Slides of the talk "Gauge-Higgs Unification"

$SU(3)$ Generators and Gauge Fields

Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

gauge fields in $SU(3)_W$:

$$\frac{1}{2} A_\mu^a \lambda^a = \frac{1}{2} \left(\begin{array}{cc:c} A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & A_\mu^1 - i A_\mu^2 & A_\mu^4 - i A_\mu^5 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & A_\mu^6 - i A_\mu^7 \\ \hdashline A_\mu^4 + i A_\mu^5 & A_\mu^6 + i A_\mu^7 & -\frac{2}{\sqrt{3}} A_\mu^8 \end{array} \right)$$

Derivation of Effective Potential

Calculation of 1-loop effective potential (according to Antoniadis et al)

take

$$V_{eff}(\phi) = \frac{1}{2} \sum_I (-1)^{F_I} \int \frac{d^4p}{(2\pi)^4} \log\left[p^2 + M_I^2(\phi)\right]$$

Schwinger representation

$$V_{eff}(\phi) = -\frac{1}{2} \sum_I (-1)^{F_I} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4} e^{-t\left[p^2 + M_I^2(\phi)\right]}$$

$$= \frac{-1}{32\pi^2} \sum_I (-1)^{F_I} \int_0^\infty \frac{dt}{t^3} e^{-t M_I^2(\phi)}$$

$$= \frac{-1}{32\pi^2} \sum_I (-1)^{F_I} \int_0^\infty dl\, l\, e^{-M_I^2(\phi)/l}$$

with change of variables $t = 1/l$

5D case: $M_I^2(\phi) \to \left(\frac{m + a^I(\phi)}{R}\right)^2$

$$V_{eff}(\phi) = -\sum_I \sum_m (-1)^{F_I} \frac{1}{32\pi^2} \int_0^\infty dl\, l\, e^{-\frac{(m + a^I(\phi))^2}{R^2 l}}$$

- commute integral with sum over KK states
- perform Poisson resummation

$$V_{eff}(\phi) = -\sum_I (-1)^{F_I} \frac{R}{32\pi^{5/2}} \sum_m e^{2\pi i m a^I(\phi)} \int_0^\infty dl\, l^{3/2} e^{-\pi^2 l m^2 R^2}$$

- note: $m = 0$ is divergent, but independent of ϕ ⇝ not relevant

for $m \neq 0$: change variables: $l' = \pi^2 l m^2 R^2$

- integrate over l': finite ϕ-dependent result

$$V_{eff}(\phi) = -\sum_I (-1)^{F_I} \frac{\Gamma(5/2)}{32\, \pi^{13/2}} \cdot R \sum_{m \neq 0} \frac{e^{2\pi i m a^I(\phi)}}{m^5 R^5}$$

for example for massive 5D fermion (without bulk mass)

$$V_{eff}^{\psi}(\phi) \sim \frac{1}{\pi^6 R^4} \sum_{m=1}^{\infty} \frac{1}{m^5} \cos\left(2\pi m a^I(\phi)\right)$$

