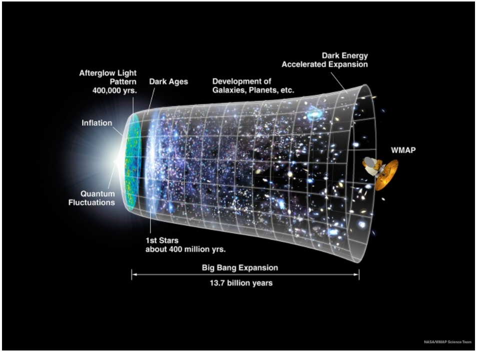
Implications of Inflation on Particle Physics

Julian Heeck

24.9.2014



The Big Bang Theory

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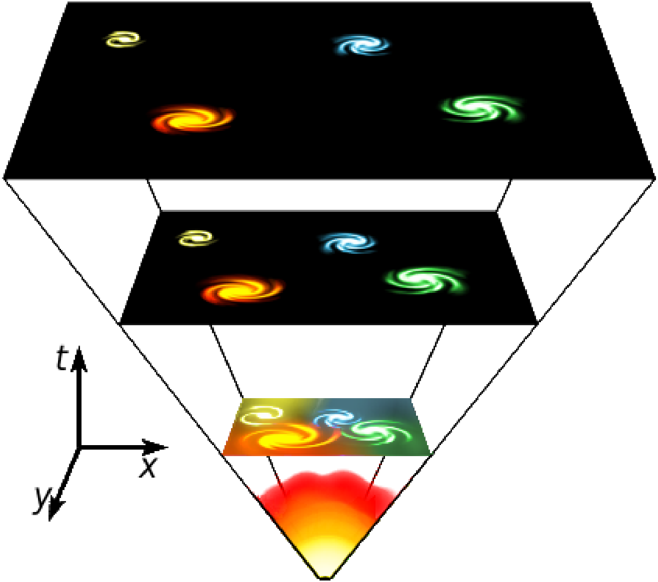

General Relativity

- Einstein's gravity:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (g^{\alpha\beta} R_{\alpha\beta}) = 8\pi G_N T_{\mu\nu} \,,$$

  a differential equation for the metric $g_{\mu\nu}$ [Ricci tensor $R_{\mu\nu}$], with Newton's constant

$$8\pi G_N = 1/M^2_{\text{Planck}} \simeq 1/(10^{19}\,\mathrm{GeV})^2 .$$

  Energy–Momentum tensor $T_{\mu\nu}$ as source for geometry.

- Observations: our Universe is flat, homogeneous, and isotropic.
  $\Rightarrow$ Friedmann–Robertson–Walker metric

$$g_{\mu\nu} = \mathrm{diag}(1, -a(t)^2, -a(t)^2, -a(t)^2)$$

  with scale factor $a(t)$.

Expanding Universe

The Universe expands, depending on the content:[1]

- Non-relativistic matter:

$$T^\mu_\nu = \rho_\mathrm{m}\,\mathrm{diag}(1,0,0,0), \quad a_\mathrm{m}(t) \propto t^{2/3}, \quad \ddot{a}_\mathrm{m}(t) < 0\,.$$

- Relativistic matter (radiation):

$$T^\mu_\nu = \rho_\mathrm{rad}\,\mathrm{diag}(1,-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},), \quad a_\mathrm{rad}(t) \propto t^{1/2}, \quad \ddot{a}_\mathrm{rad}(t) < 0\,.$$

- Cosmological constant Λ: $T_{\mu\nu} \propto g_{\mu\nu}$:

$$T^\mu_\nu = \rho_\Lambda\,\mathrm{diag}(1,1,1,1), \quad a_\Lambda(t) = \exp\left(\sqrt{\frac{\rho_\Lambda}{3M^2_\mathrm{Planck}}}\,t\right), \quad \ddot{a}_\Lambda(t) > 0\,.$$

Linear combination is used to fit CMB data.
$\Rightarrow$ Very successful Cosmological Standard Model $\underbrace{\Lambda\mathrm{CDM}}_{\Lambda+\text{ cold dark matter}}$.

