Standard Leptogenesis Scenarios

Outline:

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* Heavy neutrino decay
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  → Bounds on neutrino masses
* Washout processes
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  → Inverse decays / scattering processes
  → Simplified Boltzmann equations
* Flavour effects
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* Casas-Ibarra parameterization
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  → Phenomenology (with flavour effects)
* General comments / conclusion

1. Introduction

(primordial abundances of light elements $\propto \eta$ & CMB spectrum fluctuations)

- Evidence for baryon asymmetry, as discussed by Kher Sham.

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} = \eta_B \sim 6 \times 10^{-10}$$

[note, also can use

$$Y_{\Delta B} \equiv \frac{n_B - n_{\bar{B}}}{s} = (8.75 \pm 0.23) \times 10^{-11}$$

(equivalent)

$$s = g_* \left(\frac{2\pi^2}{45}\right) T^3$$

$\hookrightarrow$ entropy density]

- Sakharov's conditions
  - → Baryon number violation
  - → C and CP violation
  - → Non-equilibrium (thermal)

⇒ How does leptogenesis fulfill these criteria?

→ The seesaw mechanism requires the existence of N_R (as many as required) & Majorana nature

→ Extra Majorana phases are source of CP violation

→ Heavy neutrino decay, CP asymmetry leads to lepton number asymmetry ($N_R \to l_L H$, $N_R \to \bar{l}_L H^+$)

→ Decay is out of equilibrium, ie $\Gamma_D < H$

1.1 Seesaw mechanism

- Add 3 heavy N_R to SM

$$\mathcal{L}_Y = f_{ij} \bar{e}_{R_i} l_{L_j} H^+ + h_{ij} \bar{N}_{R_i} l_{L_j} H - \frac{1}{2} (M_R)_{ij} \bar{\nu}^c_{R_i} \nu_{R_j} + h.c.$$

- can choose basis where f_{ij} & M_{ij} are diagonal, so h_{ij} is complex matrix

→ 9 phases (-3 into l_L) = 6 phases & 6 mixing angles (biunitary trafo.)

- Now integrate out heavy N_R
  - → Find E.OM for N_R, neglecting kinetic terms

$$\left(\text{ie} \quad \partial_\mu \frac{\partial \mathcal{L}_Y}{\partial(\partial_\mu N_R)} - \frac{\partial \mathcal{L}_Y}{\partial N_R} = 0\right)$$

  - → Substitute back into $\mathcal{L}_Y$

$$\Rightarrow \mathcal{L}_{Y_{eff}} = f_{ii}\,\bar{e}_{R_i} l_{L_i} H^+ + \frac{1}{2}\sum_k h^T_{ik} h_{kj} l_{L_i} l_{L_j} \frac{H^2}{(M_R)_k} + h.c$$

$$\xrightarrow{SSB} \; -\frac{1}{2} l_L^T m_\nu l_L$$

$v = \langle H \rangle$

where $m_\nu = -m_D^T M_R^{-1} m_D$ and $m_D = h v$

m_ν is effective Majorana mass matrix, shows up in neutrino oscillation experiments, $0\nu\beta\beta$ decay etc.

m_ν has $6-3=3$ phases & 3 mixing angles

- CP phases from 6 to 3 ... how to connect the leptogenesis phases (in h_{ij}) with low energy CPV?

Alternatively, can write seesaw as:

$$m_\nu^{D+M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad \text{in } \mathcal{L}_{Y(\nu)} = h_{ij}\bar{\nu}_{R_i} l_{L_j} H - \frac{1}{2}(M_R)_{ij}\bar{\nu}^c_{R_i}\nu_{R_j} + h.c$$

then with $m_D = hv \ll M_R$, get

$$m_\nu(\text{light}) \simeq -m_D^T M_R^{-1} m_D \quad \& \quad M_{N}(\text{heavy}) \simeq M_R$$

1.2 Casas-Ibarra Parameterization

- Introduce here, as it relates to seesaw, and is a useful way to separate low & high energy parameters (ie "parameterizing our ignorance")

- we have
$$m_\nu = m_D{}^T M_R^{-1} m_D \quad \text{(ignoring sign conventions)}$$
$$= h^T M_R^{-1} h \, (v^2)$$

$$\Rightarrow \quad m \equiv \frac{m_\nu}{v^2} = h . M_R^{-1} h$$

Now we use a bottom-up approach, ie start with low energy (instead of M_R at Majorana scale & RGE run down)

- Choose basis where M_R is diagonal, ie $M_R \to D_{M_R}$

- $D_m = U^T m \, U$ (U is PMNS matrix)

$$\Rightarrow D_m = U^T h^T D_{M_R}^{-1} h \, U$$
$$= U^T h^T (D_{M_R})^{-\frac{1}{2}} (D_{M_R})^{-\frac{1}{2}} h \, U$$