Effective Potential

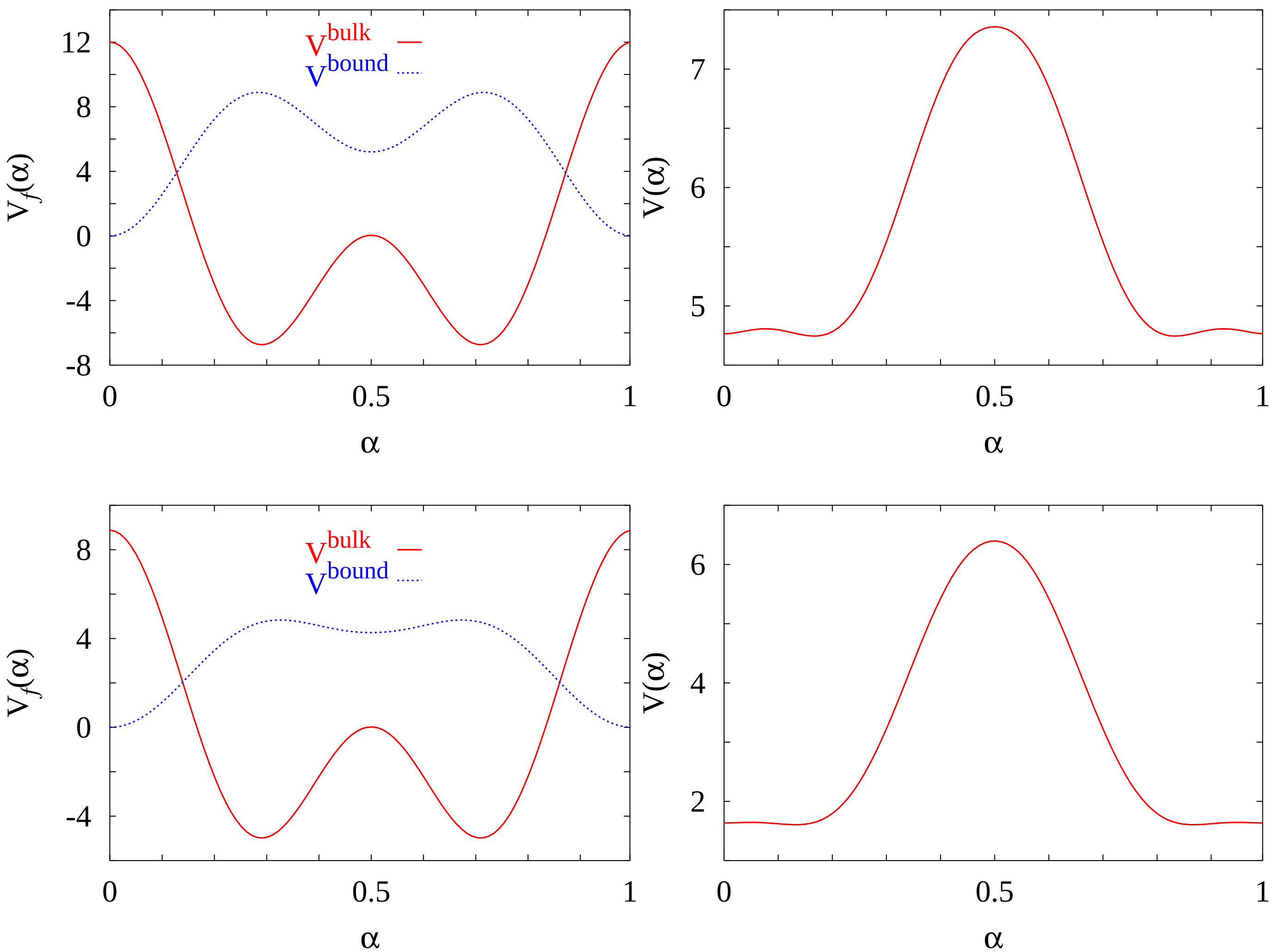

Figure 1: Different contributions to the effective potential (in arbitrary units): the bulk and boundary fermion contributions (upper left) and the full potential (upper right) for $\lambda = 1.57$, $\epsilon_1 = 3.1$, $\epsilon_2 = 0.7$ and $\delta = 0$; the bulk and boundary fermion contributions (lower left) and the full potential (lower right) for $\lambda = 1.83$, $\epsilon_1 = 6.4$, $\epsilon_2 = 6.1$ and $\delta = 1$.
Taken from *C.A. Scrucca et al.*.

