GLoBES workshop

On the interpretation of sensitivity limits of future experiments

Thomas Schwetz

CERN

based on hep-ph/0612223

Outline

- Introduction
  Remarks on the “standard” way to compute sensitivities
- Generalized definition of sensitivities
- The sensitivity to θ_{13}
  Monte Carlo simulation of D-Chooz and T2K
- Sensitivity to CP violation
  at the example of T2HK
- Summary

The "standard" way to calculate sensitivities

Calculation of event rates for given experiment:

$$N_i(\boldsymbol{\theta}) = \Phi \cdot \sigma \cdot R \cdot \epsilon \cdot P(\boldsymbol{\theta})$$

Φ: neutrino flux
σ: detection cross section
R: energy resoltion
ϵ: efficiencies

$P(\boldsymbol{\theta})$: 3-flavour osc. prob., $\boldsymbol{\theta} = (\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{23}, \theta_{13}, \delta)$

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assume “true values” $\hat{\boldsymbol{\theta}}$ and

calculate “data” for these true values: $\hat{N}_i = N_i(\hat{\boldsymbol{\theta}})$

$\chi^2(\boldsymbol{\theta}; \hat{\boldsymbol{\theta}}) \quad \rightarrow \quad$ allowed regions for $\boldsymbol{\theta}$

The "standard" way to calculate sensitivities

Example: Sensitivity to θ_{13} at 3σ:

Looking for the value of $\theta_{13}^{\mathrm{true}}$, for which $\theta_{13} = 0$ can be excluded at 3σ:

$$\chi^2(\theta_{13} = 0; \hat{\theta}_{13} = \theta_{13}^{\mathrm{true}}) = 9$$

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This can be done conveniently with GLoBES, but ...

The "standard" way to calculate sensitivities

What is the precise statistical meaning of such sensitivities?

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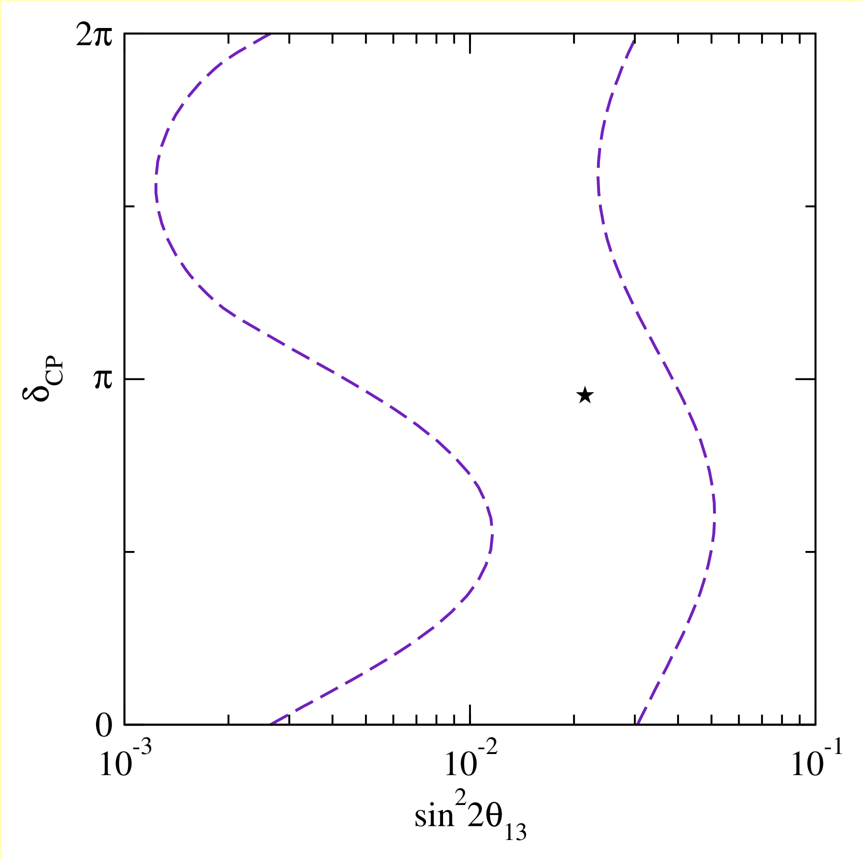
What is the precise statistical meaning of such sensitivities?

- Statistical fluctuations are not taken into account, $\chi^2 = 0$ at the best fit point $\equiv$ true values
- Gaussian approximation for calculating allowed regions, e.g., $\Delta\chi^2 = 9$ for a 3σ intervall