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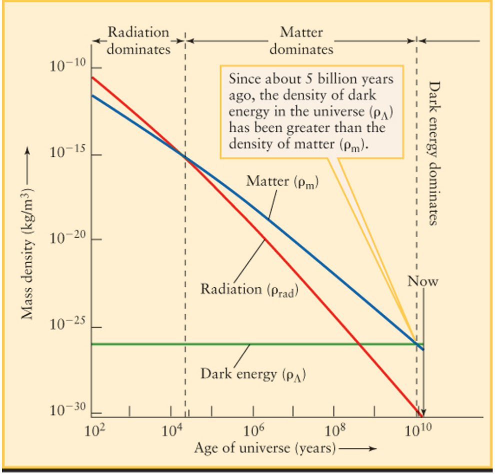
[1] A *static* solution can be constructed, but is highly finetuned/unstable.

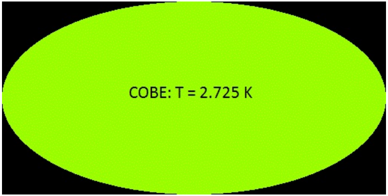
Expanding Universe

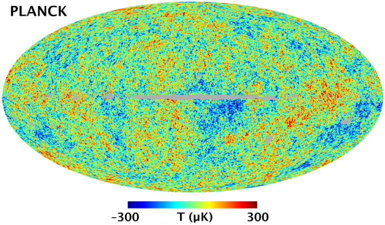
$$\rho_{\text{rad}} \propto a^{-4}\,, \quad \rho_{\text{m}} \propto a^{-3}\,, \quad \rho_\Lambda = \text{const.}$$



Cosmic Microwave Background

- 400 000 years after the Big Bang ($T \simeq 1\,\mathrm{eV}$), electrons and protons form *neutral* hydrogen.
  $\Rightarrow$ Opaque plasma (our Universe) becomes clear.
- Photons from this surface of last scattering are the CMB.

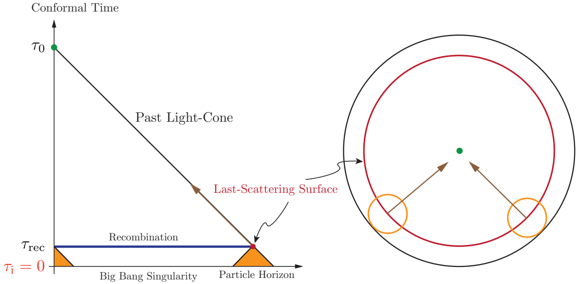




Problems of Big Bang Theory

Dynamics work great, but initial conditions on $\rho(\mathbf{x})$ seem finetuned.

- (Flatness problem.)
- Horizon problem: evolving backwards, the CMB consists of 10^5 *causally disconnected patches*.[2]



Cosmic background radiation has same temperature in opposite directions, but they couldn't ever talk to each other!

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[2] Baumann, arXiv:0907.5424

What is Inflation?

What is Inflation?

- Start Universe with exponential expansion à la Λ:

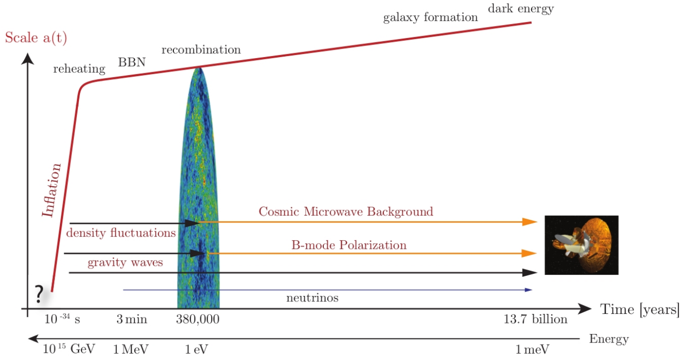
$$a_\Lambda(t) = \exp\left(\sqrt{\frac{\rho_\Lambda}{3}}\,t\right) .$$

- If long enough ($a_\text{end} \gtrsim e^{60} a_\text{start}$), a small causally connected patch is blown up to our Universe!
- (Flatness problem solved, too.)
- Strong dependence on the inital $\rho(\mathbf{x})$ is *smoothed out*.

