Introduction to Baryogenesis and Leptogenesis

prepared by K.S.Lim
MPIK Division for Particle and Astroparticle Physics
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1 Introduction

- Since antiparticles were first predicted and observed, it has been clear that there exist high degree of particle-antiparticle symmetry. This observation is a stark contradiction to the phenomena of everyday and cosmological evidence, particularly our universe consists of almost entirely matter with little primordial antimatter.
- The evidence of universe devoid of antimatter comes in different scale. Universe consists of matter up to Hubble-scale.
- Quantitative estimate of relative abundance of baryonic matter and antimatter may be obtained from standard cosmology.
- Primordial nucleosynthesis allows accurate predictions of cosmological abundances of all light elements with only one input parameter, the baryon to entropy ratio:

$$\eta = \frac{n_b - n_{\bar{b}}}{s} \tag{1}$$

  and η is measured to be $1.5 \times 10^{-10} < \eta < 7 \times 10^{-10}$
- While the success of standard cosmology is encouraging, there remains the question of initial conditions. The values required for many parameters are extremely unnatural in the sense that tiniest deviation will lead to a universe different from what we observe.
- The generation of observed value of η in this context is refered to as baryogenesis. Three necessary conditions for generation of baryonic asymmetry were identified and known as the Sakharov criteria:
  - Violation of baryon number symmetry
  - Violation of C and CP
  - Departure from thermal equilibrium

2 Vacuum structure of $SU(2)$

- Consider the Yang-Mills $SU(2)$ gauge theory.

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \tag{2}$$

- We demand that the boundary condition for pure Yang-Mills field is $F_{\mu\nu}(x) \to 0$, $\forall x \in S^3$

- For some open connected neighbourhood of x, $A_\mu(x)$ is a pure gauge

$$A_\mu(x) = (\partial_\mu U(x))\, U^{-1}(x) \tag{3}$$

A vacuum state can be represented by the matrix $U(x) \in SU(2)$, $\forall x \in S^3$. Since $SU(2) \cong S^3$, $U(x)$ can be considered as a map from 3-sphere to an element in $SU(2)$ with a suitable boundary condition.

- The vacuum state can be classified as $\Pi_3(S^3) = \mathbb{Z}$. The degenerate vacuum state are physically equivalent but topologically distinct. Topologically the vacuum state can be divided into different homotopy classes, classified by integer winding number:

$$n(U) = \frac{1}{24\pi^2} \oint d\sigma_\mu\, \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left[(\partial_\nu U)U^{-1}(\partial_\rho U)U^{-1}(\partial_\sigma U)U^{-1}\right] \tag{4}$$

Note that this definition is group measure invariant.

- Generally one can assign the topological charge Q of configuration to a four dimension field configuration with finite Euclidean action.

$$\begin{aligned} Q &= -\frac{1}{16\pi^2} \int d^4x\, \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) && (5) \\ &= -\frac{1}{16\pi^2} \int d^4x\, \partial_\mu K^\mu && (6) \end{aligned}$$

with $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ and K^μ defined as $K^\mu = -2\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(A^\nu\partial^\rho A^\sigma + \frac{2}{3}A^\nu A^\rho A^\sigma)$.

- The Chern-Simon number is defined by

$$\begin{aligned} N_{CS} &= -\frac{1}{16\pi^2} \int d^3x\, K^0 && (7) \\ &= -\frac{1}{8\pi^2} \int d^3x\, \epsilon^{ijk}\mathrm{tr}(A^i\partial^j A^k + \frac{2}{3}A^i A^j A^k) && (8) \end{aligned}$$

By doing a large gauge transformation, we see that N_{CS} is changed by a unit of winding number.

- Between 2 neighbouring vacuum state, there must be an energy barrier, since they cannot be transformed into one another without passing through non-vacuum state. Transition from one vacuum state to another can happen due to quantum tunneling. In this sense N_{CS} can be regarded as a parameter for different vacuum state.

$$\begin{aligned} Q(t) &= \frac{1}{16\pi^2} \int_0^t dt \int d^3x\, \partial_\mu K^\mu && (9) \\ &= N_{CS}(t) - N_{CS}(0) && (10) \end{aligned}$$

Notice that Chern-Simon number is not gauge invariant, but the topological charge is. The solution to this field configuration is called the instanton.

3 Chiral anomaly and baryon/lepton number violation

- Consider the lagrangian for massless Dirac field, coupled to $U(1)$ gauge field.

$$\mathcal{L} = \overline{\Psi} \not{D} \Psi - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} \tag{11}$$

  This lagrangian is invariant under local $U(1)$ gauge transformation. However, it is also invariant under the axial $U(1)$ transformation.

$$\Psi(x) \to e^{-i\alpha\gamma_5} \Psi(x) \tag{12}$$

  The associated axial current should be conserved according to Noether theorem. But it has an anomalous divergence:

$$\partial_\mu j^\mu_A = -\frac{1}{16\pi^2} \mathrm{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) \tag{13}$$

  This chiral anomaly arises from the fact that the integral measure $\mathcal{D}\Psi\mathcal{D}\overline{\Psi}$ is not invariant under chiral transformation.

- If we couple the fermions to $SU(2)$ gauge theory ala Standard Model, we would obtain the anomalous divergence for baryonic and leptonic currents. $B-L$ is conserved in this case. Since $\Delta L = \Delta B = N_f(N_{CS}(t_f) - N_{CS}(t_i))$, we expect a contribution of 12 fermions lead by the $SU(2)$ instanton for $\Delta N_{CS} = 1$.

- At zero temperature, B and L are violated respectively by the tunneling of instanton. However, the tunneling rate is heavily surpressed, $\Gamma(T=0) \approx e^{-2S_E} \approx 10^{-170}$.