... with high-rank bulk fermions

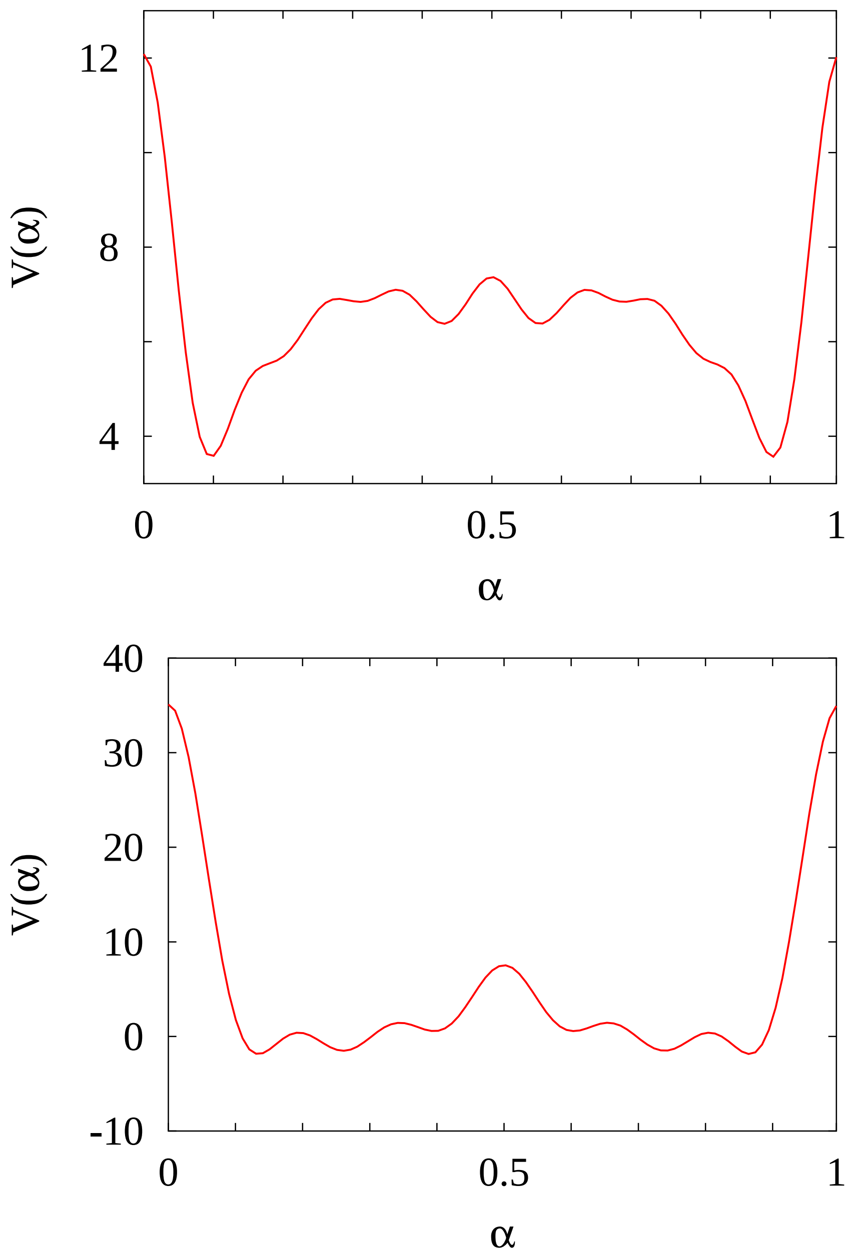

Figure 2: The full effective potential (in arbitrary units) in the presence of high-rank bulk fermions. Left: $r = 8$, $\lambda = 3.47$, $\epsilon_1 = \epsilon_2 = 9$ and $\delta = 0$, resulting in $m_H = 112$ GeV and $1/R = 830$ GeV. Right: $r = 6$, $\lambda = 2.23$, $\epsilon_1 = 7$, $\epsilon_2 = 1$ and $\delta = 1$, resulting in $m_H = 104$ GeV and $1/R = 600$ GeV. Taken from *C.A. Scrucca et al.*.