Brief (10^{-36}–$10^{-33}\,\mathrm{s}$) but extreme inflationary period nicely explains *flatness* and *smoothness* of our Universe.

What is Inflation not?

Brief (10^{-36}–$10^{-33}\,\mathrm{s}$) inflationary period explains smoothness etc.



Main problem: how to turn it off.

- Can not just take constant Λ, need "$\Lambda \to 0$" at end of inflation.

The Inflaton

A simple scalar field (the inflaton) can mimick Λ.

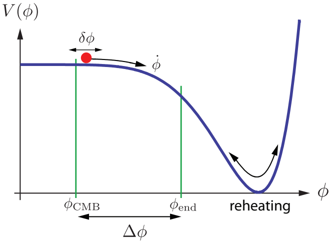
- Action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] .$$

- Energy–Momentum tensor of *homogeneous* inflaton

$$T^\mu_\nu = \rho \, \mathrm{diag}(1, -w, -w, -w) \,, \quad w \equiv \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} \,.$$

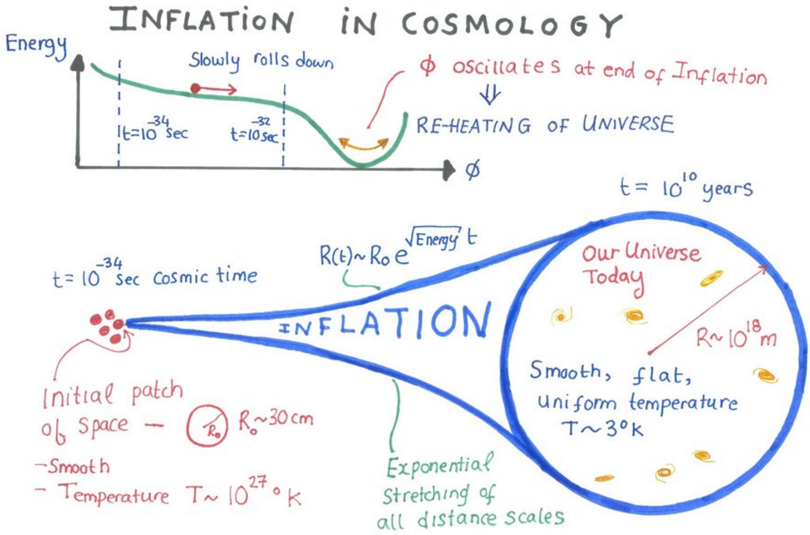
- Looks like Λ for $\dot\phi^2 \ll V(\phi)$, but like matter for $\dot\phi^2 \gg V(\phi)$.



Inflaton falls down potential ⇒ Inflation. Gains kinetic energy ⇒ Inflation stop.

Summary so far

Exponential expansion after Big Bang:[3]



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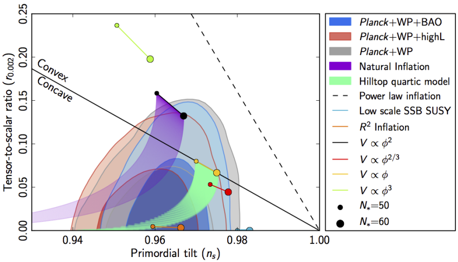
[3] http://www.strings.ph.qmul.ac.uk

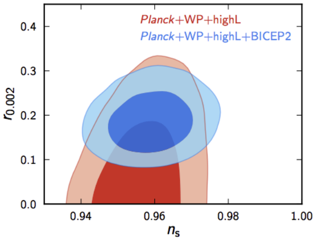
The Inflaton II

More features of the inflaton:

