Electroweak Baryogenesis

Outline

1) Introduction (+ Reminder)
2) Electroweak Phase Transition
3) Calculating the Baryon Asymmetry

1) Introduction

- $\eta = \frac{n_B}{s} \sim 10^{-10}$ one problem - many solutions
(show list Shaposhnikov 09)

- Fact: The (minimal) Standard Model automatically satisfies Sakharov conditions:

  - CP violation
    ↳ Complex phase in CKM (Alina)

  - Baryon Number Violation
    ↳ anomaly: $\partial_\mu J^\mu_B = \frac{g^2}{32\pi^2} \mathrm{Tr}(W\tilde{W})$ (Khoi-Shan)

  - Thermal Non-Equilibrium
    ↳ electroweak phase transition

2. Electroweak Phase Transition

2.1 The Nature of the Transition

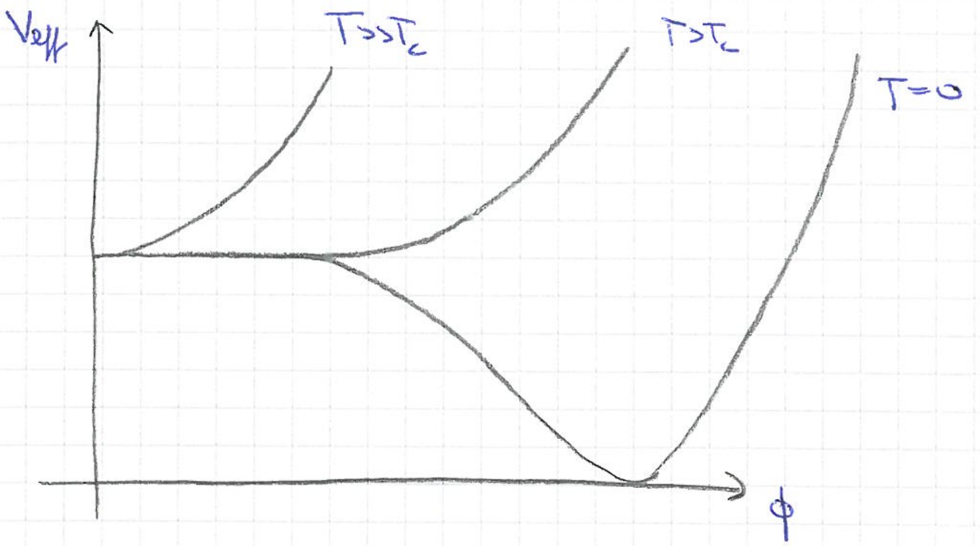
- @ $T \sim v \sim (100\,\text{GeV})$: $\Gamma_{\not{B}} \gg H$
- expansion of universe does not provide sufficient departure from equilibrium
- need additional source: <u>phase transition</u>

- order parameter : $\langle\varphi\rangle$

  first order : $\langle\varphi\rangle$ changes discontinuously at T_c

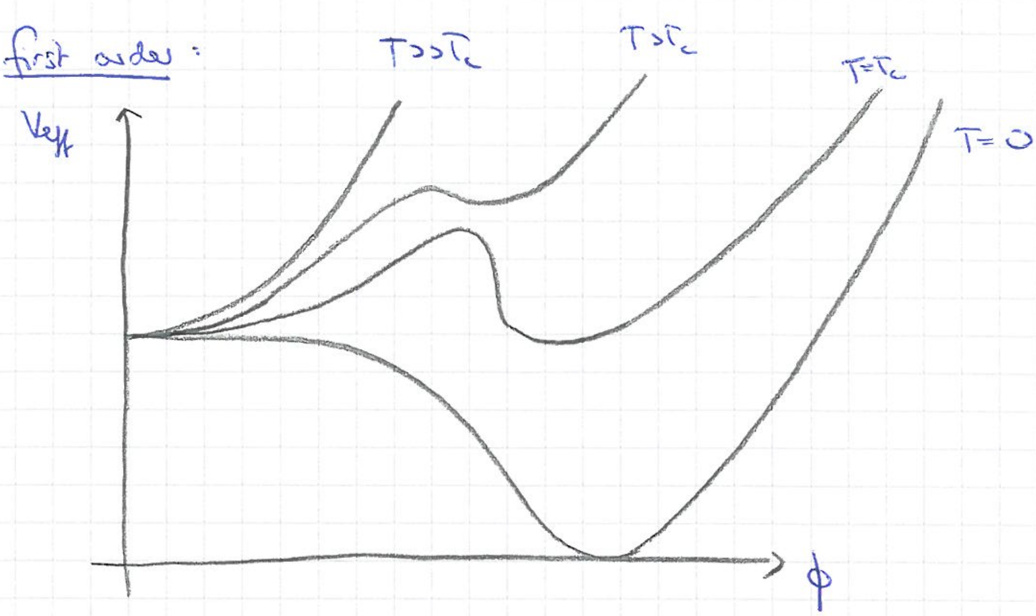
  second order : $\langle\varphi\rangle$ changes continuously at T_c