The impact of statistical fluctuations

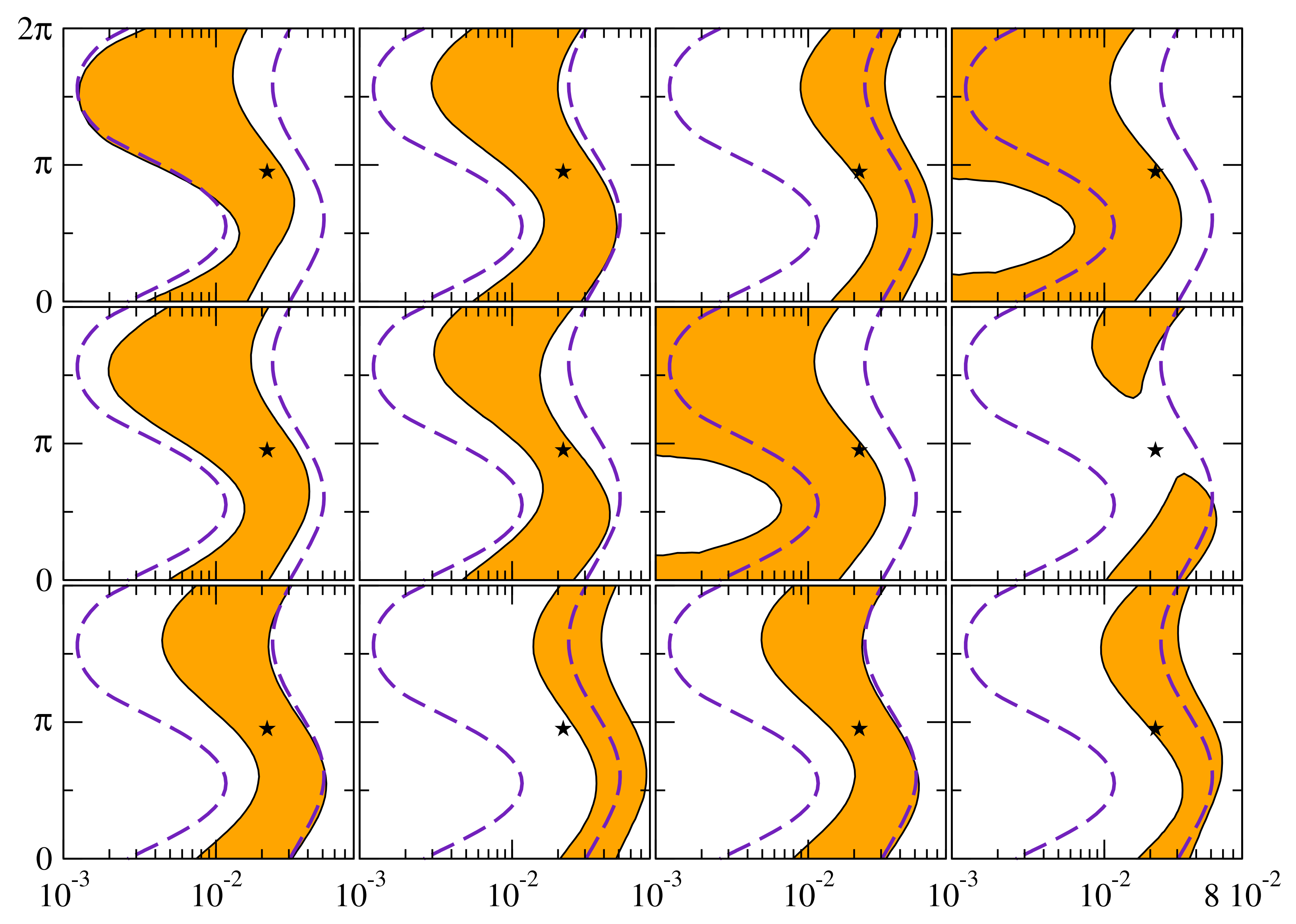
Example: **T2K**, true $\sin^2 2\theta_{13} = 0.02$, $\delta = \pi$:

GLoBES will give you:



$\longrightarrow$ Sensitivity of an “average” experiment

The impact of statistical fluctuations



Generalized definition of sensitivity

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The correct question to ask should be:

What is the value of $\theta_{13}^{\text{true}}$, for which $\theta_{13} = 0$ can be excluded at the $100(1-\alpha)\%$ CL with a probability P_{disc}?

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What is the value of $\theta_{13}^{\rm true}$, for which $\theta_{13} = 0$ can be excluded at the $100(1-\alpha)\%$ CL with a probability $P_{\rm disc}$?

- have to quote two numbers, α (CL) and $P_{\rm disc}$
- at fixed $\theta_{13}^{\rm true}$, decreasing α (incr. the CL) decreases $P_{\rm disc}$
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The correct question to ask should be:

What is the value of $\theta_{13}^{\text{true}}$, for which $\theta_{13} = 0$ can be excluded at the $100(1 - \alpha)\%$ CL with a probability P_{disc}?

- have to quote two numbers, α (CL) and P_{disc}
- at fixed $\theta_{13}^{\text{true}}$, decreasing α (incr. the CL) decreases P_{disc} at fixed α (fixed CL), decreasing $\theta_{13}^{\text{true}}$ decreases P_{disc}
- the implicit assumption for the "standard" sensitivity is that using data without fluctuations describes an "average" experiment, which should correspond to $P_{\text{disc}} = 50\%$.
- "Good" sensitivity means high CL and high P_{disc}.

