Some Comments on the LFV Lectures

Toshihiko Ota[1]
[1] *Max–Planck–Institut für Kernphysik,*
Postfach 10 39 80, 69029 Heidelberg, Germany[*]

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I would like to make some small comments about the LFV lectures 27 Feb - 1 Mar 2007.

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I. DIRECT AND INDIRECT CP VIOLATION

I would like to give one example for the direct and indirect CP violation in the Keon system following the Michael's explanation. First of all, some definitions: $K^0(\bar{s}d)$ and $\bar{K}^0(s\bar{d})$ are the CP conjugate states;

$$\mathrm{CP}|K^0\rangle = -|\bar{K}^0\rangle, \tag{1}$$

where the minus sign is just coming from the convention. The CP eigenstate of the Keon system is made as

$$|K_{\mp}\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\bar{K}^0\rangle\right). \tag{2}$$

Since the two(three)-π mode is CP-even(-odd), if CP is conserved, K_- cannot decay into 2π mode. This makes the life-time of K_- be longer than that of K_+ because the phase space of the 3π decay mode is much smaller than that of the 2π's. Therefore, we can identify the long-life Keon (K_L) is the CP odd state if CP is conserved.

Here, we consider the leptonic decay of K_L: it can decay into $\pi^- e^+ \nu_e$ and $\pi^+ e^- \bar{\nu}_e$. If the CP is conserved, $K_L(= K_-)$ includes K_0 and $\bar{K}_0$ with the same weight $(1/\sqrt{2})$ and the decay amplitude of the CP conjugate processes $\langle \pi^- e^+ \nu_e | \mathcal{H}_{\mathrm{int}} | K^0 \rangle (\equiv \mathcal{A}_+)$ and $\langle \pi^+ e^- \bar{\nu}_e | \mathcal{H}_{\mathrm{int}} | \bar{K}^0 \rangle (\equiv \mathcal{A}_-)$ are the same. Therefore, the decay rate must be the same in the both processes;

$$|K_L\rangle = |K_-\rangle = \begin{cases} \frac{1}{\sqrt{2}}|K^0\rangle \xrightarrow{\mathcal{A}_+} \pi^- e^+ \nu_e \\ \frac{1}{\sqrt{2}}|\bar{K}^0\rangle \xrightarrow{\mathcal{A}_- = \mathcal{A}_+} \pi^+ e^- \bar{\nu}_e \end{cases}. \tag{3}$$

The CP conservation concludes the fact

$$\Gamma(K_L \to \pi^- e^+ \nu_e) = \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e). \tag{4}$$

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[*] Electronic address: `toshi@mpi-hd.mpg.de`

However, the CP symmetry is violated and actually the violation of the above relation is established experimentally. As you can see in Eq.(3), there are two ways to violate the relation Eq.(4); (i) K_L includes K^0 and $\bar{K}^0$ with different weight, i.e., K_L is no longer the CP eigenstate K_-, and (ii) the decay amplitude $\mathcal{A}_+$ is not the same as $\mathcal{A}_-$[1];

$$|K_-\rangle \neq |K_L\rangle = \begin{cases} p|K^0\rangle \xrightarrow{\mathcal{A}_+} \pi^- e^+ \nu_e \\ q(\neq p)|\bar{K}^0\rangle \xrightarrow{\mathcal{A}_- \neq \mathcal{A}_+} \pi^+ e^- \bar{\nu}_e \end{cases} . \tag{5}$$

the effect coming from (i) ($p \neq q$) is called as *the indirect CP violation*, and that of (ii) ($\mathcal{A}_+ \neq \mathcal{A}_-$) is called as *the direct CP violation*, i.e., the indirect CP concerns with the mixing (oscillation) stuffs, and the direct CP relates to the decay amplitudes.

In the Keon system, the indirect CP violation dominates whole the CP violation effect (the ratio between the direct and the indirect CP effect is mentioned as ϵ'/ϵ and it is very small[1]), therefore, people first tried to explain the CP violation only by modifying the K^0-$\bar{K}^0$ mixing long long ago, e.g., the superweak model, it *was* maybe reasonable to introduce the *not-yet-found* third generation and the CKM matrix at the time.

II. PARAMETERS OF THDM

Since Alex explained quite nicely, I would like to add only one thing.