- Quantum fluctuations nicely reproduce CMB perturbations, e.g. spectral index n_s.
- "Reheating" converts inflaton energy to Standard Model particles, fills Universe!
- The scale of inflation can be observed from the ratio of tensor-to-scalar fluctuations in CMB:

$$V^{1/4} \sim 2 \times 10^{16}\,\mathrm{GeV} \left(\frac{r}{0.2}\right)^{1/4}, \quad r \equiv \frac{\Delta_t^2(k)}{\Delta_s^2(k)}\,.$$





This Talk

- We assume here that inflation has been experimentally confirmed. This is arguable only possible for large r, so we take BICEP2 as a guide:[4]

$$r \simeq 0.2\,, \quad V^{1/4} \simeq 2 \times 10^{16}\,\mathrm{GeV}\,, \quad H \simeq 10^{14}\,\mathrm{GeV}\,.$$

- BICEP2 might be wrong![5]
- Inflaton mass and reheating temperature are model dependent, but we assume similarly high values.

Scale of inflation far beyond collider-testable $10^4\,\mathrm{GeV}$. Surely impact on particle physics?!

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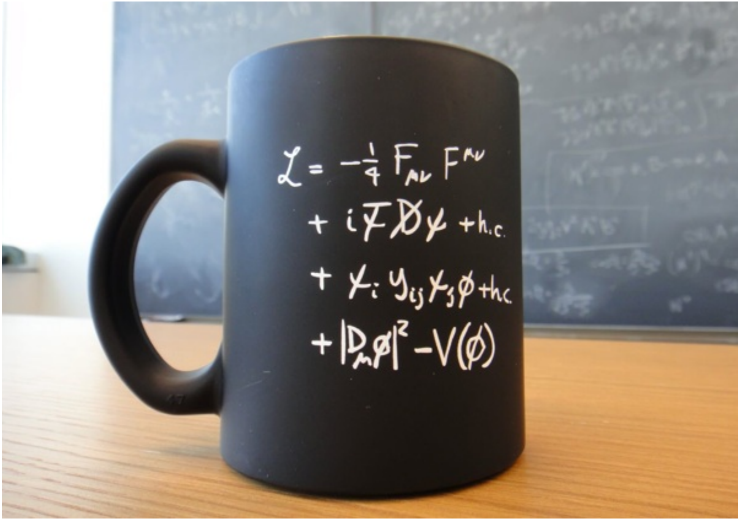
[4] BICEP2: Phys.Rev.Lett. 112 (2014) 241101, arXiv:1403.3985, $\mathcal{O}(700)$ citations.

[5] Planck: arXiv:1409.5738: B-modes just dust.

Standard Model of Particle Physics

Beautiful and simple:



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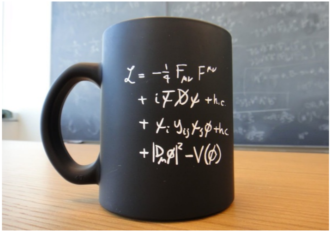

Not shown:

- Gauge group $SU(3)_{\text{color}} \times SU(2)_{\text{isospin}} \times U(1)_{\text{hypercharge}}$;
  $\Rightarrow 8 + 3 + 1$ spin-1 bosons with field strength $F_{\mu\nu}$,
- Three copies of spin-$\frac{1}{2}$ Weyl fields (families/generations) in rep.

$$\Psi_{1,2,3} \sim \underbrace{(\mathbf{3}, \mathbf{2}, \tfrac{1}{6}) \oplus (\mathbf{3}, \mathbf{1}, -\tfrac{2}{3}) \oplus (\mathbf{3}, \mathbf{1}, \tfrac{1}{3})}_{\text{quarks}} \oplus \underbrace{(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}) \oplus (\mathbf{1}, \mathbf{1}, 1)}_{\text{leptons}},$$

- One complex spin-0 field $\phi \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ which breaks $SU(2) \times U(1) \to U(1)_{\text{EM}}$ via $\langle \phi \rangle \simeq 250\,\mathrm{GeV}$.
- About 18 free parameters, all measured as of 2013 (Higgs mass).

