Dark Matter in the MSSM: Contents

(A. Merle)

Dark Matter in the MSSM (A. Merle)

1. Introduction:

- supersymmetry (SUSY) is the "classical" example for a theory beyond the Standard Model (SM)
- some reasons for its popularity:
  - the maximal symmetry a theory can have together with Poincaré symmetry and internal symmetries is one that exchanges fermions and bosons (Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula theorem)
  - provides a solution for the hierarchy problem (quadratic divergence of the Higgs mass)
  - R-parity conservation ⇒ provides Dark Matter candidates ⇒ later
- MSSM: minimal supersymmetric standard model

2. A short review of the MSSM:

- basic principle of SUSY: bosons ⇔ fermions
- building blocks:
  - chiral supermultiplets: complex scalar ϕ ⇒ spin 0
    Weyl fermion ψ ⇒ spin $\frac{1}{2}$
    complex auxiliary field F
    - used for: quarks, leptons, Higgses
    - maximal Lagrangian allowed by SUSY:

$$\mathcal{L}_{chiral} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

↳ with: (just labels the fields)

$$\mathcal{L}_{free} = -(\partial^\mu \phi^{i*})(\partial_\mu \phi_i) - i\psi^{i\dagger}\bar{\sigma}^\mu \partial_\mu \psi_i + F^{i*}F_i\,, \quad \bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$$

$$\mathcal{L}_{int} = \left(-\frac{1}{2}\frac{\delta^2 W}{\delta\phi_i\,\delta\phi_j}\psi_i\psi_j + \frac{\delta W}{\delta\phi_i}F_i\right) + c.c.$$

- superpotential: $W = L^i\phi_i + \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$

  if ϕ_i is a singlet

②

× integrating out the auxiliary field $\Rightarrow$ E.O.M.: $F_i = -\frac{\partial W}{\partial \phi^{i*}}$

× example: fermion & scalar mass for only one ϕ and one ψ

□ fermion mass in: $-\frac{1}{2}\frac{\partial^2 W}{\partial \phi^2}\psi^2$, $\frac{\partial^2 W}{\partial \phi^2} = \frac{\partial^2}{\partial \phi^2}\left(\frac{1}{2}M\phi^2\right) + \ldots = M + \ldots$

$\Rightarrow$ mass term: $-\frac{1}{2}M\psi^2$

$\Rightarrow$ fermion has mass M

□ boson mass in: $\frac{\delta W}{\delta \phi}F = -\left|\frac{\partial W}{\partial \phi}\right|^2 = -|M\phi + \ldots|^2 = -M^2\phi^2$

$\Rightarrow$ boson has mass M

□ BUT: this is not found in nature $\Rightarrow$ SUSY must be broken

• gauge supermultiplets: massless gauge boson $A^a_\mu \Rightarrow$ spin 1
2-component Weyl "gaugino" $\lambda^a \Rightarrow$ spin $\frac{1}{2}$
real auxiliary field D^a (adjoint rep., too)

× used for: gauge bosons

× maximal lagrangian allowed by SUSY:

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} - i\lambda^a\bar{\sigma}^\mu D_\mu\lambda^a, \quad D_\mu = \partial_\mu - igA^a_\mu T^a$$

× covariant derivatives $\Rightarrow$ mixing terms:

$$\mathcal{L}_{mix} = -\sqrt{2}g\,(\phi^* T^a\psi)\lambda^a - \sqrt{2}g\,\lambda^{a\dagger}(\psi^\dagger T^a\phi) + g(\phi^* T^a\phi)D^a$$

generators of the gauge group

• important quantity: scalar potential ($\mathcal{L} = -V$!)

$$V(\phi_i, \phi_i^*) = F^{i*}F_i + \frac{1}{2}\sum_a D^a D^a = \frac{\partial W}{\partial \phi^{i*}}\frac{\partial W}{\partial \phi_i} + \frac{1}{2}g_a^2(\phi^{i*}T^a\phi_i)^2$$

"F-term" "D-term"

• soft breaking terms: involve only superpartners $\Rightarrow$ break SUSY

× these terms parameterize the lack of knowledge of the exact SUSY-breaking mechanism