The MSSM is a kind of the Two-Higgs-Doublet Model (THDM) but a constrained THDM. In the MSSM (at the tree level), the Higgs sector is characterized with only three parameters, vev, $\tan\beta$, and the CP odd Higgs mass m_A, i.e., the masses of m_h m_H, the mixing angle $\sin(\alpha-\beta)$ etc are determined by the three parameters because the Higgs quartic couplings are written by the gauge couplings and they are related with each other. Many observations suggest the decoupling limit $\sin(\alpha-\beta) \xrightarrow{m_A\to\infty} -1$ if the MSSM is true.

In the THDM, all the quadratic couplings (masses) m_{1-3} and the quartic couplings λ_{1-5} are the independent parameters, and these eight parameters are re-written by the physical parameters, i.e., vev, $\tan\beta$, $\sin\alpha$, masses of the Higgs bosons $m_{h,H,A,H^\pm}$, and one additional mass parameter which is proportional to the mass of the $\mathbb{Z}_2$ symmetry violation (concretely the mass parameter of the $\Phi_u^\dagger\Phi_d$ term). Therefore, the non-decoupling limit $\sin(\alpha-\beta) \sim -1$ in the THDM could survive by adjusting the mass parameters independently. The constraints for the THDM parameters are very mild, and they are constrained by, e.g., the perturbativity (no too large λ's), the vacuum stability (the potential must not fall down at somewhere faraway) and so on. In both the MSSM and the THDM, the $b \to s\gamma$ process constrains the mass of the charged Higgs boson. Since the $H^\pm$-loop picks up the CKM mixing and induce the $b \to s\gamma$ process, even when we do not introduce the additional flavour violating interaction in the quark sector. To avoid this, we have to put the charged Higgs mass more than about 350 GeV or so[2]. In the MSSM, this is also realized only in the decoupling limit because $m_H^\pm \sim m_A$.

III. LFV IN MSSMNR

In the MSSM with right-handed neutrinos, only the source of the lepton flavour violation is the mixing of the left-handed slepton mass matrix $(m^2_{\tilde{l}})_{\alpha\beta}$ which is induced by the neutrino Yukawa coupling Y_ν, as Andreas explained. This is actually the reason why only the diagram which contains only one of the chirality component of the initial muon (shown as the left-plot in Fig.1) can affect the $\mu \to e\gamma$ process. However, you may have a complain: we can also draw the diagram which contains the other chirality component of the muon by using the mixing of $(m^2_{LL})_{e\mu} \simeq (m^2_{\tilde{l}})_{e\mu}$ which is shown as the right-plot in Fig.1, and it looks similar to the diagram which can contribute. Why does not this diagram contribute? — They are different in the sense of *the chirality flip* of the slepton[3]. The flavour structure of the m^2_{LR} term is determined by the A-term and the Yukawa matrix for the charged lepton, i.e., it is proportional to the Yukawa coupling of the charged lepton. Therefore, we have to pick up the chirality flip term as a s-muon, not

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[1] Equation (5) is maybe too much simplified. From this, you may think that since the decay amplitudes $\mathcal{A}_\pm$ are different only in the phase, it cannot affect the calculation of the decay rate (=the absolute value square of the amplitude). In the calculation of the decay rate of $K_L \to \pi^- e^+ \nu_e$, the interference term between $K_L \to K^0 \to \pi^- e^+ \nu_e$ and $K_L \to \bar{K}^0 \xrightarrow{\text{osc}} K^0 \to \pi^- e^+ \nu_e$ appears and it depends on the phase.

[2] In the MSSM, it is possible that the loop including a s-top cancels out the charged-Higgs loop, and this constraint could be relaxed.

[3] Here *the chirality* means the chirality of the superpartner (fermion) field.

a s-electron (see e.g. Fig.2 and Eq.(26) in Ref.[2]). This is similar to the SM case in which the chirality flip is done out side of the loop by picking up the muon mass, not electron mass.

You may further have a question, can the diagram in which the chirality flip is done outside of the loop by the muon mass contribute to this process? It can contribute and it also contains only μ_R (in the $m_e = 0$ limit), but in many cases it becomes sub-dominant. More details are shown in Ref.[2], and Ref.[3] includes much more but maybe too much.