... with localized gauge kinetic terms

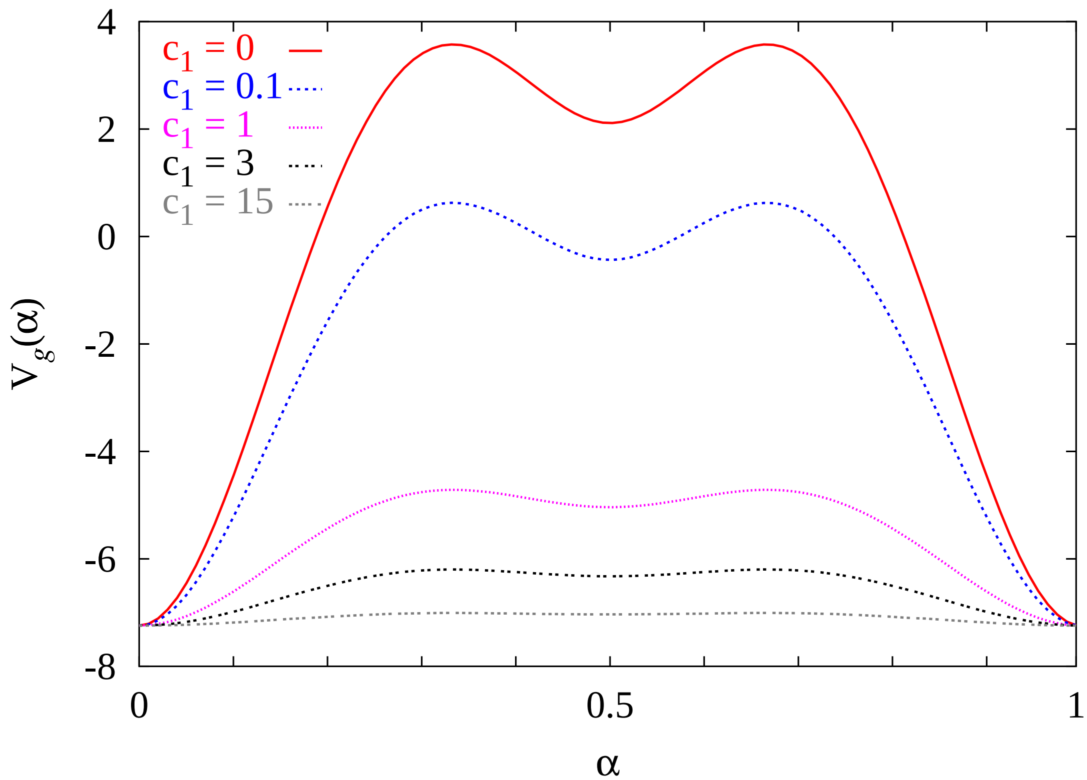

Figure 3: Gauge contribution to the effective potential (in arbitrary units) in the presence of localized gauge kinetic terms, with $c_2 = 0$ and increasing values of c_1.
Taken from *C.A. Scrucca et al.*.