- Multiply by $(D_m)^{-\frac{1}{2}}$ on left and right

$$\Rightarrow \mathbb{1} = \underbrace{D_m^{-\frac{1}{2}} U^T h^T D_{M_R}^{-\frac{1}{2}}} \, \underbrace{D_{M_R}^{-\frac{1}{2}} h \, U D_m^{-\frac{1}{2}}}$$
$$= R^T R$$, where $R = D_{M_R}^{-\frac{1}{2}} h \, U D_m^{-\frac{1}{2}}$, a complex orthoganal matrix

$$\Rightarrow \boxed{h = \sqrt{D_{M_R}} \, R \sqrt{D_m} \, U^+ = \frac{1}{v} \sqrt{D_{M_R}} \, R \sqrt{D_{m_\nu}} \, U^+}$$

$h \Rightarrow$ function of D_{m_ν}, U (low energy parameters)

and D_{M_R}, R (high energy parameters)

* In general there is no connection between these two sets of parameters (except for specific models, ie LR symmetry) (Tibor's talk)

* One can diagonalize h by a biunitary trafo

$$h = V_R^{\nu\dagger} \mathrm{diag}(h_1, h_2, h_3) V_L^\nu \quad \text{(in basis where } f \text{ \& } M_R \text{ diagonal)}$$

from Casas-Ibarra,

$$h h^\dagger v^2 = \left(D_{M_R}^{1/2} R D_{m_\nu}^{1/2} U^\dagger \right)\left(U D_{m_\nu}^{1/2} R^\dagger D_{M_R}^{\frac{1}{2}} \right)$$

$$= D_{M_R}^{\frac{1}{2}} R D_{m_\nu} R^\dagger D_{M_R}^{\frac{1}{2}}$$

$$= \left[V_R^{\nu\dagger} \mathrm{diag}(\underbrace{h_1^2, h_2^2, h_3^2}_{\text{real eigenvalues}}) V_R^\nu \right] \cdot v^2$$

$\Rightarrow$ This shows that the phases of R are related to the phases in the RH sector, ie in V_R^ν

* The product $hh^\dagger$ will be important for leptogenesis.

* Side note:

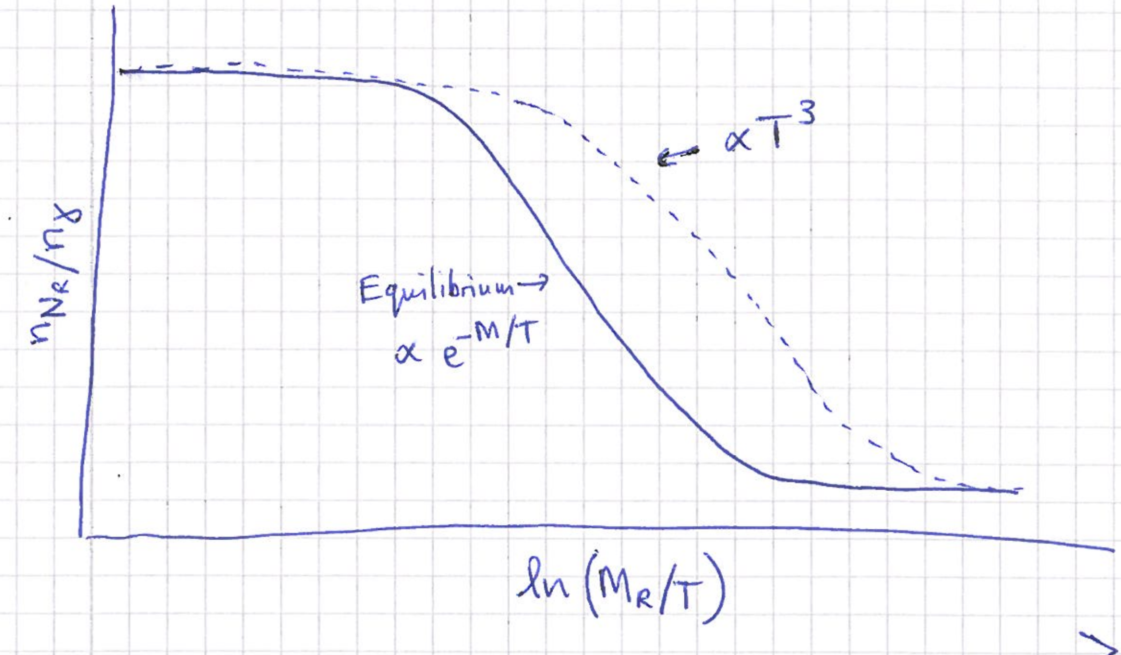
$\Rightarrow$ In LFV, the product $h^\dagger h$ (its off-diagonal elements) are important, which allows for combined phenomenology in certain scenarios. (ie SUSY models)

1.2 Out of equilibrium decay

* The heavy N_R's need to decay out of equilibrium to fulfill Sakharov's conditions.