<u>Second order:</u>



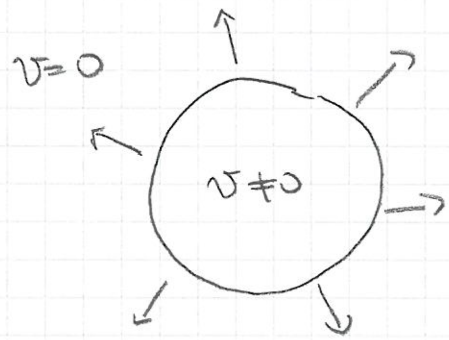
- extremum at $\phi = 0$ becomes local maximum at T_c.
- field rolls classically to new minimum at $\varphi \neq 0$
- insufficient to lead to relevant baryon number production (Kuzmin et. al., '85)

first order:



- at $T = T_c$ two energetically equally favoured minima separated by energy barrier

- at $T \lesssim T_c$: - quantum tunneling → nucleation of bubbles of true vacuum in sea of false
  - have to overcome surface tension
  - collapse again (for $T \sim T_c$)

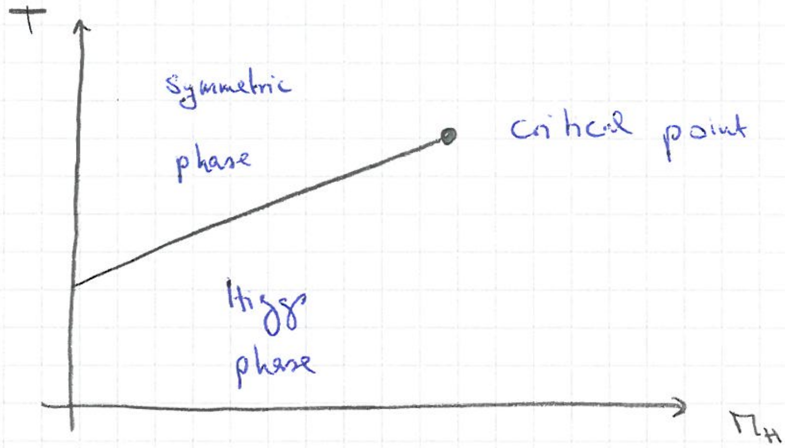


- at $T \ll T_c$: - critical bubbles form that grow and fill all of space, collide
  - rapid change of order parameter → significant departure from thermal equilibrium

- after EW phase transition, no sphaleron washout ⇒ strongly 1st order $\boxed{\frac{\phi(T)}{T} \gtrsim}$

Phase Diagram of Electroweak Theory

Result:



• important tool: finite temperature effective potential

[$T \neq 0$ QFT:

$$O_\beta(x_1,\dots,x_n) = \frac{\mathrm{tr}\; e^{-\beta H}\, O(x_1,\dots,x_n)}{\mathrm{tr}\; e^{-\beta H}}$$ Gibbs average

[M. Sha, Phys. Rep. '89]

$$\mathrm{tr}\; e^{-\beta H} A(x_1,t_1)\, B(x_2,t_2) \dots C(x_n,t_n)$$

$$= \mathrm{tr}\; e^{-\beta H} e^{iHt_1} A(x_1,0)\, e^{-iHt_1}\, e^{iHt_2} B(x_2,0)\, e^{-iHt_2} \dots e^{iHt_n} C(x_n,0)\, e^{-iHt_n}$$

* symmetry: $\boxed{it_n \to it_n + \beta}$

• periodic / anti-periodic in euclidian time $\tau = it$

$$\chi(x,\tau) = \begin{cases} 2n\pi T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

$$\chi(x,\tau) = \sum_{n=-\infty}^{\infty} \chi_n(x)\, e^{i\omega_n \tau}$$

$$\omega_n = \begin{cases} 2n\pi T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

