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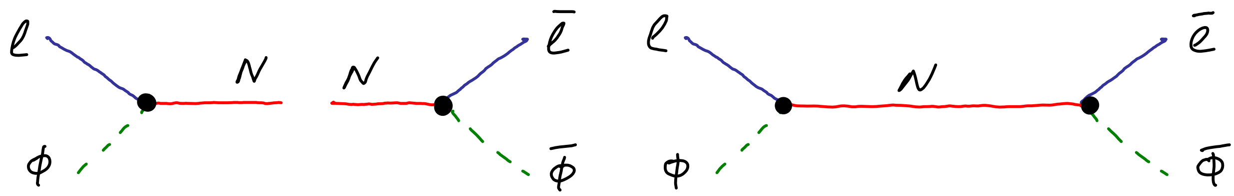
Advanced Topics in Leptogenesis

Notiztitel

1. Lepton number violating processes

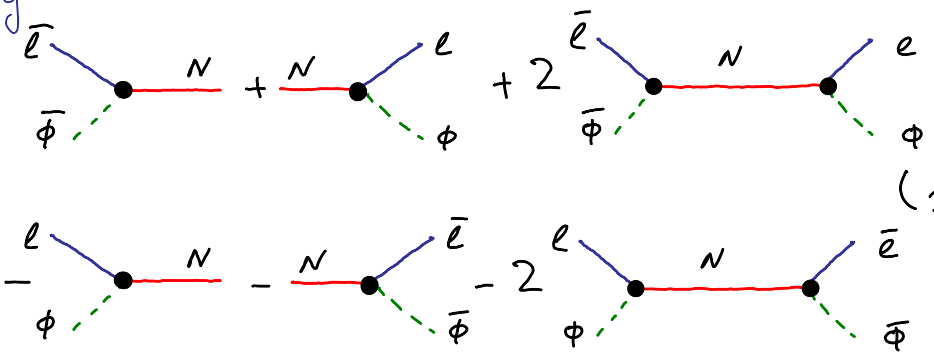
- The three most important processes which contribute to the generation and washout of the asymmetry are the decay and inverse decay of the heavy Majorana neutrino and two-body scattering mediated by the heavy neutrino.



- The inverse decay and decay process violate lepton number by one unit, whereas the scattering by two units. Schematically the equation for the asymmetry generation:

lepton asymmetry

$$\frac{1}{a^3}\frac{d}{dt}(a^3 L) =$$



(1)

scale factor, describes expansion of the Universe.

- Connecting the heavy neutrino lines in the first two diagrams we see, that an inverse decay immediately followed by decay is equivalent to two-body scattering process, where the intermediate state is on the mass shell. Therefore, if we use full amplitude for the scattering process, we will consider the same physical process – inverse decay immediately followed by decay – twice. This leads to generation of the asymmetry even in thermal equilibrium, which contradicts to one of the Sakharov conditions. This is reffered to as the double-counting problem of the canonical approach. It is solved by subtracting contribution of the real intermediate state from the scattering amplitude.

Problem 1

- It is important to note here, that the initial states are always 'weighted' by f, where f is the Bose-Einstein or Fermi-Dirac distribution function, and the final states are 'weighted' by $(1 \pm f)$. In the canonical approach RIS subtraction is only possible if we neglect the quantum statistical factors and replace the BE and FD distributions by the Boltzmann equation.

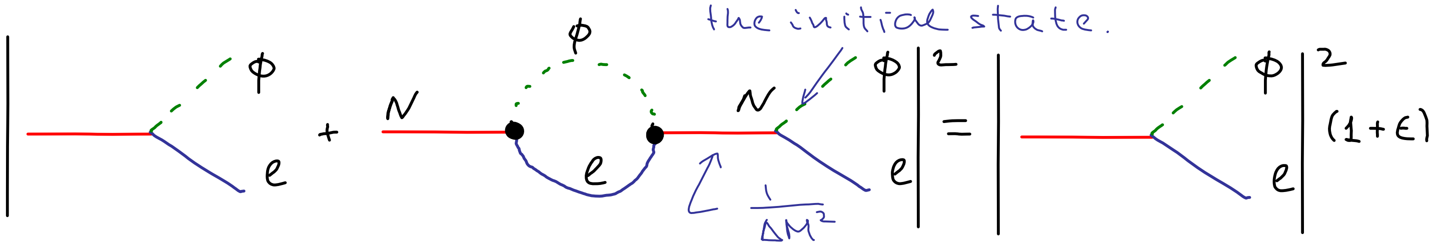
Problem 2

- Another question that arises here: can one assign a distribution function to the intermediate neutrino? After all, the scattering process takes place in a hot plasma.