Effective potential with anti-periodic fermions

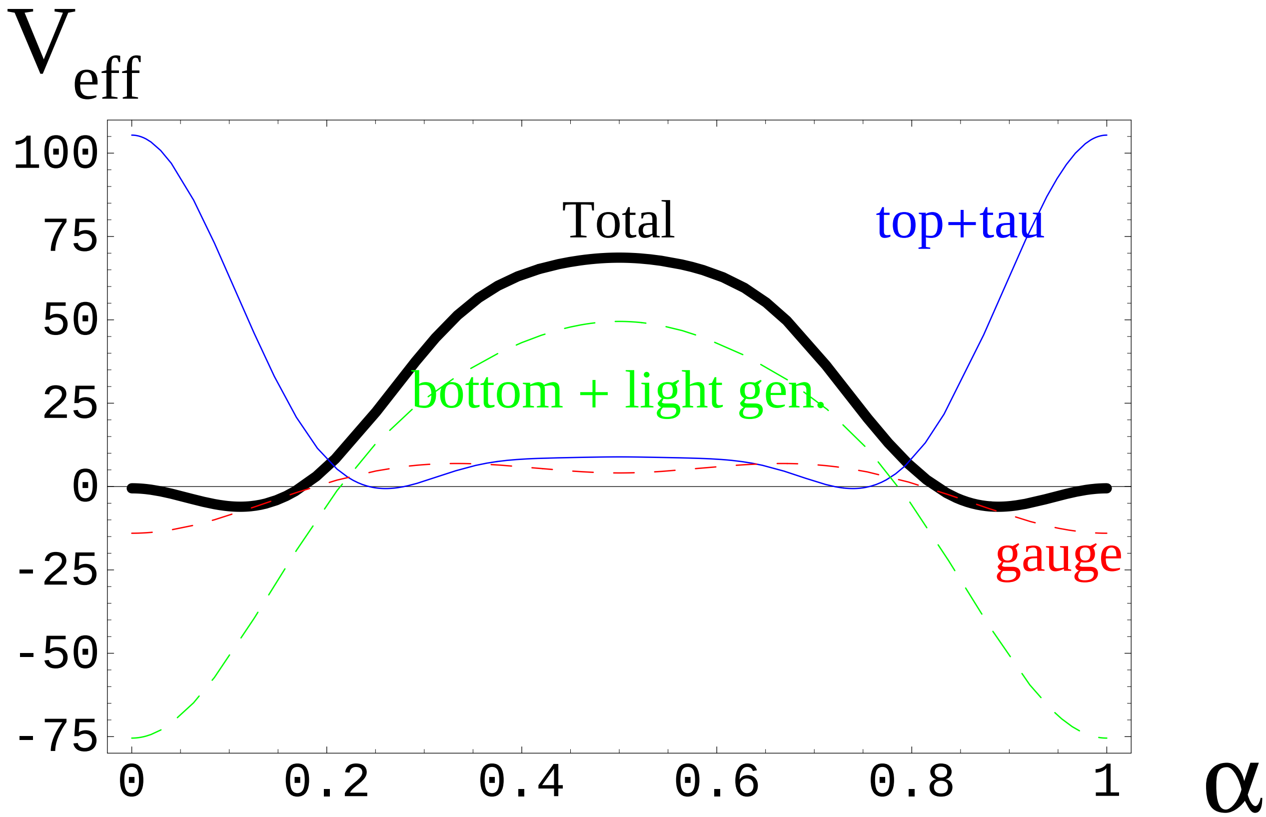

Figure 4: Plot of the Higgs potential (in arbitrary units) from the gauge bosons (dashed-red), tau and top (blue), twisted fermions (dashed-green), and the total (**thick black**), for one light generation with $\kappa_l = 3$. The other parameters are like described in the text. Taken from *G. Cacciapaglia et al.*.

