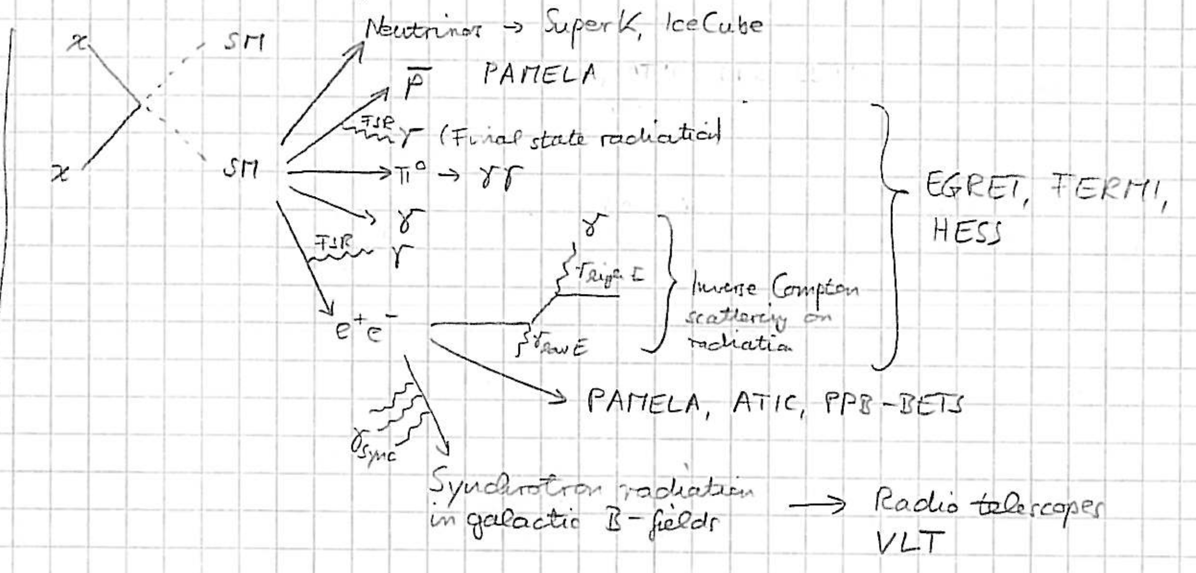
Annihilating DM

Outline

1. Probes of annihilating DM
   (e^+e^-, ICS γ-rays, bremsstrahlung, synchrotron radiation, final state radiation, secondary π^0 decays, cosmology)
2. Constraints on annihilating DM
3. Boost factors

1. Probes of annihilating DM



1.1 Cosmology

Kolb-Turner
Dan Hooper's TASI Lectures

- Relic density

Boltzmann eq.: $$\frac{d\overset{\text{WIMP number density in physical coordinates (not comoving)}}{n_\chi}}{dt} + \underbrace{3Hn_\chi}_{\text{Hubble expansion}} = -\underset{\sigma_{\chi\bar{\chi}\to SM\,\overline{SM}}}{\langle\sigma v\rangle}\left(n_\chi^2 - \overset{\text{equilibrium density}}{n_\chi^{eq\,2}}\right)$$

↳ drives n_χ into equilibrium if $\langle\sigma v\rangle$ is large enough.

Rule of thumb: $\Omega_{\text{WIMP}} \sim \frac{10^{-26}\,\text{cm}^3\,\text{s}^{-1}}{\langle\sigma v\rangle}$ (for GeV – TeV scale WIMPs with weak scale interactions)

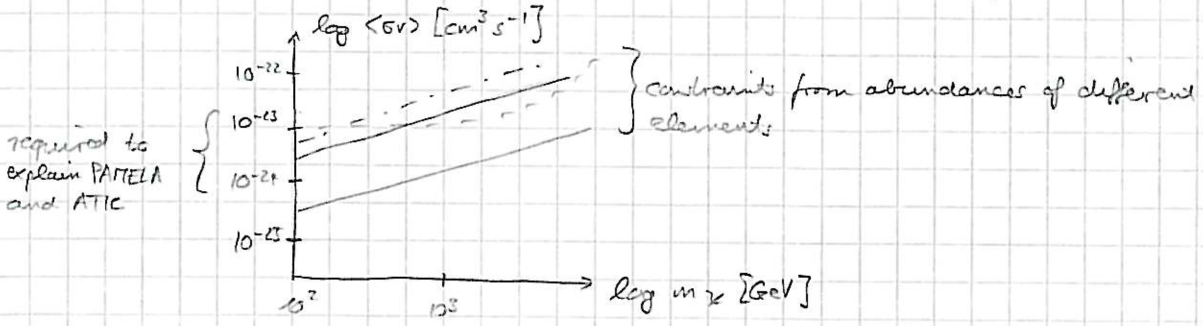
Problem: Fits to PAMELA / ATIC / PPB-BETS data require $\langle\sigma v\rangle \sim 10^{-24} - 10^{-23}\,\text{cm}^3\,\text{s}^{-1}$

Ways out: – Late time boost of $\langle\sigma v\rangle$ (see below)
– Non-thermal production
(e.g. late decay of long-lived particle)

Hisano et al.
0901.3582

- BBN

  - Injection of energy (photons) → Photodissociation of produced nuclei
  - For hadronic annihilation modes: Injection of hadrons



Approach for deriving such bounds:
Set up Boltzmann equations containing all relevant processes and solve them numerically.

1.2 Astrophysics

Cirelli et al.
0811.3744

- γ-ray flux from DM annihilations:

$$\frac{d\Phi_\gamma}{d\Omega\, dE} = \frac{1}{2}\, \frac{r_\odot}{4\pi}\, \underbrace{\frac{\rho_\odot^2}{m_\chi^2}}_{\text{(DM number density)}^2}\, J \sum_f \langle\sigma v\rangle_f \frac{dN_\gamma^f}{dE}$$

distance ⊙-GC

DM density at ⊙
remember: $\Gamma \sim \rho^2$

produced γ spectrum (normalised to $\int dN_\gamma^f/dE \cdot dE$ = # of produced γ's per annihilation)

avoid double counting of annihilations

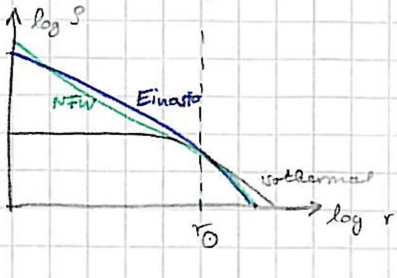
divide by solid angle to get diff. Φ_γ

sum over all γ production channels

$$J = \int_{\text{line of sight}} \frac{ds}{r_\odot} \left(\frac{\rho(r)}{\rho_\odot}\right)^2$$