Matsubara frequencies]

effective potential

- $V_{eff}(\varphi_{cl}) \equiv -\frac{1}{V_4} \Gamma[\varphi_{cl}]$ allows one to determine the vacuum of a theory, taking all quantum fluctuation into account:

$$\boxed{\frac{\partial}{\partial \varphi_{cl}} V_{eff}(\varphi_{cl}) = 0}$$

- can evaluate V_{eff} in perturbation theory:

example:

$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$

$$\varphi(x) = \varphi_{cl} + \chi(x)$$

T=0: $V^{(1)}(\varphi_{cl}) =$

$$= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 + m^2(\varphi_{cl})\right)$$

$$m^2(\varphi_{cl}) = 3\lambda\varphi_{cl}^2 - \mu^2$$

T≠0: $k_0 \to 2\pi n T$; $\int dk_0 \to 2\pi T \sum_{n=-\infty}^{\infty}$

$$V^{(1)}(\varphi, T) = \frac{T}{2(2\pi)^3} \sum_{n=-\infty}^{\infty} \int d^3k \ln\left[(2\pi T n)^2 + |\vec{k}|^2 + 3\lambda\varphi^2 - \mu^2\right]$$

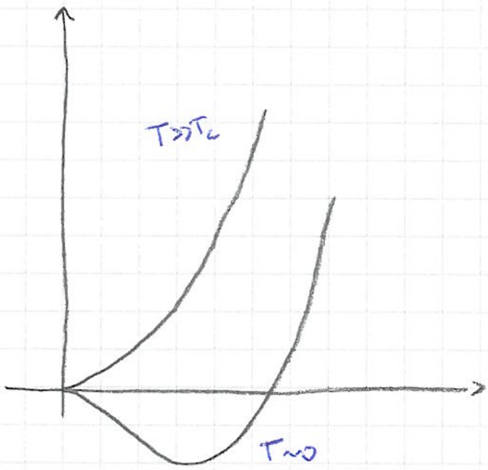
after renormalization and expansion in $\frac{m^2(\varphi_{cl})}{T^2} \ll 1$:

$$\boxed{V_{eff}^{(1)}(\varphi, T) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \underbrace{\frac{m^2(\varphi)}{24}T^2}_{\text{Higgs mass correction}} - \underbrace{\frac{\pi^2}{90}T^4}_{\text{irrelevant vacuum energy}}}$$

• $V_{eff}^{(1)}(\varphi,T) = \underbrace{\left(-\frac{\mu^2}{2} + \frac{1}{8}\lambda T^2\right)}_{\mu_{eff}^2}\varphi^2 + \dots$

$\rightarrow$ $T \to \infty$: $\mu_{eff}^2 > 0$ no symmetry breaking

$T \to 0$: $\mu_{eff} < 0$ symmetry breaking

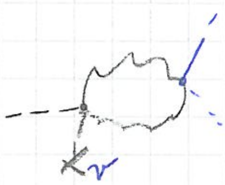


• second order phase transition
• no energy barrier in this toy model.

FTEP for standard electroweak theory:

$$\boxed{\begin{aligned} V_{eff}^{(1)}(\varphi,T) &= \left(\frac{3}{32}g^2 + \frac{\lambda}{4} + \frac{m_t^2}{4v^2}\right)\left(T^2 - T_*^2\right)\varphi^2 \\ &\quad - \frac{3g^2}{32\pi^2}T\varphi^3 + \frac{\lambda}{4}\varphi^4 \end{aligned}}$$

$\varphi \equiv \sqrt{\Phi^\dagger\Phi}$, m_t = top mass, g $SU(2)_L$ gauge coupling

•  : gauge fields make ~~linear~~ cubic term in potential

$\Rightarrow$ energy barrier between vacua

$\Rightarrow$ first order phase transition

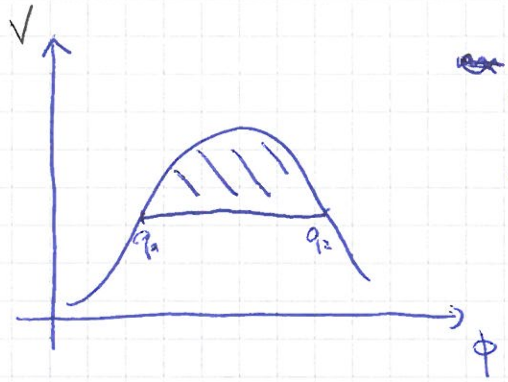
Γ

- tunneling rate may be approximated by WKB

$$\Gamma = |A_0^2|\, e^{-B_0}$$

$$B_0 = 2 \int_{q_1}^{q_2} dq \sqrt{2(V(q)-E)}$$

$V(q_1) = V(q_2) = E$ $\qquad$ $q = \phi$



- $B_0 \sim \lambda^{-2} \gamma^2$
- the larger λ/γ the weaker the phase transition $\lrcorner$
- no 1st order transition for $m_h > 72$ GeV < 114 GeV (LEP bound)