... with large $SU(3)$ representation for top

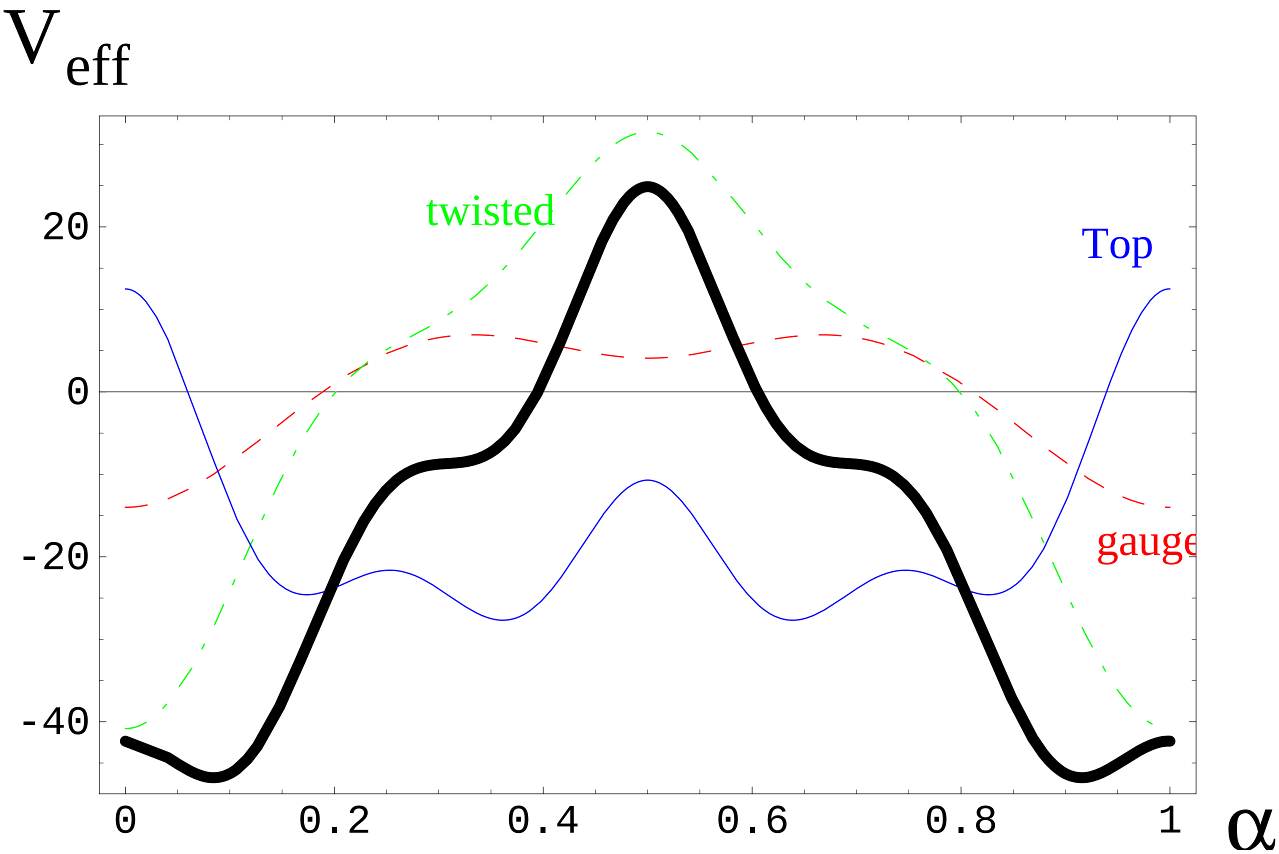

Figure 5: Plot of the Higgs potential (in arbitrary units) from the gauge bosons (red-dashed), top (blue), bottom (**3**) and tau (**10**) (green-dashed), and the total (**thick black**), for $\epsilon = 1.25$.
Taken from *G. Cacciapaglia et al.*.

Effective Potential with $SO(4,1)$ Lorentz violation (and anti-periodic fermions)