Problems of the Standard Model + Big Bang

Actual problems that require (particle physics) extensions of SM:[6]

- Neutrinos have mass.
- Dark matter.

Finetuning problems:

- Matter–antimatter asymmetry of our Universe.
- Strong CP problem:
  Why no term $\theta_{\text{CP}} G_{\mu\nu} \tilde{G}^{\mu\nu}$ on coffee mug?
- Hierarchy problem:
  Why $\langle\phi\rangle, m_{\text{Higgs}} \simeq 100\,\text{GeV} \ll M_{\text{Planck}} \simeq 10^{19}\,\text{GeV}$?
- Flavor structure, charge quantization, ...

and of course: How to include/quantize gravity?

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[6] Warning: the views expressed are those of the presenter and do not necessarily represent those of the MPG, the MPIK, or any other physicist.

Problems of the Standard Model + Inflation

Basis of this talk: **Assuming inflation is confirmed.**

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[7] Fairbairn, Hogan, PRL 112, arXiv:1403.6786.

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Matter–Antimatter Asymmetry

- You, Me, Earth, Sun made of *matter*:
  baryons (protons & neutrons) and electrons.
- Rest of (visible) Universe (probably) too.[8]
- Measurements from CMB ($T \sim 1\,\mathrm{eV}$) and during Big Bang Nucleosynthesis ($T \sim 1\,\mathrm{MeV}$) give same value:

$$\left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{photons}}}\right|_{\text{today}} \simeq 6 \times 10^{-10} \sim \left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{baryons}} + n_{\text{antibaryons}}}\right|_{T \gtrsim \mathrm{GeV}}$$

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[8] Shaposhnikov et al., arXiv:1204.4186.

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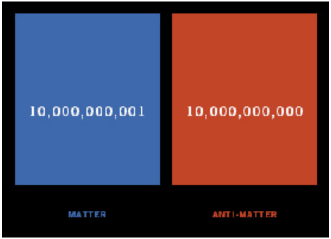

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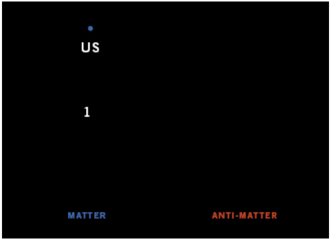

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Why? Particle physics (pretty) symmetric in particle ↔ antiparticle. . .

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[8] Shaposhnikov et al., arXiv:1204.4186.

Matter–Antimatter Asymmetry II

$$\left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{photons}}}\right|_{\text{today}} \simeq 6 \times 10^{-10} \sim \left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{baryons}} + n_{\text{antibaryons}}}\right|_{T \gtrsim \mathrm{GeV}}$$

In Big Bang theory:

- Yet another finetuned initial condition.

With inflation:

- Impossible as initial condition![9]
  - Initial baryonic charge density diluted by inflation by $(e^{60})^3$.

    ⇒ Need *really* large inital charge.

    ⇒ Gives large energy density ρ_{baryons}, dominates over ρ_{infl}.

    ⇒ No (sufficiently long) inflation. ☹

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[9] Dolgov, Phys.Rept. 222 (1992) 309-386.

Dynamical Matter–Antimatter Asymmetry

$$\left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{photons}}}\right|_{\text{today}} \simeq 6 \times 10^{-10} \sim \left.\frac{n_{\text{baryons}} - n_{\text{antibaryons}}}{n_{\text{baryons}} + n_{\text{antibaryons}}}\right|_{T \gtrsim \mathrm{GeV}}$$

With inflation:

- Has to be generated *dynamically* à la Sakharov. Requires:
  - Baryon number violation.
  - Violation of charge conjugation C and charge–parity CP.
  - Out-of-equilibrium processes.
- Qualitatively fulfilled in Standard Model + expanding Universe, but *quantitatively* way too small.