  ⇒ If the decay rate Γ_D at the time when particles become non-relativistic ($T \sim M_R$) is too slow, ie smaller than the expansion of the universe ($\Gamma_D < H$), then they decouple from the thermal bath and the decay departs from equilibrium.

  H ↑ Hubble constant



∘ Initial thermal abundance $n_{N_R} \sim n_\gamma \sim T^3$ $(T \gtrsim M_R)$

In equilibrium $n_{N_R} (= n_{N_R}) \simeq (M_R T)^{3/2} e^{-M_R/T} \ll n_\gamma$ $(T \lesssim M_R)$

However, since particles decouple while still relativistic, they populate universe at $T \simeq M_R$ with much larger abundance, ie they cannot follow the equilibrium distr. and $n_{N_R} \sim T^3$ instead, even at $T \lesssim M_R$

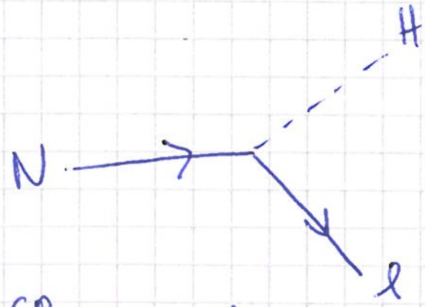
- Since $\frac{\Gamma}{H} \propto \frac{1}{M_R}$, states must be heavy.

10^{15-16} GeV for gauge bosons, 10^{10-16} GeV for scalars

$\sim 10^9$ GeV for Majorana neutrinos.

* In leptogenesis, RH neutrinos generate Y_L asymmetry at $T < M_R$ by out-of-equilibrium decays, and this is converted into ΔY_B through sphaleron interactions

2. Heavy neutrino decay



2.1 The CP asymmetry

- At the tree level, there is no CP asymmetry

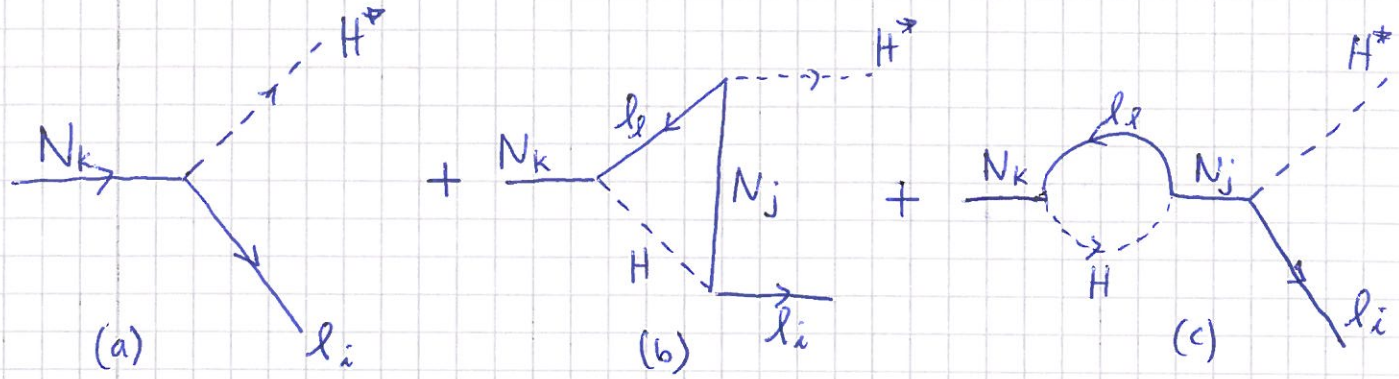
$$\Gamma_{D_i} = \sum_\alpha \left[\Gamma(N_i \to H + \ell_\alpha) + \Gamma(N_i \to \bar{H} + \bar{\ell}_\alpha) \right]$$

$$= \frac{1}{8\pi} (hh^\dagger)_{ii} M_i$$

and $\Gamma(N_i \to H + \ell_\alpha) = \Gamma(N_i \to \bar{H} + \bar{\ell}_\alpha)]$

- Asymmetry is generated at loop level.
- For simplicity, assume that N_1 decay dominates (i.e., LNV interactions of N_1 wash out asymmetry from decay of $N_{2,3}$ at $T \gg M_1$)