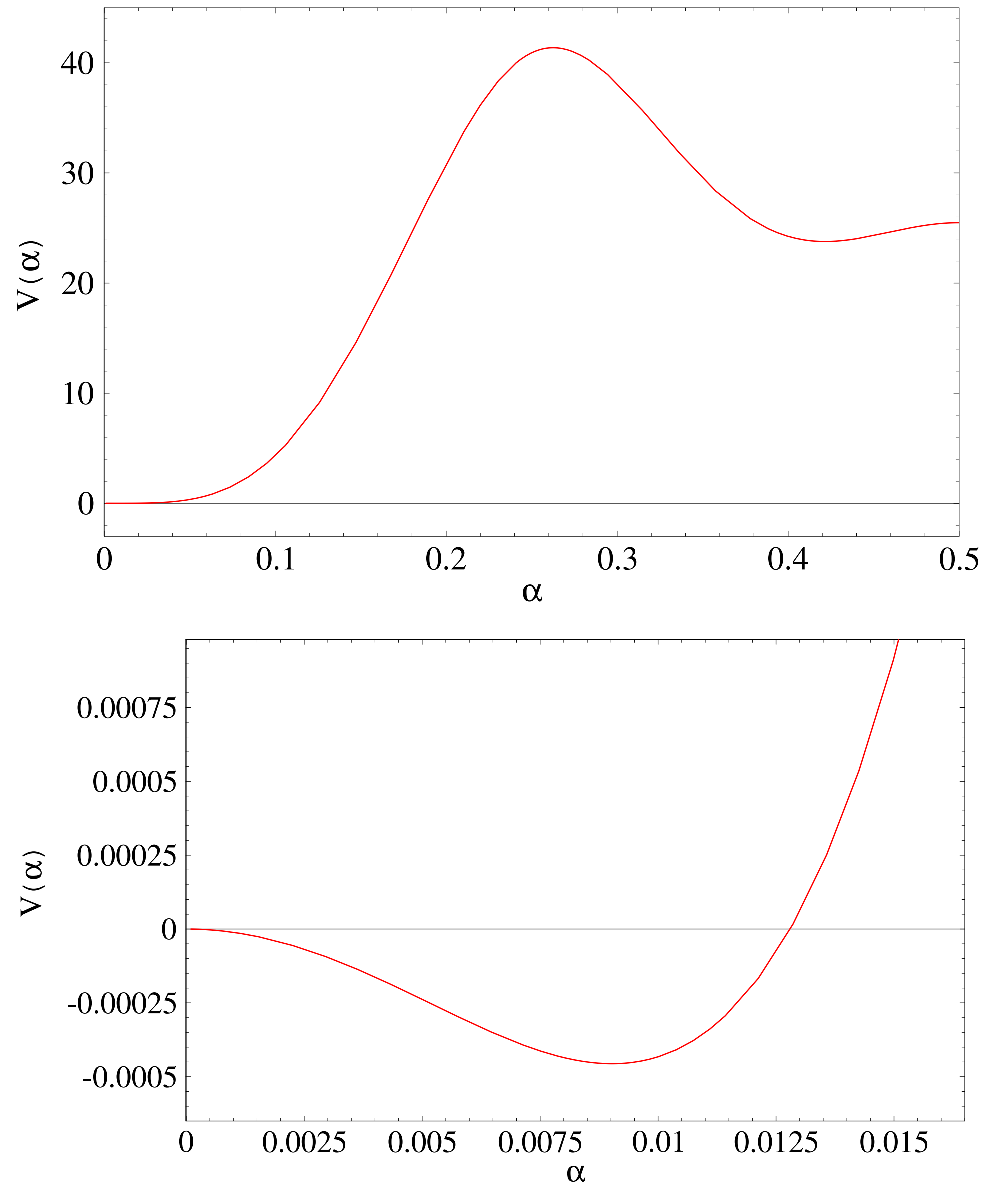

Figure 6: The Higgs potential in the $\delta = 0$ set-up, obtained with input parameters $\lambda^t = 0.99$, $\lambda^b = 6.9$, $\lambda^A = 0.24$, $k_t = 2.42$, $k_b = 2.26$, $k_A = 3.14$, $\varepsilon_1^t = 1.9$, $\varepsilon_2^t = 1.6$, $\varepsilon_1^b = 2.9$, $\varepsilon_2^b = 3.4$. Taken from *G. Panico et al.*.

χ^2 fit on the EWPT for $SU(3)_w$ model with Z_2 symmetry

Taken from *G. Panico et al.*.

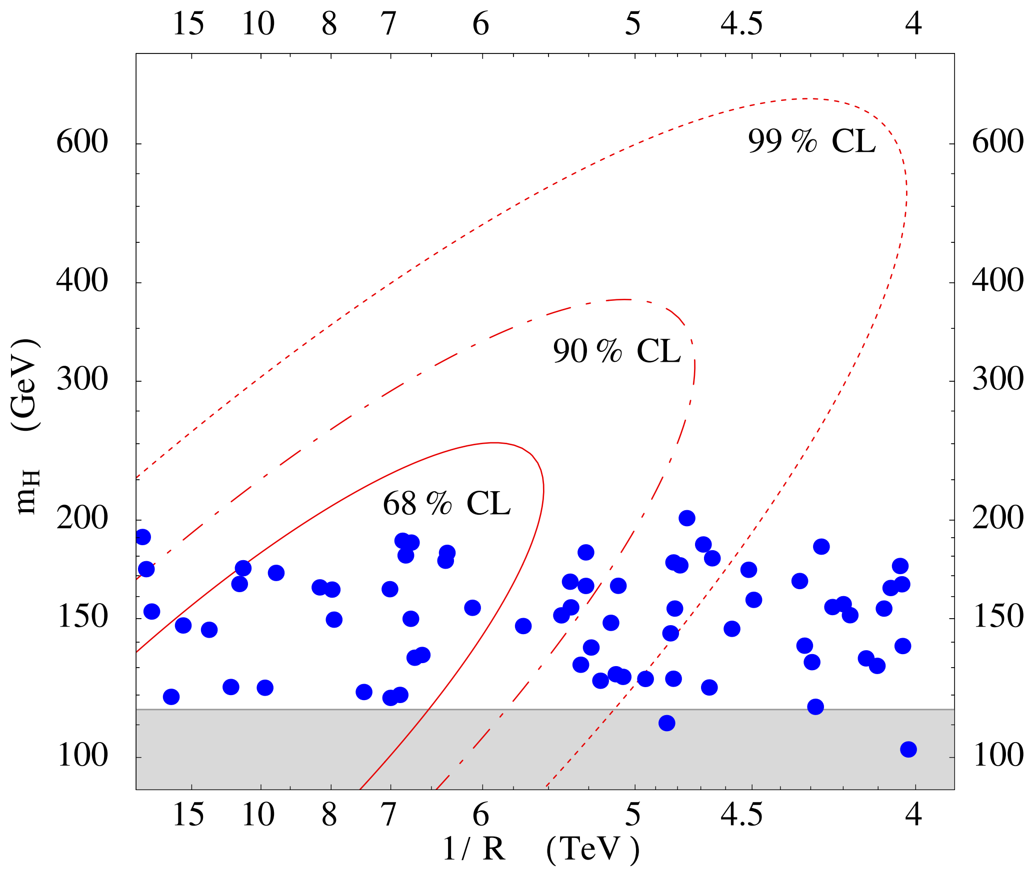

Figure 7: Constraints coming from a χ^2 fit on the EWPT. The contours represent the allowed regions in the $(1/R, m_H)$ plane at 68%, 90% and 99% confidence level (2 d.o.f.). The shaded band shows the experimentally excluded values for the Higgs mass ($m_H < 115$ GeV). The blue dots represent the predictions of our model for different values of the microscopic parameters (only points with the correct top and bottom masses are plotted).