③ • soft: only logarithmically divergent corrections to the Higgs mass

* explicit:

$$\mathcal{L}_{soft} = -\Big(\underbrace{\tfrac{1}{2} M_a \lambda^a \lambda^a}_{\text{gaugino masses}} + \underbrace{\tfrac{1}{6} a^{ijk} \phi_i \phi_j \phi_k}_{\text{scalar trilinear couplings}} + \underbrace{\tfrac{1}{2} b^{ij} \phi_i \phi_j}_{\text{scalar mixing}} + \underbrace{t^i \phi_i}_{\text{tadpole (if } \phi = \text{singlet)}}\Big) + c.c. - \underbrace{(m^2)^i_{\ j}\, \phi^{j*} \phi_i}_{\text{scalar masses}}$$

• particle content of the MSSM: SM + $\boxed{\text{Table 7.1 from Ⓢ}}$

3. SUSY renormalisation group equations (RGEs):

- like for every theory, the SUSY-parameters depend on the energy scale
- parameter:

$$t \equiv \ln\left(\frac{Q}{Q_0}\right)$$

Q: renormalization scale

Q_0: reference scale

↳ $t = 0$ for $Q = Q_0$

- example RGEs (1-loop):

• gauge couplings: $$\frac{d}{dt} g_a = \frac{b_a}{16\pi^2} g_a^3$$ $\left(b_a = \left(\frac{33}{5}, 1, -3\right) \text{ for } (U(1), SU(2), SU(3))\right)$

• gaugino masses: $$\frac{d}{dt} M_a = \frac{b_a}{8\pi^2} g_a^2 M_a$$

• scalar soft masses: complicated...

- explicit: gaugino masses

• gauge couplings: $$\int_{g_a(0)}^{g_a(t)} \frac{dg_a}{g_a^3} = \frac{b_a}{16\pi^2} \int_0^t dt \Rightarrow \left[-\frac{1}{2} g_a^{-2}\right]_{g_a(0)}^{g_a(t)} = \frac{b_a}{16\pi^2} t$$

$$\Rightarrow -\frac{1}{2}\left(g_a^{-2}(t) - g_a^{-2}(0)\right) = \frac{b_a}{16\pi^2} t \Rightarrow \frac{1}{g_a^2(0)} - \frac{1}{g_a^2(t)} = \frac{b_a}{8\pi^2} t$$

$$\Rightarrow \frac{1}{g_a^2(0)} - \frac{b_a}{8\pi^2} t = \frac{1}{g_a^2(t)} \Rightarrow g_a^2(t) = \frac{g_a^2(0)}{1 - \frac{b_a}{8\pi^2} g_a^2(0) t}$$

$$\Rightarrow \boxed{g_a(t) = \frac{g_a(0)}{\sqrt{1 - \frac{b_a}{8\pi^2} g_a^2(0)\, t}}}$$

3. R-parity & lightest supersymmetric particle (LSP):

- idea: all SM-particles (including Higgs scalars and graviton) have R-parity (+1), while all superpartners have (−1)
- reason for this introduction: by introducing R-parity conservation, one gets rid of dangerous B- & L-violating generators that could potentially violate proton decay if their coefficients are not unnaturally small
  ↳ p^+-decay is not seen in experiments
- bonus: if R-parity is conserved, the LSP is stable!
  $\Rightarrow$ ideal Dark Matter candidate
- hence: it is important to know which particle can be the LSP

4. SUSY renormalization group evolution (RGE):

- like for every theory, the SUSY-parameters depend on the energy scale
- parameter:

$$t \equiv \ln\left(\frac{Q}{Q_0}\right)$$

Q ← renormalization scale

Q_0 ← reference scale (e.g. GUT)

$\Rightarrow t = 0$ for $Q = Q_0$

- example: gauginos (for 1-loop RGE)
  - gauge couplings:

$$\frac{d}{dt} g_a(t) = \overset{\equiv c_a/2}{\frac{b_a}{16\pi^2}} g_a^3(t) \quad ; \quad c_a \equiv \frac{b_a}{8\pi^2}$$