- Then, if $\Gamma_{D_1} < H\big|_{T=M_1}$, heavy neutrinos (N_1) cannot follow equilibrium distribution. When they eventually decay, they produce a CP asymmetry as follows:



$$\epsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to l_\alpha H) - \Gamma(N_1 \to \bar{l}_\alpha \bar{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to l_\alpha H) + \Gamma(N_1 \to \bar{l}_\alpha \bar{H})\right]}$$

no flavour index, as we sum over flavours here

$$\simeq \frac{1}{8\pi} \frac{1}{(hh^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left\{(hh^\dagger)^2_{1i}\right\} \cdot \left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right]$$

where $f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$ is vertex correction (b)

and $g(x) = \frac{\sqrt{x}}{1-x}$ is from the self-energy diagram, in the limit $|M_i - M_1| \gg |\Gamma_i - \Gamma_1|$

<u>Note</u>: ϵ comes from interference of tree-level & one-loop amplitudes

See Ref. [4]

ie $\mathcal{M} = \underset{\substack{\uparrow \\ \text{matrix element}}}{\mathcal{M}_0} + \mathcal{M}_1 = c_0 \underset{\substack{\uparrow \\ \text{ampl}}}{A_0} + \underset{\substack{\uparrow \\ \text{coupling (complex)}}}{c_1} A_1$ and $\bar{\mathcal{M}} = c_0^* \bar{A}_0 + c_1^* \bar{A}_1$

$$\epsilon \text{ "}\propto\text{" } \frac{\int |c_0 A_0 + c_1 A_1|^2 - \int |c_0^* A_0 + c_1^* A_1|^2}{2\int |c_0 A_0|^2} \Longrightarrow \frac{\mathrm{Im}\{c_0 c_1^*\}}{|c_0|^2} \frac{2\int \mathrm{Im}\{A_0 A_1^*\}}{\int |A_0|^2}$$

- Assuming hierarchical RH ν's $(M_1 << M_{2,3})$, we have $x >> 1$, so

$$f(x) + g(x) \simeq -\frac{3}{2}\frac{1}{\sqrt{x}} - \frac{5}{6x^{3/2}} + \dots \quad \text{(in SM)}$$

(in MSSM this is 2 times larger, since N_1 decays to slepton & Higgsino as well)

$$\Rightarrow \boxed{\epsilon_1 \simeq -\frac{3}{16\pi}\frac{1}{(hh^\dagger)_{11}}\sum_{i=2,3} \mathrm{Im}\left\{(hh^\dagger)^2_{1i}\right\}\frac{M_1}{M_i}} \quad (*)$$

* Note: If N_j & N_k in self-energy diagram are degenerate, there is a resonant enhancement, which can allow M_1 to be much lower while still generating sufficient asymmetry.
  $\Rightarrow$ Resonant leptogenesis (Tibor's talk)

- To prevent decay being washed out by inverse decay & scattering, the decay must be out of equilibrium.

ie $\Gamma_{D_1} < H\big|_{T=M_1}$

Now $\Gamma_{D_1} = \frac{1}{8\pi}(hh^\dagger)_{11} M_1$ and $H(T=M_1) = 1.66\, g_*^{1/2} \frac{T^2}{M_{Pl}}\Big|_{T=M_1}$

# of relativistic degrees of freedom $= 106.75$ (SM)

Define $\tilde{m}_1 \equiv \sum \tilde{m}_{\alpha\alpha} \equiv \sum_\alpha \frac{(hh^\dagger)_{\alpha 1} v^2}{M_1} = 8\pi \frac{v^2}{M_1^2}\Gamma_D$

one-flav approx

as "effective light ν mass"

and $m_* \equiv 8\pi \frac{v^2}{M_1^2} H\Big|_{T=M_1} \simeq 1.1 \times 10^{-3}\ \mathrm{eV}$

as "equilibrium ν mass"

- The out of equilibrium condition $\Gamma_{D_1} < H$ is equivalent to $\tilde{m}_1 < m_* \simeq 1.1 \times 10^{-3}$ eV, in rough agreement with the mass scale of light neutrinos (see comments later)
- The final lepton asymmetry depends on the amount of washout, ie processes like inverse decays and scattering which can destroy the asymmetry created in decays. (See the next section)

parameterisation of washout

$$\Longrightarrow \quad \boxed{Y_L \equiv \frac{n_L - \bar{n}_L}{s} = \kappa \frac{\epsilon_1}{g_*}}$$

entropy ($s = 7.04 n_\gamma$ in present epoch)

and finally $\boxed{Y_B \equiv \frac{n_B - \bar{n}_B}{s} = c\, Y_{B-L} = \frac{c}{c-1} Y_L}$

(via sphaleron processes (kher Sham's talk))

- Before discussing washout processes, we discuss various bounds on neutrino masses.