[$\frac{\rho^2}{m^2}\langle\sigma v\rangle$ ... # of annihilations per time interval

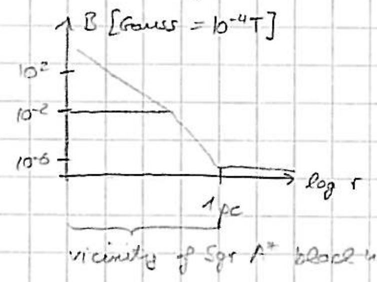
- Must make assumptions on halo profile



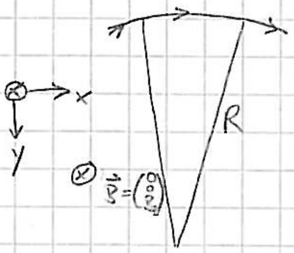
- Can use dwarf galaxies (disrupted by multiple passages through Milky Way disk → normal matter stripped off → DM dominated with few astrophysical backgrounds)

- Synchrotron radiation (radio waves)

  - Galactic B field



- Power of synchrotron radiation



$$\frac{dE}{dt} = -\frac{q^2|\ddot{\vec{r}}|^2}{6\pi}$$

(derived as follows:
- compute 4-potential of moving charge using Maxwell's equations
  $\rightarrow$ Liénard-Wiechert potentials
- go to instantaneous rest frame of the charge $\rightarrow$ simple expression for $\vec{E}$ and $\vec{B}$
- Compute Poynting vector $\vec{S} = \frac{1}{4\pi}\vec{E} \times \vec{B}$
  $\rightarrow$ e.m. energy current density
  $\Rightarrow$ integrate over $d\Omega$ $\Rightarrow \frac{dE}{dt}$)

Force on electron in its instantaneous rest frame S'

$$\vec{F}' = m_e \ddot{\vec{r}}' = e\vec{E}'$$

$\vec{E}' \mathrel{\hat{=}} \vec{E}$-field in S', given by

$$\vec{E}' = (0;\ -v\gamma B;\ 0)$$

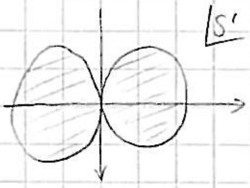
$$\Rightarrow |\ddot{\vec{r}}'| = \frac{e\gamma v B}{m_e}$$

$$\hookrightarrow \left(\frac{dE}{dt}\right)' = -\frac{q^2 e^4 \gamma^2 v^2 B^2}{6\pi\, m_e^2} = \frac{dE}{dt}$$ (since $\frac{dE}{dt}$ is Lorentz-invariant)

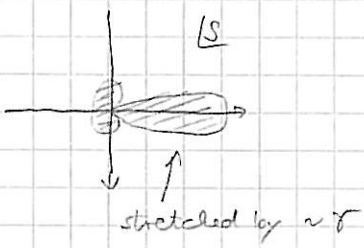
Longair:
High energy astrophysics

- Frequency of synchrotron radiation:

Instantaneous rest frame:



Laboratory frame



→ observer sees only short pulse of radiation when the dumbbell passes his line of sight.

Opening angle of dumbbell:

S' : $\sim \frac{\pi}{4} \sim O(1)$

S : $\sim 1/\gamma$

⇒ particle moves along fraction $\sim \frac{1}{\gamma}$ of its orbit while aligned with the observer

$$\Rightarrow \Delta\tau = \frac{1}{\gamma\omega_e} = \frac{1}{\omega_e'}$$

ω_e': particle's rotation frequency in its own instantaneous rest frame

ω_e: particle's rotation frequency in lab frame

Distance between leading and trailing edge of light pulse:

$$\Delta t \simeq \frac{1}{\omega_e'} - \frac{1}{\omega_e'} \cdot v \qquad \Big[1 - v = \frac{(1-v)(1+v)}{1+v} = \frac{1}{\gamma^2(1+v)} \simeq \frac{1}{2\gamma^2}$$

$$\simeq \frac{1}{\omega_e'} \cdot \frac{1}{2\gamma^2}$$

Typical scale for synchrotron radiation frequency

$$\nu \sim \frac{1}{\Delta t} \sim \gamma^2 \omega_e' \sim \gamma^3 \omega_e$$

Exact result:

$$\nu \sim \frac{3}{2} \gamma^2 \omega_e' \sin\alpha$$

α: angle between particle's velocity and $\vec{B}$-field

Numerology:

$\omega' = \frac{eB}{\gamma m}$ (Jackson, sec. 12.2)

$m = 511\ \text{keV}$

$B = 10^{-2}\ G$

$\gamma = E/m = 100\ \text{GeV}/m_e$

$$\Rightarrow \nu \sim \text{MHz} \cdot \left(\frac{B}{G}\right) \cdot \left(\frac{E}{m_e}\right)^2$$

- e^+e^-, $\bar{p}$, etc.

show PAMELA/ATIC results

2. Constraints on annihilating DM

- Cirelli et al., 0811.3744, figs 3 + 4
  (includes only bremsstrahlung and fragmentation of hadrons produced in the annihilations → high E γ's)

- Cirelli et al., 0904.3830, figs. 3 + 4
  (ICS → GeV – 10 GeV γ's)

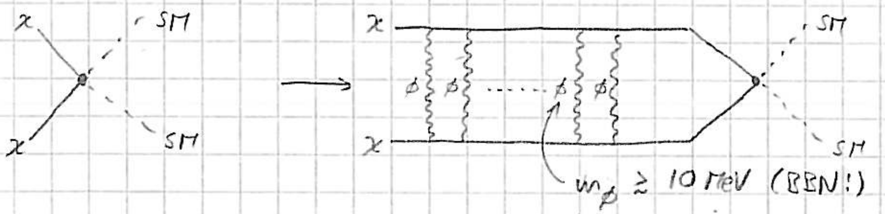
3. Boost factors

↳ Explain why DM annihilation is strong today, but weaker during earlier epochs (e.g. no additional energy source allowed during BBN)