- Note that in the canonical approach the self-energy is computed using vacuum Feynman rules and therefore does not take the medium effects into account. This problem is partially resolved by thermal field theory.

- At tree-level the above processes are lepton number violating, but CP-conserving. That is, although each individual process generates or washes out the asymmetry, on average number of the produced leptons is exactly equal to the number of the produced antileptons and no asymmetry is generated.

  To leading order in the Yukawa couplings CP-violation in the system is generated by one-loop self-energy diagram: propagator evaluated on the mass-shell of the initial state.



  Here ϵ is the CP-violating parameter. For the decay into the antiparticles it enters (by CPT theorem) with the oposite sign. The larger ϵ is, the more asymmetry can be produced.

- The ammount of the generated asymmetry is fixed by the observations. But the CP-violating parameter depends on many quantities which are not fixed by the experiment. This makes viable at least two options:

Weak washout

- ϵ is small

Strong washout

- ϵ is large

- the generated asymmetry is not washed out

- most of the generated asymmetry is washed out.

Efficiency of the washout processes depends on many factors and in particular on the rate of the Universe expansion $H \propto T^2/M_{Pl}$. This is easy to understand: if the Universe expands slowly, the lepton number violating processes are almost in equilibrium and in equilibrium the asymmetry must vanish. Dynamically this is ensured by the washout processes.

- Scale, at which the asymmetry is generated is essentially set by the mass of the RH neutrino. That is, the expansion rate $H \propto M_N^2/M_{Pl}$ and the choice of the regime is equivalent to the choice of the heavy neutrino mass scale.

- Are there any physical arguments to preffer one or the other regime? There are pro- and contra arguments:

a) large M_N make see-saw mechanism natural

b) in supersymmetric extensions of the SM high reheating temperatures lead to an overproduction of gravitinos and therefore low M are preferable.

c) light RH neutrino can potentially be produced in accelerator experiment.

d) In some models the Yukawa couplings are constrained by some symmetries and quasidegenerate mass spectrum is the only way to get the right asymmetry.

Problem 3

- The one-loop approximation is fine as long as the Majorana neutrino mass spectrum is strongly hierarchical, i.e. ΔM^2 is large. If this is not the case, we have to take into account also higher order contributions - resum loops.

- There is an immediate solution to this problem. Let us consider the sum

$$i\Delta(p) = \frac{i}{p^2-\mu_0^2+i\varepsilon} + \frac{i}{p^2-\mu_0^2+i\varepsilon}\left(-i\Sigma(p^2)\right)\frac{i}{p^2-\mu_0^2+i\varepsilon} + \ldots =$$

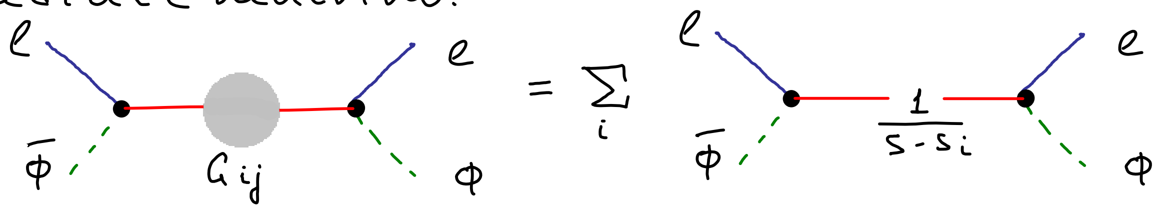
$$= \frac{i}{p^2-\mu_0^2+i\varepsilon}\left[\frac{1}{1+i\Sigma(p^2)\dfrac{i}{p^2-\mu_0^2+i\varepsilon}}\right] = \frac{i}{p^2-\mu_0^2-\Sigma(p^2)+i\varepsilon}. \quad (2)$$

As one can see, $\Delta(p)$ is a solution of Schwinger-Dyson equation:

$$\Delta^{-1}(p^2) = \Delta_0^{-1}(p^2) - \Sigma(p^2) \quad (3)$$

where Δ is a full propagator and Δ_0 is the tree-level propagator.

- How can we use it for calcullation of the CP-violating parameter? To this end we should consider the scattering process with on-shell intermediate neutrino.



In this case the total amplitude is a product of the inverse decay amplitude, propagator of the quasiparticle excitation and the decay amplitude. It is important that all components of the propagator have poles on the mass shell of the quasiparticle excitations, $G_{ij} = \sum_\alpha \frac{A^\alpha_{ij}}{s - s_\alpha}$ so that both the diagonal and off-diagonal components of the full propagator contribute.