↳ $b_a = \frac{33}{5}, 1, -3$ for $U(1), SU(2), SU(3)$

↳ solution:

$$\int_{g_a(0)}^{g_a(t)} g_a^{-3}\, dg_a = \frac{c_a}{2}\int_0^t dt$$

$$\Rightarrow -\frac{1}{2}\left(g_a^{-2}(t) - g_a^{-2}(0)\right) = \frac{c_a}{2} t \Rightarrow \frac{1}{g_a^2(0)} - \frac{1}{g_a^2(t)} = c_a t$$

$$\Rightarrow \frac{1}{g_a^2(t)} = \frac{1 - c_a g_a^2(0) t}{g_a^2(0)} \Rightarrow \boxed{g_a(t) = \frac{g_a(0)}{\sqrt{1 - c_a g_a^2(0) t}}}$$

- gaugino masses:

$$\frac{d}{dt} M_a(t) \overset{!}{=} c_a\, g_a^2(t)\, M_a(t) = -\frac{-c_a g_a^2(0)}{1-c_a g_a^2(0)\,t}\, M_a(t)$$

$$\Rightarrow \int_{M_a(0)}^{M_a(t)} \frac{dM_a}{M_a} = -\int_0^t \frac{-c_a g_a^2(0)}{1-c_a g_a^2(0)\,t}\, dt \Rightarrow \ln\left(\frac{M_a(t)}{M_a(0)}\right) = -\ln\left(1-c_a g_a^2(0)\,t\right)$$

$$\Rightarrow \boxed{M_a(t) = \frac{M_a(0)}{1-c_a g_a^2(0)\,t}}$$

- both together: $\frac{g_a^2(t)}{M_a(t)} = \frac{g_a^2(0)}{M_a(0)}$ (for each gauge group separately)
- unification:
  - gauge couplings: $g_a(0) \equiv g_u$ @ GUT-scale
  - mSUGRA (hypothesis): common fermion masses @ GUT-scale $\Rightarrow M_a(0) = m_{1/2}$

$\Rightarrow$ then: $\forall a: \boxed{M_a(t) = m_{1/2} \cdot \frac{g_a^2(t)}{g_u^2}}$

$\Rightarrow$ the larger the gauge coupling at the weak scale, the heavier the corresponding gaugino

- experiment: $g_1 = 0.46$, $g_2 = 0.65$, $g_3 = 1.22$

$\Rightarrow$ $\boxed{\underbrace{\text{Bino} < \text{Wino}}_{\text{neutral LSP should involve those guys}} \ll \text{Gluino}}$

↳ intuitive reason: larger couplings $\Rightarrow$ loop-corrections to the sparticle mass more important

- sfermions:
  - more complicated $\Rightarrow$ $\boxed{\text{S } (7.59) - (7.67)}$
  - observations:
    - QCD-corrections largest $\Rightarrow$ squarks heavy
    - D-term contributions: smallest for $Q=0$
    - usually lightest: sneutrino (from ν_L or N_R)
- example plot: $\boxed{\text{Figure 7.4 from S}}$

6

- @GUT scale: scalars < gauginos << Higgses (example)
- @weak scale: 1 Light Higgs
  Bino is the lightest
  sneutrino only mildly heavier

5. SUSY-candidates for Dark Matter:

- neutralino:
  - mass eigenstate of Bino, neutral Wino, & Higgsinos
  - different possibilities (e.g. Bino-like LSP)
  - most promising candidate ⇒ might even explain DAMA
- sneutrino:
  - pure MSSM: excluded by direct & indirect searches (O)
  - extensions: still possible
- gravitino:
  - for the inclusion of gravity
  - problem: physics close to the Planck-scale not well known (J)
  - very weak coupling ⇒ very long-lived
    ⇒ barely detectable
- axino:
  - axion-superpartner
  - weak couplings ⇒ long-lived even when heavier than the LSP
  - difficult to detect