With inflation, we also need new physics to explain baryon asymmetry!

"Typical" model:

- Heavy $M \gtrsim 10^9\,\mathrm{GeV}$ new particle decays slowly (compared to expansion of Universe) into SM particles, CP/C violated in loops.
- Usually requires Universe to have been hotter than $\sim 10^9\,\mathrm{GeV}$, fits nicely to high inflation/reheating scale.

The Strong *CP* Problem

We know *CP* is violated in weak interactions, so why not write down

$$\mathcal{L} \supset \frac{\theta}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \,,$$

with $SU(3)$ field-strength tensor $G_{\mu\nu}$?

- Measurements of neutron's electric dipole moment: $|\theta| < 10^{-11}$.

One possible explanation: Peccei–Quinn symmetry $U(1)_{\mathrm{PQ}}$:

- Promote θ to a field that relaxes to $\langle\theta\rangle = 0$.
- Light field (axion) can be *cold* dark matter.

The Strong *CP* Problem II

- More accurately, QCD allows

$$\mathcal{L}_{\text{QCD}} \supset \mathrm{Re}(m_q e^{i\phi})\,\overline{q}q + \mathrm{Im}(m_q e^{i\phi})\,\overline{q}\gamma_5 q + \frac{\theta}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}\,.$$

- Get rid of stupid γ_5 mass term via $q \to e^{-i\gamma_5\phi/2} q$.

BUT:

- Path integral measure *not* invariant under $q \to e^{-i\gamma_5\phi/2} q$! Anomaly effectively shifts

$$\mathcal{L}_{\text{QCD}} \to\; m\,\overline{q}q + + \frac{\theta+\phi}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}\,.$$

- The physical parameter is $\overline{\theta} \equiv \theta + \arg\det M_q$. Why small?

Peccei–Quinn symmetry $U(1)_{\text{PQ}}$

Kim–Shifman–Vainshtein–Zakharov model:

- Introduce new $SU(3)_C$-charged "quarks" Q and SM-singlet complex scalar σ

$$\mathcal{L} = y\,\overline{Q}Q\,\sigma - V(\sigma, H) + \frac{\overline{\theta}}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu} + \mathcal{L}_{\text{QCD}}\,,$$

  with $U(1)$ symmetry $Q \to e^{-i\alpha\gamma_5/2}$, $\sigma \to e^{i\alpha}$.
  Rest of SM uncharged.

- Mexican-hat potential for σ: VEV $f_a \gg 100\,\mathrm{GeV}$:

$$\sigma = (f_a + \tilde{\sigma}(x))\, e^{ia(x)/f_a}\,.$$

- Higgs-like real scalar $\tilde{\sigma}$ is heavy (and ignored).
- Make heavy mass of Q real:

$$\overline{\theta}_{\text{eff}} = \overline{\theta} + \frac{1}{f_a} a(x)\,.$$

- Strong *CP* angle dynamical!

The Axion

- Pseudoscalar axion $a(x)$ *would be* massless Goldstone if the global $U(1)$ were real symmetry.
- $U(1)_{\text{PQ}}$ is anomalous (that is the whole problem) ⇒ Goldstone theorem not applicable.
- QCD instantons generate potential for pseudo-Goldstone axion a:

$$V(a) = m_\pi^2 f_\pi^2 \left[1 - \cos\left(\overline{\theta} + a/f_a\right)\right].$$

$\Rightarrow$ Physical $\overline{\theta}_{\text{eff}}$ relaxes to zero dynamically! Strong CP solved.

- Axion mass

$$m_a \simeq \frac{m_\pi f_\pi}{f_a} \simeq 10^{-5}\,\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right).$$

- Axion couplings all suppressed by PQ-breaking scale f_a:

$$\mathcal{L}_a \supset -\left(\frac{\partial_\mu a}{f_a}\right)\overline{q}\gamma_\mu\gamma_5 q + \frac{g^2}{32\pi^2}\left(\frac{a}{f_a}\right) G_{\mu\nu}\tilde{G}^{\mu\nu}.$$

- Astrophysics: $10^9\,\mathrm{GeV} \lesssim f_a \lesssim 10^{17}\,\mathrm{GeV}$.