- The expression for the CP-violating parameter which comes from the solution of the SD-equation reads:

$$\epsilon_i = -\frac{|g_j|^2}{16\pi} \mathrm{Im}\left(\frac{g_i g_j^*}{g_i^* g_j}\right) \frac{M_j^2 - M_i^2}{(M_j^2 - M_i^2)^2 + M_j^2 \Gamma_j^2} \quad (4)$$

Breit-Wigner propagator evaluated on the mass shell of the decaying particles

- Let us summarize at this point:

Problem	Canonical	Thermal	Kadanoff-Baym
- double counting	✓	✓	?
- resonance effects	✓	✗	?
- medium effects	✗	✓	?

2. Kadanoff-Baym equations

- The Schwinger-Dyson equation above depends only on the Lorentz-invariant momentum transfer squared p^2 and is valid only in vacuum. One could write it in the coordinate representation:

  vacuum is translationaly invariant.

$$\Delta^{-1}(x-y) = \Delta_0^{-1}(x-y) - \Sigma(x-y) \qquad (5)$$

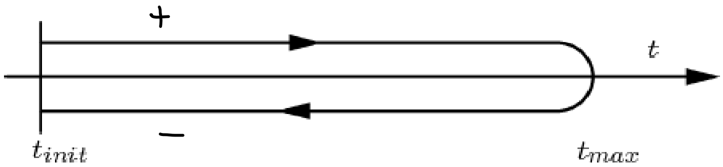
  Upon a Fourier transformation with respect to $x-y$ it reverts to the one above.

- Medium is not Lorentz-invariant, since there is a distinguished refference frame - the rest frame of the medium.

- A generalization of the Schwinger-Dyson equation must depend on two space-time arguments

$$\Delta^{-1}(x,y) = \Delta_0^{-1}(x,y) - \Sigma(x,y) \qquad (6)$$

  It turns out, that one more generalization is required. One has to introduce Keldysh-Schwinger contour



  and require that time components of x and y can lie on the positive or negative branch of the time contour.

- This equation is difficult to deal with. It is convenient to introduce spectral function and statistical propagator:

$$G_{ij}(x,y) = G_F^{ij}(x,y) - \tfrac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) G_\rho^{ij}(x,y) \tag{7}$$

- Upon substitution to the SD equation we obtain a system of KB equations for the spectral function and statistical propagator:

$$[\Box_x + M_i^2] G_F^{ij}(x,y) = \int_0^{y^0} \mathscr{D}^4 z \, \Pi_F^{ik}(x,z) G_\rho^{kj}(z,y) - \int_0^{x^0} \mathscr{D}^4 z \, \Pi_\rho^{ik}(x,z) G_F^{kj}(z,y) \,, \tag{8a}$$

$$[\Box_x + M_i^2] G_\rho^{ij}(x,y) = \int_{x^0}^{y^0} \mathscr{D}^4 z \, \Pi_\rho^{ik}(x,z) G_\rho^{kj}(z,y) \,. \tag{8b}$$

- Let us now clarify, what spectral function and statistical propagator are. To this end let us consider single weakly interacting field in equilibrium. Solution of the KB equation in this case:

$$G_\rho = (2\pi)\,\mathrm{sign}(p^0)\,\delta(p^2 - m^2) \tag{9a}$$

$$G_F = [1/2 + f(x,p)]\,G_\rho \tag{9b}$$

where f is the Bose-Einstein distribution function. For weakly interacting systems close to equilibrium one can use these expressions as an Ansatz.

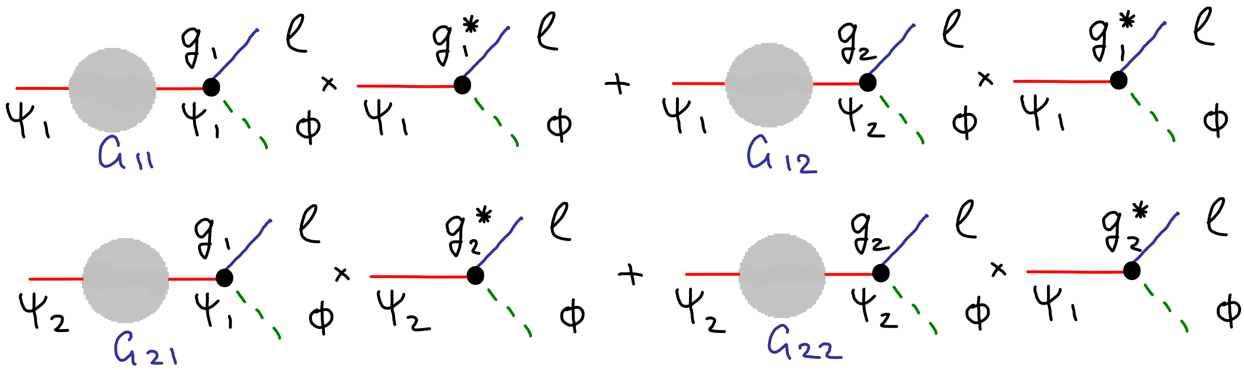
3. CP-violating parameter and the decay width

