Introduction to Inflation

Miguel Campos

MPI für Kernphysik
&
Heidelberg Universität

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The Cosmological Principle

“When viewed on a large enough scale, the distribution of matter in the universe is homogeneous and isotropic”.
This premise is reflected in the Friedmann-Robertson-Walker metric, which is the starting point of standard cosmology:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{1}$$

When used in the Einstein’s field equation we obtain:

$$\begin{aligned} H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \\ \left(\frac{\ddot{a}}{a}\right) &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned} \tag{2}$$

Some remarks

- Ordinary matter obeys the strong energy condition:

$$\rho + 3p \geq 0 \tag{3}$$

- The Friedmann equations for normal matter imply $\ddot{a} < 0$ and then the existence of a singularity in the finite past:

$$a(t \equiv 0) = 0.$$

- During conventional Big bang expansion the *comoving Hubble radius* $(aH)^{-1}$ grows monotonically with time.

Flatness

If we consider the Friedmann equation for a homogeneous and isotropic spacetime

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{4}$$

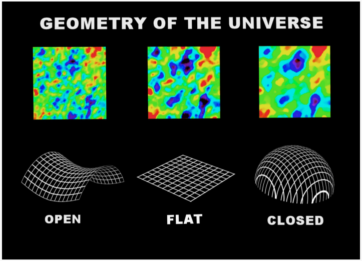
and define the density parameter as:

$$\Omega(a) \equiv \frac{8\pi G\rho}{3H^2} \tag{5}$$

we find that

$$\Omega(a) - 1 = \frac{-k}{(aH)^2} \tag{6}$$

$\Omega \to 1 \Rightarrow$ Flat space.



Horizons

Defining the *particle horizon*, which is the distance travelled by light since the beginning of the universe, as

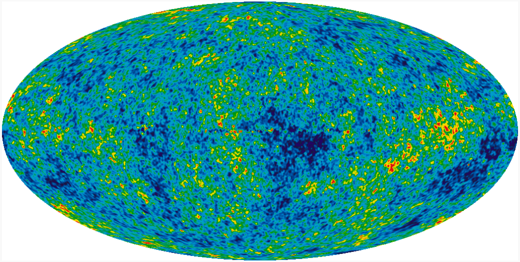
$$d_H(t) \equiv a(t) \int_{t_*}^{t} \frac{dt'}{a(t')} \tag{7}$$

for a radiation-dominated universe we have that

$$a \propto t^{1/2} \Rightarrow d_H \propto t \tag{8}$$

The horizon grows monotonically with time which implies that comoving scales entering the horizon today have been far outside the horizon at CMB decoupling.

According to standard cosmology, distant points in the sky have never communicated. How can the temperatures of the cosmic background in these points be so similar (identical to a part in a hundred thousand)?



Monopoles

In Grand Unified Theories a simple symmetry group is spontaneously broken at a high energy ($\approx 10^{16}$ GeV) to the gauge symmetry of the SM. The scalar fields that break the symmetry can be left in configurations that carry non-zero magnetic charge and that can not be smoothed out through any continuous processes.
Quantitatively, at 10^{16} GeV the ratio of monopoles to photons is roughly 10^{-9}. If monopoles did not find each other to annihilate (finite horizon size) this ratio would remain constant to the present, but with at least 10^{9} microwave photons per nucleon today, this would give one monopole per nucleon.

Driving forces

As we established, the expansion of the universe can only decelerate for ordinary matter or radiation.

In this sense it is hard to see how the big-bang ever happened. In the standard cosmology the initial expansion and high temperature is just assumed.

Inflation: Solutions

We can express the *comoving horizon* τ in function of the comoving Hubble radius:

$$\tau = \int_{t_*}^{t} \frac{dt'}{a(t')} = \int_{a_*}^{a} d\ln a' \left(\frac{1}{a' H} \right) . \tag{9}$$

The fundamental difference between these two quantities is that if particles are separated by distances greater than τ, they *never* were is causal contact, but if they are separated by distances greater than $(aH)^{-1}$ they are not in contact *now*.

We can solve the problems of standard cosmology if we assume that the Hubble radius experienced an early stage of rapid shrinking. This implies that the scale factor of the universe *inflates*.

Flatness solution.
Considering again:

$$|1-\Omega| = \frac{1}{(aH)^2} \tag{10}$$

we see that the decreasing of the Hubble radius automatically brings Ω to unity, hence the universe towards flatness.

Horizons solution.
A period of shrinking Hubble radius implies that in the very early universe large scales could have been causally connected, generating the nearly thermal homogeneity that we observe in the CMB.
During inflation ($H \approx$const.), the scale factor is:

$$a(\tau) = -\frac{1}{H\tau} \tag{11}$$

and the singularity is pushed back to $-\infty$.

Driving forces solution.
In the period of inflation the universe is dominated by a field with an anomalous equation of state:

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \Rightarrow \rho + 3p < 0 \tag{12}$$

this field, which violates the strong energy condition, drives the accelerated expansion of the universe.