The Axion in Cosmology

- At $T \gg \Lambda_{\text{QCD}} \simeq 200\,\text{MeV}$: axion potential is flat, massless axion frozen.
  $\Rightarrow$ Vacuum-misalignment angle $\theta_i \equiv a_i/f_a$.
- At

  $$T_f \simeq 620\,\text{MeV} \left(\frac{10^{12}\,\text{GeV}}{f_a}\right)^{1/6}$$

  we have $3H(T_f) = m_a(T_f)$ and the axion starts to oscillate around zero:

  $$\frac{\ddot{a}}{f_a} + 3H(T)\,\dot{a} + m_a^2(T)\,a = 0\,.$$

  $\Rightarrow$ coherent oscillations with $n_a \propto a_{\text{scale factor}}^{-3}$: cold dark matter.

Two possibilities: Peccei–Quinn phase transition before inflation ($f_a > H_{\text{inflation}} \simeq 10^{14}\,\text{GeV}$) or after ($f_a < H_{\text{inflation}}$).

Peccei–Quinn Phase Transition before Inflation

$f_a > H_\text{inflation}/2\pi$:

- Vacuum-misalignment angle $\theta_i \equiv a_i/f_a$ identical in whole Universe.
- Quantum fluctuations of massless axion $\sqrt{\langle\delta_a^2\rangle} = H_\text{inflation}/2\pi$ generate isocurvature power spectrum

$$\Delta_a(k) = \frac{H^2_\text{inflation}}{\pi^2}\theta_i^2 f_a^2\,.$$

- Same θ_i sets dark matter abundance...
- PLANCK constrains isocurvature:[10]

$$\frac{\Omega_\text{axion}}{\Omega_\text{dark matter}} \lesssim 4\times 10^{-12}\left(\frac{f_a}{10^{16}\,\mathrm{GeV}}\right)^{5/6}\left(\frac{0.2}{r}\right)$$

High-f_a axion ruled out as dark matter!

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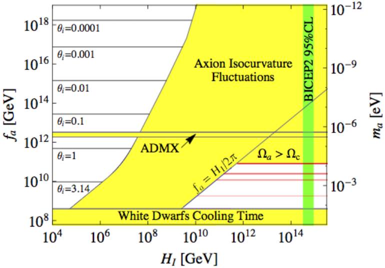
[10] Marsh et al., PRL 113, arXiv:1403.4216.

Peccei–Quinn Phase Transition after Inflation

$f_a < H_\text{inflation}/2\pi$:[11]

- No isocurvature modes.
- Axion potential with different initial θ_i; require average over Hubble volume: $\langle\theta_i^2\rangle \simeq 2.67\,\pi^2/3$.
- Abundance from misalignment: $\Omega_a^\text{mis} h^2 \simeq 2(f_a/10^{12}\,\mathrm{GeV})^{7/6}$.
- Axionic strings not inflation-diluted; contribute to abundance.[12]



QCD-axions can still be dark matter for $m_a = 71 \pm 2\,\mu\mathrm{eV}\,(\alpha^\text{dec}+1)^{6/7}$.

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[11] Visinelli & Gondolo, PRL 113, arXiv:1403.4594.

[12] But impossible to calculate. . . Relative contribution α^dec between 0.16 and 186.

Summary

- Inflation solves *and creates* some problems.
- Experimental observation of high inflation scale has impact on particle physics:
  - Baryon asymmetry *definitely* of dynamical origin.
  - Hierarchy problem more real (still just finetuning).
  - Axion solution to strong CP problem narrowed down.
  - (Axion-like particles are constrained, too.)
  - Vaccum stability (potentially) changes.
  - . . .
- Also impact on cosmology of course. (Branimir's talk)

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