Arkani-Hamed

3.1 Sommerfeld enhancement

Idea: DM experiences attractive potential when non-relativistic



$m_\phi \gtrsim 10$ MeV (BBN!)
but still light to get sufficient enhancement

Contact interaction: $H_{ann} = \mathcal{U}_{ann}\, \delta^{(3)}(\vec{x})$

$$\Rightarrow \Gamma \sim |\Psi_k(0)|^2 \equiv \underbrace{S}_{\text{Enhancement}} \cdot |\underbrace{\Psi_k^{(0)}(0)}_{\text{WF at } V \equiv 0}|^2$$

$\equiv 1$ for $\Psi_k^{(0)}(0) = e^{ikz}$

Schrödinger equation:

$$E_{\vec{k}}\, \Psi_k(\vec{x}) = \frac{\vec{k}^2}{2M} \Psi_k(\vec{x}) = -\frac{\vec{\nabla}^2}{2M} \Psi_{\vec{k}}(\vec{x}) + V(\underbrace{r}_{=|\vec{x}|})\, \Psi_{\vec{k}}(\vec{x})$$

at $r \to \infty$, just ordinary E_{kin}

Write $\Psi_k(\vec{x}) = \sum_\ell A_\ell\, P_\ell(\cos\Theta)\, R_{k\ell}(r)$ (for axisymmetric solutions)

Legendre polynomial

$$\Rightarrow \frac{\vec{k}^2}{2M} R_{k\ell}(r) = -\frac{1}{2M}\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr} R_{k\ell}(r)\right) + \left(\frac{\ell(\ell+1)}{2mr^2} + V(r)\right) R_{k\ell}(r)$$

or, with $R_{k\ell}(r) \equiv \frac{\chi_{k\ell}(r)}{r}$

$$\frac{\vec{k}^2}{2M}\chi_{k\ell}(r) = -\frac{1}{2M}\frac{d^2}{dr^2}\chi_{k\ell}(r) + \frac{\ell(\ell+1)}{2mr^2}\chi_{k\ell}(r) + V(r)\chi_{k\ell}(r)$$

For $\ell > 0$, $\chi_{k\ell}(r)$ must decrease at least like r^2 to ensure that the probability density ($\sim r^2 R_{k\ell}^2(r) \sim \chi_{k\ell}^2(r)$) remains finite (l.h.s. finite → ℓ term must not blow up)

⇒ To compute $\Psi_k(0)$, we need to consider only the $\ell = 0$ case.

$$S = |A_0|^2 \cdot \left|\lim_{r\to 0} \frac{\chi_{k0}(r)}{r}\right|^2 = |A_0|^2\, |\chi'_{k0}(r)|^2$$

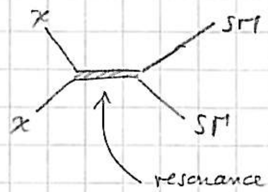
Example: Yukawa potential $V(r) = -\frac{\alpha}{2r} e^{-m_\phi r}$

$$\hookrightarrow \left(\frac{k^2}{2M} + \frac{\alpha}{2r} e^{-m_\phi r}\right) \chi_{k0}(r) = -\frac{1}{2M}\frac{d^2}{dr^2} \chi_{k0}(r)$$

Rewrite using $\varepsilon_\phi \equiv \frac{m_\phi}{\alpha M}$; $\varepsilon_v \equiv \frac{v}{\alpha}$

3.2 Resonance enhancement

Guo et al., 0901.1450
Ibe et al., 0812.0072



resonance of mass $\frac{M}{2m_\chi} = 1 - \varepsilon$

$$\Rightarrow \sigma_{ann} \sim \frac{M^2\Gamma^2}{(E_{cm}^2 - M^2)^2 + M^2\Gamma^2}$$

↳ As long as $E_\chi \gg m_\chi$ → out of resonance

3.3 Dark matter clumps

Assume local DM density peaks
→ greatly enhanced $\langle \sigma v \rangle$ due to $\Gamma \sim \rho^2$

Problem:
- Does not come out of simulations of galaxy formation
- DM self-interactions must not be too strong (from cosmology)

Literature

- Kolb, Turner: "The early universe"
  (Westview Press, 1990, ISBN 0-201-62674-8)
  THE textbook about cosmology

- Dan Hooper: "TASI 2008 lectures on Dark Matter"
  arXiv: 0901.4090
  Summer school lecture about different aspects of
  DM physics

- Hisano, Kawasaki, Kohri, Moroi, Nakayama:
  "Cosmic rays from Dark Matter annihilation and big-bang
  nucleosynthesis"
  arXiv: 0901.3582
  Limits on annihilation cross sections from BBN
  (energy injection, hadron injection must be small!)

- Bertone, Cirelli, Strumia, Taoso:
  "Gamma ray and radio tests of the e^+e^- excess
  from DM annihilations"
  arXiv: 0811.3744
  Constraints on $\langle \sigma v \rangle_{ann}$ from high-E γ ray observation
  (limit bremsstrahlung and hadron dissociation to γ's)
  and radio / microwave observations (limit synchrotron
  emission from DM decay products). Tension with
  PAMELA / ATIC is found

- Longair: "High energy astrophysics"
  Cambridge University Press, 1981 (1^{st} ed.), 1992/1994 (2^{nd} ed.)
  ISBN 0-521-38773-5 (Vol 1); ISBN 0-521-43439-4 (Vol 2)
  Nice textbook about high-E processes in astrophysics, e.g.
  generation of synchrotron radiation by high-E particles