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- Let us denote the result of the experiment by $x^{\rm obs}$. Then, $x > 0$ is discovered at 3σ if $x^{\rm obs} > 3\sigma$.
- As a function of the true value $x^{\rm true}$ the probability to discover $x > 0$ at 3σ is given by

$$P_{\rm disc} = P\left[x^{\rm obs} \geq 3\sigma \mid x^{\rm true}\right] = \int_{3\sigma}^{\infty} dx\, G(x;\, x^{\rm true}, \sigma)$$

$$= \frac{1}{2}\left[1 - {\rm erf}\left(\frac{3\sigma - x^{\rm true}}{\sqrt{2}\sigma}\right)\right]$$

Generalized definition of sensitivity

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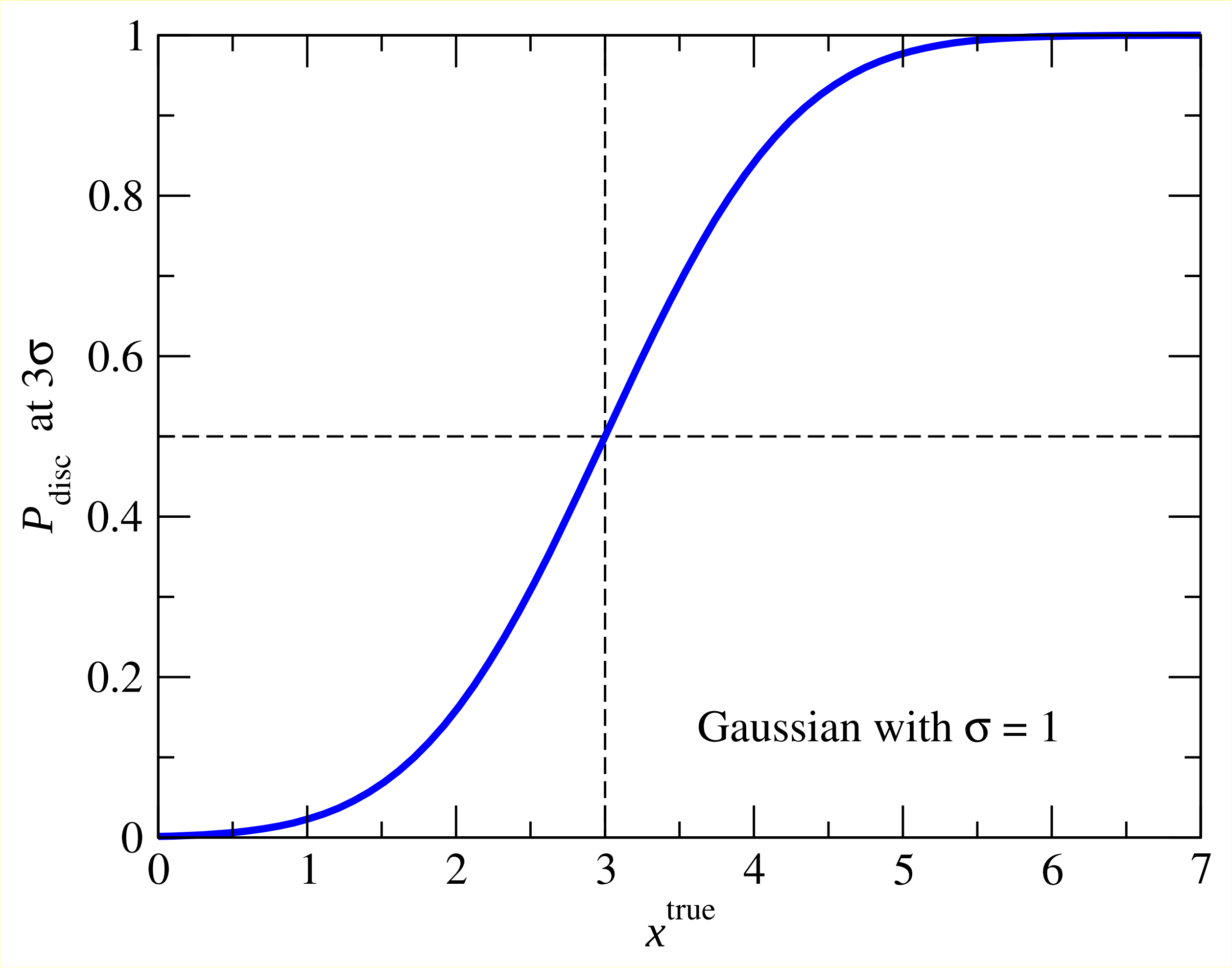

Remark on hypothesis testing

"Statistics language":

In testing a hypothesis H_0 we can make two kinds of errors:

- Error of type 1: reject H_0 although it is true
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In our case: H_0: $\theta_{13} = 0$

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- Type 1: "discover" non-zero θ_{13} although the true value is 0.
- Type 2: accept $\theta_{13} = 0$ although the true value is non-zero.

The probability to make an error of type 1 is α

per definition of a $100(1-\alpha)\%$ CL intervall

The probability to make an error of type 2 is $(1 - P_{\rm disc})$

Thanks to Toshihiko Ota for pointing this out to me!

Generalized definition of sensitivity

To answer this question in realistic situations it is necessary to perform a Monte Carlo simulation, i.e., simulate many artificial data sets for a given experiment, including statistical fluctuations:

Calculate prediction $N_i(\hat{\boldsymbol{\theta}})$ for some “true values” $\hat{\boldsymbol{\theta}}$.
The “data” D_i, which go into the χ^2 are obtained by throwing a Poisson variable with mean $N_i(\hat{\boldsymbol{\theta}})$.

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Unfortunately, GLoBES is too slow for this task.