6. DAMA & neutralinos: (H)

- DAMA:
  - direct detection experiment
  - method: weakly interacting massive particles (WIMPs) are expected to scatter from the target nuclei in the detector
  - observation: annual modulation of the signal
    ↳ could be explained by the variation of the Earth's velocity with

respect to the Dark Matter halo

⇒ elastically ~~...~~ scattering WIMP of several GeV

- problem: null results by CDMS, CRESST, XENON, ...
  - 2-bin analysis of DAMA ⇒ okay even with these constraints
  - finer binning ⇒ difficult
- further information: Dark Matter annihilation
  - WIMPs can be gravitationally captured in the Sun (and evaporate for small masses)
  - if the annihilate, they will give a high energy neutrino-signal (among others) ⇒ visible in Super K ⇒ can constrain DAMA
- $\boxed{\text{plots}}$: 2BIN-analysis, $\sigma v = 3 \cdot 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$, spin-independent elastic CS
  - original one: hatched ≙ allowed by DAMA → from (H)
  - brown: still allowed by CRESST & Co.
  - purple: rest after SK-constraints (constraints vanish for small WIMP-masses due to evaporation)
- ~~...~~ conclusions for SUSY:
  - MSSM + ~~...~~ neutralino in DAMA-range ⇒ will be overproduced in the early Universe
    ↳ reason: LEP ⇒ Higgses, charginos, sfermions > 100 GeV ⇒ cannot effectively mediate ~~...~~ neutralino annihilation
  - MSSM + singlet: very light (1-10 GeV) neutralinos can be produced in acceptable quantities by exchange of a relatively light pseudo-scalar Higgs (and cross section okay for DAMA)

7. References:

(S) SUSY-primer: hep-ph/9709356

(J) ~~...~~ Jungman et al.: Phys. Rep. 267 (1996) 195-373

(O) Olive: hep-ph/0412054

(H) Hooper et al.: 0808.2464

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

Taken from hep-ph/9709356 (SUSY-Primer).

$$K_1 \approx 0.15 m_{1/2}^2, \qquad K_2 \approx 0.5 m_{1/2}^2, \qquad K_3 \approx (4.5 \text{ to } 6.5) m_{1/2}^2$$

$$\Delta_\phi = (T_{3\phi} g^2 - Y_\phi g'^2)(v_d^2 - v_u^2) = (T_{3\phi} - Q_\phi \sin^2\theta_W)\cos(2\beta)\, m_Z^2$$

$$m_{\tilde{d}_L}^2 = m_0^2 + K_3 + K_2 + \frac{1}{36} K_1 + \Delta_{\tilde{d}_L}$$

$$m_{\tilde{u}_L}^2 = m_0^2 + K_3 + K_2 + \frac{1}{36} K_1 + \Delta_{\tilde{u}_L}$$

$$m_{\tilde{u}_R}^2 = m_0^2 + K_3 \qquad + \frac{4}{9} K_1 + \Delta_{\tilde{u}_R}$$

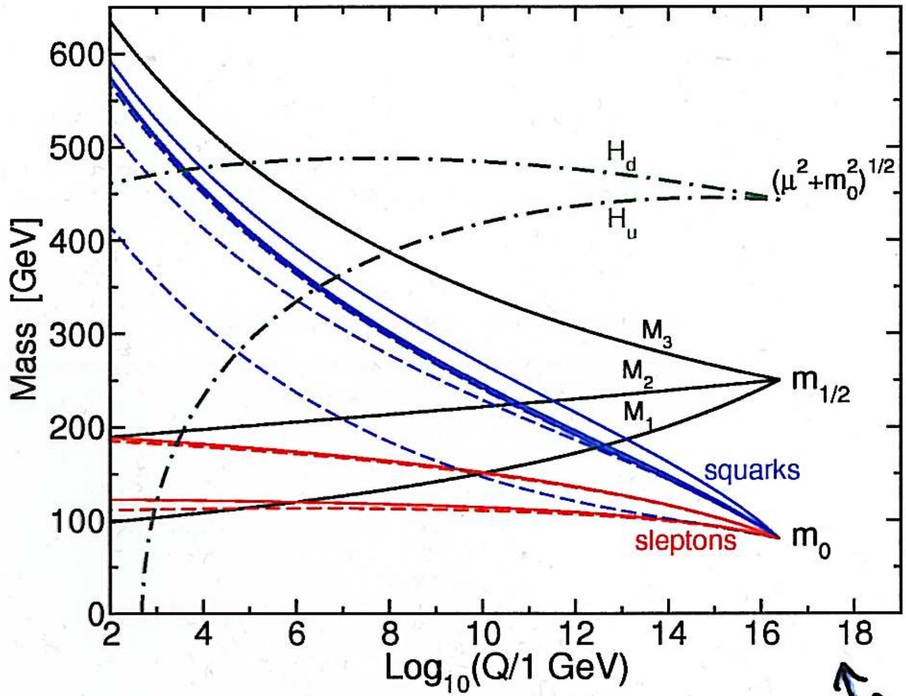
$$m_{\tilde{d}_R}^2 = m_0^2 + K_3 \qquad + \frac{1}{9} K_1 + \Delta_{\tilde{d}_R}$$