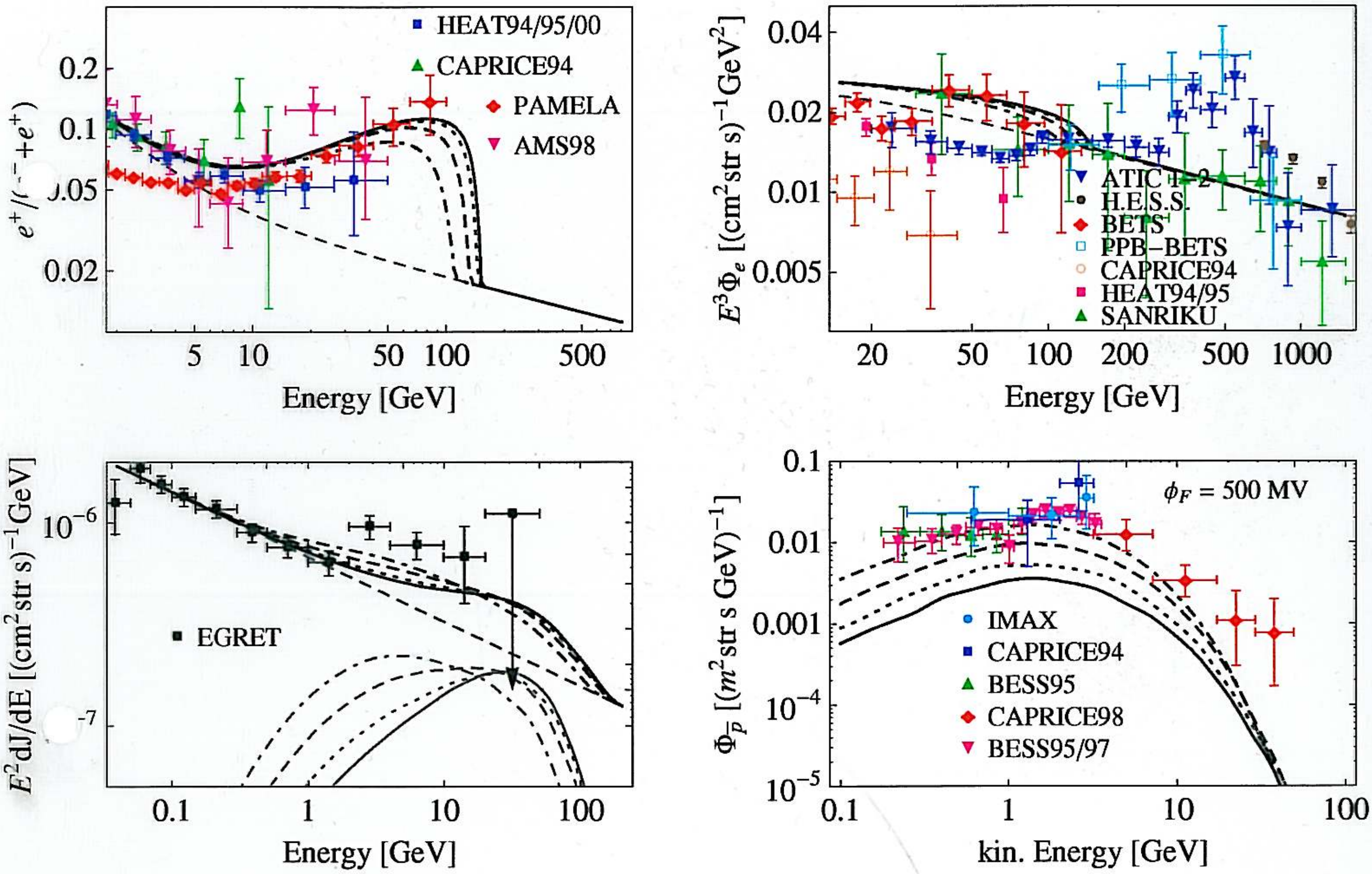
- Adriani et al.: "Observation of an anomalous positron
  abundance in the cosmic radiation"
  arXiv: 0810.4995
  The paper about the PAMELA e^+e^- result

- Cirelli, Panci: "Inverse Compton constraints on dark matter
  e^+e^- excess"
  arXiv: 0904.3830
  The title says everything. ICS constraints are about as
  strong as those from bremsstrahlung / synchrotron
  radiation (arXiv: 0811.3744)

- Arkani-Hamed, Finkbeiner, Slatyer, Weiner: "A theory
  of dark matter"
  arXiv: 0810.0713
  An attempt to explain all hints towards DM in a
  single model. Good appendix on Sommerfeld enhancement

- Ibe, Murayama, Yanagida: "Breit-Wigner enhancement of dark matter annihilation"
  arXiv: 0812.0072

- Guo, Wu: "Enhancement of dark matter annihilation via Breit-Wigner resonance."
  arXiv: 0901.1450


HEAT94/95/00
CAPRICE94
PAMELA
AMS98
0.2
0.1
0.05
0.02
5
10
50
100
500
Energy [GeV]
e+/(e−+e+)
0.04
0.02
0.01
0.005
20
50
100
200
500
1000
Energy [GeV]
E3Φe [(cm2str s)−1GeV2]
ATIC 1+2
H.E.S.S.
BETS
PPB−BETS
CAPRICE94
HEAT94/95
SANRIKU
E2dJ/dE [(cm2str s)−1GeV]
10−6
10−7
EGRET
0.1
1
10
100
Energy [GeV]
0.1
0.01
0.001
10−4
10−5
ϕF = 500 MV
IMAX
CAPRICE94
BESS95
CAPRICE98
BESS95/97
Φp̄ [(m2str s GeV)−1]
0.1
1
10
100
kin. Energy [GeV]