Wrote a speed optimized code for D-Chooz, T2K, and T2HK, cross-checked the "standard sensitivities" with GLoBES.

Generalized sensitivity to θ_{13} for D-Chooz and T2K

Sensitivity to θ_{13}

First, define a criterion to “discover” $\theta_{13} > 0$:

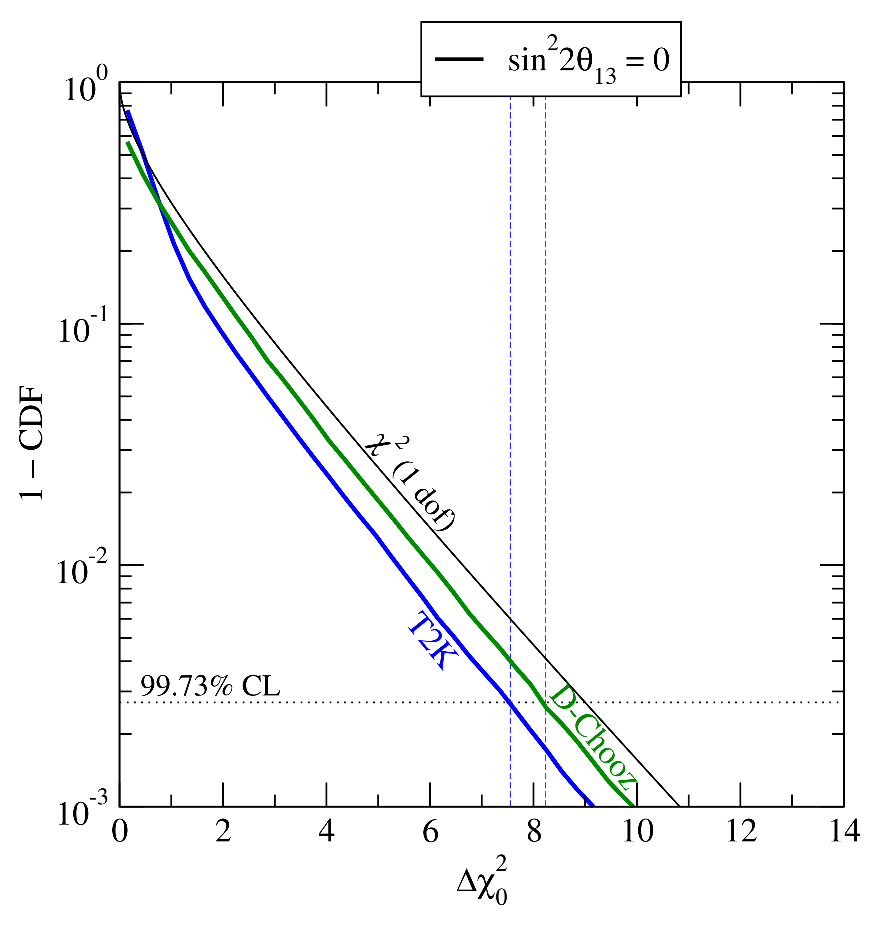
$$\chi^2(\theta_{13} = 0) - \chi^2_{\rm min} \equiv \Delta\chi^2 > \lambda(\alpha)$$

Calculation of $\lambda(\alpha)$:

- Set $\theta_{13}^{\rm true} = 0$, simulate many experiments, and calculate for each experiment $\Delta\chi^2$.
- Determine the value $\lambda(\alpha)$ by requiring that a fraction α of all experiments has $\Delta\chi^2 > \lambda(\alpha)$.

Sensitivity to θ_{13}

Calculation of $\lambda(\alpha)$ for $\alpha = 0.0027$:



$$\lambda(\alpha)_{\mathrm{T2K}} = 7.55$$

$$\lambda(\alpha)_{\mathrm{DC}} = 8.23$$

$$\lambda(\alpha)_{\mathrm{Gauss}} = 9$$

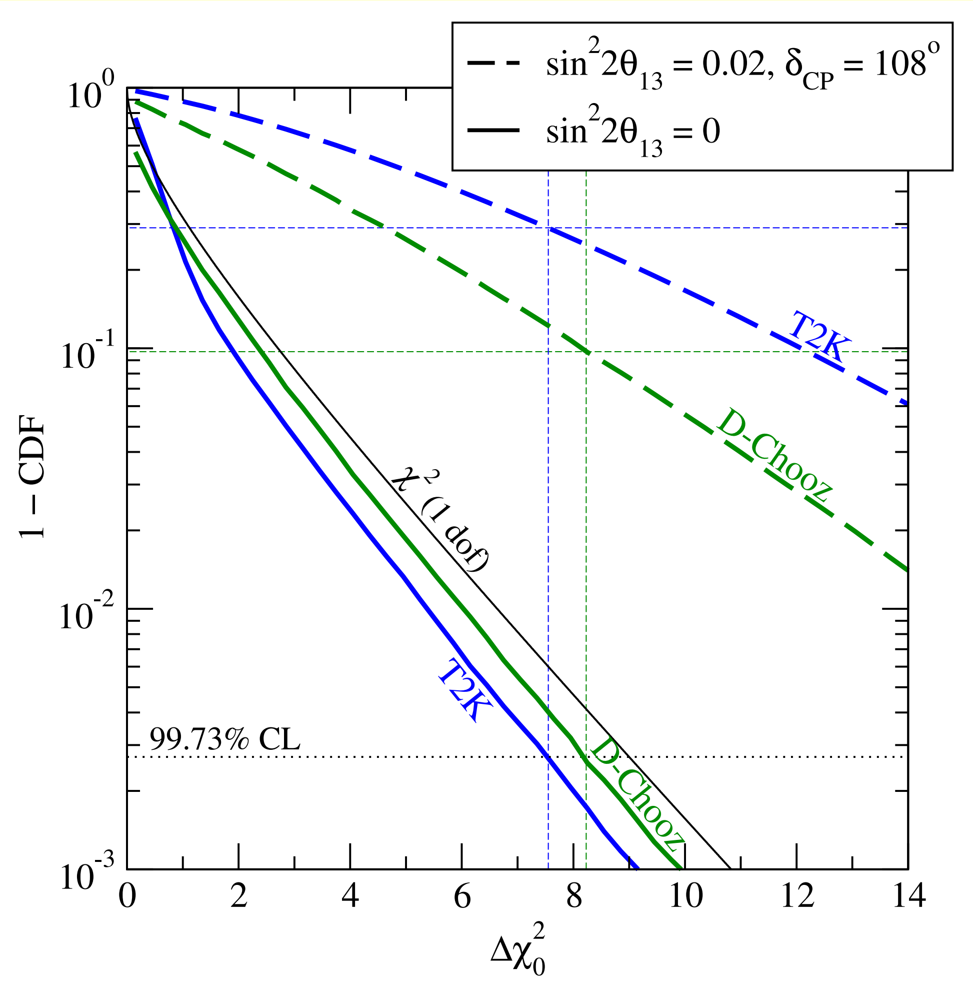
Sensitivity to θ_{13}

Second, calculation of $P_{\rm disc}$:

- Set $\theta_{13}^{\rm true} > 0$, simulate many experiments, and calculate for each experiment $\Delta\chi^2$.
- $P_{\rm disc}$ for the $100(1-\alpha)\%$ CL is given by the fraction of experiments which have $\Delta\chi^2 > \lambda(\alpha)$.
- Repeat this for each value of $\theta_{13}^{\rm true}$.

Sensitivity to θ_{13}

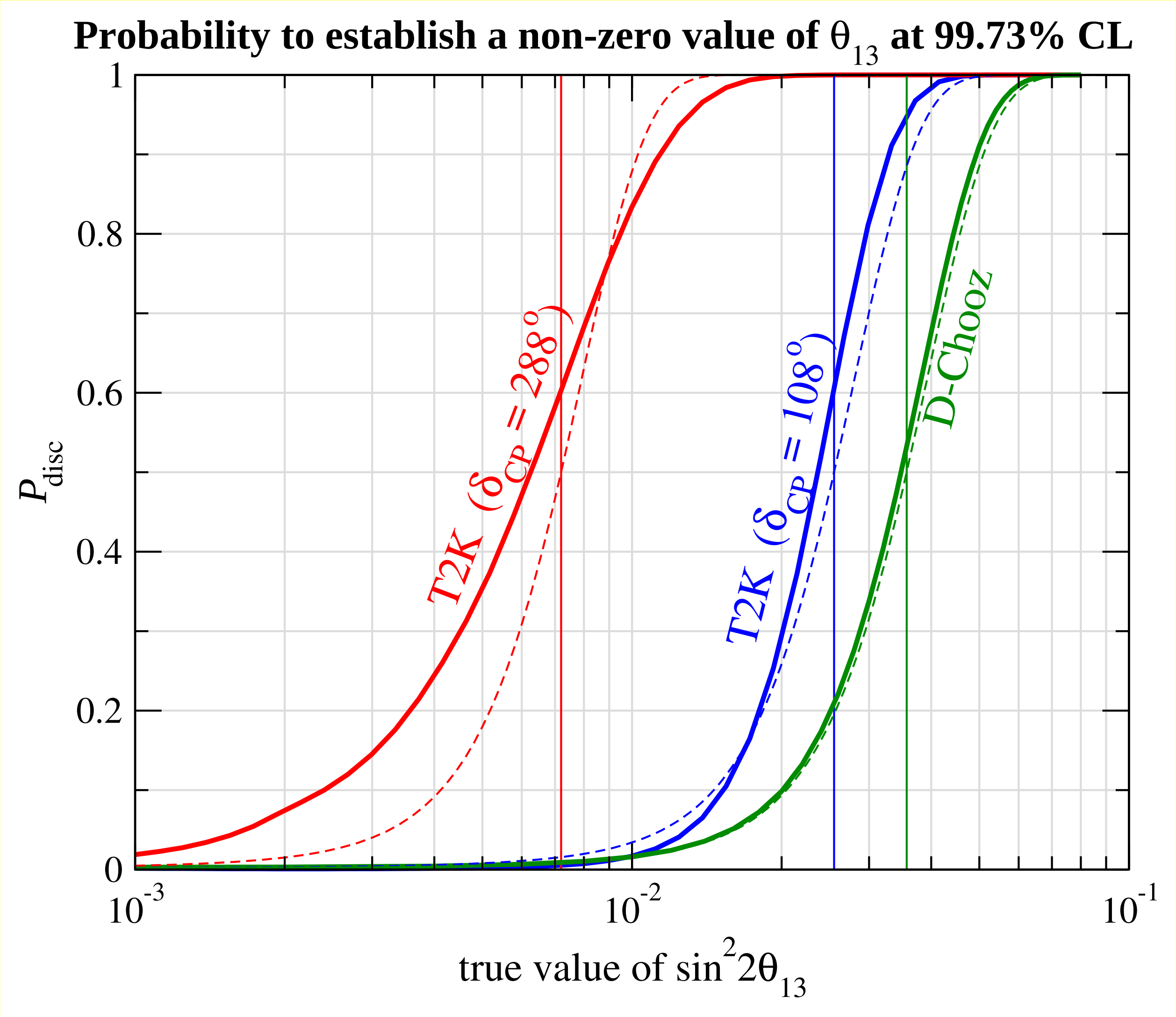
Calculation of $P_{\rm disc}$ at a given true value of θ_{13}:



$$P_{\rm disc}({\rm T2K}) = 29\%$$

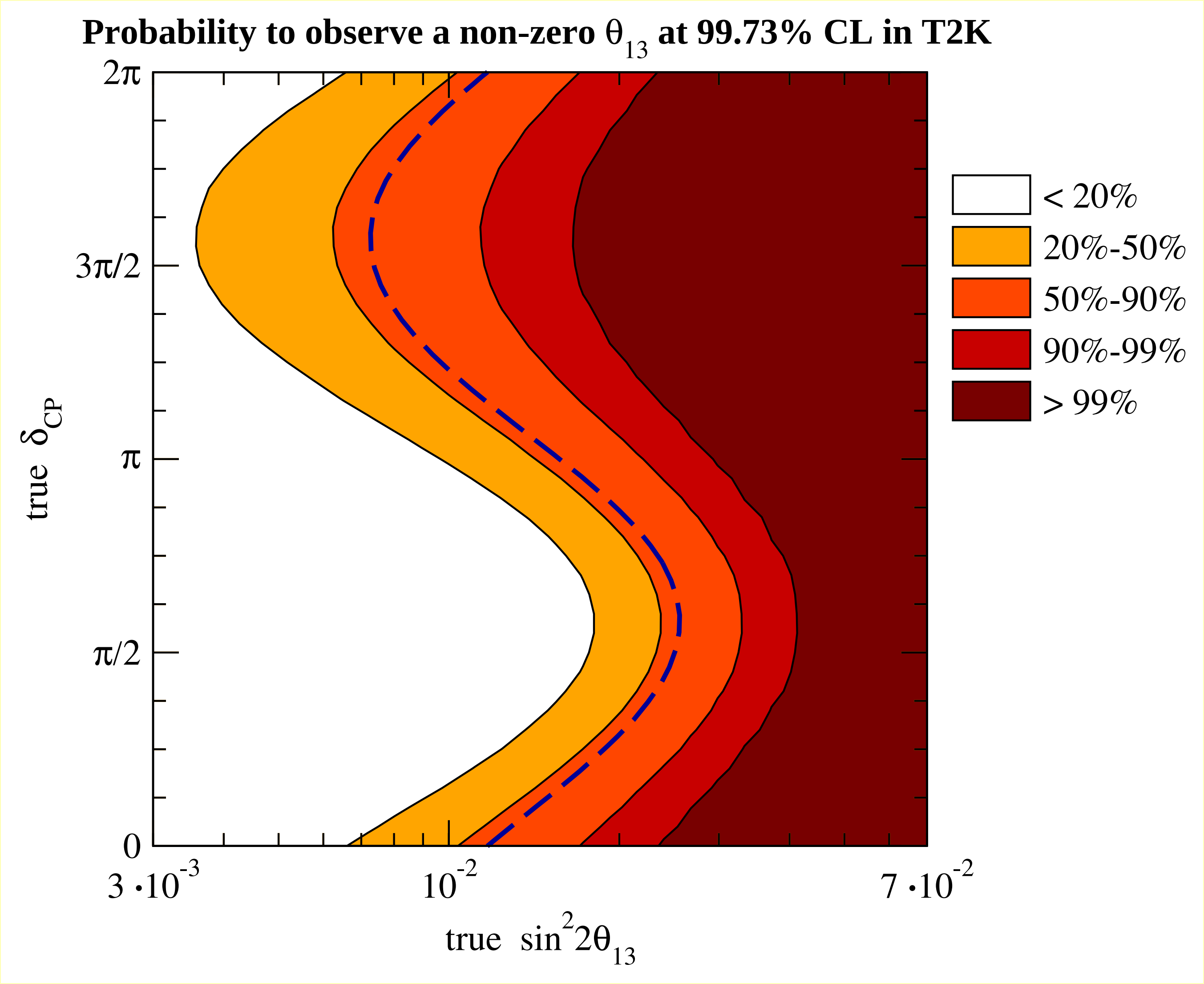
$$P_{\rm disc}({\rm DC}) = 9.7\%$$

Sensitivity to θ_{13}



dashed: Gaussian approximation

Sensitivity to θ_{13}



dashed: “standard sensitivity”

Sensitivity to θ_{13}

Side-remark:

For this plot

$$41 \times 41 \times 10^5 \approx 1.7 \times 10^8$$

fits have been performed $\Rightarrow$

Total calculation time:

$$T \simeq 2\,\mathrm{days} \times \left(\frac{\text{time for one } \chi^2 \text{ minimisation}}{10^{-3}\,\mathrm{sec}} \right)$$

Generalized sensitivity to CPV for T2HK

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For given values of $\theta_{13}^{\text{true}}$ and δ^{true}, what is the probability P_{disc} that CPV can be established at the $100(1-\alpha)\%$ CL?