$$m_{\tilde{e}_L}^2 = m_0^2 \qquad + K_2 + \frac{1}{4} K_1 + \Delta_{\tilde{e}_L}$$

$$m_{\tilde{\nu}}^2 = m_0^2 \qquad + K_2 + \frac{1}{4} K_1 + \Delta_{\tilde{\nu}}$$

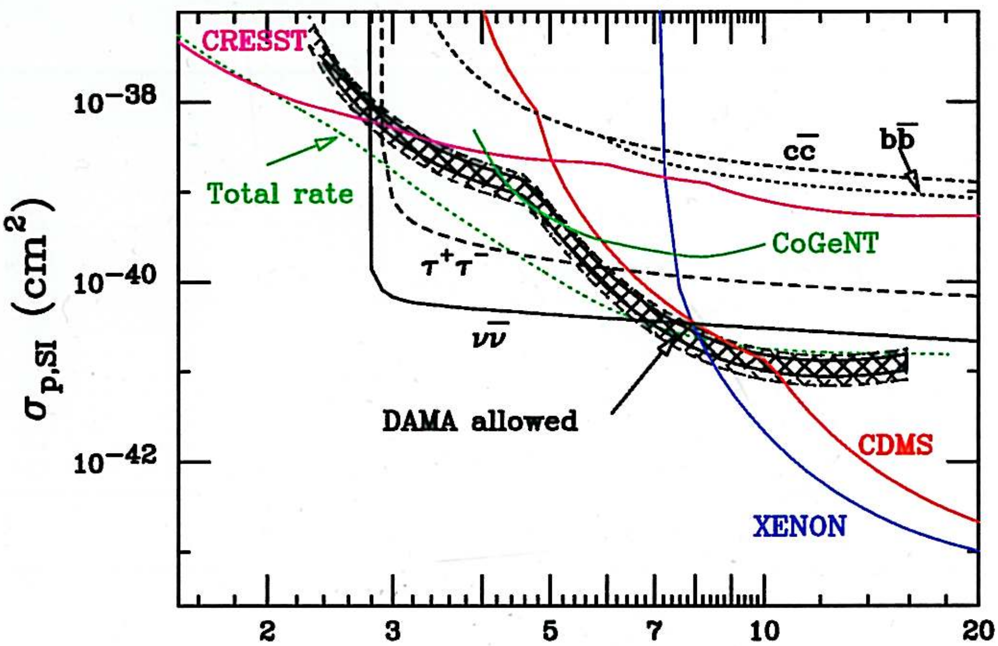
$$m_{\tilde{e}_R}^2 = m_0^2 \qquad + K_1 + \Delta_{\tilde{e}_R}$$

Taken from hep-ph/9709356 (SUSY-Primer).