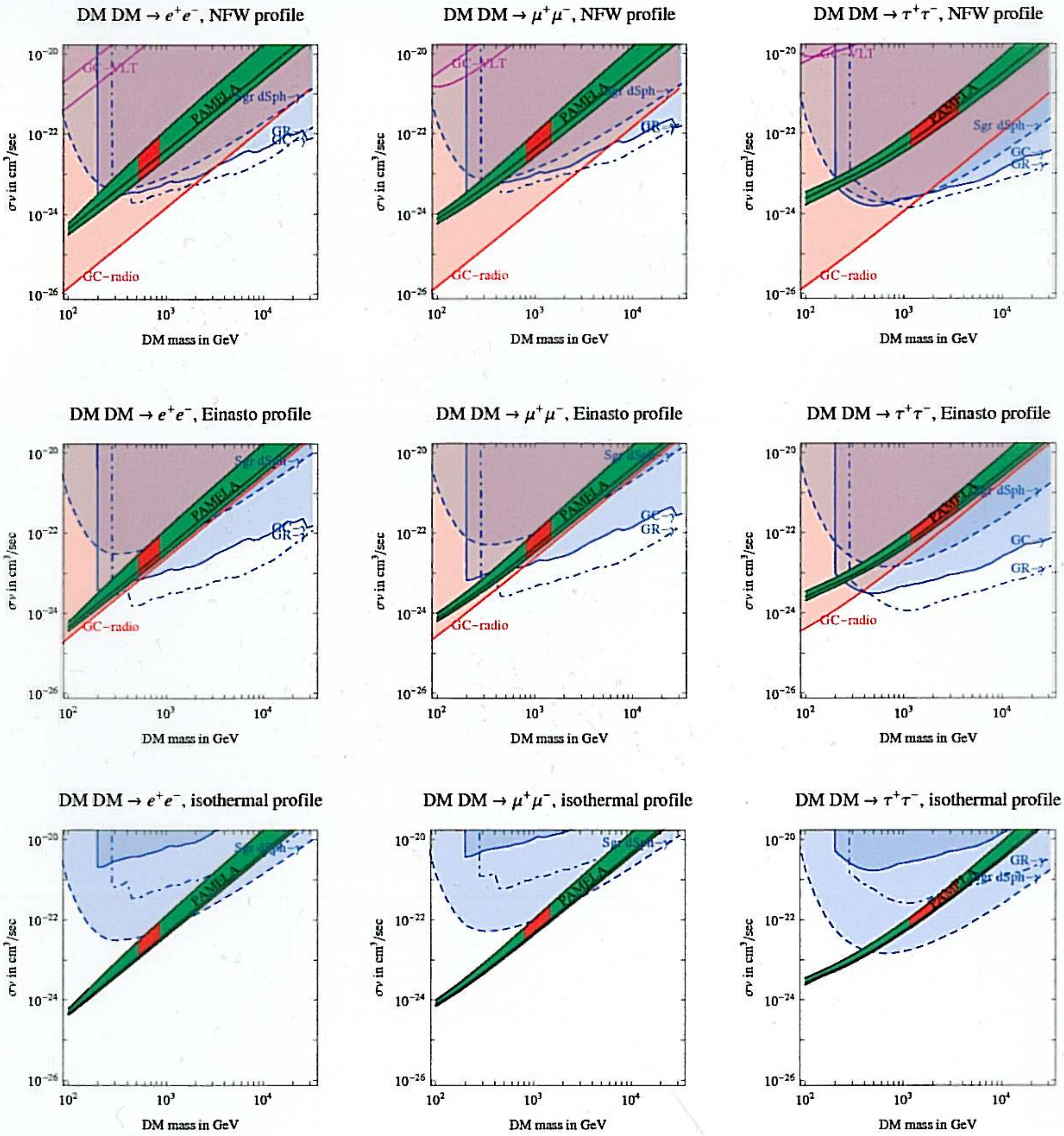
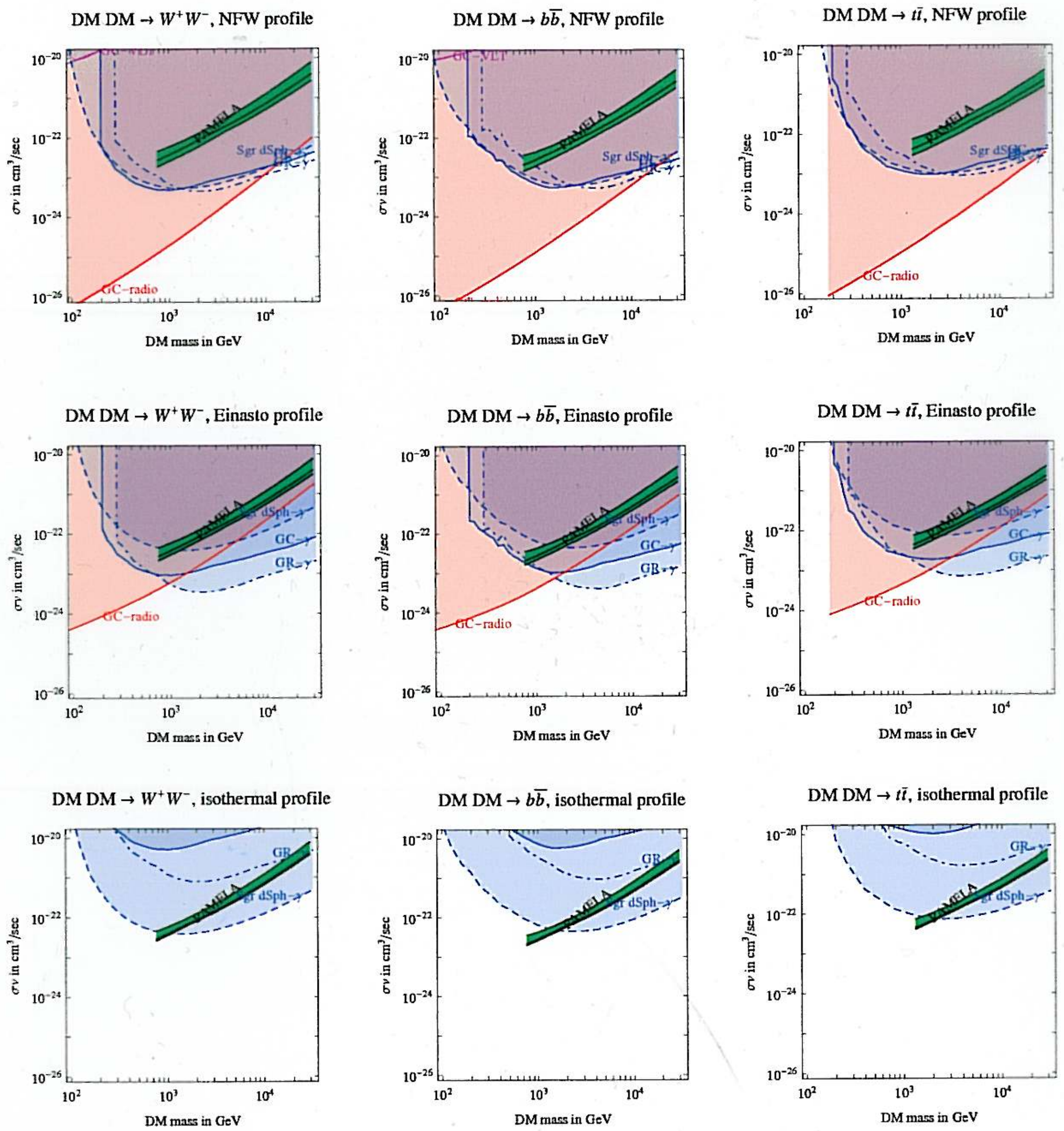