2.2 Bounds on neutrino masses ("Davidson-Ibarra" bound)

- In case with strong hierarchy of RH ν & ϵ_1 dominated asymmetry, can get bounds on ϵ_1 & M_1 (with decay $T \gtrsim 10^{12}$)
- Eq. (*) can be rewritten as

$$\epsilon_1 \simeq -\frac{3}{16\pi} \frac{M_1}{v^2} \frac{1}{(hh^\dagger)_{11}} \mathrm{Im}\{(h\, m_\nu^\dagger h^T)_{11}\}$$

Using the Casas-Ibarra parameterization

$$h = \frac{1}{v}\sqrt{D_{M_R}}\, R \sqrt{D_{m_\nu}}\, U^\dagger = \frac{1}{v}\sqrt{M_R}\, R\sqrt{m_\nu}\, U^\dagger$$

(diagonal M_R, m_ν)

one gets

$$\epsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i m_i^2 \, \mathrm{Im}\,(R_{1i}^2)}{\sum_i m_i |R_{1i}|^2}$$

From orthogonality, $\sum_i R_{1i}^2 = 1$, it follows

$$\boxed{\epsilon_1 \lesssim \frac{3}{16\pi}\frac{M_1(m_3 - m_2)}{v^2}} \simeq \frac{3}{16\pi}\frac{M_1 m_3}{v^2} \quad \text{for hierarchical LH } \nu\text{'s}$$

$(m_3 \approx \sqrt{\Delta m_A^2})$

from here,

$$M_1 \geqslant \eta_B \frac{1-c}{c}\left[\frac{n_{\nu_R}+n_{\bar\nu_R}}{s}\frac{3}{16\pi}\frac{m_3}{v^2}K\right]^{-1}$$

$\eta_B \simeq 6.1 \times 10^{-10}$ $\Rightarrow$ sets a lower limit on ϵ_1

$m_3 \simeq \sqrt{\Delta m_A^2} \simeq 0.05\,\mathrm{eV}$

$K \lesssim 1$ from solving Boltzmann Eqs.

- If RH ν's thermally produced, then $M_1 \gtrsim 2 \times 10^9$ GeV can be obtained, since $\eta_{B-L} \lesssim 10^{-(2-3)}\epsilon_1$

Note: The reheat temp. after inflation must be at least as large as M_1, i.e. $T_{RH} > 10^8 - 10^{10}$ GeV. This is a problem in SUSY models as $T_{RH} < 10^{8-9}$ GeV from WMAP constraints on gravitino production. If T_{RH} ~~there~~ is too big there is an overproduction of gravitinos.

"the gravitino problem" $\Rightarrow$ non-standard scenarios

- Also, can show from orthogonality of R that

$$\tilde{m}_1 = \sum_{\alpha} \frac{(hh^{\dagger})_{\alpha 1} v^2}{M_1} > m_{lightest}$$

and $\tilde{m}_1 \leq m_3$

ie $m_1 \leq \tilde{m}_1 \leq m_3$

↖ eff. mass from N_1 decay.

→ generally speaking, by requiring that washout effects are weak, one gets $\tilde{m}_1 \lesssim 0.1 - 0.2$

also, if $\Delta L = 2$ washout effects lead to successful leptogenesis, $\sqrt{m_1^2 + m_2^2 + m_3^2} \lesssim (0.1 - 0.2)\,\mathrm{eV}$

$\Rightarrow$ $0.05 \lesssim m_3 \lesssim 0.15\,\mathrm{eV}$ [7] (if η_B from N_1-dominated L.G.)

or $10^{-3} \lesssim m_i \lesssim 0.1\,\mathrm{eV}$ [3]

This is known as the "leptogenesis conspiracy", ie, leptogenesis requires that ν mass scale be of the same order as indicated by other experiments, ie oscillations, beta decay, $0\nu\beta\beta$ and cosmology.

3. Washout processes & Boltzmann equations

3.1 Washout regimes

Blah In general, amount of washout (K), is determined by

$$r \equiv \frac{\Gamma_1}{H|_{T=M_1}} = \frac{\tilde{m}_1}{m_*}$$

There are different regimes, corresponding to weak or strong washout:

(1) If $r \ll 1$ for $T_D \lesssim M_R$ (weak washout), then

$$\frac{\Gamma_{ID}}{H} \sim \left(\frac{M_R}{T}\right)^{3/2} e^{-M_R/T} \cdot r \qquad \text{(inverse decays)}$$

and $$\frac{\Gamma_S}{H} \sim \alpha \left(\frac{T}{M_R}\right)^5 \cdot r \qquad \text{(scattering)}$$

$\Rightarrow$ both are negligible, so the asymmetry from N_1 decay survives. Below T_D, N_1 decay out of equilibrium, with n_{N_1} a thermal distibution

(2) If $r \gg 1$, (strong washout), the N_1 & $\bar{N}_1$ abundance follows equilibrium distribution, and any asymmetry is washed out.