- one may also use $m \ll T$ to build effective theory in 3D that only contains bosonic zero modes
- gauge boson masses $\sim gT$ $\rightarrow$ integrate out

$$\mathcal{L} = \frac{1}{4} W^a_{ij} W^a_{ij} + \frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^\dagger D_i \phi + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2$$

Γ

exercise: Work out the 3D theory and show that it leads to the same results. $\lrcorner$

- for $m_h \gg m_{W,Z}$ PT perturbation theory is unreliable at critical temperature $\rightarrow$ lattice field theory

- $V_{eff}(\phi, T) = \underbrace{(\mu^2 + \alpha T^2)}_{\text{neglect}} \phi^2 - \gamma T \phi^3 + \frac{\lambda}{4} \phi^4 + ..$

$$\Rightarrow \quad \boxed{\frac{\phi}{T} \sim \frac{\gamma}{\lambda}} \qquad \gamma \sim g^2$$

$$\frac{\phi}{T} > 1 \Rightarrow \lambda \lesssim g^2 \rightsquigarrow m_h \lesssim \mathcal{O}(10)\ \text{GeV}.\ (72\ \text{GeV})$$

- MSSM: $\gamma \sim y^3$ loop

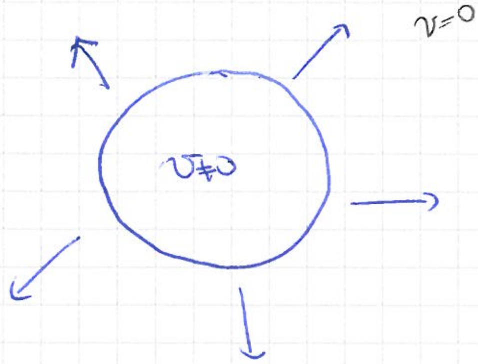
- NMSSM: $\gamma \sim A_\lambda$ tree

MSSM: need light scalar for 1st order phase transition

$\rightarrow$ light, 3rd generation, right handed scalar involved

$\rightarrow$ light stop $\underline{t_L \sim t_R}$

2.2 Dynamics of the Transition in the Early Universe



- 'critical bubbles' are able to overcome surface tension and grow
- complicated problem, numerical solutions needed
- relevant time scale

$$\tau_{form} \propto \exp\left(F_c/T\right)$$

$$F_c[\varphi(r)] = \int dr\, r^2 \left[\frac{1}{2}\left(\varphi'(r)\right)^2 + V_{eff}(\varphi, T)\right]$$

with $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\varphi'\right) - \frac{\partial V_{eff}}{\partial \varphi} = 0$$

and the boundary cond.:

$$\lim_{r\to\infty} \varphi(r) = 0$$

$$\varphi(0) = v(T) = \text{minim of } V_{eff}(\varphi, T)$$

- example: $$V_{eff}(\phi, T_c) = \frac{\lambda}{4}(\phi - v)^2\phi^2$$

$$\hookrightarrow \phi = \frac{v}{2}\left[1 + \tanh\left(-r v \sqrt{\frac{\lambda}{8}}\right)\right]$$