Figure 3: *We compare the region favored by PAMELA (green bands) and ATIC (red regions within the bands) with the bounds from HESS observations of the Galatic Center [29] (blue continuous line), Galactic Ridge [41] (blue dot-dashed), and SgrDwarf [42] (blue dashed) and of observations of the Galactic Center at radio-frequencies* $\nu = 408\,\mathrm{GHz}$ *by Davies et al. [51] (red lines) and at* $\nu \sim 10^{14}\,\mathrm{Hz}$ *by VLT [52] (upper purple lines, when present, for equipartition and constant magnetic field). We considered DM annihilations into* e^+e^- *(left column),* $\mu^+\mu^-$ *(middle),* $\tau^+\tau^-$ *(right), unity boost and Sommerfeld factors and the NFW (upper row), Einasto (middle), isothermal (lower) MW DM density profiles and the NFW (upper), large core (middle and lower) Sgr dSph DM density profiles.*


DM DM → W^+W^-, NFW profile

DM DM → $b\bar{b}$, NFW profile

DM DM → $t\bar{t}$, NFW profile

DM DM → W^+W^-, Einasto profile

DM DM → $b\bar{b}$, Einasto profile

DM DM → $t\bar{t}$, Einasto profile

DM DM → W^+W^-, isothermal profile

DM DM → $b\bar{b}$, isothermal profile

DM DM → $t\bar{t}$, isothermal profile

Figure 4: *As in the previous fig. 3, but for the cases of DM annihilations into* W^+W^- *(left),* $b\bar{b}$ *(middle),* $t\bar{t}$ *(right).*

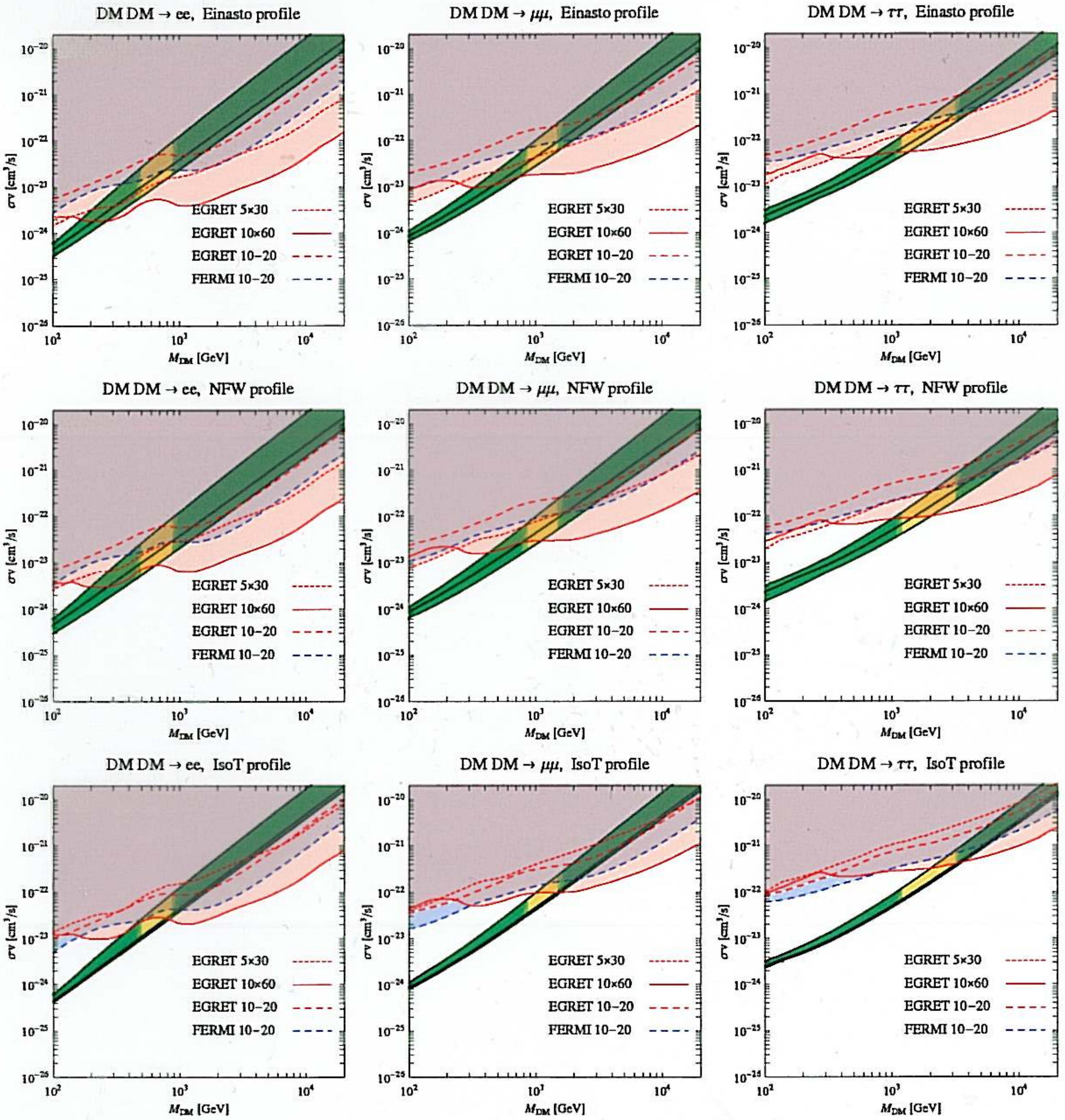


Figure 3: *The regions favored by PAMELA (green bands), containing in particular the areas favored by PAMELA+ATIC (yellow areas), are compared with the bounds from ICS secondary radiation. The first column of panels refers to DM annihilations into* e^+e^-*, the second into* $\mu^+\mu^-$ *and the third into* $\tau^+\tau^-$*; the three rows assume respectively an Einasto, an NFW and an isothermal profile. In each panel, the bounds from EGRET data in the '5×30' region are plotted with a short dashed red line, those from EGRET data in the '10×60' region with a solid red line and those from EGRET data in the '10–20' strips with a dashed red line. The preliminary FERMI bounds in the '10–20' strips are plotted with a dashed blue line.*