Sensitivity to CPV

"Standard" sensitivity to CPV at 3σ:

Scan true values $\hat{\theta}_{13}$ and $\hat{\delta}$, and check whether

$$\chi^2(\theta_{13}, \delta_{\rm CPC}; \hat{\theta}_{13}, \hat{\delta}) > 9 \quad \text{with} \quad \delta_{\rm CPC} = 0, \pi$$

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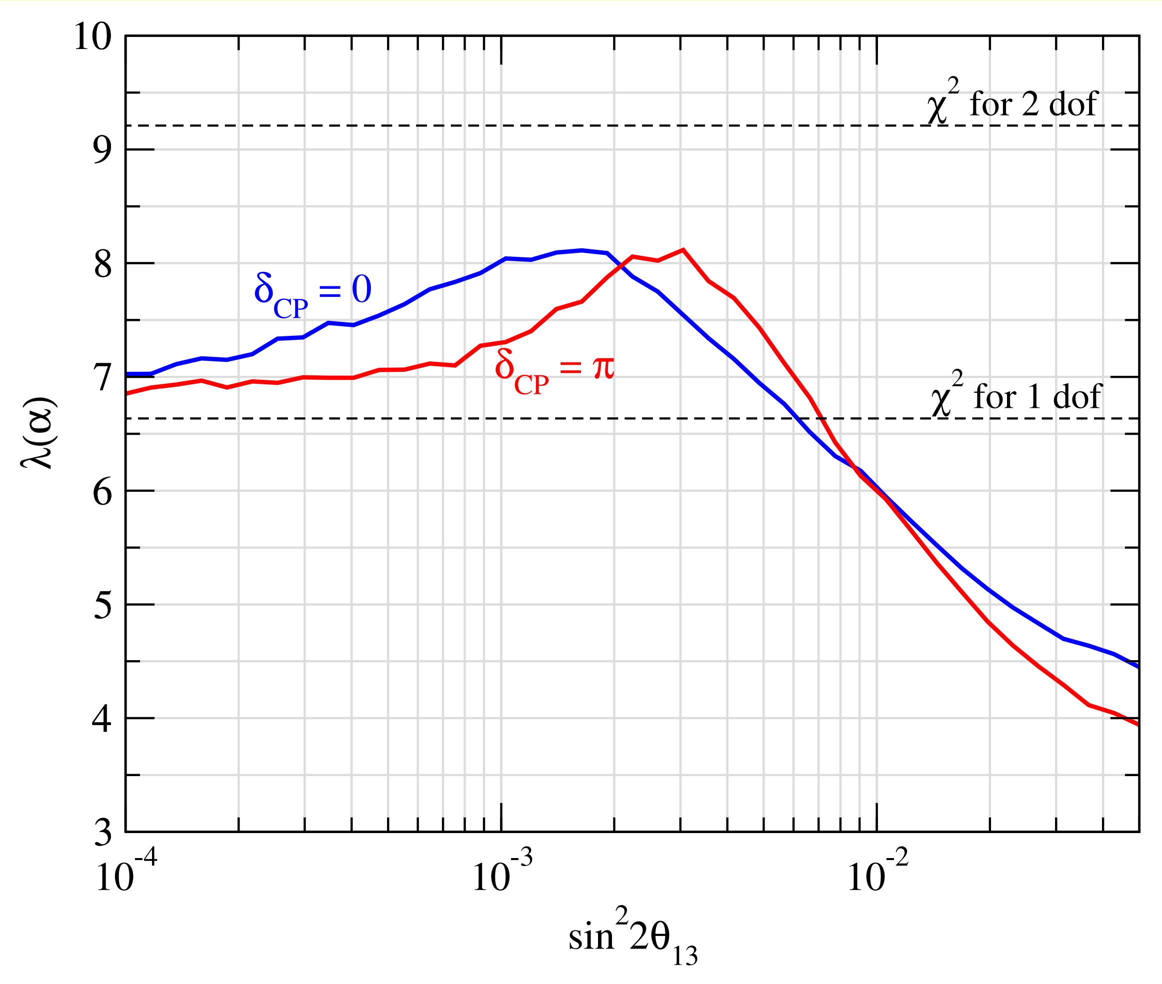
Now:

$$\chi^2(\theta_{13}, \delta_{\rm CPC}) - \chi^2_{\rm min} > \lambda(\alpha; \theta_{13}, \delta_{\rm CPC})$$

The cut-value λ depends on the parameter values!