Monopoles solution.

An exponential expansion occurring before the production of monopoles would have greatly reduced the monopole to photon ratio.

In particular, the searches of monopoles establish that there are fewer than 10^{-39} monopoles per photon. In order for inflation to reduce the ratio by a factor of 10^{-30} it must have increased the horizon by a factor 10^{10}, which is equivalent to a number of e-foldings of 23.

Inflation: Conditions

Starting from:

$$\frac{d}{dt}(aH)^{-1} = \frac{-\ddot{a}}{(aH)^2} \tag{13}$$

we see that a shrinking comoving Hubble radius implies an accelerated expansion:

$$\frac{d^2a}{dt^2} > 0 \tag{14}$$

Through

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon) \quad \text{with} \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \tag{15}$$

we see that acceleration corresponds to $\epsilon < 1$. We can also express this quantity as

$$\epsilon = -\frac{d\ln H}{d\mathcal{N}} \tag{16}$$

Inflation ends when $\epsilon \approx 1$.

$\mathcal{N}$ is the number of e-folds of inflationary expansion. We can define it formally as:

$$\mathcal{N} \equiv \int_t^{t_f} H dt. \tag{17}$$

In order to solve the flatness problem Ω is required to be $|\Omega_f - 1| \lesssim 10^{-60}$ after the end of inflation. Meanwhile the ratio at the beginning and the end of this process is

$$\frac{|\Omega_f - 1|}{|\Omega_i - 1|} \simeq \left(\frac{a_i}{a_f}\right)^2 = e^{-2\mathcal{N}_i} \tag{18}$$

where we have used the fact that H is nearly constant during inflation. Assuming that $|\Omega_i - 1|$ is of order unity the number of e-foldings is required to be $\mathcal{N} \gtrsim 60$ to solve the flatness problem.

Guth's Old Theory

Inhomogeneous bubbles

In Guth's original work [1981], a scalar field was initially trapped in a local minimum of some potential, and then leaked through the potential barrier and rolled toward a true minimum of the potential.

It was soon realized that this idea did not work.

The transition could not have occurred everywhere simultaneously, but here and there in small inhomogeneous bubbles of true vacuum. At first Guth thought the bubbles would have merged, leading to our homogeneous universe, but they would have moved too fast away from each other to ever have fused.

Linde et al's New Theory

Slow-roll Inflation

In the new version due to Linde and to Albrecht and Steinhardt [1982] there is a scalar field φ, known as the *inflaton* which at some early time takes a value at which the potential $V(\varphi)$ is large but quite flat. The scalar field *rolls very slowly* and the universe experiences an exponential inflation before the field changes very much. Eventually the field energy is converted into ordinary particles filling the bubble. This stage occurring right after inflation is usually called *reheating*.

Inflationary Dynamics

Let us consider a homogeneous single scalar field φ whose potential energy can lead to the accelerated expansion of the universe. The energy density and the pressure of the inflaton are given by

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{19}$$

Then, from the condition of acceleration $\ddot{a} > 0$ we obtain:

$$\dot{\varphi}^2 < V(\varphi) \tag{20}$$

the potential energy of the inflaton dominates over the kinetic energy

If we use the condition $\dot{\varphi}^2 \ll V(\varphi)$ we obtain the condition for a *de Sitter space*:

$$p \approx -\rho \tag{21}$$

Replacing this into the continuity equation:

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0 \tag{22}$$

$$\begin{aligned} \Rightarrow \dot{\rho} &\approx 0 \\ \rho &= \text{constant} \end{aligned} \tag{23}$$

Free lunch!

This implies that the energy density remain nearly constant as the universe expands. Unlike ordinary matter or radiation, scalar field potential energy is neither diluted nor redshifted by cosmic expansion: the inflationary *free lunch*.

Finally, using this fact in the Friedmann equation:

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right) &\propto \rho = \text{constant} \\ &\Rightarrow a(t) \propto e^{Ht}\end{aligned} \tag{24}$$

and we obtain an *exponentially* accelerated expansion.

New puzzles

What is φ?

So far, no clear candidate has emerged. The very simplest models are compatible with observation, but the field φ is *ad hoc* and introduced mainly to drive inflation.

Hundreds of models have been proposed satisfying particle physics and cosmological constraints but none of them is quantum-mecanically consistent, due to the problem of infinities in quantum gravity.

Why did it start out up the hill?

In the simplest approach to inflation, one just assumes the desired result by starting the field up the hill, but when one allows the initial conditions to be really random, huge spatial gradients occur, leading to inhomogeneous initial conditions.

Higher order corrections.

The calculations of quantum fluctuations in inflation yield approximate scale invariance, as required by the data. However these corrections are performed in the lowest order in perturbation theory. At the first non-trivial order, non-renormalizable divergences appear.

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