- By coupling the field theory to thermal bath, one can obtain a higher rate of anomalous process through sphaleron. Note that as sphaleron conserves $B-L$, phase transition at equilibrium will destroy any $B+L$ should $B-L$ was zero before the phase transition, thus washing out all the baryon asymmetry.

- Sphaleron is a classical static solution of field configuration, which is a saddle point in energy functional. The energy of sphaleron E_{sp} is given as the infimum of the maximum energies for loops in a non-contractible homotopy class H, and it possesses topological charge of $\frac{1}{2}$.

- For temperature greater than the temperature of electroweak phase transition, the rate of baryon number violating events can be very profound, and it has been given as

$$\frac{\Gamma_{B+L}}{V} \approx \alpha_W^5 T^4 \ln(1/\alpha_W) \tag{14}$$

4 Relation between Baryon and Lepton Asymmetries

- In a weakly coupled plasma, one can assign a chemical potential μ to each of the quark, lepton and Higgs fields. For a non-interacting gas of massless particles the asymmetry in particle and antiparticle number densities is given by:

$$n_i - \bar{n}_i = \frac{gT^3}{6} \begin{cases} \beta\mu_i + \mathcal{O}((\beta\mu_i)^3) & \text{, fermions} \\ 2\beta\mu_i + \mathcal{O}((\beta\mu_i)^3) & \text{, bosons} \end{cases} \tag{15}$$

- The effective 12 fermions interaction induced by $SU(2)$ instanton implies:

$$\sum_i (3\mu_{q_i} + \mu_{l_i}) = 0 \tag{16}$$

One has to take $SU(3)$ QCD instanton process into account, which generate an effective interaction between left-handed and right-handed quark.

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0 \tag{17}$$

- The third condition, valid at all temperatures, arises from requirement that the total hypercharge of the plasma vanishes.

$$\sum_i (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{l_i} - \mu_{e_i} + \frac{2}{N_f}\mu_H) = 0 \tag{18}$$

The Yukawa interactions, supplemented by gauge interactions, yield relations between chemical potentials of left-handed and right-handed fermions. We assume equilibrium between different type of generations, $\mu_{q_i} = \mu_q$, $\mu_{l_i} = \mu_l$.

$$\mu_q - \mu_H - \mu_d = 0\,, \qquad \mu_q + \mu_H - \mu_u = 0\,, \qquad \mu_l - \mu_H - \mu_e = 0 \tag{19}$$

- And finally using the relation of B and L we get:

$$B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i}) \tag{20}$$

$$L_i = 2\mu_{l_i} + \mu_{e_i}\,, \qquad L = \sum_i L_i \tag{21}$$

Solving B and L under the constrains given above, one obtains

$$B = \frac{8N_f + 4}{22N_f + 13}(B - L)\,, \quad L = \left(\frac{8N_f + 4}{22N_f + 13} - 1\right)(B - L) \tag{22}$$

The value of $B - L$ at time t_f, where Leptogenesis process is completed, determines the value of the baryon asymmetry today.

$$B(t_0) = \frac{8N_f + 4}{22N_f + 13}(B - L)(t_f) \tag{23}$$

5 Recipe for Leptogenesis

- First one relies on a leptonic-violating process, such as adding a right-handed Majorana particle and letting it decay before the electroweak phase transition. In this sense, we violate $B - L$.
- Washout of lepton asymmetry occurs. Unfortunately this unwanted process happens and one needs to solve the Boltzmann equation to determine the decay and inverse decay of right-handed neutrinos.
- Conversion of remaining lepton asymmetry to baryonic asymmetry due to sphaleron.

6 Departure from Thermal Equilibrium

- I only discuss the scenario of out-of-equilibrium decay mechanism as the condition for departure from thermal equilibrium.
- Consider the scenario when the decay rate Γ_X of superheavy particle X at time they become nonrelativistic is much smaller than the expansion rate of universe, then the X particles cannot decay on the time scale of expansion and so they remain abundant as photon for $T \leq M_X$. Therefore they populate the universe with an abundance which is larger than the equilibrium one.
- This overabundance is the departure from thermal equilibrium needed to produce a final nonvanishing baryon asymmetry when X undergoes B and $CP-$ violating decays.

References

[1] W. Buchmuller, R.D. Peccei, and T. Yanagida. Leptogenesis as the origin of matter. *Ann.Rev.Nucl.Part.Sci.*, 55:311–355, 2005.

[2] Ta-Pei Cheng and Ling-Fong Li. *Gauge Theory of elementary particle physics*. Oxford University Press, USA, 1988.

[3] Frère. Introduction to Baryo- and Leptogenesis. *Surveys in High Energy Physics*, 20(1):59–87, 2006.

[4] M. Fukugita and T. Yanagida. Baryogenesis Without Grand Unification. *Phys.Lett.*, B174:45, 1986.

[5] Frans R. Klinkhamer and N.S. Manton. A Saddle Point Solution in the Weinberg-Salam Theory. *Phys.Rev.*, D30:2212, 1984.

[6] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov. On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe. *Phys.Lett.*, B155:36, 1985.

[7] N.S. Manton. Topology in the Weinberg-Salam Theory. *Phys.Rev.*, D28:2019, 1983.

[8] R. Rajaraman. *Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory*. North Holland, 2nd repr. 1989 edition, 1987.

[9] S. Skadhauge. Sphalerons and electroweak baryogenesis. 1996. Hardcopy at DESY.

[10] Mark Trodden. Electroweak baryogenesis. *Rev.Mod.Phys.*, 71:1463–1500, 1999.

[11] Mark Trodden. Baryogenesis and leptogenesis. page L018, 2004.