Sensitivity to CPV

$\lambda(\alpha; \theta_{13}, \delta_{\rm CPC})$ for the 99% CL ($\alpha = 0.01$) for T2HK:



Sensitivity to CPV

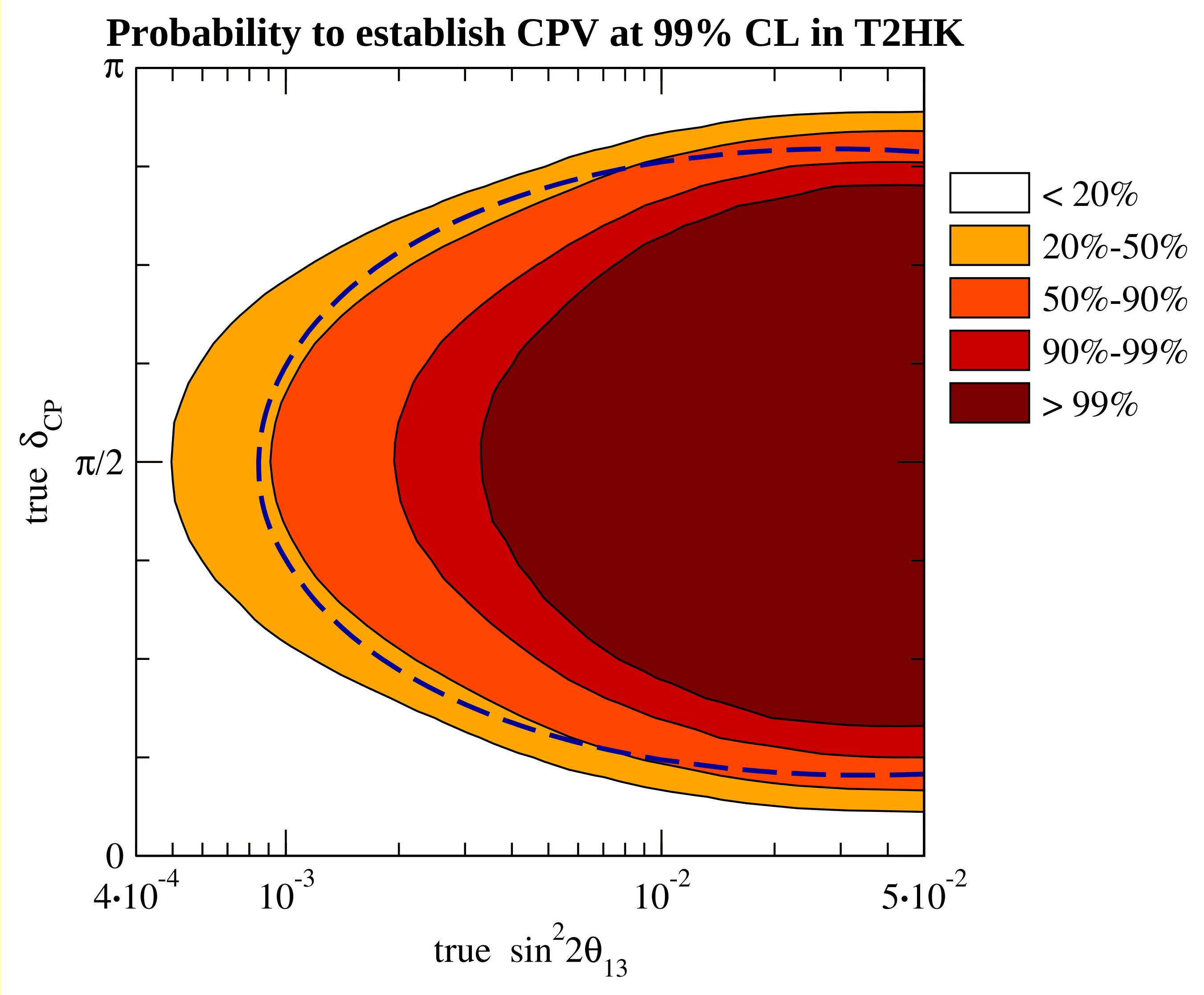
Calculation of $P_{\rm disc}$:

- Fix $\theta_{13}^{\rm true}, \delta^{\rm true}$ and simulate many experiments.
- $P_{\rm disc}$ for the $100(1-\alpha)\%$ CL is given by the fraction of experiments for which

$$\chi^2(\theta_{13}, \delta_{\rm CPC}) - \chi^2_{\rm min} > \lambda(\alpha; \theta_{13}, \delta_{\rm CPC})$$

  for all values of θ_{13}.
- Repeat this for each point in the $\theta_{13}^{\rm true}, \delta^{\rm true}$-plane.

Sensitivity to CPV



dashed: “standard” sensitivity

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- To estimate the sensitivity of future experiments to a given true value of a parameter one should quote two numbers: the CL at which we want to make the discovery, and the probability $P_{\rm disc}$ that we actually will make this discovery.

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- To estimate the sensitivity of future experiments to a given true value of a parameter one should quote two numbers: the CL at which we want to make the discovery, and the probability $P_{\rm disc}$ that we actually will make this discovery.
- For DC, T2K, and T2HK I have checked by explicit MC simulation that “standard sensitivities” correspond roughly to an “average” experiment with $P_{\rm disc} \approx 50\%$.
- It is OK to use GLoBES! ;-))

  However, one has to be aware of the correct interpretation of the “standard sensitivities”: The regions with high $P_{\rm disc}$ can be much smaller!