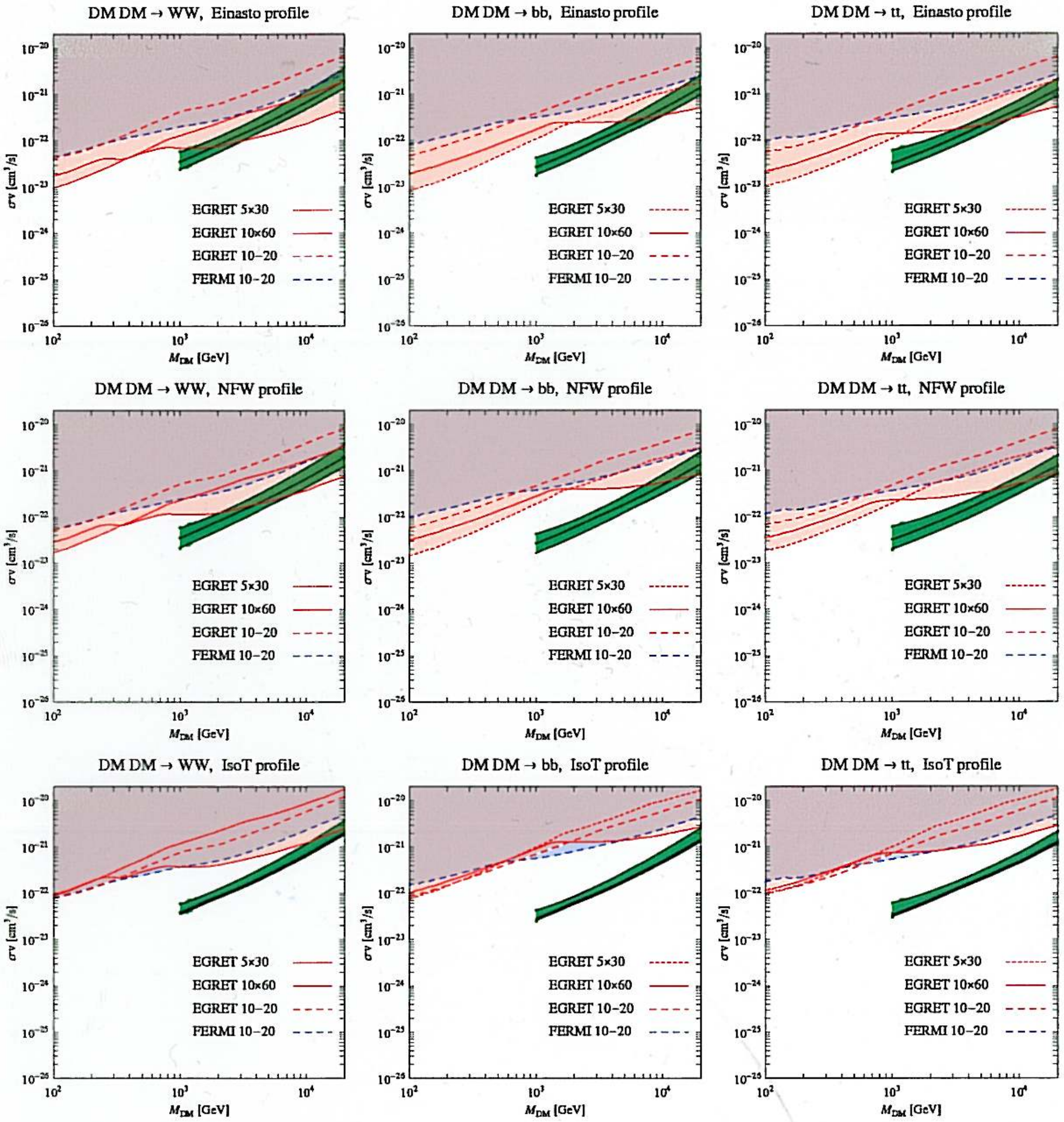


Figure 4: *As in the previous fig. 3, but for* W^+W^-, $b\bar{b}$ *and* $t\bar{t}$ *annihilation channels. Since a DM particle fitting the PAMELA data has to be multi-TeV, the green bands start at larger masses. There is no possibility to fit the ATIC data in these channels.*