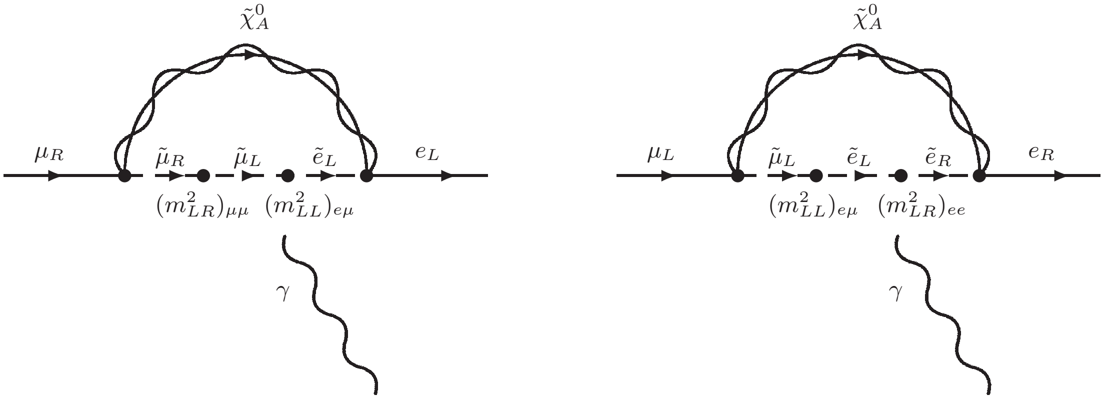

FIG. 1: Diagram contributes to the $\mu \to e\gamma$ process in the MSSMNR (left), and that does not contribute (right). Here the arrows denote the direction of the particle number's current, not the chirality like the figures in Ref.[2].

IV. MSUGRA BOUNDARY CONDITION

When people study the LFV in the SUSY(-GUT) models, almost all the people adopt the mSUGRA scenario. However, as far as I understand, it is just an assumption. The slepton mass matrix is induced by the effective interaction with the hidden sector as

$$c_{ij} \frac{Z^\dagger Z}{M_*^2} L_i^\dagger L_j \Big|_{\theta^2 \bar{\theta}^2} \xrightarrow{\langle Z \rangle = F_Z} (m^2_{\tilde{l}})_{ij} = c_{ij} \frac{|F_Z|^2}{M_*^2} \tag{6}$$

where Z is the chiral superfield living in the Hidden sector and the F-term of it takes vev F_Z and breaks the SUSY, and the breaking is mediated by some messenger whose mass is M_*. Sometimes, people say, in the case where the SUSY breaking is mediated by the gravity, the gravity is the flavour blind ... but even in the gravity mediation case, the soft mass could obtain the complicated flavour structure, e.g., Eq.(31.7.44)-(31.7.48) and the paragraph above Eq.(31.7.52) in Ref.[4]. Furthermore, maybe we can say, if the SUSY breaking is mediated by really *flavour blind* something, then the flavour structure of the slepton mass matrix could be democratic (which is also rank 1 matrix), not necessary to be proportional to unity.

However, it is also true that there are some ways to realize the mSUGRA type flavour structure. For example, if gauge bosons mediate the SUSY breaking, since the generation means just a copy for the gauge interactions, they can mediate the braking with the same way in each flavour and it could realize the flavour unity structure. Some people consider that the SUSY breaking is propagated from the other brane from ours (the *sequestered* hidden sector), to induce the flavour unity structure... although I am not sure. We cannot say which one is preferable, the SUSY breaking mechanism itself is a controversial topic. Then, you may think that the prediction with the mSUGRA type boundary condition seems to be meaningless. However, we can maybe say, the mSUGRA scenario gives the bottom-line of the magnitude of the LFV process in the SUSY(-GUT) model. The bottom-line means, even if the flavour violation is strongly suppressed by some mechanism at the fundamental scale, if the SUSY(-GUT) is true, then the LFV should appear at least with the magnitude which the mSUGRA scenario predicts. Therefore, no LFV is not fatal for the SUSY(-GUT), but it could be one of the circumstantial evidence for the non-SUSY(-GUT). It is like the proton decay bound in the GUT.

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