Introduction to EDs

- T. G. Rizzo, *Pedagogical Introduction to Extra Dimensions*, hep-ph/0409309.
- M. Quiros, *New Ideas in Symmetry Breaking*, hep-ph/0302189.
- C. Csaki, *TASI lectures on extra dimensions and branes*, hep-ph/0404096.
- R. Sundrum, *To the fifth Dimension and Back*, hep-th/0508134.

Models of Gauge-Higgs Unification

- Ignatios Antoniadis, K. Benakli, M. Quiros, *Finite Higgs mass without supersymmetry*, New J. Phys. **3**, 20 (2001), hep-th/0108005: contains derivation of the effective potential.
- Claudio A. Scrucca, Marco Serone, Luca Silvestrini, *Electroweak symmetry breaking and fermion masses from extra dimensions*, Nucl. Phys. B **669**, 128 (2003), hep-ph/0304220: presents the $SU(3)_w$ model in 5 dimensions with the gauge-Higgs unification, treats the problem of the Higgs potential in the minimal model as well as in extension with large $SU(3)_w$ representations in the extra dimension and brane localized kinetic terms for the gauge fields.
- Giacomo Cacciapaglia, Csaba Csaki, Seong Chan Park, *Fully radiative electroweak symmetry breaking*, JHEP **0603**, 099 (2006), hep-ph/0510366: also studies the $SU(3)_w$ model in 5 dimensions and the Higgs potential; comes to the same conclusion as the paper above and tries to improve the Higgs potential, i.e. α small and $m_H > 115\,\text{GeV}$, and the top mass prediction with the use of anti-periodic fermions and large representations in the bulk which couple to the top which is mainly a bulk fermion in this setup.
- Giuliano Panico, Marco Serone, Andrea Wulzer, *A Model of electroweak symmetry breaking from a fifth dimension*, Nucl. Phys. B **739**, 186 (2006), hep-ph/0510373: study of $SU(3)_w$ including $SO(4,1)$ Lorentz violating kinetic terms for the bulk fermions (and anti-periodic fermions)
- Giuliano Panico, Marco Serone, Andrea Wulzer, *Electroweak Symmetry Breaking and Precision Tests with a Fifth Dimension*, Nucl. Phys. B **762**, 189 (2007), hep-ph/0605292: analysis of EWPO in the most elaborate $SU(3)_w$ model with the conclusion that the compactification scale $1/R$ needs to be larger than $4.7\,\text{TeV}$ and the Higgs mass (accidentally) turns out to be smaller than $200\,\text{GeV}$ all the time.
- further literature, for example:
  - Nobuhito Maru, Kazunori Takenaga, *Effects of bulk mass in gauge-Higgs unification*, Phys. Lett. B **637**, 287 (2006), hep-ph/0602149: some general study about the role of the bulk mass for the Higgs potential in the $SU(3)_w$ model.

– Naoyuki Haba, Yutaka Hosotani, Yoshiharu Kawamura, Toshifumi Yamashita, *Dynamical symmetry breaking in gauge Higgs unification on orbifold*, Phys. Rev. D **70**, 015010 (2004), hep-ph/0401183: some more general calculation of the Higgs potential in the $SU(3)_w$ as well as in a GUT $SU(6)$ model.

– C.S. Lim, Nobuhito Maru, *Calculable One-Loop Contributions to S and T Parameters in the Gauge-Higgs Unification.*, Phys. Rev. D **75**, 115011 (2007), hep-ph/0703017: some calculation of S and T parameter.

– G. Martinelli, M. Salvatori, C.A. Scrucca, L. Silvestrini, *Minimal gauge-Higgs unification with a flavor symmetry*, JHEP **0510**, 037 (2005), hep-ph/0503179: some $SU(3)_w$ model with a flavor symmetry $SU(2)_F$ in the bulk.

– Nikos Irges, Francesco Knechtli, Magdalena Luz, *Higgs mechanism in five-dimensional gauge theories*, 0709.4549 [hep-lat] : study the effective potential for the Higgs field with an explicit cutoff as well as on the lattice and compare their results with the usual one from the one-loop Coleman-Weinberg potential. This paper contains further references to the works of the authors.