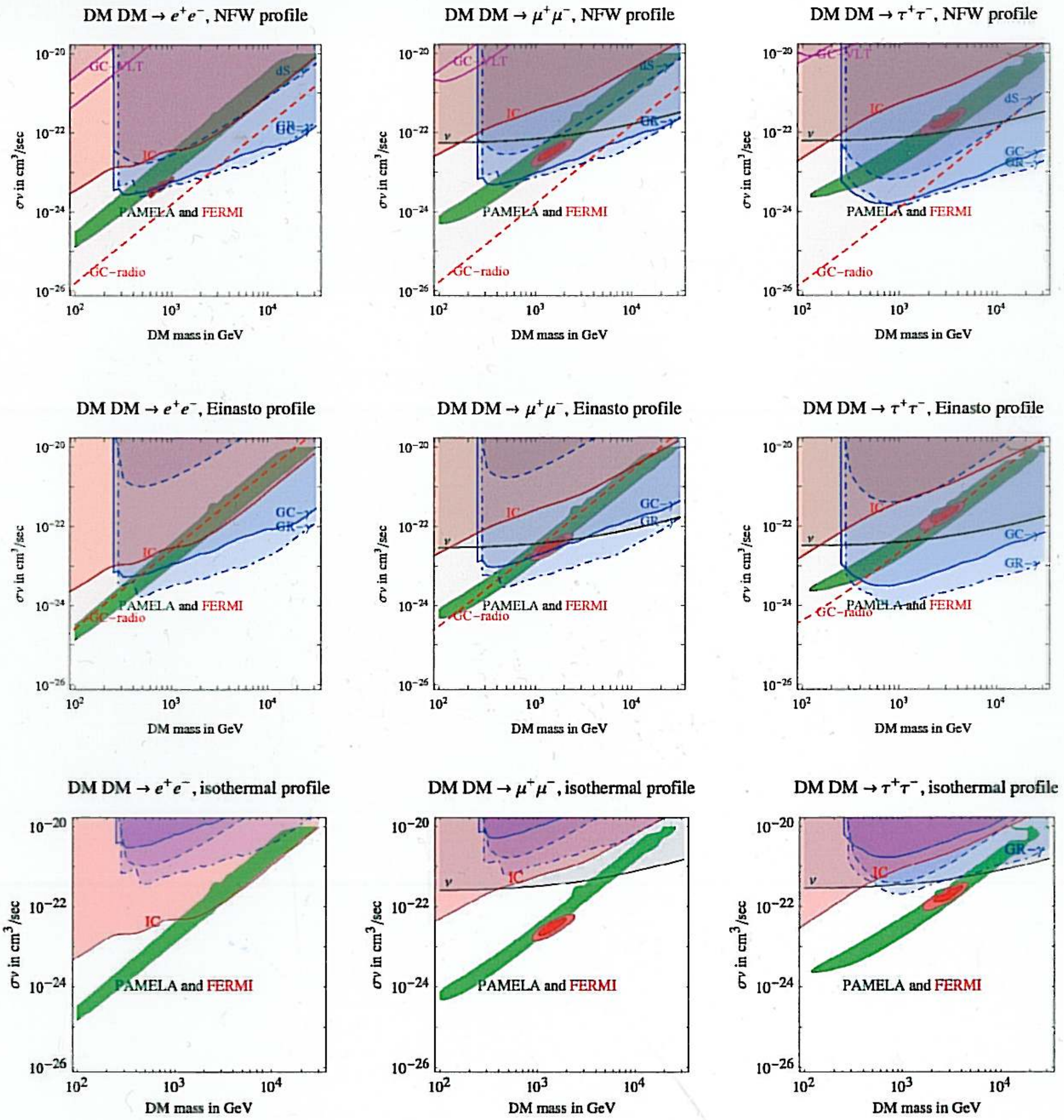
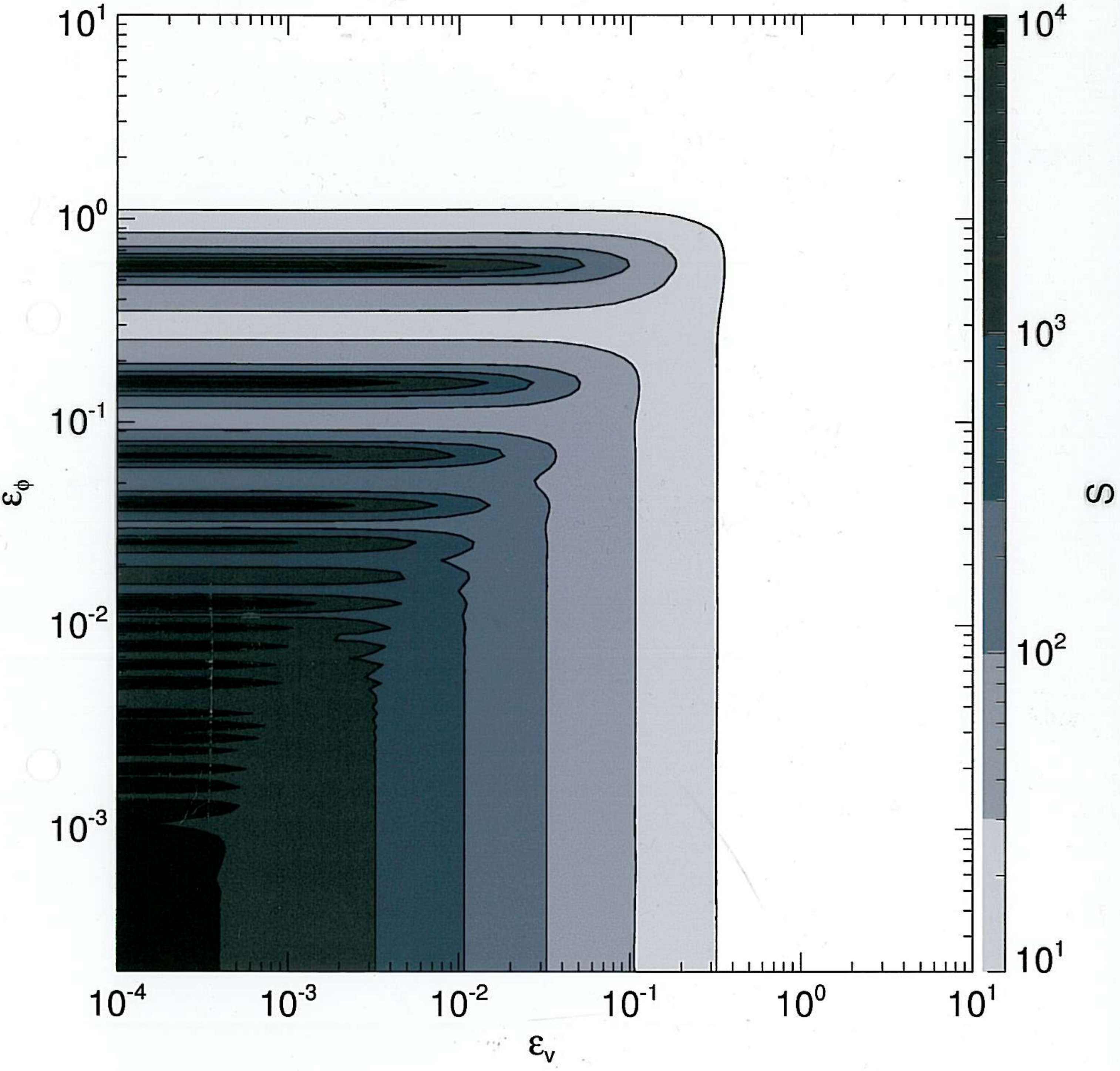


Figure 6: **Direct DM annihilation.** *We compare the region favored by PAMELA (green bands) and by PAMELA, FERMI and HESS observations (red ellipeses) with HESS observations of the Galatic Center [19] (blue continuous line), Galactic Ridge [20] (blue dot-dashed), and spherical dwarfes [21, 22] (blue dashed), FERMI observations in the '*10° ÷ 20°*' region and of observations of the Galactic Center at radio-frequencies* $\nu = 408\,\mathrm{GHz}$ *[44] (dashed red lines) and at* $\nu \sim 10^{14}\,\mathrm{Hz}$ *by VLT [45] (upper purple lines, when present, for equipartition and constant magnetic field). See discussion in the text for remarks regarding the validity of the constraints. We considered DM annihilations into* e^+e^- *(left column),* $\mu^+\mu^-$ *(middle),* $\tau^+\tau^-$ *(right), unity boost and Sommerfeld factors and the NFW (upper row), Einasto (middle), isothermal (lower) DM density profiles in the Milky Way.*



10^1
10^0
10^{-1}
10^{-2}
10^{-3}
ε_ϕ
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
ε_v
10^4
10^3
10^2
10^1
S