* For $1 < r < 10$ the asymmetry could be sizable, so to quantify this one needs to solve the Boltzmann equations.

A rough approximation is given by. (Kolb & Turner)

$$10^6 \lesssim r : \; K = (0.1 r)^{\frac{1}{2}} e^{-\frac{4}{3}(0.1)^{1/4}} \qquad (< 10^{-7})$$

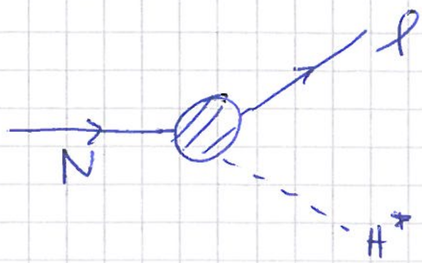
$$10 \lesssim r \lesssim 10^6 : \; K = \frac{0.3}{r(\ln r)^{0.8}} \qquad (10^{-2} \sim 10^{-7})$$

$$0 \lesssim r \lesssim 10 : \; K = \frac{1}{2\sqrt{r^2+9}} \qquad (10^{-1} \sim 10^{-2})$$

3.2 Inverse decays / scattering & Boltzmann equations

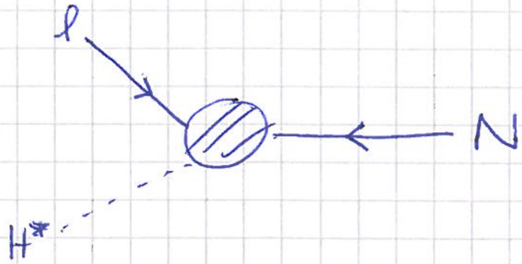
1) Decay of N

$N \to \ell + H$, $N \to \bar{\ell} + \bar{H}$



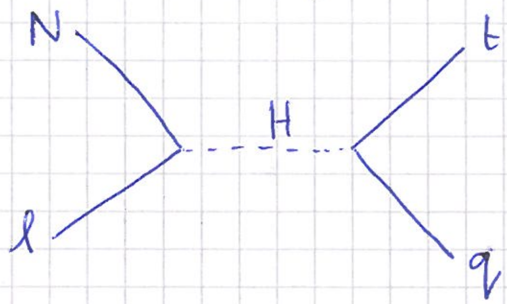
2) Inverse decay of N

$\ell + H \to N$, $\bar{\ell} + \bar{H} \to N$

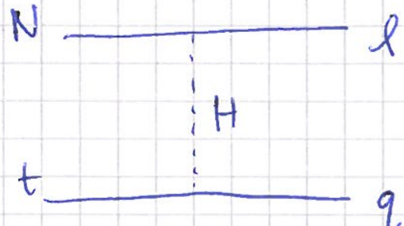


3) 2-2 scattering

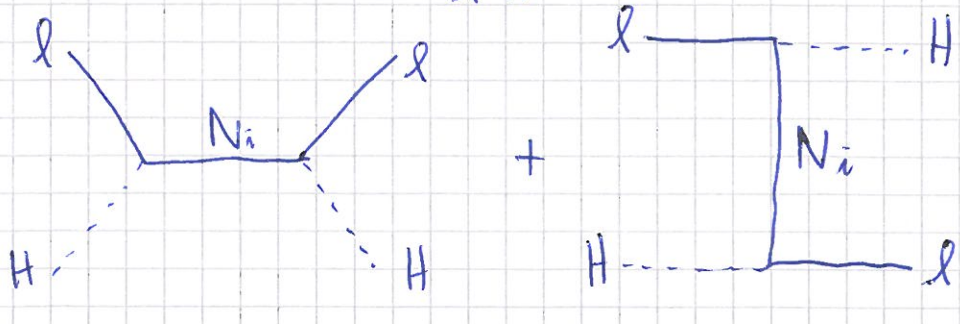
$\Delta L = 1$: $N_1 \ell \leftrightarrow t\bar{q}$, $N_1 \bar{\ell} \leftrightarrow t\bar{q}$ (s-channel)



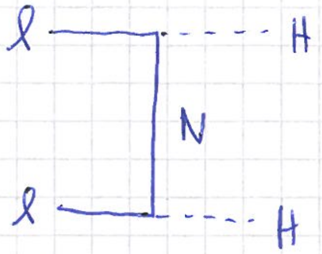
$N_1 t \leftrightarrow \bar{\ell} q$, $N_1 t \leftrightarrow \ell \bar{q}$ (t-channel)



$\Delta L = 2$: $\ell H \leftrightarrow \bar{\ell} \bar{H}$



$$\bar{\ell}\bar{\ell} \leftrightarrow H H, \quad \ell\ell \leftrightarrow \bar{H}\bar{H}$$



* In order for successful leptogenesis, at

  $T \gtrsim M_1$: $\Delta L = 1$, $\Delta L = 2$ processes strong enough to keep N_1 in equilibrium

  but for $T \lesssim M$: the washout processes must be weak enough so that decay of N_1 generates asymmetry!