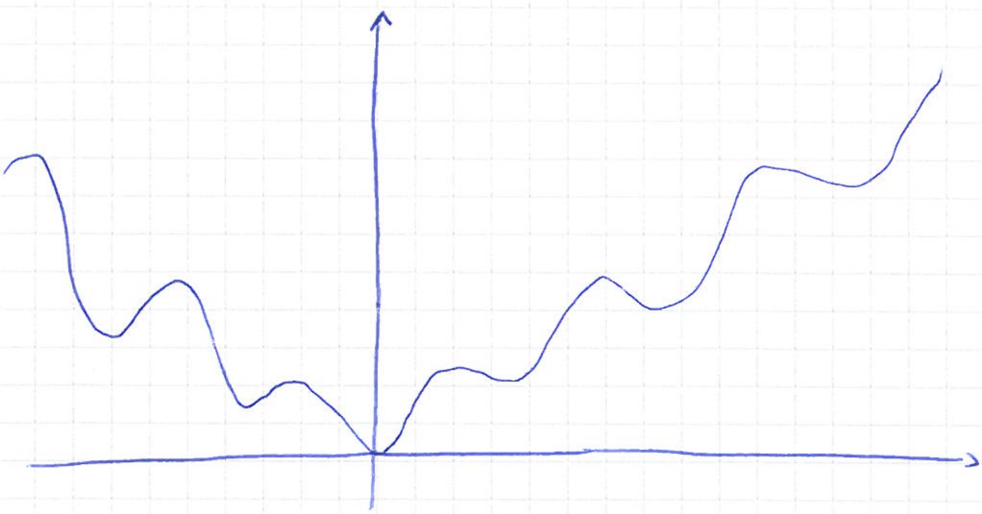
$$\delta_w \equiv \left(\sqrt{8/\lambda}\right)/v$$ model dependent

3. Calculating the Baryon Asymmetry

Potential Energy along "sphaleron" trajectory

$$\Delta B = N_f N_{CS} = N_f \frac{g^2}{32\pi^2} \int d^3x \, \varepsilon^{ijk} \, \mathrm{tr}\left(h \partial_i h^{-1} h \partial_j h^{-1} h \partial_k h^{-1} \right)$$

- this does not include energy of baryons:



$$\rightarrow \quad \boxed{\frac{dn_B}{dt} = 3\left(\Gamma_+ - \Gamma_-\right) = -3\,\frac{\Gamma_a}{T}\,\Delta F}$$

ΔF = free energy difference between neighboring minima $\left(= 3 \frac{\partial F}{\partial B}\right.$, $\Delta B = 3$ between neighboring vacua$\left.\right)$

Γ_a = rate per unit volume for fluctuations in absence of bias

$$\Gamma_a = \begin{cases} \gamma (\alpha_W T)^{-3} M_W^7 \, e^{-E_{sp}/T} & \text{(broken phase)} \\ \\ K \, (\alpha_W T)^4 & \text{(symmetric phase)} \end{cases}$$

~~3.1~~ 3.1 CP violation

- $n_{B/s} \simeq (0.6 - 1.0)\, 10^{-10}$

$$S = \frac{2\pi^2 g_*}{45} T^3 \qquad g_* \approx 100$$

$$\Gamma_a \sim \alpha_W^4 \sim 10^{-6}$$

↳ additional suppression $\lesssim 10^{-2}$ from CP violation

↳ any ~~CP~~ effect has to include all 3 generations

→ large suppression due to small off-diagonal elements, $\delta_{CP} \simeq 10^{-20}$,

→ new source of CP violation needed!

- Jarlskog determinant:

$$J_{CP} = S_{12} S_{13} S_{23} C_{12} C_{23} C_{13} \sin\delta$$

$$= \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})$$

↳ M. G. Schmidt et al. 2009

↳ if one assume the observable to be polynomial in quark masses

$$\Delta_{CP} = v^{-12} \operatorname{Im} \det [m_u m_u^\dagger, m_d m_d^\dagger] = J\, v^{-12} \underbrace{\prod_{i<j} (\tilde m_{u,i} - \tilde m_{u,j}) \prod_{i<j} (\tilde m_{d,i}^2 - \tilde m_{d,j})}_{\sim 10^{-19}}$$

3.2 Time scales

1) thermalization time τ_T : how fast do particles in cosmic plasma equilibrate

2) Higgs time scale $\tau_H \simeq \phi/\dot{\phi}$ governs the departure from equilibrium

3) τ_{sp} : sphaleron time scale gives the rate of baryon violation in symmetric phase

[4) Expansion rate H^{-1} << others irrelevant]

* $\tau_{sp}^{-1} \simeq \alpha_w^4 T = 1.3 \cdot 10^{-6}\ T$

* $\tau_T^{-1} \equiv \ell_T^{-1} \simeq \begin{cases} 0.25\ T & q \\ 0.08\ T & w, z, \ell \end{cases}$

* $\tau_H^{-1} = \dot{\phi}/\phi \simeq \frac{v_w}{\delta_w} \simeq (0.01 - 1.0)\,T$ model dependent

2 regimes:

- adiabatic regime : $\tau_T \ll \tau_H$
  thermal equilibrium maintained as bubble wall passes by

- non-adiabatic regime : $\tau_T \gg \tau_H$

3.3 Adiabatic "thick wall" regime

- treat plasma in the bubble as being in quasi-static thermal equilibrium with a classical time-dependent field
- plasma is not in chemical equilibrium, as baryon violating reactions are slower than τ_H^{-1}
- treat this deviation by introducing chemical potentials for slowly varying quantities

- toy model: $\psi\ (B=1)$, $\chi(B=-1)$, $\phi(B=2)$ B conserved

  - Yukawa coupling $\bar{\psi}\phi\chi$ [$\partial_t\theta \hat{=} \mu_B$]
  - During phase transition $\phi(t)=\rho(t)\,e^{i\theta(t)}$
    $\psi, \chi \to e^{i\theta B/2}$
    $$\Rightarrow \mathcal{L}_{kin} \to \mathcal{L}_{kin} + \tfrac{1}{2}\dot{\theta}\left(\bar{\psi}\gamma^0\psi - \bar{\chi}\gamma^0\chi\right) = \mathcal{L}_{kin} + \tfrac{1}{2}\dot{\theta}\, n_B$$

$\mu_B = \dot{\theta}\mathcal{N}$ fudge factor

  - as bubble wall has passed $\dot{\theta}$ returns to zero, Higgs vev turns on, Γ_a goes rapidly to zero
    $\Rightarrow$ Baryon number drops out of equilibrium and remain to present epoch

  - Crude estimate: $\langle\phi\rangle_{co} \simeq \frac{7gT}{2\pi} = 0.7T$ "cut-off value"

  - Integrate $\frac{dn_B}{dt} = -g\frac{\Gamma_a}{T}\frac{\partial F}{\partial B} = -g\frac{\Gamma_a \mu_B}{T}$

$$\Rightarrow n_B = -\frac{g\mathcal{N}}{T}\int_{\infty}^{t_{co}} dt\ \dot{\theta}\ \Gamma_a(\phi(t)) \simeq -\frac{g\mathcal{N}\Gamma_a}{T}\Delta\theta$$

where we have treated Γ_a as step function, vanishing for $\langle\phi\rangle \geq \langle\phi\rangle_{co}$

- given $s = (2\pi^2 g_* / 45)\, T^3$ & $\Gamma_a \sim k(\alpha_w T)^4$

$$\Rightarrow \boxed{\frac{n_B}{s} = -k \left(\frac{w}{0.1}\right)\left(\frac{100}{g_*}\right)\left(\frac{\Delta\theta}{\pi}\right) \cdot 3 \cdot 10^{-7}} \quad \text{(adiabatic)}$$

$w \sim 0.1 - 1$, $k \sim \mathcal{O}(1)$

3.4 Electroweak Baryogenesis in the MSSM

- in MSSM no $\cancel{CP}$ in Higgs potential

- $$\left[\bar{U} \lambda_u Q H + \bar{D} \lambda_D Q H' + \bar{E} \lambda_E L H' + |\mu| e^{-i\phi_B} H H\right]_F$$

$$\Big[+ m_{3/2} \Big[|A| e^{i\phi_A} (\bar{U} \xi_u Q H + D \xi_D Q H' + \bar{E} \xi_E L H') + |\mu_B| H H' \Big]_A \Big]$$

$$\psi_i \to U_{ij}(x)\, \psi_j$$

$$\mathcal{L}_{kin} \to \mathcal{L}_{kin} + \bar{\psi} \gamma^\mu (U^+ i \partial_\mu U) \psi$$

$$\leadsto \quad n_B = \frac{3}{T} \int_0^{\langle\phi\rangle_0} d\phi \; \mathrm{tr}\; U^+(\phi) \frac{i\, dU(\phi)}{d\phi} t_{wk}^2 \, \Gamma_a(\phi)$$

t_{wk}^2 SU(2) Casimir matrix

- n_B/s acceptable for certain regions in parameter space $\Rightarrow$ <u>testable</u>!

- light stop for strongly 1st order phase transition $(y^3 T \phi^3)$
- $\cancel{CP}$: light $\tilde{W}_1$ $(M_{2,M} \lesssim 500\,\text{GeV}$ & $\text{Arg}(M_2\mu) \gtrsim 0.1)$

- Constraints from e^- EDM, $b \to s\gamma$
- testable at LHC, ILC