- To illustrate how this formalism can be applied to analyse the generation of the asymmetry let us consider a simple toy model of leptogenesis:

$$
\begin{aligned}
\mathcal{L} = &\ \frac{1}{2}\partial^\mu\psi_i\partial_\mu\psi_i - \frac{1}{2}M_i^2\psi_i\psi_i + \partial^\mu\bar{b}\partial_\mu b - m^2\bar{b}b \\
&- \frac{\lambda}{2!2!}(\bar{b}b)^2 - \frac{g_i}{2!}\psi_i bb - \frac{g_i^*}{2!}\psi_i\bar{b}\bar{b} + \mathcal{L}_{rest}\,, i = 1,2
\end{aligned}
\qquad (10)
$$

where the scalar fields ψ_i model the heavy Majorana neutrinos and the complex scalar b the leptons.

- The processes which contribute to generation and washout of the asymmetry are:



Their contribution to the RHS of the Boltzmann equation are:

'leptons'
$$
\Sigma_{\gtrless}(X,p) = -\int d\Pi_k d\Pi_q (2\pi)^4\delta^g(k-q-p) \times g_i^* g_j\, G^{ij}_{\gtrless}(X,k) D_{\lessgtr}(X,q)\,, \qquad (11a)
$$

'antileptons'
$$
\bar{\Sigma}_{\gtrless}(X,p) = -\int d\Pi_k d\Pi_q (2\pi)^4\delta^g(k-q-p) \times g_i g_j^*\, G^{ij}_{\gtrless}(X,k) \bar{D}_{\lessgtr}(X,q)\,, \qquad (11b)
$$

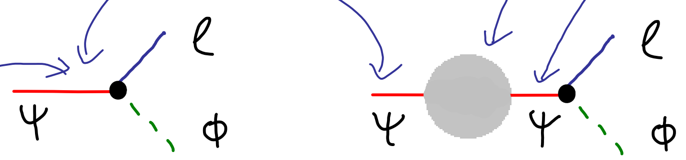
Here $G_<$ corresponds to the decay and $G_>$ to the inverse decay of the heavy neutrino.

- In equilibrium or very close to equilibrium the derivatives in the KB equations vanish and from an integro-differential equation it becomes an algebraic equation, which is relatively easy to solve. The solution reads:

decay → ← scattering

amplitude modulo squared →
$$\hat G_{\gtrless} = \frac{[\hat I - \hat{\mathcal G}_R \hat\Pi'_R][\hat{\mathcal G}_{\gtrless} - \hat{\mathcal G}_R \hat\Pi'_{\gtrless} \hat{\mathcal G}_A][\hat I - \hat\Pi'_A \hat{\mathcal G}_A]}{\det[\hat I - \hat{\mathcal G}_R\hat\Pi'_R]\det[\hat I - \hat\Pi'_A\hat{\mathcal G}_A]} \qquad (12)$$

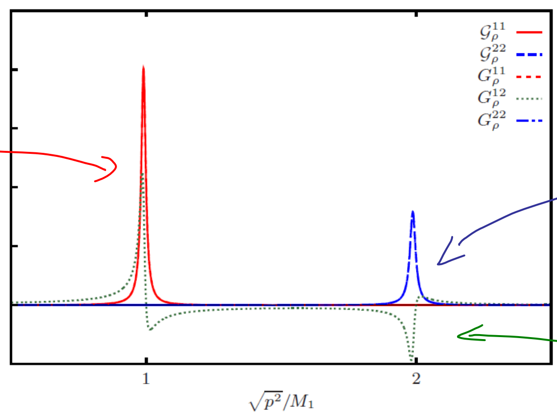
'tree-level' contribution.



