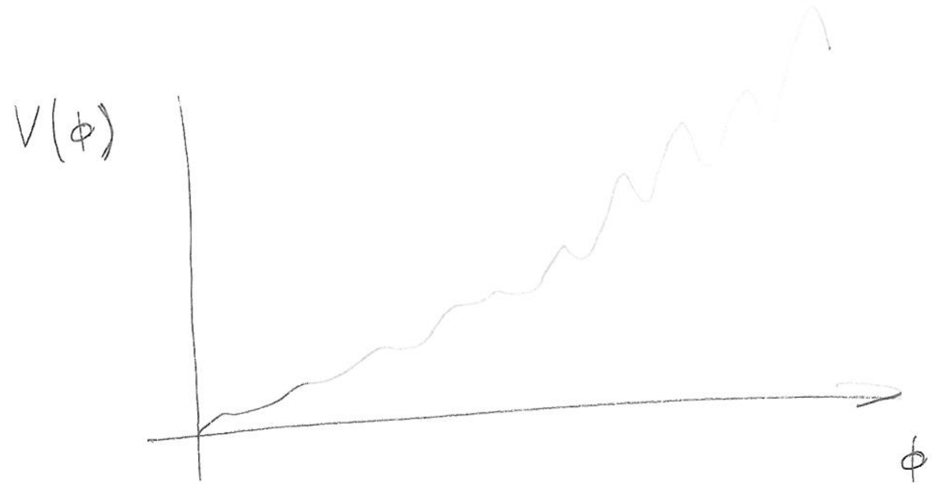
$$\frac{\Delta V}{V} = C_1 \frac{V''}{M_{PL}^2} + C_2 \frac{V}{M_{PL}^4}$$

- no gravity back reaction
- no graviton problem
- UV gravity maybe a problem

$|\Phi|^n$

$$V(\phi,\theta) = f_A(\phi,\theta)\, V(|\Phi|)$$

$$f(\phi,\theta) = 1 + \frac{A}{2}\sin\left(\frac{|\phi|^n}{\Lambda^n} + \theta\right)$$



- two fields