* Boltzmann eqns (in the one-flav. approximation)

$$\frac{dN_{N_1}}{dz} = -(D+S)\left(N_{N_1} - N_{N_1}^{eq}\right) \quad (1)$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 D\left(N_{N_1} - N_{N_1}^{eq}\right) - W N_{B-L} \quad (2)$$

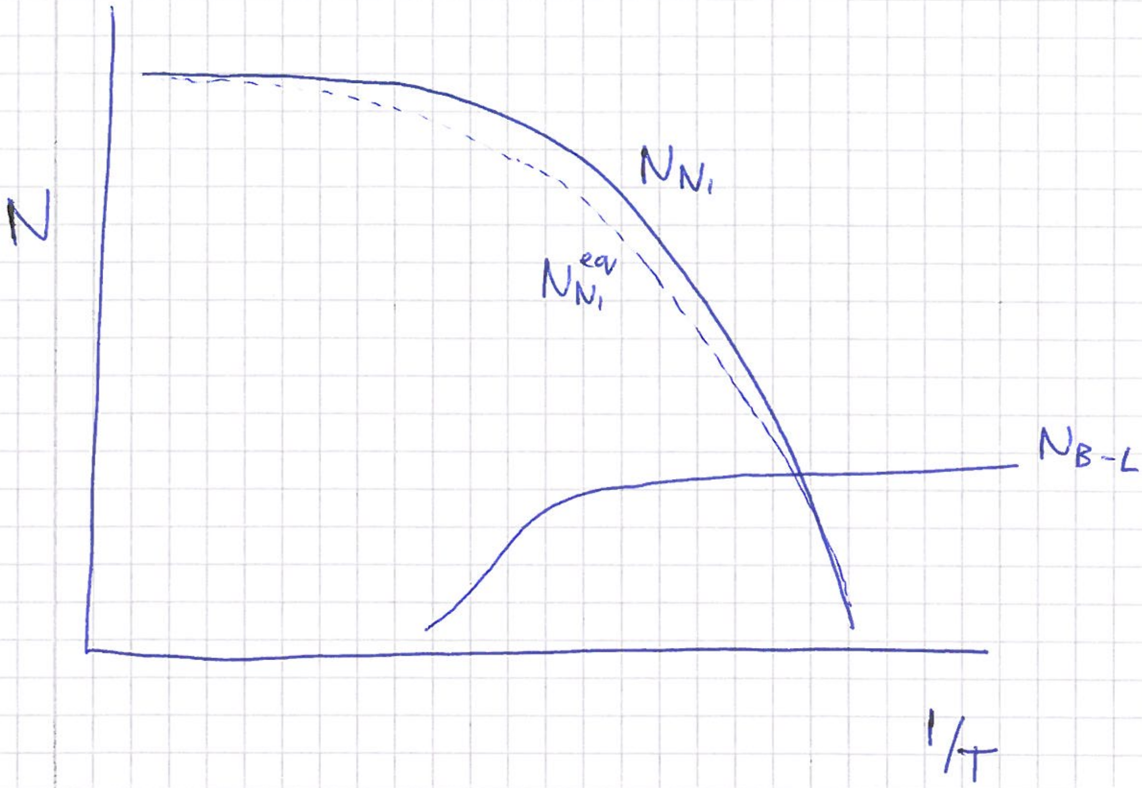
where $(D, S, W) \equiv \dfrac{(\Gamma_D, \Gamma_S, \Gamma_W)}{Hz}$, $z = \dfrac{M_1}{T}$

Γ_D: decay & inverse decay

Γ_S: $\Delta L = 1$ scattering

Γ_W: $\Delta L = 1, \Delta L = 2$, inverse decay

* N_1 abundance affected by D & S (decay, inverse decay, $\Delta L = 1$)
* From eq. (2), see that washout processes (W) affect the final asymmetry created by decay of N_1



* Note:

This treatment is oversimplified, especially without taking flavour into account. The decays, inverse decays and washout processes all depend on flavour, ie the decays can produce different flavours of leptons & vice versa for inverse decays.