Here $\hat{\mathcal G}_{\gtrless}$ describe on-shell initial (decay) and final (inverse decay) terms, and $\hat{\mathcal G}_R$ and $\hat{\mathcal G}_A$ describe the intermediate heavy Majorana neutrino. Finaly $\Pi_{R/A}$ are off-diagonal (in the flavor space) self-energies which correspond to to the self-energy loop.

- It is instructive to plot dependence of the propagator components on the momentum:

diagonal →



diagonal

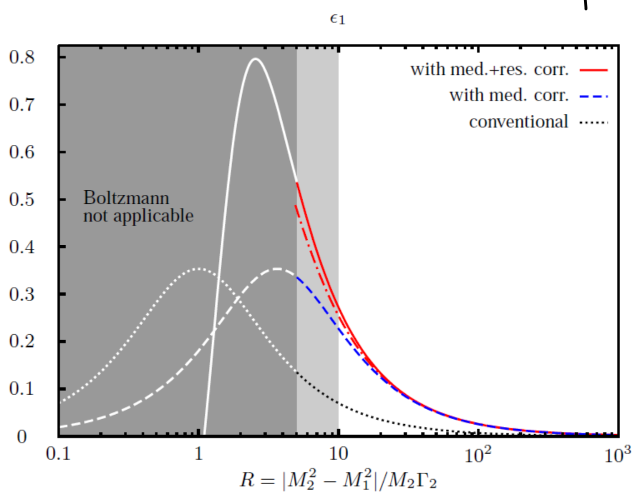
off-diagonal

As we see, the off-diagonals peak at the same positions as the diagonals. This allows us to describe the system in terms of two quasi-particle excitations with definite decay widths and CP-violating parameters. Note that is not always passible. If the diagonals overlap, then the off-shell effects play an important role and a more general treatment is needed.

- After substituting explicit expression for the self-energy we obtait the following result for the CP-violating parameter:

$$\epsilon_i = -\frac{|g_j|^2}{16\pi}\mathrm{Im}\left(\frac{g_i g_j^*}{g_i^* g_j}\right)\times \frac{\Delta_{ij}-\delta_{ij}}{c_{CP}^2[\Delta_{ij}-\delta_{ij}]^2 + s_{CP}^2\left[\Delta_{ij}^2 + (M_j\Gamma_j)^2 L_\rho^2\right]}\cdot L_\rho \quad (13)$$

which is more complicated than the cannonical one. Here is a numerical comparison:



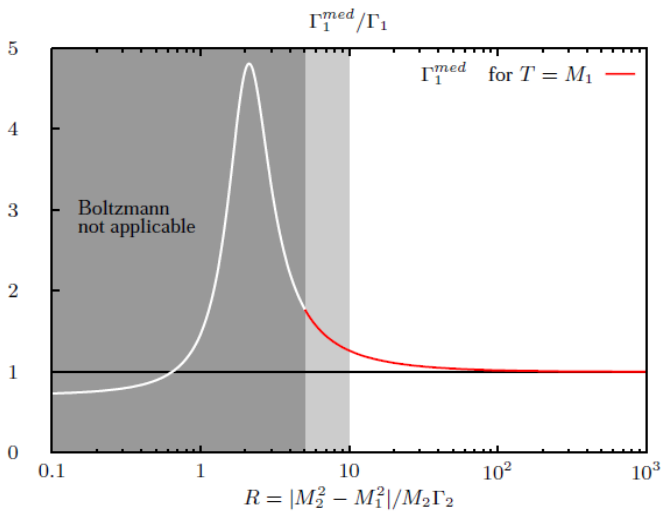
There is a clear difference between the conventional result and the result based on the KB approach.

- We have not yet considered the denominator of (12)

It modulates the overall 'strength' of the lepton-number violating processes and can therefore be interpreted as a correction to the in-medium decay width:

$$\Gamma_i^{med} = \Gamma_i \times \frac{c_{CP}^2[\Delta_{ij} - \delta_{ij}]^2 + s_{CP}^2\,[\Delta_{ij}^2 + (M_j\Gamma_j)^2\,L_\rho^2]}{[\Delta_{ij} - 2c_{CP}^2\delta_{ij}]^2 + [s_{CP}^2(M_j\Gamma_j L_\rho) + c_{CP}^2\delta_{ij}^2/(M_j\Gamma_j L_\rho)]^2}. \qquad (14)$$

Hier is the numerical result:



The enhancement of the decay width is moderate, but visible.

4. Summary

Problem	Canonical	Thermal	Kadanoff-Baym
- double counting	✓	✓	✓
- resonance effects	✓	✗	✓
- medium effects	✗	✓	✓
- enhancement of Γ	✓	✓	✓