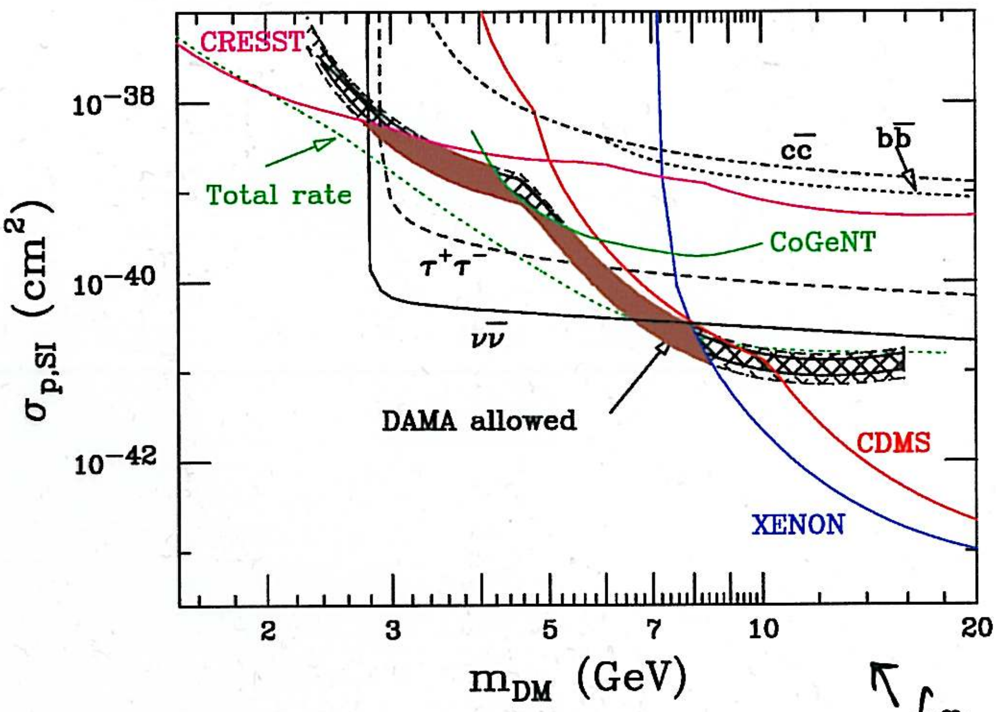
Mass [GeV]
Log10(Q/1 GeV)
Hd
Hu
(μ2+m02)1/2
M3
M2
M1
m1/2
squarks
sleptons
m0

from S

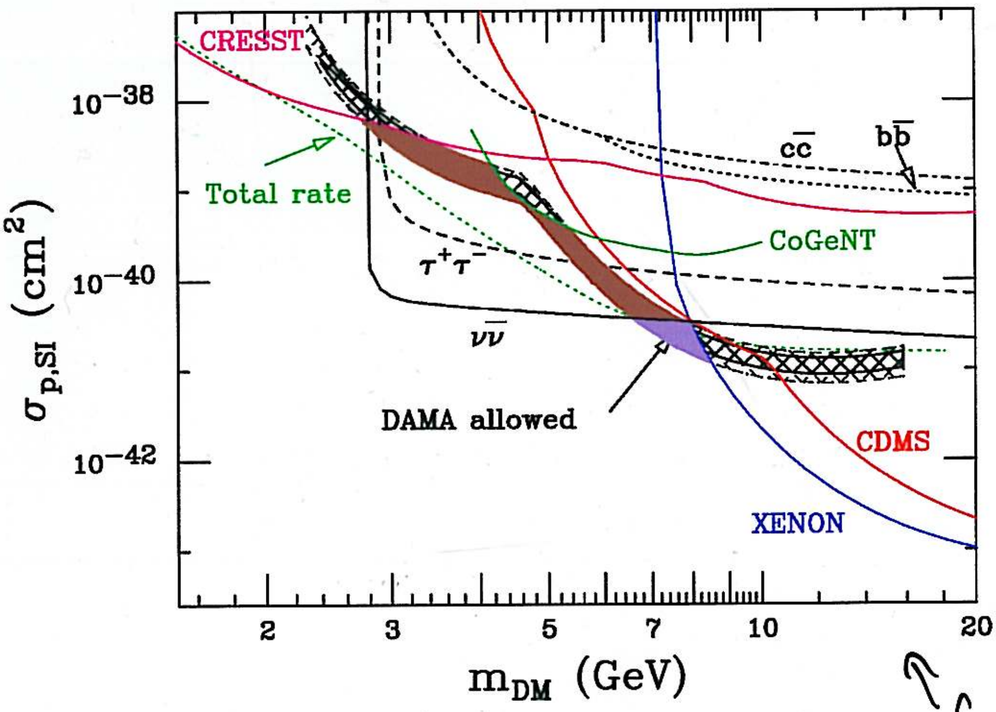


CRESST
Total rate
cc̄
bb̄
CoGeNT
τ+τ−
νν̄
DAMA allowed
CDMS
XENON
σp,SI (cm2)
mDM (GeV)

from H



from (H)
(modified by me)



from (H)
(modified by me)