3.3 Flavour effects

$$\epsilon_1 = \sum_{\alpha=e,\mu,\tau} \epsilon^{\alpha\alpha}$$ → CP asymm. in flavour α

→ If N_R's are hierarchical, the Yukawa interactions of e, μ & τ reach equilibrium at different temperatures.

$$h^2 M_{Pl} = T_{eq}$$

τ : $T_{eq} \sim 10^{12}$ GeV ; μ : $T_{eq} \sim 10^{9}$ GeV

* If leptogenesis occurs at $T \sim M_1 > 10^{12}$ GeV, then all 3 flavours of Yukawa interactions are out of equilibrium, and there are no flavour effects. (Washout factors are also universal)
* If $T \sim M_1 < 10^{12}$ GeV, (generally the case), then 2 flavours are distinguishable (3 flavours for $T < 10^9$ GeV)

⇒ BE's must be modified to include flav. dependence

$$\Rightarrow \frac{dY^{\alpha\alpha}}{dz} = \frac{z}{sH(M_1)}\left[\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon^{\alpha\alpha}\left(\gamma_D^{\alpha\alpha} + \gamma_{\Delta L=1}\right) - \frac{Y^{\alpha\alpha}}{Y_L^{eq}}\left(\gamma_D^{\alpha\alpha} + \gamma_{\Delta L=1}\right)\right]$$

⇒ This can enhance ~~asymm~~ amount of leptogenesis by factor of 2 to 3. (eg. washout is less efficient, since only "eats" certain flavours)

→ BE's become matrix equations (coupled), & flavour effects depend on the basis one chooses. (The BE's are not covariant in lepton flavour space). For example, if $T_{LG} < 10^{12}$ GeV, then h_τ interactions choose a physical flavour basis

→ See plots, where $K_{\alpha\alpha} \equiv \tilde{m}_{\alpha\alpha}/m_*$ $\left(\tilde{m}_{\alpha\alpha} \equiv |h_{1\alpha}|^2 \frac{v^2}{M_1}\right)$

[⇒ Flav. effects can enhance baryon asymmetry]

[More details in Ch. 9 of Ref. [4] & also [8,9]]

4 Phenomenology with leptogenesis

- As discussed, leptogenesis chooses a mass scale for LH neutrinos. Here one can compare the mass scale from low-energy experiments.
  - ⇒ Note; D-I bound changes in flavoured calculation, and so models can be tuned to work for $m_\nu \lesssim$ few eV
- CP asymmetry in flavoured case can be written as

$$\epsilon_\alpha = -\frac{3 M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{\frac{1}{2}} m_\rho^{3/2} U_{\alpha\beta}^* U_{\alpha\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}$$

and the trace over $\alpha = e, \mu, \tau$ gives ϵ_1

⇒ Presence of U_{PMNS} shows some connection with low-energy CP violation. However, any value of PMNS phase can make leptogenesis work, so one can only make model-dependent statements.

⇒ See plots from Ref. [13], here one can assume R real (CP conserved in RH sector), so that Baryon asym. is generated only by low energy phases.
(With other assumptions about neutrino mass spectrum etc.)

⇒ This framework is further complicated by Type II seesaw (Tibor) ie, if R is real

$$\epsilon_\alpha \propto \sum_{\beta,\rho>\beta}' \sqrt{m_\beta m_\rho}\,(m_\rho - m_\beta)\underbrace{R_{1\beta}R_{1\rho}}\,\mathrm{Im}\left(U^*_{\alpha\beta}U_{\alpha\rho}\right)$$

↳ R must be real & diagonal for $\epsilon_\alpha = 0$

→ Can also study cases where R is complex, parameterize it ito three complex rotations

$$R = R_{12}R_{13}R_{23} \quad , \text{with } w_{ij} = \rho_{ij} + i\sigma_{ij}$$

a complex angle

[See ref. 12]

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(5) Conclusion

→ Leptogenesis & seesaw mechanism provide a "nice" explanation for light ν mass & BAU. (adding SUSY leads to gravitino problem)

→ Testable? N_R too heavy / h_{ij} too small, also the η_L is hard to measure (can't even observe relic ν background, so $\mathcal{O}(10^{-10})$ fluctuations hopeless!)

⇒ Indirectly: can look for Majorana nature in $0\nu\beta\beta$

→ LNV, 1st Sakh. condition

OR CPV in ν superbeam experiments.

→ Falsifiable?

⇒ no $0\nu\beta\beta$ → Dirac particles (but can have Dirac leptogenesis... Tibor)

⇒ ν mass scale → KATRIN / cosmology

⇒ if m_ν not in 0.1–0.2 eV range, type I ruled out!?

$\Rightarrow$ LHC

↳ can test EW baryogenesis

↳ SUSY or no SUSY!

↳ limit / constrain leptogenesis

↳ may play more fundamental role in determining m_ν mechanism? ; perhaps rule out type I?

(*) However, seesaw (type I) is perhaps the most natural & straightforward explanation of η_B and m_ν, but with limited predictive power.